Quantum-information processing with a single photon by an input-output process with respect to low-Q cavities

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Both cavity quantum electrodynamics and photons are promising candidates for quantum information processing. We consider a combination of both candidates with a single photon going through spatially separate cavities to entangle the atomic qubits, based on the input-output process of the cavities. We present a general expression for the input-output process regarding the low-Q cavity confining a single atom, which works in a wide range of parameters. Focusing on low-Q cavity case, we propose some schemes for quantum information processing with Faraday rotation using single photons, which is much different from the high-Q cavity and strong-coupling cases.

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I. INTRODUCTION

Over past decades, cavity quantum electrodynamics (QED) has become an important platform to demonstrate quantum characteristics of atoms and photons [1]. Some remarkable experiments, such as strong coupling of the trapped atoms with cavity [2,3], conditional logic gating [4], efficient generation of single photons [5,6], and so on have been performed successfully.

With atoms strongly interacting with the local cavity mode as quantum nodes and the photons flying between different nodes as quantum bus, we may set up a quantum network, which has been considered as a promising way to scaling of the qubit system for large-scale quantum information processing. Many theoretical [7–12] and experimental works [3,13] have been done in this direction and some advances have been achieved over recent years. Among the theoretical work mentioned above, the single photon experiencing input-output process of a cavity [10,11] is of particular interests, in which the atomic qubits could be entangled by a single moving photon and the successful implementation is monitored by a click of the detector.

The present paper will focus on the input-output process regarding optical cavities. Motivated by a very recent experiment with microtoroidal resonator (MTR) [14], we intend to carry out some quantum information processing (QIP) tasks by means of the input-output process relevant to optical cavities with low-Q factors. We have noticed that the proposals [10,11] required the cavities to be with high quality and with strong coupling to the confined atoms, otherwise the schemes would not work well or would be pretty inefficient. Some simulation has been done to check how well the cavity input-output process works [12]. It was shown that the gating time should be much shorter than the decay time of the cavity if we expect to have the gating with high success probability [12]. Evidently, it is not an easy experimental task to meet those requirements because the efficient output of photons, to some extent, implies the larger cavity decay rate, i.e., the cavity with a relatively lower-Q factor. In this sense, the recent achievement of the MTR gives us hopes to solve the problem. Although it is still of large decay rate (i.e., called "bad" cavity in [14]) and moderate coupling to the atom, the MTR, with individual photons input and output through a microresonator, has explicitly shown the effect of the photon blockade. So this MRT seems a promising candidate setup to be meeting the requirements in those QIP schemes [10,11].

Our present work will, however, show the possibility of accomplishing some interesting QIP tasks with the currently achieved MTR. The key step is to design a scheme for entangling two atoms confined respectively in two spatially separate cavities with low-Q factors. So our work is actually not only relevant to the MTR, but also related to any single-sided optical cavities with one wall perfectly reflective but the other partially reflective [15]. As a result, the MTR and the single-sided optical cavity will be mentioned alternately in what follows. We will first present an analytical expression for the reflection rate of the input-output process, which works for a wide range of parametric variation, from weak to strong-coupling regimes, and in the presence or absence of the confined atom. Then we will try to use a single photon to entangle two atoms confined respectively in two spatially separate cavities, based on which further QIP tasks could be carried out. We argue that QIP with single photons is an efficient way in the low-Q cavity situation, which is considerably different from the high-Q cavity and strong-coupling cases.

Different from the previous schemes [10,11] with photonic polarization unchanged or changed by a phase π, the large cavity decay and moderate coupling in our case lead to a certain angle rotation of the photonic polarization after the input-output process, which is called Faraday rotation. The

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Faraday rotation was originally studied in a resonant medium with Zeeman level splitting under the radiation of linearly polarized light [16] and has been recently observed experimentally in cold atom and quantum dot systems [17–20]. The key point for the Faraday rotation is the birefringent propagation of the light through the medium. By using the values from [14], we will show the desired Faraday rotation for our purpose can be obtained in a two-mode cavity by using single photons with suitable frequencies.

The paper is organized as follows. In Sec. II, we introduce a general input-output relation for a single photon with respect to a low-Q optical cavity confining a single atom under Jaynes-Cummings model. We show in Sec. III the Faraday rotation in a two-mode cavity holding a three-level atom by Λ-type configuration. Based on the Faraday rotation, we may entangle remote atoms by a single photon, as shown in Sec. IV. Section V is devoted to the application of the generated entanglement. Finally, we end with some discussion and a summary in Sec. VI.

II. INPUT-OUTPUT RELATION UNDER JAYNES-CUMMINGS MODEL

In this section, we present the basic input-output relation for a cavity coherently interacting with a trapped two-level atom. Under the Jaynes-Cummings model, we have the following Hamiltonian:

$$H = \frac{\hbar\omega_0}{2}\sigma_z + \hbar\omega_c a^\dagger a + i\hbar g(a\sigma_+ - a^\dagger\sigma_-), \tag{1}$$

where a and $a^\dagger$ are the annihilation and creation operators of the cavity field with frequency ω_c, respectively; σ_z, σ_+, and σ_- are, respectively, inversion, raising, and lowering operators of the two-level atom with frequency difference ω_0 between the two levels.

Consider a single-photon pulse with frequency ω_p input in an optical cavity. The pulse can be expressed by $|\Psi_p\rangle = \int_0^T f(t)a_{\rm in}^\dagger(t)dt|\text{vac}\rangle$ [10], where $f(t)$ is the normalized pulse shape as a function of t, T is the pulse duration, $a_{\rm in}^\dagger(t)$, a one-dimensional field operator, is the cavity input operator satisfying the commutation relation $[a_{\rm in}(t), a_{\rm in}^\dagger(t')] = \delta(t-t')$, and we denote the vacuum of all the optical modes by $|\text{vac}\rangle$. In the rotating frame with respect to the frequency of the input pulse, the quantum Langevin equation of the cavity mode a driven by the corresponding cavity input operator $a_{\rm in}(t)$ is [21]

$$\dot{a}(t) = -\left[i(\omega_c - \omega_p) + \frac{\kappa}{2}\right]a(t) - g\sigma_-(t) - \sqrt{\kappa}a_{\rm in}(t), \tag{2}$$

where κ is the cavity damping rate. Moreover, the atomic lowering operator also obeys a similar equation and, in the rotating frame of the frequency ω_p, we have

$$\dot{\sigma}_-(t) = -\left[i(\omega_0 - \omega_p) + \frac{\gamma}{2}\right]\sigma_-(t) - g\sigma_z(t)a(t) + \sqrt{\gamma}\sigma_z(t)b_{\rm in}(t), \tag{3}$$

where $b_{\rm in}(t)$, with the commutation relation $[b_{\rm in}(t), b_{\rm in}^\dagger(t')] = \delta(t-t')$, is the vacuum input field felt by the two-level atom and γ is the decay rate of the two-level atom. The input and output fields of the cavity are related by the intracavity field as [21]

$$a_{\rm out}(t) = a_{\rm in}(t) + \sqrt{\kappa}a(t). \tag{4}$$

Now assuming a large enough κ to make sure that we have a weak excitation by the single-photon pulse on the atom initially prepared in the ground state, i.e., keeping $\langle\sigma_z\rangle = -1$ throughout our operation, we can adiabatically eliminate the cavity mode and arrive at the input-output relation of the cavity field,

$$r(\omega_p) = \frac{\left[i(\omega_c - \omega_p) - \frac{\kappa}{2}\right]\left[i(\omega_0 - \omega_p) + \frac{\gamma}{2}\right] + g^2}{\left[i(\omega_c - \omega_p) + \frac{\kappa}{2}\right]\left[i(\omega_0 - \omega_p) + \frac{\gamma}{2}\right] + g^2}, \tag{5}$$

where $r(\omega_p) \equiv \frac{a_{\rm out}(t)}{a_{\rm in}(t)}$ is the reflection coefficient for the atom-cavity system, and we have assumed in Eq. (5) that the input field of the two-level atom, $b_{\rm in}(t)$, as a vacuum field, gives negligible contribution to the output cavity field $a_{\rm out}(t)$. Equation (5) is a general expression for various cases. For example, in the case of $g=0$, Eq. (5) can recover the previous result for an empty cavity [21],

$$r_0(\omega_p) = \frac{i(\omega_c - \omega_p) - \frac{\kappa}{2}}{i(\omega_c - \omega_p) + \frac{\kappa}{2}}. \tag{6}$$

Equation (5) also fits very well the results in [10,11]: If g is dominant with respect to other parameters, $r(\omega_p)$ would be 1, implying that the input photon remains unchanged when it is output. Using the models in [10,11], we could explain the dominant g case as that due to the strong resonant coupling between the cavity field and the atom, the energy levels of the cavity will be shifted by the large vacuum Rabi splitting, yielding a large detuning between the dressed cavity mode and the single photon which is of the same frequency as that of the original cavity. Equivalently the total system can be seen as a photonic pulse interacting with a far-detuned bare cavity. As a result of Eq. (6) we have the reflective coefficient being 1. This implies that the photon enters and then leaks out of the cavity without being absorbed by the cavity mode. In contrast, in the case that the cavity mode is far detuned with respect to the confined atom, no level shift would occur in the cavity. Since the single photon is of the same frequency as that of the cavity, we can equivalently see the model as the photon interacting resonantly with a bare cavity, which yields $r_0(\omega_p) = -1$ from Eq. (6). From Eq. (5) the large detuning between the cavity mode and the atom means $g=0$. As a result, we have $r(\omega_p) = -1$. In this sense, we may argue that the controlled phase flip designed in [10,11] seems a special case of Eq. (5).

In fact, the key condition for Eq. (5) is $\langle\sigma_z\rangle = -1$. So as long as κ is large enough, which makes $\langle\sigma_z\rangle = -1$ always satisfied, Eq. (5) should work well even if g is bigger than κ. As this condition was met in [10,11], it is not strange that we could apply Eq. (5) to some physics in [10,11]. For more general cases, we plot Fig. 1 by Eq. (5) with the values from [14] for the absolute value and the phase shift of the reflection coefficient. In the case of an empty cavity, the pulse will experience a perfect reflection in the whole frequency re-

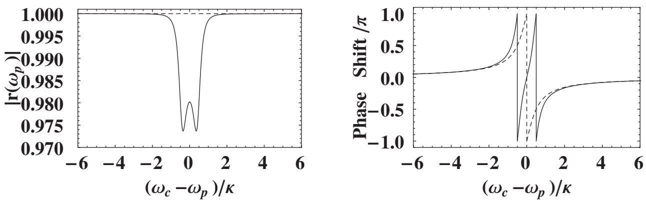

FIG. 1. The absolute value and phase shift of the reflection coefficient $r(\omega_p)$ as functions of the detuning between the input pulse and the cavity modes, with (solid line) and without (dashed line) the presence of the atom, where $\omega_0=\omega_c$, $\gamma/\kappa=0.01$, and $g/\kappa=0.5(g=0)$ for solid line (for dashed line).

gime, i.e., $|r(\omega_p)|=1$. The phase shift is $\pm\pi$ at $\omega_p=\omega_c$, but reduces to zero rapidly with the frequency of the input pulse deviating from the resonant point. In the case of an atom presented in the cavity, the coupling between the cavity field and the atom will shift the cavity mode, which leads to the vacuum splitting. Such a splitting also makes the reflection coefficient a corresponding splitting, as shown in the absolute value and the phase shift of the reflective coefficient in Fig. 1. So it is the interaction between the cavity field and the atom that induces weak absorption of the photon with detuned frequency by the cavity mode and also decreases the reflection rate $|r(\omega_p)|$. However, as shown in Fig. 1, even for a bad cavity with strong damping, the decrease in the reflection rate is very small so that we can still have $|r(\omega_p)|\approx 1$.

III. FARADAY ROTATION OF THE PHOTONIC POLARIZATION

In the preceding section, we did not consider the polarization degrees of freedom of the single photon. In this section, we show that with the general input-output relation, we can obtain a rotation regarding the polarization of the single-photon pulse after the input-output process, known as the Faraday rotation [18].

To explain the mechanism of Faraday rotation in cavity QED, we may consider the atom with the level structure as in Fig. 2. The states $|0\rangle$ and $|1\rangle$ correspond to the Zeeman sublevels of an alkali atom in the degenerate ground state, and $|e\rangle$ is the excited state. We assume that the transitions of $|e\rangle\leftrightarrow|0\rangle$ and $|e\rangle\leftrightarrow|1\rangle$ are due to the coupling to two degenerate cavity modes a_L and a_R with left (L) and right (R)

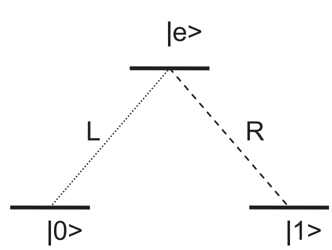

FIG. 2. The relevant atomic structure subject to a bimodal cavity field, where the lower levels are Zeeman sublevels of the ground state and the upper level is the excited one. The dashed line represents the R circularly polarized mode and the dotted line represents the L circularly polarized mode.

circular polarization, respectively. If the atom is initially prepared in $|0\rangle$, the only possible transition is $|0\rangle\rightarrow|e\rangle$, which implies that only the L circularly polarized single-photon pulse $|L\rangle$ will take action. So from Eq. (5) we have the output pulse related to the input one as $|\Psi_{\rm out}\rangle_L=r(\omega_p)|L\rangle\approx e^{i\phi}|L\rangle$ with ϕ the corresponding phase shift determined by the parameter values. It also means that an input R circularly polarized single-photon pulse $|R\rangle$ would only sense the empty cavity. As a result, the corresponding output governed by Eq. (6) is $|\Psi_{\rm out}\rangle_R=r_0(\omega_p)|R\rangle=e^{i\phi_0}|R\rangle$ with ϕ_0 a phase shift different from ϕ. Therefore, for an input linearly polarized photon pulse $|\Psi_{\rm in}\rangle=\frac{1}{\sqrt{2}}(|L\rangle+|R\rangle)$, the output pulse is

$$|\Psi_{\rm out}\rangle_-=\frac{1}{\sqrt{2}}(e^{i\phi}|L\rangle+e^{i\phi_0}|R\rangle). \qquad (7)$$

The polarization degrees of freedom of a linearly polarized optical field can be characterized by the Stokes vector $\mathbf{S}=(S_x,S_y,S_z)$ with [22],

$$S_x=\frac{1}{2}(a_L^\dagger a_R+a_R^\dagger a_L),$$

$$S_y=\frac{1}{2i}(a_L^\dagger a_R-a_R^\dagger a_L),$$

$$S_z=\frac{1}{2}(a_L^\dagger a_L-a_R^\dagger a_R), \qquad (8)$$

where $a_k(a_k^\dagger)$ with $k=L$ or R is the annihilation (creation) operator regarding different polarization. It is easily verified that $|\Psi_{\rm in}\rangle=\frac{1}{\sqrt{2}}(|L\rangle+|R\rangle)$ corresponds to $\mathbf{S}_{\rm in}=\frac{1}{2}(1,0,0)$ and Eq. (7) could be rewritten as $\mathbf{S}_{\rm out}=\frac{1}{2}[\cos(\phi_0-\phi),\sin(\phi_0-\phi),0]$, for which we define $\Theta_F^-=\phi_0-\phi$ to be Faraday rotation.

Similarly, if the atom is initially prepared in $|1\rangle$, then only the R circularly polarized photon could sense the atom, whereas the L circularly polarized photon only interacts with the empty cavity. So we have

$$|\Psi_{\rm out}\rangle_+=\frac{1}{\sqrt{2}}(e^{i\phi_0}|L\rangle+e^{i\phi}|R\rangle), \qquad (9)$$

where the Faraday rotation is $\Theta_F^+=\phi-\phi_0$.

IV. ENTANGLEMENT GENERATION OF THE ATOMIC STATES BY FARADAY ROTATION

Using the Faraday rotation introduced in Sec. III, we could generate entanglement between the atoms in spatially separate cavities with low-Q factors, as plotted in Fig. 3(a). We consider the level structure of each trapped atom as in Fig. 3(b), where $|g_i\rangle$ and $|e_i\rangle$ $(i=\pm 1,0)$ are the degenerate Zeeman sublevels of a typical alkali atom with $F=1$. As the atom is in resonance with the cavity modes, the possible cavity-mode-induced transitions are $|g_{-1}\rangle\leftrightarrow|e_0\rangle$ and $|g_0\rangle\leftrightarrow|e_{+1}\rangle$ (or $|g_{+1}\rangle\leftrightarrow|e_0\rangle$ and $|g_0\rangle\leftrightarrow|e_{-1}\rangle$) by absorbing or emitting a L (or R) circularly polarized photon. The transitions $|g_i\rangle\rightarrow|e_i\rangle(i=\pm 1,0)$ can be realized by a classical laser pulse. To entangle two atoms confined in spatially separate

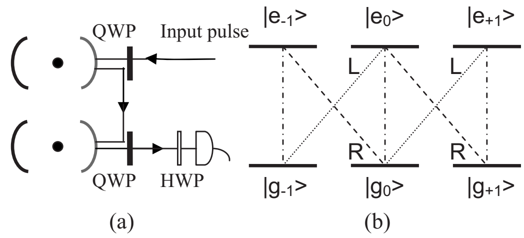

FIG. 3. (a) Schematic for the generation of entangled atomic states in fiber-connected cavities by Faraday rotation, where the atoms are resonantly coupled to the cavities, respectively, and the input photon pulse is detuned from the cavity modes. The bold lines represent quarter-wave plates (QWPs), which achieve $|L\rangle \leftrightarrow |h\rangle$ and $|R\rangle \leftrightarrow |v\rangle$ of the photon. HWP denotes a half-wave plate. (b) The relevant atomic structure is subject to the bimodal cavity field, where the dashed line represents the R-polarized mode, the dotted line represents the L-polarized mode and the dot-dashed line denotes the laser pump pulse.

locations, we encode the qubits in $|g_{-1}\rangle$ and $|g_{+1}\rangle$. As shown in Fig. 3(a), we input a single-photon pulse in superposition of horizontal and vertical polarizations, i.e., $|\Psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}}(|h\rangle + |v\rangle)$ [which changes to $|\Psi_{\text{in}}\rangle = \frac{1}{\sqrt{2}}(|L\rangle + |R\rangle)$ after going through the quarter-wave plate (QWP)] into the first cavity, and then we direct the output pulse by a fiber to the second cavity. The detection of the output photon from the second cavity, assisted by a QWP and a half-wave plate (HWP), would yield the atoms to be entangled.

Specifically if the two atoms are prepared in $|\psi_n\rangle = \alpha_n|g_{-1}\rangle_n + \beta_n|g_{+1}\rangle_n = \alpha_n|0\rangle_n + \beta_n|1\rangle_n$ with $n=1,2$ for different cavities, the photonic input and output regarding the first cavity yield

$$|\Psi_{\text{in}}\rangle|\psi_1\rangle \to \alpha_1|0\rangle_1|\Psi_{\text{out}}\rangle_- + \beta_1|1\rangle_1|\Psi_{\text{out}}\rangle_+, \tag{10}$$

which means an entanglement between the photonic and atomic qubits due to different Faraday rotations. The photon going in and then reflected out of the second cavity corresponds to

$$\begin{aligned}[\alpha_1|0\rangle_1|\Psi_{\text{out}}\rangle_- + \beta_1|1\rangle_1|\Psi_{\text{out}}\rangle_+]|\psi_2\rangle &\to \alpha_1\alpha_2\frac{1}{\sqrt{2}}(e^{i(\phi+\phi')}|h\rangle \\ &+ e^{i(\phi_0+\phi_0')}|v\rangle)|0\rangle_1|0\rangle_2 + \beta_1\beta_2\frac{1}{\sqrt{2}}(e^{i(\phi_0+\phi_0')}|h\rangle + e^{i(\phi+\phi')} \\ &\times|v\rangle)|1\rangle_1|1\rangle_2 + \alpha_1\beta_2\frac{1}{\sqrt{2}}(e^{i(\phi+\phi_0')}|h\rangle + e^{i(\phi_0+\phi')}|v\rangle) \\ &\times|0\rangle_1|1\rangle_2 + \beta_1\alpha_2\frac{1}{\sqrt{2}}(e^{i(\phi_0+\phi')}|h\rangle + e^{i(\phi+\phi_0')}|v\rangle)|1\rangle_1|0\rangle_2,\end{aligned} \tag{11}$$

where the actions of the QWPs have been included. As we may adjust the frequency of the input pulse to $\omega_p = \omega_c - \kappa/2$, we actually have $\phi = \phi' = \pi$ from Eq. (5) with $g = \kappa/2$ and $\omega_0 = \omega_c$, and we have $\phi_0 = \phi_0' = \pi/2$ from Eq. (6) with $\omega_0 = \omega_c$. So the output state becomes

$$\begin{aligned}&\frac{1}{\sqrt{2}}(|h\rangle - |v\rangle)(\alpha_1\alpha_2|0\rangle_1|0\rangle_2 - \beta_1\beta_2|1\rangle_1|1\rangle_2) \\ &- i\frac{1}{\sqrt{2}}(|h\rangle + |v\rangle)(\alpha_1\beta_2|0\rangle_1|1\rangle_2 + \beta_1\alpha_2|1\rangle_1|0\rangle_2).\end{aligned} \tag{12}$$

After the output photon goes through a HWP, which makes $(|h\rangle + |v\rangle)/\sqrt{2} \to |h\rangle$ and $(|h\rangle - |v\rangle)/\sqrt{2} \to |v\rangle$ [23], we may detect the photon in the $|h\rangle$ state, yielding a projection onto the atomic state as

$$|\Phi\rangle_{12} = \frac{1}{N_1}[\alpha_1\beta_2|0\rangle_1|1\rangle_2 + \beta_1\alpha_2|1\rangle_1|0\rangle_2], \tag{13}$$

where N_1 is the normalization constant. Alternatively, we may also detect the photon in the $|v\rangle$ state, yielding

$$|\Phi'\rangle_{12} = \frac{1}{N_2}[\alpha_1\alpha_2|0\rangle_1|0\rangle_2 - \beta_1\beta_2|1\rangle_1|1\rangle_2], \tag{14}$$

with N_2 the normalization constant. The atomic states we obtained in Eqs. (13) and (14) are entangled with arbitrary amount of entanglement determined by α_1, α_2, β_1, and β_2.

It is straightforward to extend above operations to the cases involving three atoms. For example, if the output photon pulse from the second cavity is directed to get in and then reflected out of the third cavity, we can obtain the atomic state,

$$\begin{aligned}|\Phi\rangle_{123} = \frac{1}{N_1'}[&\alpha_1\alpha_2\alpha_3|0\rangle_1|0\rangle_2|0\rangle_3 - \beta_1\beta_2\alpha_3|1\rangle_1|1\rangle_2|0\rangle_3 \\ &- \alpha_1\beta_2\beta_3|0\rangle_1|1\rangle_2|1\rangle_3 - \beta_1\alpha_2\beta_3|1\rangle_1|0\rangle_2|1\rangle_3],\end{aligned} \tag{15}$$

corresponding to the output state of the photon with polarization $(|h\rangle + i|v\rangle)$, and

$$\begin{aligned}|\Phi'\rangle_{123} = \frac{1}{N_2'}[&\beta_1\beta_2\beta_3|1\rangle_1|1\rangle_2|1\rangle_3 - \alpha_1\alpha_2\beta_3|0\rangle_1|0\rangle_2|1\rangle_3 \\ &- \beta_1\alpha_2\alpha_3|1\rangle_1|0\rangle_2|0\rangle_3 - \alpha_1\beta_2\alpha_3|0\rangle_1|1\rangle_2|0\rangle_3],\end{aligned} \tag{16}$$

with the output state as $(|h\rangle - i|v\rangle)$. By using another HWP (with a different tilted angle from the previously used one) to achieve $(|h\rangle + i|v\rangle)/\sqrt{2} \to |h\rangle$ and $(|h\rangle - i|v\rangle)/\sqrt{2} \to |v\rangle$, we distinguish the states of the output pulse by the single-photon detector. Therefore, we can generate the three-qubit entangled states in Eqs. (15) and (16) at our will. Evidently, the scheme can be generalized to the situation involving more atoms.

V. APPLICATION OF THE GENERATED ENTANGLEMENT

In this section, we carry out some quantum information processing tasks using the generated atomic entanglement. The key integrant is the efficient conversion between the quantum nodes and the flying qubits. We will first show a conversion from the atomic entanglement to the photonic entanglement by some single-qubit rotations on the atoms. Then we transfer any unknown photonic state to the atom trapped in a distant cavity and transfer an unknown state from one atom to another.

A. Entanglement conversion from atoms to photons

Supposing that the atoms in two separate cavities have been entangled as in Eq. (13) with $\alpha=\alpha_1\beta_2/N_1$ and $\beta=\beta_1\alpha_2/N_1$, we apply two π-polarized classical laser pulses simultaneously on the two atoms, and the states of the atoms change as

$$\alpha|0\rangle_1|1\rangle_2+\beta|1\rangle_1|0\rangle_2 \rightarrow \alpha|e_{-1}\rangle_1|e_{+1}\rangle_2+\beta|e_{+1}\rangle_1|e_{-1}\rangle_2. \quad (17)$$

Due to the atomic decay subject to the cavity modes, the de-excitation from $|e_{-1}\rangle$ (or $|e_{+1}\rangle$) to $|g_0\rangle$ produce a R (or L) circularly polarized photon. Then Eq. (17) becomes

$$\alpha|e_{-1}\rangle_1|e_{+1}\rangle_2+\beta|e_{+1}\rangle_1|e_{-1}\rangle_2 \rightarrow |g_0\rangle_1|g_0\rangle_2(\alpha|R\rangle_1|L\rangle_2 + \beta|L\rangle_1|R\rangle_2), \quad (18)$$

where $|L\rangle_k$ (or $|R\rangle_k$) is the generated photonic state with L (or R) polarization from the kth cavity, and the photon will change the polarization to $|h\rangle_k$ (or $|v\rangle_k$) after going through the QWP. In Eqs. (17) and (18), we have omitted the photonic states in vacuum for simplicity. The two equations above clearly show that the entanglement is converted from the atomic states to the states of the emitting photons, which is actually a source of entangled photons and will be used for further quantum information processing mission.

B. State transfer from photonic qubit to atomic qubit via entanglement

Our another scheme is to use the atomic entanglement generated above for state transfer from a photon to an atom in distance. Suppose that the atoms have been prepared in a maximally entangled state $\frac{1}{\sqrt{2}}(|0\rangle_1|1\rangle_2+|1\rangle_1|0\rangle_2)$. As we want to transfer an unknown photonic state $(x|h\rangle+y|v\rangle)(|x|^2+|y|^2=1)$ via the atomic entanglement from one side to another, the photon only needs to go through one of the cavities. First we input the single photon to the cavity in its local side (for convenience, we call it as first cavity and the other one the second cavity), the state of the total system changes as

$$\frac{1}{\sqrt{2}}(x|h\rangle+y|v\rangle)(|0\rangle_1|1\rangle_2+|1\rangle_1|0\rangle_2) \rightarrow \frac{1}{\sqrt{2}}(-x|h\rangle|0\rangle_1|1\rangle_2 + ix|h\rangle|1\rangle_1|0\rangle_2+iy|v\rangle|0\rangle_1|1\rangle_2-y|v\rangle|1\rangle_1|0\rangle_2), \quad (19)$$

where the input-output related Faraday rotations and the action of QWPs have been considered. Then the output pulse from the first cavity goes through a HWP, which makes $(|h\rangle+|v\rangle)/\sqrt{2}\rightarrow|h\rangle$ and $(|h\rangle-|v\rangle)/\sqrt{2}\rightarrow|v\rangle$, followed by a detection. Besides, to recover the photonic state in the second atom, we have to make Hadamard operation and a σ_z measurement on the first atom. Depending on different measurement results regarding the photon and the first atom, we perform different single-qubit operations M_i on the second atom, i.e.,

$$h;+:\quad M_1=-i\exp\left(-i\tfrac{\pi}{4}\sigma_x\right),$$

$$h;-:\quad M_2=-i\sigma_y\exp\left(-i\tfrac{\pi}{4}\sigma_x\right),$$

$$v;+:\quad M_3=-i\sigma_y\exp\left(i\tfrac{\pi}{4}\sigma_x\right),$$

$$v;-:\quad M_4=i\exp\left(i\tfrac{\pi}{4}\sigma_x\right), \quad (20)$$

and the photonic state on the second atom is recovered as $|\psi_f\rangle=\frac{1}{\sqrt{2}}(x|0\rangle+y|1\rangle)$.

C. State transfer from one atomic qubit to another via entanglement

With similar steps in above subsection, we can also transfer states between two separate atoms. Starting from Eq. (12) with $\alpha_2=\beta_2=1/\sqrt{2}$, we transfer an arbitrary state $(\alpha_1|0\rangle_1+\beta_1|1\rangle_1)$ of the first atom to the second. To this end, we perform a Hadamard gate on the first atom in Eq. (12), which yields

$$\frac{1}{2\sqrt{2}}(|h\rangle-|v\rangle)[|0\rangle_1(\alpha_1|0\rangle_2-\beta_1|1\rangle_2)+|1\rangle_1(\alpha_1|0\rangle_2+\beta_1|1\rangle_2)] - i\frac{1}{2\sqrt{2}}(|h\rangle+|v\rangle)[|0\rangle_1(\beta_1|0\rangle_2 + \alpha_1|1\rangle_2)+|1\rangle_1(-\beta_1|0\rangle_2+\alpha_1|1\rangle_2)]. \quad (21)$$

By detecting the photon polarization and the states of the first atom, the second atom would be projected onto four corresponding states with equal probability. After the local operations conditioned on the measurement results, the unknown state, i.e., $(\alpha_1|0\rangle_2+\beta_1|1\rangle_2)$, on the second atom is reconstructed.

VI. DISCUSSION AND SUMMARY

Equation (5) is the key result of the present paper, based on which we have made the schemes for entanglement generation and state transfer. It is easy to check that Eq. (5) fits very well the numerical results in [10]. As an analytical expression working for a wide range of parametric values, Eq. (5) should be very useful in the study of cavity QED.

The entanglement generation and state transfer in Secs. IV and V are sketched for the low-Q cavities, such as the achieved MTR [14] or the single-sided cavity [15]. One point we have to mention is that we have not yet figured out the controlled logic gate between the two distant atomic qubits based on the Faraday rotation, although we could achieve entanglement in between. So we could not achieve teleportation between the two atoms from the conventional viewpoint [24]. Anyway we have shown the possibility to transfer an atomic or a photonic qubit state to a distant atomic qubit. Different from the standard teleportation steps, we need local operations on the qubits on both sides as well as classical information to achieve the transfer of a quantum state.

The entanglement of two distant atomic qubits by a single photon in our scheme is more efficient than that by interference of two photons emitted from two respective atoms [8]. In the latter case, the entanglement between the two atomic qubits relies on the two leaking photons reaching at the beam splitter simultaneously, as well as the high efficiency of the two detectors, which strongly restricts the success rate in real implementation. In contrast, as only a single photon is involved in our scheme, there is no requirement for simultaneous detection. So our scheme should be more efficient than those using two detectors in the case of the large inefficiency of current single-photon detector.

The imperfection in our scheme is the photon loss, which is also a problem in previously published schemes with photon interference. The photon loss occurs due to the cavity mirror absorption and scattering, the fiber absorption, and the inefficiency of the detector. As the successful detection of the photon ensures the accomplishment of our implementation, the photon loss actually only affects the efficiency of the scheme, but not the fidelity of the entanglement generation. Moreover, even if we implement the scheme with low success rate, due to the highly efficient single-photon source, such as 10 000 single photons per second [6], we are able to accomplish our schemes within a short time. We may simply assess the implementation of our schemes, where the failure rate due to atomic decay is about $\gamma/g=2\%$. Moreover, the current dark count rate of the single-photon detector is about 100 Hz, which can reduce the efficiency of our scheme by a factor of 10^{-4}. Other imperfection rate, regarding the photon absorption by the fiber and the scattering of the cavity mirror, can be assumed to be 6%. Thus the success rate of our implementation should be $(1-2\%)^2\times 10^{-4}\times(1-6\%)=0.009\%$, where the square is due to two atoms involved. By using the generation rate $1\times 10^4\ \mathrm{s}^{-1}$ of single photons, the two atoms can be entangled within 2 s, provided that the two atoms are not very distant, i.e., without considering the time of the photon traveling in between. Evidently, our scheme is in principle scalable with the single photon going through the spatially separate cavities one by one. However, with more atoms (confined in cavities) involved, the photon loss would be more serious and thereby it will take a longer time for a successful event.

Before ending the paper, we would like to reiterate the difference of our present work from the relevant work published previously [10–12]. The previous schemes prefer to work in strong-coupling condition, i.e., $g \gg \kappa, \gamma$. The fidelity of the operations decreases significantly with the decrease in g/κ. In contrast, our study focuses on the input-output process of the low-Q cavity. We have not only analytically presented an important expression for the photonic reflection, which straightforwardly leads to our understanding of Faraday rotation, but also demonstrated a way to entanglement generation and state transfer in the case of low-Q cavity. We argue the impossibility of controlled-NOT gating in the case under our consideration, which is different from the strong-coupling or other cases considered in [10–12]. Meanwhile, we have shown the preference with a single photon to carry out QIP tasks.

In summary, the general reflection rate of input-output process we have analytically presented can be applied to different cases for cavity QED: with and without atoms confined, and with strong or weak coupling, and particularly useful for weak-coupling case. Based on this general reflection rate and a recently achieved MTR, we have proposed a scheme using cavities with low-Q factors to entangle distant atoms by a single photon, to generate entangled photons, and to transfer quantum state to a distant qubit. We argue that our work would be useful for QIP in cavity QED with current technology.

Note added. Recently we became aware of a recent work [25] with some similarities for quantum dots in micropillar cavities. Our study on the cavity-atom system, strongly relevant to the latest progress of cavity QED experiments, is feasible with current technology.

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