Generalized model of the quantum measurement process in an environment

Masashi Ban, Sachiko Kitajima, and Fumiaki Shibata
Graduate School of Humanities and Sciences, Ochanomizu University, 2-1-1 Ohtsuka, Bunkyo-ku, Tokyo 112-8610, Japan
(Received 11 January 2009; published 25 March 2009)

A generalized model of the quantum measurement process is considered, in which the relaxation of the detector system interacting with an environment can be treated. When the effect of the environment on the detector system is described by the quantum Markovian process, the model can be solved exactly. The time evolution of the particle and detector is investigated in detail. It is shown that the decay of quantum coherence of the particle becomes negligible when the relaxation time of the detector system is sufficiently short. Furthermore the stochastic model is found, which is equivalent to the proposed model with respect to the particle state.

DOI: 10.1103/PhysRevA.79.032113 PACS number(s): 03.65.Yz, 03.65.Ta

I. INTRODUCTION

The characteristic features of quantum mechanics are the complementarity of wavelike and particlelike properties of a particle and the state reduction caused by a quantum measurement [1]. When we observe particlelike (wavelike) properties of a quantum-mechanical particle, wavelike (particlelike) properties are inevitably destructed by the observation. Then the decoherence of the particle takes place in a quantum measurement process. Exactly solvable models are very significant in order to understand the measurement-induced decoherence since they exclude the effects of approximations. The Coleman-Hepp model [2–5] and its generalized version [6–8], which are exactly solvable, provide us the comprehensive understanding of the quantum measurement processes (see Ref. [9] for review). In these models, however, the composite system of the particle (a measured system) and the detector system (a measuring apparatus) are isolated from a surrounding environment. Hence the detector state remains unchanged after the interaction with the particle. If the models can be generalized such that the detector system is placed under the influence of an environment, the generalized model would be more important as a model of the quantum measurement process. Such an extension of the model is the main purpose of this paper. When the influence of the environment on the detector system is described by the quantum Markovian process [10,11], the generalized model can be solved exactly. In this paper, we solve the model by means of nonequilibrium thermofield dynamics (NETFD) [12–17].

The Coleman-Hepp model for the quantum measurement process is exactly solvable [2–5], where a propagating particle Q described by a ultrarelativistic Hamiltonian $H_Q = vP$ with momentum operator P and velocity parameter $v(\approx c)$ interacts with a one-dimensional array of N spins with magnitude 1/2, called a detector system D. Then the total Hamiltonian of the Coleman-Hepp model is provided by

$$H = H_Q + H', \tag{1}$$

with

$$H' = \sum_{k=1}^{N} \hbar \Omega_k (X - x_k) \sigma_x^{(k)}, \tag{2}$$

where X is a position operator of the propagating particle, which satisfies the canonical commutation relation $[X, P] = i\hbar$, $\sigma_x^{(k)}$ is the Pauli matrix of the kth spin of the detector system, and the real parameter x_k stands for the position of the kth spin. In Eq. (2), the function $\Omega_k(x)$ characterizes the strength of the interaction between the particle and the kth spin. In the original Coleman-Hepp model, neither the energy of the detector system D nor the energy exchange between the particle and detector are taken into account.

Nakazato and Pascazio [6,7] have generalized the Coleman-Hepp model so that the detector energy and the energy exchange can be taken into account. Remarkably the generalized Coleman-Hepp model is still exactly solvable. The Hamiltonian of the generalized Coleman-Hepp model is described by the Hamiltonian,

$$H = H_0 + H_1, \tag{3}$$

where the free Hamiltonian H_0 and the interaction Hamiltonian H_1 are given by

$$H_0 = H_Q + H_D = vP + \frac{1}{2}\sum_{k=1}^{N} \hbar \omega_k \sigma_z^{(k)}, \tag{4}$$

$$H_1 = \sum_{k=1}^{N} \hbar \Omega_k (X - x_k)[\sigma_+^{(k)} e^{-i\omega_k X/v} + \sigma_-^{(k)} e^{i\omega_k X/v}], \tag{5}$$

with $\sigma_\pm^{(k)} = (\sigma_x^{(k)} \pm i\sigma_y^{(k)})/2$. In this model, the kth spin of the detector system is excited by absorbing the energy $\hbar\omega_k$ from the propagating particle and it relaxes into the ground state by emitting the energy $\hbar\omega_k$ into the particle. Hiyama and Takagi [8] have further generalized the 1/2-spin detector system into the detector system with arbitrary spins. In their model, the Pauli matrices $\sigma_z^{(k)}$ and $\sigma_\pm^{(k)}$ in Eqs. (4) and (5) are replaced with arbitrary spin operators $S_z^{(k)}$ and $S_\pm^{(k)} = S_x^{(k)} \pm iS_y^{(k)}$. Moreover, Hiyama and Takagi [8] have considered the model where the detector system consists of N harmonic oscillators. In this model, which is called the boson-detector (BD) model, the free Hamiltonian of the particle and detector system is given by

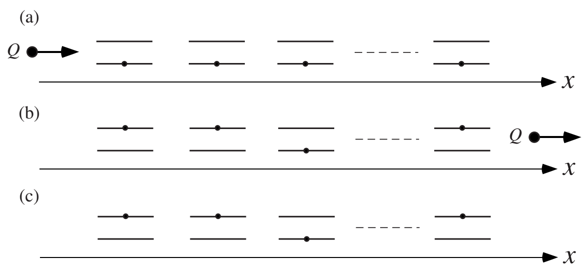

FIG. 1. The schematic representation of the state change in the detector system in the generalized Coleman-Hepp model. (a) The detector system is prepared in an initial state, e.g., the ground state, before the interaction with the particle. (b) The detector system is excited by the propagating particle. (c) The detector system excited by the particle remains unchanged after the interaction since the detector system is isolated.

$$H_0 = H_Q + H_D = vP + \sum_{k=1}^{N} \hbar\omega_k a_k^\dagger a_k, \tag{6}$$

and the interaction Hamiltonian is

$$H_1 = \sum_{k=1}^{N} \hbar\Omega_k(X - x_k)[a_k^\dagger e^{-i\omega_k X/v} + a_k e^{i\omega_k X/v}], \tag{7}$$

where a_k and $a_k^\dagger$ are the annihilation and creation operators of the kth harmonic oscillator of the detector system. This model is also exactly solvable. The slightly different model in which the interaction Hamiltonian H_1 is replaced with

$$H_1 = \sum_{k=1}^{N} \hbar\Omega_k(X - x_k)(a_k^\dagger + a_k), \tag{8}$$

has been considered by Kobayashi [18]. The properties of the generalized Coleman-Hepp model and the boson-detector model have been investigated in detail by means of the nonequilibrium statistical method [19–23].

In the generalized Coleman-Hepp model described by Eq. (3) with Eqs. (4) and (5) or the boson-detector model by Eqs. (6) and (7), the composite system of the particle Q and detector D is isolated from a surrounding environment. Thus the quantum state of the detector system D remains unchanged after the interaction with the propagating particle Q. For example, we suppose that the detector system in the generalized Coleman-Hepp model is initially prepared in the ground state [Fig. 1(a)].

Some of the detector spins are excited by the interaction with the particle [Fig. 1(b)]. After the interaction, the excited spins stay in the excited state [Fig. 1(c)]. In the real world, however, the excited spins inevitably relax to the equilibrium state under the influence of an environmental system. Therefore, in this paper, we further generalize the model so that the relaxation of the detector system caused by a surrounding environment can be treated. When the effect of the environment on the detector system is described by the quantum Markovian process, the generalized model can be still exactly solvable.

This paper is organized as follows. In Sec. II, we propose a generalized version of the boson-detector model and obtain the time evolution of the particle-detector system by means of nonequilibrium thermofield dynamics [12–14], which is abbreviated as NETFD in the rest of this paper. In Sec. III, we investigate the decay of the quantum coherence of the propagating particle. We will show that the decay of the quantum coherence becomes negligible when the relaxation of the detector caused by the environment is very significant. In Sec. IV, we will find the stochastic model which is equivalent to the generalized model with respect to the particle dynamics. In Sec. V, we give the concluding remarks.

II. DISSIPATIVE MODEL AND ITS SOLUTION

A. Boson-detector model interacting with an environment

In this section, the boson-detector model is generalized so that the detector system placed under the influence of an environment can be treated. In the generalized model, after the interaction with the particle, the detector system relaxes into the equilibrium state by the interaction with the environment. To make the model analytically tractable, we assume that the detector system is a one-dimensional array of N harmonic oscillators [8]. The generalized model is described by the Hamiltonian,

$$H_T = H + H_E + H_{DE}, \tag{9}$$

where $H = H_0 + H_1$ is the particle-detector Hamiltonian with H_0 and H_1 being given by Eqs. (6) and (7), H_E is the environmental Hamiltonian, and H_{DE} is the interaction Hamiltonian between the detector system and environment.

We assume that the effect of the environment on the detector is described by the quantum Markovian process. Let $W(t)$ be a density operator of the particle Q and detector system D. Since the quantum Markovian process can be described by the superoperator $\mathcal{L}$ of the Lindblad form [10,11], we obtain the quantum master equation for the density operator $W(t)$ of the particle Q and detector D,

$$\frac{\partial}{\partial t}W(t) = -\frac{i}{\hbar}[H, W(t)] + \mathcal{L}W(t), \tag{10}$$

with

$$\mathcal{L}W(t) = \sum_{k=1}^{N} \kappa_k(\bar{n}_k + 1)\{[a_k, W(t)a_k^\dagger] + [a_k W(t), a_k^\dagger]\} + \sum_{k=1}^{N} \kappa_k \bar{n}_k\{[a_k^\dagger, W(t)a_k] + [a_k^\dagger W(t), a_k]\}, \tag{11}$$

where κ_k is the damping parameter of the kth oscillator of the detector system and $\bar{n}_k = (e^{\hbar\omega_k/k_B T} - 1)^{-1}$ with T being the absolute temperature of the environment. In the rest of this paper, we assume that the detector is initially prepared in the vacuum state, the density matrix of which is given by

$$W_D = \bigotimes_{k=1}^{N} |0_k\rangle\langle 0_k|, \tag{12}$$

with $a_k|0_k\rangle = 0$. The temperature of the environment is $T = 0$ and thus $\bar{n}_k = 0$. Then, in the generalized model, the detector

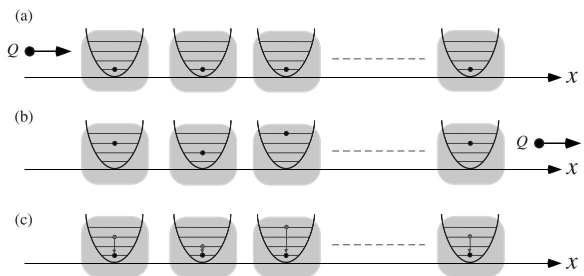

FIG. 2. The schematic representation of the state change in the detector system in the generalized boson-detector model with dissipation. (a) The detector system is prepared in the vacuum state before the interaction with the particle. (b) The detector system is excited by the propagating particle. (c) After the interaction, the detector system relaxes into the vacuum state under the influence with the environment of $T=0$.

system that is initially prepared in the vacuum state [Fig. 2(a)] is excited by the interaction with the propagating particle [Fig. 2(b)]. After the interaction, the detector system evolves into the vacuum state under the influence of the environment.

B. Time evolution of the particle-detector system

Now, using the method of NETFD, we solve the quantum master equation given by Eq. (10) with Eq. (11) and $\bar{n}_k=0$. The initial condition is assumed to be

$$W(0) = W_Q \otimes W_D, \tag{13}$$

where W_D is the density operator of the detector system in the vacuum state, given by Eq. (12), and W_Q represents the initial state of the particle. Introducing the interaction picture by $W'(t)=e^{(it/\hbar)H_0}W(t)e^{-(it/\hbar)H_0}$, we can obtain the equation of motion for $W'(t)$,

$$\frac{\partial}{\partial t}W'(t) = -\frac{i}{\hbar}[H_1'(t),W'(t)] + \mathcal{L}W'(t), \tag{14}$$

with

$$\mathcal{L}W'(t) = \sum_{k=1}^{N} \kappa_k\{[a_k, W'(t)a_k^\dagger] + [a_k W'(t), a_k^\dagger]\}, \tag{15}$$

and

$$H_1'(t) = \sum_{k=1}^{N} \hbar\Omega_k(X+vt-x_k)[a_k^\dagger e^{-i\omega_k X/v} + a_k e^{i\omega_k X/v}]. \tag{16}$$

Here it is important to note that the commutation relation $[H_1'(t),H_1'(t')]=0$ holds for any times t and t'. To solve Eq. (14), it is convenient to use the method of NETFD, which is briefly summarized in the Appendix.

In NETFD, the density operator $W'(t)$ acting on the Hilbert space $\mathcal{H}$ of the particle-detector system is represented by the state vector $|W'(t)\rangle$ of the extended Hilbert space $\mathcal{H}\otimes\tilde{\mathcal{H}}$ (see Appendix). The state vector $|W'(t)\rangle$ is subject to the equation of motion,

$$\frac{\partial}{\partial t}|W'(t)\rangle = -\frac{i}{\hbar}[H_1'(t) - \tilde{H}_1'(t)]|W'(t)\rangle + \hat{\Pi}|W'(t)\rangle, \tag{17}$$

where $\tilde{H}_1'(t)$ is the tilde conjugate of $H_1'(t)$, namely,

$$\tilde{H}_1 = \sum_{k=1}^{N} \hbar\Omega_k(\tilde{X}-x_k)[\tilde{a}_k^\dagger e^{i\omega_k\tilde{X}/v} + \tilde{a}_k e^{-i\omega_k\tilde{X}/v}], \tag{18}$$

and the Lindblad operator $\hat{\Pi}$ with $\bar{n}_k=0$ in NETFD is given by

$$\hat{\Pi} = -\sum_{k=1}^{N} \kappa_k(a_k^\dagger a_k + \tilde{a}_k^\dagger\tilde{a}_k - 2a_k\tilde{a}_k). \tag{19}$$

The operator $\hat{X}$ is the tilde conjugation of the position operator, and $\tilde{a}_k$ and $\tilde{a}_k^\dagger$ are the tilde conjugate of the annihilation and creation operators. Furthermore, setting $|W''(t)\rangle = e^{-\hat{\Pi}t}|W'(t)\rangle$, we obtain

$$\begin{aligned}\frac{\partial}{\partial t}|W''(t)\rangle &= -\frac{i}{\hbar}e^{-\hat{\Pi}t}[H_1'(t) - \tilde{H}_1'(t)]e^{\hat{\Pi}t}|W''(t)\rangle \\ &= -i\sum_{k=1}^{N}\{\Omega_k(X+vt-x_k)[b_k(t)e^{i\omega_k X/v} + b_k(t)e^{i\omega_k X/v}] \\ &\quad - \Omega_k(\tilde{X}+vt-x_k)[\tilde{b}_k(t)e^{-i\omega_k\tilde{X}/v} + \tilde{b}_k^\dagger(t)e^{-i\omega_k\tilde{X}/v}]\} \\ &\quad \times|W''(t)\rangle,\end{aligned} \tag{20}$$

where the operators $b_k(t)=e^{-\hat{\Pi}t}a_k e^{\hat{\Pi}t}$, $b_k(t)=e^{-\hat{\Pi}t}a_k^\dagger e^{\hat{\Pi}t}$, $\tilde{b}_k(t) = e^{-\hat{\Pi}t}\tilde{a}_k e^{\hat{\Pi}t}$, and $\tilde{b}_k^\dagger(t)=e^{-\hat{\Pi}t}\tilde{a}_k^\dagger e^{\hat{\Pi}t}$ are calculated to be

$$b_k(t) = e^{-\kappa_k t}a_k, \tag{21}$$

$$b_k^\dagger(t) = e^{-\kappa_k t}\tilde{a}_k + e^{\kappa_k t}(a_k^\dagger - \tilde{a}_k), \tag{22}$$

$$\tilde{b}_k(t) = e^{-\kappa_k t}\tilde{a}_k, \tag{23}$$

$$\tilde{b}_k^\dagger(t) = e^{-\kappa_k t}a_k + e^{\kappa_k t}(\tilde{a}_k^\dagger - a_k). \tag{24}$$

Substituting these equations into Eq. (20), we can obtain the equation of motion for the state vector $|W''(t)\rangle$,

$$\begin{aligned}\frac{\partial}{\partial t}|W''(t)\rangle &= -i\sum_{k=1}^{N}[A_k(t)a_k + B_k(t)a_k^\dagger - \tilde{A}_k(t)\tilde{a}_k - \tilde{B}_k(t)\tilde{a}_k^\dagger] \\ &\quad \times|W''(t)\rangle,\end{aligned} \tag{25}$$

where the operators $A_k(t)$ and $B_k(t)$ are given by

$$\begin{aligned}A_k(t) &= \Omega_k(X+vt-x_k)e^{-\kappa_k t}e^{i\omega_k X/v} - \Omega_k(\tilde{X}+vt-x_k) \\ &\quad \times(e^{-\kappa_k t} - e^{\kappa_k t})e^{i\omega_k\tilde{X}/v},\end{aligned} \tag{26}$$

$$B_k(t) = \Omega_k(X+vt-x_k)e^{\kappa_k t}e^{-i\omega_k X/v}, \tag{27}$$

and $\tilde{A}_k(t)$ and $\tilde{B}_k(t)$ are the tilde conjugate of $A_k(t)$ and $B_k(t)$. It should be noted that $A_k(t)$, $B_k(t)$, $\tilde{A}_k(t)$, and $\tilde{B}_k(t)$ are commutable operators, e.g., $[A_k(t),B_k(t')]=0$ for any times t and t'.

To solve Eq. (25), we remember the operator identity for annihilation and creation operators,

$$T\exp\left\{-i\int_0^t dt'[f(t')a+g(t')a^\dagger]\right\} = \exp\left\{-\frac{1}{2}\int_0^t dt_1\int_0^{t_1} dt_2[f(t_1)g(t_2)-f(t_2)g(t_1)]\right\} \times\exp\left\{-i\int_0^t dt'[f(t')a+g(t')a^\dagger]\right\}, \quad (28)$$

where the symbol "T" stands for the chronological ordering of operators from the right to the left. Using this identity, we can obtain the solution of Eq. (25),

$$|W''(t)\rangle = e^{-(1/2)[\mathcal{G}(t)+\tilde{\mathcal{G}}(t)]}\mathcal{D}(t)\tilde{\mathcal{D}}(t)|W(0)\rangle, \quad (29)$$

where the operators $\mathcal{G}(t)$ and $\mathcal{D}(t)$ are given by

$$\mathcal{G}(t) = \sum_{k=1}^{N}\int_0^t dt_1\int_0^{t_1} dt_2[A_k(t_1)B_k(t_2)-A_k(t_2)B_k(t_1)], \quad (30)$$

$$\mathcal{D}(t) = \exp\left(-i\sum_{k=1}^{N}[\mathcal{A}_k(t)a_k+\mathcal{B}_k(t)a_k^\dagger]\right), \quad (31)$$

with

$$\mathcal{A}_k(t) = \int_0^t dt' A_k(t'), \quad \mathcal{B}_k(t) = \int_0^t dt' B_k(t'). \quad (32)$$

Therefore we finally obtain the state vector $|W(t)\rangle$ that represents the quantum state of the particle and detector system of the generalized boson-detector model,

$$|W(t)\rangle = e^{-(it/\hbar)(H_0-\tilde{H}_0)}e^{\hat{\Pi}t}e^{-(1/2)[\mathcal{G}(t)+\tilde{\mathcal{G}}(t)]}\mathcal{D}(t)\tilde{\mathcal{D}}(t)|W(0)\rangle. \quad (33)$$

C. Relaxation of the detector state

Using the result obtained above, we investigate the time evolution of the detector system which is initially prepared in the vacuum state. In NETFD, the initial state of the detector is given by

$$|W_D(0)\rangle = |\mathbf{0},\tilde{\mathbf{0}}\rangle = \bigotimes_{k=1}^{N}|0_k,\tilde{0}_k\rangle, \quad (34)$$

with $a_k|0_k,\tilde{0}_k\rangle=\tilde{a}_k|0_k,\tilde{0}_k\rangle=0$. For the initial state $|W_Q(0)\rangle$ of the propagating particle, $P_Q(x)=\langle x,\tilde{x}|W_Q(0)\rangle=\langle x|W_Q(0)|x\rangle$ is the position probability density that the particle is located at the initial time ($t=0$). Using the completeness relation of the position eigenstate of the particle, we can obtain the quantum state of the detector system at time t from Eq. (33),

$$\begin{aligned}|W_D(t)\rangle &= \int_{-\infty}^{\infty} dx\langle x,\tilde{x}|W(t)\rangle \\ &= e^{-(it/\hbar)(H_D-\tilde{H}_D)}e^{\hat{\Pi}t} \times\int_{-\infty}^{\infty} dx\langle x-vt,\widetilde{x-vt}|e^{-1/2[\mathcal{G}(t)+\tilde{\mathcal{G}}(t)]}\mathcal{D}(t)\tilde{\mathcal{D}}(t)|W(0)\rangle \\ &= e^{-(it/\hbar)(H_D-\tilde{H}_D)}e^{\hat{\Pi}t}\int_{-\infty}^{\infty} dx\ \ e^{-1/2[\mathcal{G}(t)+\tilde{\mathcal{G}}(t)]}|_{X=\tilde{X}=x-vt} \times \mathcal{D}(t)|_{X=\tilde{X}=x-vt}\tilde{\mathcal{D}}(t)|_{X=\tilde{X}=x-vt}\langle x-vt,\widetilde{x-vt}|W(0)\rangle \\ &= e^{-(it/\hbar)(H_D-\tilde{H}_D)}e^{\hat{\Pi}t}\int_{-\infty}^{\infty} dx\ \ e^{-1/2[\mathcal{G}(t)+\tilde{\mathcal{G}}(t)]}|_{X=\tilde{X}=x-vt} \times \mathcal{D}(t)|_{X=\tilde{X}=x-vt}\tilde{\mathcal{D}}(t)_{X=\tilde{X}=x-vt}P_Q(x-vt)|\mathbf{0},\tilde{\mathbf{0}}\rangle \\ &= e^{-(it/\hbar)(H_D-\tilde{H}_D)}e^{\hat{\Pi}t}\int_{-\infty}^{\infty} dx\ \ P_Q(x)e^{-1/2[\mathcal{G}(t)+\tilde{\mathcal{G}}(t)]}|_{X=\tilde{X}=x} \times \mathcal{D}(t)|_{X=\tilde{X}=x}\tilde{\mathcal{D}}(t)|_{X=\tilde{X}=x}|\mathbf{0},\tilde{\mathbf{0}}\rangle. \end{aligned} \quad (35)$$

Here we define the c-number functions $C_k(x,t)$ and $\mathcal{C}_k(x,t)$ by

$$C_k(x,t) = \Omega_k(x+vt-x_k)e^{\kappa_k t-i\omega_k x/v}, \quad (36)$$

$$\mathcal{C}_k(x,t) = \int_0^t dt' C_k(x,t'). \quad (37)$$

Using the function $\mathcal{C}_k(x,t)$, we find that $\mathcal{D}(t)|_{X=\tilde{X}=x}$ becomes the displacement operator,

$$\mathcal{D}(t)|_{X=\tilde{X}=x} = \exp\left(-i\sum_{k=1}^{N}[\mathcal{C}_k^*(x,t)a_k+\mathcal{C}_k(x,t)a_k^\dagger]\right). \quad (38)$$

Furthermore it is easy to check that $\mathcal{G}(t)|_{X=\tilde{X}=x}$ is identically zero,

$$\mathcal{G}(t)|_{X=\tilde{X}=x} = 0. \quad (39)$$

Therefore we obtain the state vector of the detector system,

$$|W_D(t)\rangle = e^{-(it/\hbar)(H_D-\tilde{H}_D)}e^{\hat{\Pi}t}\int_{-\infty}^{\infty} dxP_Q(x)\bigotimes_k|-i\mathcal{C}_k(x,t)\rangle \times|\widetilde{-i\mathcal{C}_k(x,t)}\rangle, \quad (40)$$

where $|-i\mathcal{C}_k(x,t)\rangle$ is the Glauber coherent state of the kth oscillator of the detector. It is well-known fact that the environment with $T=0$ changes the coherent state as $|\alpha_k\rangle \to |\alpha_k e^{-\kappa_k t}\rangle$ during time t. Therefore we finally obtain the quantum state of the detector which interacts with the propagating particle under the influence of the environment,

$$|W_D(t)\rangle = \int_{-\infty}^{\infty} dxP_Q(x)\bigotimes_k|\alpha_k(x,t)\rangle|\widetilde{\alpha_k(x,t)}\rangle, \quad (41)$$

with

$$\alpha_k(x,t) = -i\mathcal{C}_k(x,t)e^{-i\omega_k t-\kappa_k t}. \quad (42)$$

In the density-matrix representation, the detector state $W_D(t)$ is given by

$$W_D(t) = \int_{-\infty}^{\infty} dx \ P_Q(x) \bigotimes_k |\alpha_k(x,t)\rangle\langle\alpha_k(x,t)|, \quad (43)$$

which is the statistical mixture of the time-dependent coherent states.

To see the property of the coherent state $|\alpha_k(x,t)\rangle$ of the kth oscillator of the detector system, we consider the case of the function $\Omega_k(x)$ which characterizes the interaction between the propagating particle and detector oscillator is proportional to the δ function,

$$\Omega_k(x) = v\beta_k\delta(x), \quad (44)$$

which means that the kth oscillator interacts with the particle only when it passes through at position x. In this case, the complex amplitude of the coherent state of the kth oscillator becomes

$$\alpha_k(x,t) = \begin{cases} 0 & (t < t_k) \\ \beta_k e^{-\kappa_k(t-t_k)}e^{-i\omega_k(t+x/v)-i\pi/2} & (t > t_k), \end{cases} \quad (45)$$

where $t_k=(x_k-x)/v$ is the time that the particle initially located at the position x passes through the site of the kth oscillator. The meaning of the result is clear. The kth oscillator is in the vacuum state before the interaction with the particle ($t<t_k$). The interaction at the time t_k excites the oscillator into the coherent state with amplitude $|\beta_k|$. After the interaction, the amplitude decays exponentially to zero and finally the oscillator returns to the vacuum state. This is just the situation depicted in Fig. 2.

III. DECAY OF QUANTUM COHERENCE

To investigate the decay of the quantum coherence of the propagating particle in the generalized boson-detector model, we consider the Mach-Zehnder interferometer depicted in Fig. 3. We denote as $|1_\mu\rangle$ (or $|0_\mu\rangle$) the state that there is one (or no) particle on the path $\mu(=a,b)$ of the interferometer. We also set $|+\rangle=|1_a\rangle\otimes|0_b\rangle$ and $|-\rangle=|0_a\rangle\otimes|1_b\rangle$. In the figure, we assume that the particle enters the interferometer from the input port a. In general, just after the particle appears at the output of the first beam splitter and before interacting with the detector, the particle state is written as

$$W_Q(0) = \rho_{\text{path}} \otimes \rho_Q, \quad (46)$$

with

$$\rho_{\text{path}} = S_{++}|+\rangle\langle+| + S_{+-}|+\rangle\langle-| + S_{-+}|-\rangle\langle+| + S_{--}|-\rangle\langle-|, \quad (47)$$

and ρ_Q is the initial state of the particle. The expansion coefficients $S'_{jk}s$ satisfy

$$S_{++}+S_{--}=1, \quad S_{\pm,\pm}\geq 0, \quad S_{+-}=S^*_{-+}, \quad S_{++}S_{--}\geq |S_{+-}|^2. \quad (48)$$

When we use a half mirror and enter the particle from the input port a, we have $\rho_{\text{path}}=|\psi\rangle\langle\psi|$ with $|\psi\rangle=(|+\rangle+|-\rangle)/\sqrt{2}$.

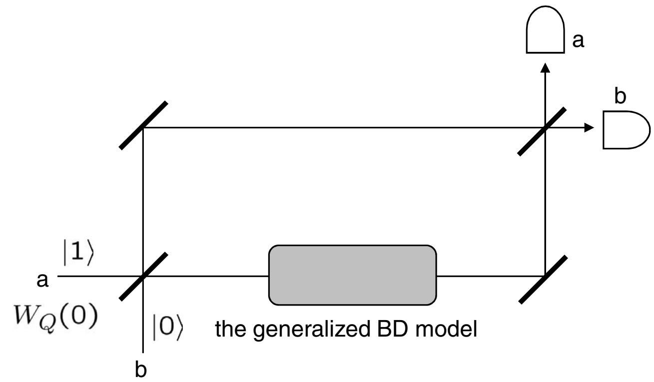

FIG. 3. The schematic representation of the Mach-Zehnder interferometer. The particle interacts with the detector only when it propagates on the path a. The particle-detector system on the path a is described by the generalized BD model. The particle detectors are placed at the output ports of the interferometer in order to observe the fringe visibility. In this figure, it is assumed that the particle enters the interferometer from the input port a.

Since the particle interacts with the detector only when it propagates on the path a, the Hamiltonian H_0+H_1 of the particle-detector system is modified as $H_0+P_+\otimes H_1$ with the projection operator $P_+=|+\rangle\langle+|$. Using the result derived in the previous section, we can obtain the quantum state of the particle and detector system in NETFD at time t before entering the second beam splitter,

$$|W(t)\rangle = e^{-(it/\hbar)(H_0-\tilde{H}_0)}e^{\hat{\Pi}t}e^{-(1/2)[\mathcal{G}(t)+\tilde{\mathcal{G}}(t)]}\mathcal{D}(t)\tilde{\mathcal{D}}(t)|W(0)\rangle, \quad (49)$$

with

$$\mathcal{D}(t) = \exp\left(-i\sum_{k=1}^{N}\left[\int_0^t dt' A_k(t')a_k + \int_0^t dt' B_k(t')a_k^\dagger\right]\right), \quad (50)$$

$$\mathcal{G}(t) = \sum_{k=1}^{N}\int_0^t dt_1\int_0^{t_1} dt_2[A_k(t_1)B_k(t_2) - A_k(t_2)B_k(t_1)], \quad (51)$$

where the operators $A_k(t)$ and $B_k(t)$ are given by

$$A_k(t) = P_+\Omega_k(X+vt-x_k)e^{-\kappa_k t}e^{i\omega_k X/v} - \tilde{P}_+\Omega_k(\tilde{X}+vt-x_k) \times(e^{-\kappa_k t}-e^{\kappa_k t})e^{i\omega_k\tilde{X}/v}, \quad (52)$$

$$B_k(t) = P_+\Omega_k(X+vt-x_k)e^{\kappa_k t}e^{-i\omega_k X/v}, \quad (53)$$

which are equal to those given by Eqs. (26) and (27) if the projection operator P_+ is replaced with the identity.

We first eliminate the information of the particle from the quantum state $|W(t)\rangle$ given by Eq. (49). Using the completeness of the position eigenstates, we obtain the reduced quantum state of the particle path and detector,

$$|\hat{W}(t)\rangle = \int_{-\infty}^{\infty} dx \langle x,\tilde{x}|W(t)\rangle = e^{-(it/\hbar)(H_D-\tilde{H}_D)} e^{\hat{\Pi}t} \int_{-\infty}^{\infty} dx \;\; P_Q(x) e^{-(1/2)[\mathcal{G}(t)+\tilde{\mathcal{G}}(t)]}|_{X=\tilde{X}=x} \times \mathcal{D}(t)|_{X=\tilde{X}=x} \tilde{\mathcal{D}}(t)|_{X=\tilde{X}=x} |\rho_{\text{path}}\rangle \otimes |\mathbf{0},\tilde{\mathbf{0}}\rangle, \tag{54}$$

where $P_Q(x)=\langle x|\rho_Q|x\rangle$ is the initial probability density of the particle position. The operator $\mathcal{D}(t)|_{X=\tilde{X}=x}$ is given by

$$\mathcal{D}(t)|_{X=\tilde{X}=x} = \exp\left\{ -i\sum_{k=1}^{N} [\tilde{P}_+\mathcal{C}^*_{+k}(x,t) + (P_+ - \tilde{P}_+)\mathcal{C}^*_{-k}(x,t)]a_k - i\sum_{k} P_+\mathcal{C}_{+k}(x,t)a_k^\dagger \right\}, \tag{55}$$

with

$$\mathcal{C}_{\pm k}(x,t) = \int_0^t dt'\,\Omega_k(x+vt'-x_k)e^{\pm\kappa_k t' - i\omega_k x/v}, \tag{56}$$

and the operator $\mathcal{G}(t)|_{X=\tilde{X}=x}$ is provided by

$$\mathcal{G}(t)|_{X=\tilde{X}=x} = -2\mathcal{M}(x,t)(P_+ - P_+\tilde{P}_+), \tag{57}$$

with

$$\mathcal{M}(x,t) = \sum_{k=1}^{N}\int_0^t dt_1 \int_0^{t_1} dt_2 \Omega_k(x+vt_1-x_k)\Omega_k(x+vt_2 - x_k)\sinh \kappa_k(t_1-t_2), \tag{58}$$

where we have used the fact that P_+ is the projection operator, namely, $P_+^2=P_+$. If we further eliminate the information of the particle path from the quantum state $|\hat{W}(t)\rangle$, we can obtain the detector state,

$$|W_D(t)\rangle = \langle +,\tilde{\mp}|\hat{W}(t)\rangle + \langle -,\tilde{-}|\hat{W}(t)\rangle, \tag{59}$$

which becomes identical with Eq. (40), where we have used the equality,

$$\langle +,\tilde{\mp}|(P_+ - P_+\tilde{P}_+) = \langle -,\tilde{-}|(P_+ - P_+\tilde{P}_+) = 0. \tag{60}$$

To obtain the fringe visibility of the interferometer, we eliminate the information of the detector system from the quantum state $|\hat{W}(t)\rangle$ given by Eq. (54). After some calculation, we can obtain

$$|\rho_{\text{path}}(t)\rangle = \langle \mathbf{1}|\hat{W}(t)\rangle = \int_{-\infty}^{\infty} dx \;\; P_Q(x)e^{\mathcal{M}(x,t)(P_+-\tilde{P}_+)^2}\langle \mathbf{1}|\mathcal{D}(t)|_{X=\tilde{X}=x}\tilde{\mathcal{D}}(t)|_{X=\tilde{X}=x}|\mathbf{0},\tilde{\mathbf{0}}\rangle|\rho_{\text{path}}\rangle = \int_{-\infty}^{\infty} dx \;\; P_Q(x)e^{\mathcal{M}(x,t)(P_+-\tilde{P}_+)^2}e^{-\hat{\mathcal{F}}(x,t)} \times \langle \mathbf{0}|\exp\left\{-i(P_+-\tilde{P}_+)\sum_{k=1}^{N}[\mathcal{C}_{-k}(x,t)a_k^\dagger + \mathcal{C}^*_{-k}(x,t)a_k]\right\}|\mathbf{0}\rangle|\rho_{\text{path}}\rangle, \tag{61}$$

where $|\mathbf{1}\rangle$ is given by

$$|\mathbf{1}\rangle = \bigotimes_{k=1}^{N}|1_k\rangle, \tag{62}$$

with $|1_k\rangle=\Sigma_{n_k=0}^{\infty}|n_k,\tilde{n}_k\rangle$ and $a_k^\dagger a_k|n_k,\tilde{n}_k\rangle=\tilde{a}_k^\dagger\tilde{a}_k|n_k,\tilde{n}_k\rangle =n_k|n_k,\tilde{n}_k\rangle$, The operator $\hat{\mathcal{F}}(x,t)$ is provided by

$$\hat{\mathcal{F}}(x,t) = \frac{1}{2}\sum_{k=1}^{N}\{(P_+ - P_+\tilde{P}_+)\mathcal{C}_{+k}(x,t)\mathcal{C}^*_{-k}(x,t) + (\tilde{P}_+ - P_+\tilde{P}_+)\mathcal{C}^*_{+k}(x,t)\mathcal{C}_{-k}(x,t) - (P_+-\tilde{P}_+)^2|\mathcal{C}_{-k}(x,t)|^2\}. \tag{63}$$

In deriving Eq. (61), we have used the equality $\langle \mathbf{1}|\hat{\Pi}=0$. Furthermore the vacuum expectation value in Eq. (61) is calculated to be

$$\langle \mathbf{0}|\exp\left\{-i(P_+-\tilde{P}_+)\sum_{k=1}^{N}[\mathcal{C}_{-k}(x,t)a_k^\dagger + \mathcal{C}^*_{-k}(x,t)a_k]\right\}|\mathbf{0}\rangle = \exp\left[-\frac{1}{2}(P_+-\tilde{P}_+)^2\sum_{k=1}^{n}|\mathcal{C}_{-k}(x,t)|^2\right]. \tag{64}$$

Then the quantum state of the particle path is given by

$$|\rho_{\text{path}}(t)\rangle = \int_{-\infty}^{\infty} dx P_Q(x)e^{\mathcal{M}(x,t)(P_+-\tilde{P}_+)^2}\mathcal{N}_k(x,t)|\rho_{\text{path}}\rangle, \tag{65}$$

with

$$\mathcal{N}_k(x,t) = \exp\left\{-\frac{1}{2}\sum_{k=1}^{N}[(P_+ - P_+\tilde{P}_+)\mathcal{C}_{+k}(x,t)\mathcal{C}^*_{-k}(x,t) + (\tilde{P}_+ - P_+\tilde{P}_+)\mathcal{C}^*_{+k}(x,t)\mathcal{C}_{-k}(x,t)]\right\}. \tag{66}$$

Substituting Eq. (47) into Eq. (65), we finally obtain

$$|\rho_{\text{path}}(t)\rangle = S_{++}|+,\tilde{\mp}| + S_{--}|-,\tilde{-}\rangle + S_{+-}\int_{-\infty}^{\infty} dx \;\; P_Q(x) e^{\mathcal{M}(x,t)} \times \exp\left[-\frac{1}{2}\sum_{k=1}^{N} \mathcal{C}_{+k}(x,t)\mathcal{C}^*_{-k}(x,t)\right]|+,\tilde{-}\rangle + S_{-+}\int_{-\infty}^{\infty} dx \;\; P_Q(x) e^{\mathcal{M}(x,t)} \times \exp\left[-\frac{1}{2}\sum_{k=1}^{N} \mathcal{C}^*_{+k}(x,t)\mathcal{C}_{-k}(x,t)\right]|-,\tilde{\mp}\rangle. \tag{67}$$

In the density-matrix form, the state of the particle path is given by

$$\rho_{\text{path}}(t) = S_{++}|+\rangle\langle+| + S_{--}|-\rangle\langle-| + S_{+-}\int_{-\infty}^{\infty} dx \;\; P_Q(x) e^{\mathcal{M}(x,t)} \times \exp\left[-\frac{1}{2}\sum_{k=1}^{N} \Theta_k^{(+)}(x,t)\Theta_k^{(-)}(x,t)\right]|+\rangle\langle-| + S_{-+}\int_{-\infty}^{\infty} dx \;\; P_Q(x) e^{\mathcal{M}(x,t)} \times \exp\left[-\frac{1}{2}\sum_{k=1}^{N} \Theta_k^{(+)}(x,t)\Theta_k^{(-)}(x,t)\right]|-\rangle\langle+|, \tag{68}$$

with

$$\Theta_k^{(\pm)}(x,t) = \int_0^t dt' \,\Omega_k(x + vt' - x_k) e^{\pm\kappa_k t'}. \tag{69}$$

Thus the decay of the fringe visibility of the interferometer (or equivalently the quantum coherence of the particle) is determined by the function $F(t)$,

$$F(t) = \int_{-\infty}^{\infty} dx \;\; P_Q(x) e^{\mathcal{M}(x,t)} \exp\left[-\frac{1}{2}\sum_{k=1}^{N} \Theta_k^{(+)}(x,t)\Theta_k^{(-)}(x,t)\right]. \tag{70}$$

The next task is to obtain the integrand of the decoherence function $F(t)$. First we note that

$$\frac{1}{2}\sum_{k=1}^{N} \Theta_k^{(+)}(x,t)\Theta_k^{(-)}(x,t) = \sum_{k=1}^{N} \int_0^t dt_1 \int_0^{t_1} dt_2 \Omega_k(x + vt_1 - x_k)\Omega_k(x + vt_2 - x_k)\cosh \kappa_k(t_1 - t_2). \tag{71}$$

Then using this equation and Eq. (58), we can finally obtain the decoherence function $F(t)$ of the generalized boson-detector model,

$$F(t) = \int_{-\infty}^{\infty} dx \;\; P_Q(x) \exp\left[-\frac{1}{2}\sum_{k=1}^{N} \int_0^t dt_1 \int_0^t dt_2 \Omega_k(x + vt_1 - x_k)\Omega_k(x + vt_2 - x_x) e^{-\kappa_k|t_1 - t_2|}\right]. \tag{72}$$

When we set $\kappa_k=0$ in this equation, the decoherence function $F(t)$ becomes equal to that obtained for the generalized boson-detector model. It is obvious that the inequality,

$$\int_0^t dt_1 \int_0^t dt_2 \Omega_k(x + vt_1 - x_k)\Omega_k(x + vt_2 - x_x) e^{-\kappa_k|t_1 - t_2|} \leq \left(\int_0^t dt' \,\Omega_k(x + vt' - x_k)\right)^2, \tag{73}$$

holds, which yields the inequality,

$$F(t) \geq F(t)|_{\kappa_k=0}. \tag{74}$$

This result means that the decoherence in the generalized boson-detector model is not greater than that in the conventional boson-detector model. In particular, we can see that

$$\lim_{\kappa_k\to\infty} \int_0^t dt_1 \int_0^t dt_2 \Omega_k(x + vt_1 - x_k)\Omega_k(x + vt_2 - x_x) e^{-\kappa_k|t_1 - t_2|} = 0. \tag{75}$$

It is found from this result that there is no decay of the quantum coherence in the strong damping limit for the detector oscillators.

To investigate the property of the decoherence function $F(t)$, we assume that all the oscillators of the detector are equal and the particle is initially located far from the detector so that there is no interaction at the initial time. We consider the time t after the particle interacts with all the oscillators. Then the function $F(t)$ becomes

$$F(\infty) = \exp\left[-\frac{1}{2}N\int_{-\infty}^{\infty} dt_1 \int_{-\infty}^{\infty} dt_2 \Omega(vt_1 - x)\Omega(vt_2 - x) e^{-\kappa_k|t_1 - t_2|}\right]. \tag{76}$$

We further assume that the function $\Omega(x)$ which characterizes the coupling between the particle and detector oscillator is Gaussian,

$$\Omega(x) = \frac{u}{\sqrt{2\pi\Delta^2 N}} \exp\left(-\frac{x^2}{2\Delta^2}\right), \tag{77}$$

where Δ determines the interaction range of each oscillator with the particle and u is some real constant with dimension of velocity. Then the decoherence function $F(\infty)$ becomes

$$F(\infty) = \exp\left[-\frac{b}{2\pi}\int_{-\infty}^{\infty} dx_1 \int_{-\infty}^{\infty} dx_2 e^{-(1/2)(x_1^2 + x_2^2) - a|x_1 - x_2|}\right], \tag{78}$$

where the parameters a and b are given by

$$a = \frac{\Delta\kappa}{v}, \quad b = \frac{1}{2}\left(\frac{u}{v}\right)^2. \tag{79}$$

Note that the ratio Δ/v is the interaction time of each oscillator and $1/\kappa$ is the relaxation time of the oscillator. Then the parameter a characterizes how much the oscillator decays during the interaction with the particle. Performing the integration, we obtain

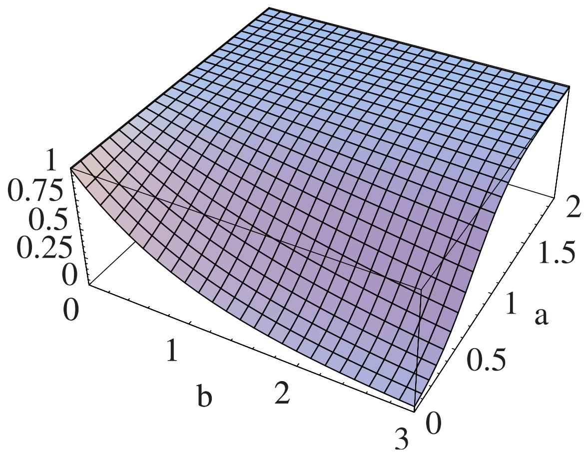

FIG. 4. (Color online) The dependence of the decoherence function $F(\infty)$ of the particle on the parameters a and b.

$$F(\infty) = \exp\{-b[1-\mathrm{erf}(a)]e^{-a^2}\}, \tag{80}$$

where $\mathrm{erf}(x)$ is the error function. This result shows that the function $F(\infty)$ rapidly approaches unity as the value of the parameter a becomes large. The decoherence function $F(\infty)$ is plotted as the function of the parameters a and b in Fig. 4.

Finally we obtain the quantum state of the particle. This can be done by eliminating the information of the particle path and detector system from the quantum state $|W(t)\rangle$ given by Eq. (49). After some calculation, we can derive the reduced quantum state of the particle,

$$\begin{aligned}|\rho_Q(t)\rangle &= \langle 1|(\langle +,\widetilde{+}| + \langle -,\widetilde{-}|)|W(t)\rangle \\ &= e^{-(itv/\hbar)(P-\widetilde{P})}[S_{--}|\rho_Q(0)\rangle + S_{++}G(X,\widetilde{X};t)|\rho_Q(0)\rangle],\end{aligned} \tag{81}$$

where the function $G(t)$ is given by

$$\begin{aligned}G(X,\widetilde{X};t) = \exp\Bigg[&-\frac{1}{2}\sum_{k=1}^{N}\int_0^t dt_1\int_0^t dt_2\Upsilon_k(X,X;t_1,t_2) \\ &-\frac{1}{2}\sum_{k=1}^{N}\int_0^t dt_1\int_0^t dt_2\Upsilon_k(\widetilde{X},\widetilde{X};t_1,t_2) \\ &+\frac{1}{2}\sum_{k=1}^{N}\int_0^t dt_1\int_0^t dt_2\Upsilon_k(\widetilde{X},X;t_1,t_2)e^{-i\omega_k(X-\widetilde{X})/v} \\ &+\frac{1}{2}\sum_{k=1}^{N}\int_0^t dt_1\int_0^t dt_2\Upsilon_k(X,\widetilde{X};t_1,t_2)e^{-i\omega_k(X-\widetilde{X})/v}\Bigg],\end{aligned} \tag{82}$$

with

$$\Upsilon_k(x,x';t_1,t_2) = \Omega_k(x+vt_1-x_k)\Omega_k(x'+vt_2-x_k)e^{-\kappa_k|t_1-t_2|}. \tag{83}$$

When we expand the initial state of the particle in terms of the position eigenstates as

$$|\rho_Q(0)\rangle = \int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dyf(x,y)|x,\widetilde{y}\rangle, \tag{84}$$

the quantum state $|\rho_Q(t)\rangle$ at time t becomes

$$\begin{aligned}|\rho_Q(t)\rangle = \int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dyf(x,y)|x+vt,\widetilde{y+vt}\rangle \\ \times[S_{--}+S_{++}G(x,y;t)].\end{aligned} \tag{85}$$

In the density-matrix representation, we have

$$\begin{aligned}\rho_Q(t) = \int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dyf(x,y)|x+vt\rangle\langle y+vt| \\ \times[S_{--}+S_{++}G(x,y;t)].\end{aligned} \tag{86}$$

Since $\lim_{\kappa_k\to\infty}G(x,y;t)=1$ is fulfilled, we obtain

$$\begin{aligned}\lim_{\kappa_k\to\infty}\rho_Q(t) &= \int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dyf(x,y)|x+vt\rangle\langle y+vt| \\ &= e^{-(itv/\hbar)P}\rho_Q(0)e^{(itv/\hbar)P},\end{aligned} \tag{87}$$

where we have used the relation $S_{++}+S_{--}=1$. This result implies that the particle propagates without interacting with the detector system in the strong damping limit for the detector oscillators.

IV. STOCHASTIC COLEMAN-HEPP MODEL

In this section, we consider the stochastic version of the Coleman-Hepp model, where the interaction Hamiltonian H' given by Eq. (2) is placed with the stochastic Hamiltonian,

$$H'(t) = \sum_{k=1}^{N}\hbar\widetilde{\Omega}_k(X-x_k)\omega_k(t). \tag{88}$$

In this equation, $\omega_k(t)$'s are the independent stochastic variables, each of which obeys the stationary Gauss-Markovian process with a zero mean value, and thus Doob's theorem provides

$$\langle\omega_k(t)\rangle = 0, \quad \langle\omega_k(t)\omega_{k'}(t')\rangle = \delta_{kk'}\Delta_k^2 e^{-|t-t'|/\tau_k}, \tag{89}$$

where $\langle\cdots\rangle$ stands for the average over the stochastic process. In this model, the effect of the detector particle located at x_k on the propagating particle is described by the random fluctuation. In the following, we renormalized $\Delta_k\widetilde{\Omega}_k(x)$ as $\Omega_k(t)$ and thus we can set $\langle\omega_k(t)\omega_{k'}(t')\rangle = \delta_{kk'}e^{-|t-t'|/\tau_k}$. To investigate the decay of the quantum coherence, we suppose the setup depicted in Fig. 3.

Then the quantum state $W(t)$ of the system obeys the Liouville-von Neumann equation,

$$\frac{\partial}{\partial t}W(t) = -\frac{i}{\hbar}[H_Q + P_+\otimes H'(t), W(t)]. \tag{90}$$

When we set $W'(t)=e^{(it/\hbar)H_Q}W(t)e^{-(it/\hbar)H_Q}$, we obtain

$$\frac{\partial}{\partial t}W'(t) = -i\sum_{k=1}^{N}\omega_k(t)[G_k(t),W'(t)] \equiv -i\sum_{k=1}^{N}\omega_k(t)G_k^{\times}(t)W'(t), \tag{91}$$

with $A^{\times}B=[A,B]$ and

$$G_k(t) = \Omega_k(X+vt-x_k)P_+. \tag{92}$$

We can easily solve the equation of motion,

$$W'(t) = \exp\left[-i\sum_{k=1}^{N}\int_0^t dt'\,\omega_k(t')G_k^{\times}(t)\right]W(0). \tag{93}$$

Averaging this equation with respect to the stochastic process, we obtain the quantum state $W'_Q(t)$ of the particle Q in the interaction picture,

$$\begin{aligned}W'_Q(t) &= \left\langle \exp\left[-i\sum_{k=1}^{N}\int_0^t dt'\,\omega_k(t')G_k^{\times}(t')\right]\right\rangle W_Q(0)\\ &= \exp\left[-\frac{1}{2}\sum_{k=1}^{N}\int_0^t dt'\int_0^t dt''\langle\omega_k(t')\omega_k(t'')\rangle G_k^{\times}(t')G_k^{\times}(t'')\right]W_Q(0)\\ &= \exp\left[-\frac{1}{2}\sum_{k=1}^{N}\int_0^t dt'\int_0^t dt'' e^{-|t'-t''|/\tau_k}G_k^{\times}(t')G_k^{\times}(t'')\right]W_Q(0),\end{aligned} \tag{94}$$

where we have used the fact that the stochastic variables obey the stationary Gauss-Markovian process.

To obtain the state of the particle path, we eliminate the information of the particle state from the quantum state $W'(t)$ given by Eq. (94). Using the completeness of the position eigenstates, we obtain

$$\begin{aligned}\rho_{\text{path}}(t) &= \int_{-\infty}^{\infty} dx\langle x|W'(t)|x\rangle\\ &= \left\langle \exp\left[-\frac{1}{2}\mathcal{A}(t;x)(P_+^{\times})^2\right]\right\rangle_x \rho_{\text{path}}(0),\end{aligned} \tag{95}$$

with

$$\begin{aligned}\mathcal{A}(t;x) = \sum_{k=1}^{N}\int_0^t dt_1\int_0^t dt_2 e^{-|t_1-t_2|/\tau_k}\Omega_k(x+xt_1-x_k)\Omega_k\\ \times(x+vt_2-x_k).\end{aligned} \tag{96}$$

In Eq. (95), $\langle\cdots\rangle_x$ stands for the average value with the probability density $P_Q(x)=\langle x|\rho_Q(0)|x\rangle$ of the particle position. Since we have the equality $(P_+^{\times})^4=(P_+^{\times})^2$, we can derive the relation,

$$e^{-(1/2)\mathcal{A}(t;x)(P_+^{\times})^2} = 1 + [e^{-(1/2)\mathcal{A}(t;x)} - 1](P_+^{\times})^2. \tag{97}$$

Using this relation and expanding the state $\rho_{\text{path}}(0)$ of the particle path as Eq. (47), we obtain

$$\begin{aligned}\rho_{\text{path}}(t) = S_{--}|-\rangle\langle-| + S_{++}|+\rangle\langle+| + F'(t)S_{+-}|+\rangle\langle-|\\ + F'(t)S_{-+}|-\rangle\langle+|,\end{aligned} \tag{98}$$

where the decoherence function $F'(t)$ of the stochastic version of the Coleman-Hepp model is given by

$$\begin{aligned}F'(t) = \int_{-\infty}^{\infty} dx\;\; P_Q(x)\exp\Bigg[-\frac{1}{2}\sum_{k=1}^{N}\int_0^t dt_1\int_0^t dt_2 e^{-|t_1-t_2|/\tau_k}\Omega_k\\ \times(x+xt_1-x_k)\Omega_k(x+vt_2-x_k)\Bigg].\end{aligned} \tag{99}$$

It is obvious that this result is equivalent to that given by Eq. (72). Therefore the generalized boson-detector model is equivalent to the stochastic Coleman-Hepp mode with the stationary Gauss-Markovian process with respect to the quantum coherence of the propagating particle. From the stochastic point of view, the Coleman-Hepp model without dissipation [6–8] corresponds to the stochastic model in which the stochastic variables have very long correlation times, that is, $\tau_k \to \infty$.

V. CONCLUDING REMARKS

In this paper, we have investigated the generalized boson-detector model, where the detector is placed under the influence of the surrounding environment. Assuming that the effect of the environmental system on the detector is described by the quantum Markovian process, we can solve the model exactly. We have obtained the time evolution of the particle and detector system by means of the method of NETFD. We have investigated the relaxation of the detector system and the decay of the quantum coherence of the propagating particle. We have found that the decay of the quantum coherence of the particle is made negligible when the relaxation of the detector caused by the environment is significant. Furthermore, with respect to the particle dynamics, we have found the stochastic model that is equivalent to the generalized boson-detector model. When the correlation time of the stochastic variable is very long, we obtain the same results as those obtained for the generalized Coleman-Hepp model without dissipation.

ACKNOWLEDGMENT

One of the authors (S.K.) greatly appreciates Toshihico Arimitsu of University of Tsukuba for stimulating discussions in the early stage of this work.

APPENDIX: NONEQUILIBRIUM THERMO FIELD DYNAMICS

In this appendix, we briefly summarize the method of NETFD [12–17]. We suppose that a physical system is described by a Hilbert space $\mathcal{H}$. Then a density operator which represents a quantum state of the system is a Hermitian operator acting on vectors of the Hilbert space $\mathcal{H}$. Any operator A acting on vectors of the Hilbert space $\mathcal{H}$ can be represented by a vector $|A\rangle$ belonging to an extended Hilbert space $\mathcal{H}\otimes\tilde{\mathcal{H}}$. In NETFD, the auxiliary Hilbert space $\tilde{\mathcal{H}}$ is called the tilde-conjugated space of the original Hilbert space $\mathcal{H}$ [15]. Hence a quantum state of the system can be given by a state vector of the extended Hilbert space $\mathcal{H}\otimes\tilde{\mathcal{H}}$. When an operator A acting on vectors of the Hilbert space $\mathcal{H}$ is given by the vector $|A\rangle$ of the Hilbert space $\mathcal{H}\otimes\tilde{\mathcal{H}}$, a product XAY can be represented by the vector $X\tilde{Y}^{\dagger}|A\rangle$, where the operator $\tilde{Y}$ acting on vectors of the Hilbert space $\tilde{\mathcal{H}}$ is the tilde conjugate of the operator Y. The tilde conjugation of operators is defined by [15]

$$(A_1A_2)^{\sim}=\tilde{A}_1\tilde{A}_2, \tag{A1}$$

$$(A^{\dagger})^{\sim}=(\tilde{A})^{\dagger}, \tag{A2}$$

$$(\tilde{A})^{\sim}=\sigma A, \tag{A3}$$

$$(a_1A_1+a_2A_2)^{\sim}=a_1^{*}\tilde{A}_1+a_2^{*}\tilde{A}_2, \tag{A4}$$

where A_k's are arbitrary operators of the Hilbert space $\mathcal{H}$, and a_k's are arbitrary c numbers. In Eq. (A3), $\sigma=1$ ($\sigma=-1$) is assigned for a bosonic (fermionic) operator A. For bosonic annihilation and creation operators a and $a^{\dagger}$ satisfying $[a,a^{\dagger}]=1$, and position and momentum operators X and P satisfying $[X,P]=i\hbar$, we have the commutation relations $[\tilde{a},\tilde{a}^{\dagger}]=1$ and $[\tilde{X},\tilde{P}]=-i\hbar$ of their tilde-conjugated operators.

Let us consider a quantum system in a quantum state described by a density operator $W(t)$, the time evolution of which obeys the Liouville-von Neumann equation,

$$\frac{\partial}{\partial t}W(t)=-\frac{i}{\hbar}[H,W(t)], \tag{A5}$$

where H is the Hamiltonian of a system. Then for the state vector $|W(t)\rangle$ which represents the density operator $W(t)$, we obtain the equation of motion $|W(t)\rangle$,

$$\frac{\partial}{\partial t}|W(t)\rangle=-\frac{i}{\hbar}\hat{H}|W(t)\rangle, \tag{A6}$$

with

$$\hat{H}=H-\tilde{H}, \tag{A7}$$

where we have used the correspondence relations, $HW(t)\leftrightarrow H|W(t)\rangle$ and $W(t)H\leftrightarrow\tilde{H}|W(t)\rangle$. When we consider an open system, the time evolution of which is described by the quantum Markovian process, the density matrix $W(t)$ obeys the quantum master equation of the Lindblad form [10,11],

$$\frac{\partial}{\partial t}W(t)=-\frac{i}{\hbar}[H,W(t)]+F\{[LW(t),L^{\dagger}]+[L,W(t)L^{\dagger}]\}$$
$$+G\{[L^{\dagger}W(t),L]+[L^{\dagger},W(t)L]\}, \tag{A8}$$

where $L=a$ for an oscillator system and $L=S_{-}(=S_x-iS_y)$ for a spin system. In this equation, G and F are real parameters. In NETFD, we obtain the equation of motion for the vector $|W(t)\rangle$,

$$\frac{\partial}{\partial t}|W(t)\rangle=-\frac{i}{\hbar}\hat{H}|W(t)\rangle+\hat{\Pi}|W(t)\rangle, \tag{A9}$$

with

$$\hat{\Pi}=F(2L\tilde{L}-L^{\dagger}L-\tilde{L}^{\dagger}\tilde{L})+G(2L^{\dagger}\tilde{L}^{\dagger}-LL^{\dagger}-\tilde{L}\tilde{L}^{\dagger}). \tag{A10}$$

The scalar product of two vectors $|A\rangle$ and $|B\rangle$ of the Hilbert space $\mathcal{H}\otimes\tilde{\mathcal{H}}$, which correspond to operators A and B of the Hilbert space, is given by the Hilbert-Schmidt product $\langle A|B\rangle=\mathrm{Tr}(A^{\dagger}B)$. Since an average value of any observable A of a physical system in a quantum state W is given by $\langle A\rangle=\mathrm{Tr}(AW)$, we have the expression $\langle A\rangle=\langle 1|A|W\rangle$ in NETFD. In this equation, the vector $|1\rangle$ is defined by

$$|1\rangle=\sum_k|e_k\rangle\otimes|\tilde{e}_k\rangle, \tag{A11}$$

where $|e_k\rangle(|\tilde{e}_k\rangle)$ is the basis vector of the Hilbert space $\mathcal{H}$ ($\tilde{\mathcal{H}}$). The vector $|1\rangle$ of the Hilbert space $\mathcal{H}\otimes\tilde{\mathcal{H}}$ corresponds to an identity operator $1=\Sigma_k|e_k\rangle\langle e_k|$ of the Hilbert space $\mathcal{H}$. For an oscillator system, we have $|1\rangle=\Sigma_{n=0}^{\infty}|n\rangle\otimes|\tilde{n}\rangle$, where $|n\rangle=(1/\sqrt{n!})a^{\dagger n}|0\rangle$ and $|\tilde{n}\rangle=(1/\sqrt{n!})\tilde{a}^{\dagger n}|\tilde{0}\rangle$ with $a|0\rangle=\tilde{a}|\tilde{0}\rangle=0$ are the Fock states. The scalar product $\langle 1|X\rangle$ in NETFD is nothing but the trace operation for the operator X. For a density operator W^{AB} of a bipartite system, the reduced density operator of the subsystem is given by performing the partial-trace operation, e.g., $W^{A}=\mathrm{Tr}_B W^{AB}$. In NETFD, the state vector that represents the reduced quantum state is provided by $|W^{A}\rangle=\langle 1_B|W^{AB}\rangle$ with $|1_B\rangle=\Sigma_k|e_k^B\rangle\otimes|\tilde{e}_k^B\rangle$, where $\{|e_1^B\rangle,|e_2^B\rangle,\ldots\}$ is a complete orthonormal system of the subsystem B. The method and applications of NETFD have been given in Refs. [12–17].

[1] A. Peres, *Quantum Theory: Concepts and Methods* (Kluwer, New York, 1993).
[2] K. Hepp, Helv. Phys. Acta **45**, 237 (1972).
[3] M. Cini, Nuovo Cimento Soc. Ital. Fis., B **73**, 27 (1983).
[4] H. Nakazato and S. Pascazio, Phys. Rev. A **45**, 4355 (1992).
[5] T. Kobayashi, Phys. Rev. A **46**, 6851 (1992).
[6] H. Nakazato and S. Pascazio, Phys. Rev. Lett. **70**, 1 (1993).
[7] H. Nakazato and S. Pascazio, Phys. Rev. A **48**, 1066 (1993).
[8] K. Hiyama and S. Takagi, Phys. Rev. A **48**, 2586 (1993).
[9] M. Namiki, S. Pascazio, and H. Nakazato, *Decoherence and Quantum Measurements* (World Scientific, Singapore, 1997).
[10] E. B. Davies, *Quantum Theory of Open Systems* (Academic, New York, 1976).
[11] H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford University Press, Oxford, 2006).
[12] T. Arimitsu and H. Umezawa, Prog. Theor. Phys. **74**, 429 (1985).
[13] T. Arimitsu and H. Umezawa, Prog. Theor. Phys. **77**, 32 (1987).
[14] T. Arimitsu and H. Umezawa, Prog. Theor. Phys. **77**, 53 (1987).
[15] H. Umezawa, *Advanced Field Theory –Micro, Macro, and Thermal Physics* (American Institute of Physics, New York, 1992).
[16] S. Chaturvedi and V. Srinivasan, J. Mod. Opt. **38**, 777 (1991).
[17] S. Chaturvedi and V. Srinivasan, Phys. Rev. A **43**, 4054 (1991).
[18] T. Kobayashi, Phys. Lett. A **185**, 349 (1994).
[19] S. Kitajima, C. Takahashi, and F. Shibata, Eur. Phys. J. D **30**, 417 (2004).
[20] S. Kitajima and F. Shibata, J. Phys. Soc. Jpn. **69**, 73 (2000).
[21] S. Kitajima and F. Shibata, J. Phys. Soc. Jpn. **69**, 2004 (2000).
[22] S. Kitajima and F. Shibata, J. Phys. Soc. Jpn. **70**, 2273 (2001).
[23] S. Kitajima and F. Shibata, J. Phys. Soc. Jpn. **72**, 1899 (2003).