

# Dispersion interaction between two atoms out of thermal equilibrium with external electromagnetic fields

Y. Sherkunov

*Department of Physics, University of Warwick, Coventry CV4 7AL, United Kingdom*

(Received 26 June 2008; revised manuscript received 12 December 2008; published 5 March 2009)

The van der Waals–Casimir-Polder potential is calculated for two atoms out of equilibrium with external thermal electromagnetic field. The effects of absorption and spontaneous or stimulated emission are shown to strongly modify the potential. We demonstrate that the potential decays like  $R^{-2}$  at large distances (much slower than at the equilibrium) can be attractive, repulsive, and resonant depending on the initial states of the atoms and electromagnetic field.

DOI: [10.1103/PhysRevA.79.032101](https://doi.org/10.1103/PhysRevA.79.032101)

PACS number(s): 12.20.-m, 34.35.+a, 42.50.Ct

## I. INTRODUCTION

The dispersion forces such as Casimir, Casimir-Polder (CP), and van der Waals forces are of particular interest not only as the beautiful manifestation of the zero-point fluctuations of electromagnetic (EM) field but also important technologically, e.g., in atomic force microscopy. Thus, the thorough understanding of the dispersion interaction under realistic conditions (e.g., in external electromagnetic fields) is required.

Recent progress has been made in understanding the non-equilibrium CP force between an atom and a substrate embedded in electromagnetic field. Antezza *et al.* [1] considered the case when the substrate and the environment were at different temperatures. They demonstrated theoretically that the nonequilibrium CP force drops like  $1/z^3$  at large distances  $z$ , which is slower than the equilibrium force ( $1/z^4$ ). Moreover the nonequilibrium force can be either attractive or repulsive depending on the ratio of the substrate temperature to the environment one. These results were verified experimentally [2] and excellent agreement with theoretical predictions has been achieved.

There are results for the CP force between an atom and a substrate if the substrate is in the equilibrium with the environment, but the atom is not [3,4]. It has been shown that if the atom is not thermalized, one should take into account possible absorption and spontaneous or stimulated emission of a thermal photon by the atom. These contributions result in the strong modification of the CP force even if the atom is in its ground state.

Recent theoretical and experimental progress in nonequilibrium Casimir physics inspired us to consider the nonequilibrium dispersion interaction between two atoms in EM field.

If two atoms are in the equilibrium with thermal EM field at temperature  $T$  the interaction potential is well known [5],

$$U_{\text{eq}} = -\frac{2T}{R^6} \sum_{n=0}^{\infty} (1 - \delta_{n,0}/2) \alpha_A(i\xi_n) \alpha_B(i\xi_n) \times e^{-2\xi_n R} (\xi_n^4 R^4 + 2\xi_n^3 R^3 + 5\xi_n^2 R^2 + 6\xi_n R + 3) \quad (1)$$

where  $\alpha_{A(B)}$  is the polarizability of atom  $A$  ( $B$ ),  $R$  is the distance between the atoms, and  $\xi_n = 2\pi nT$  is the Mazubara frequency. For short distances  $R$  ( $R \ll \lambda$ , where  $\lambda$  is the characteristic wavelength of atomic transitions) the retardation

and thermal effects are negligible, and one finds the temperature-independent London–van der Waals potential  $U \propto R^{-6}$ . For the distances large compared with  $\lambda$ , but still short compared with the parameter  $\lambda_T = c/T$ ,  $\lambda \ll R \ll \lambda_T$ , the retardation effects are significant, but thermal effects are still negligible. In this case one obtains the temperature-independent Casimir-Polder potential  $U \propto R^{-7}$ . The thermal effects are significant if  $R \gg \lambda_T$ . In this case the potential is  $U \propto TR^{-6}$ . The equilibrium potential is always attractive. In this paper we study the dispersion interaction between two atoms, provided they are *not* in the equilibrium with external thermal EM field.

## II. INTERACTION BETWEEN A GROUND-STATE ATOM AND A MAGNETODIELECTRIC MEDIUM IN ELECTROMAGNETIC FIELD

We start with the CP interaction between a single ground-state atom (atom  $A$ ) at a position  $\mathbf{R}_A$  and an arbitrary dielectric medium in the presence of thermal EM field. The field and the medium are not necessarily at thermal equilibrium. For the sake of simplicity we consider a two-level atom. Generalization to a multilevel atom is straightforward. Following the earlier treatment for a dipole interaction of a ground-state atom with a dielectric medium in EM vacuum [6], we generalize our previous results to the case of EM field surrounding the atom and the medium using the Keldysh Green's function method [7,8]. The derivation consists of two main steps. First, we derive the interaction potential in terms of the Green's tensors of the EM field surrounding the atom. Then we calculate the Green's tensors for the EM field modified by the medium. We suppose that the number of photons at the transition frequencies of atom  $A$  is negligible  $N(\omega_A) \approx 0$ , where  $\omega_A$  is the transition frequency of atom  $A$ . Thus, atom  $A$  does not transfer to its excited state. In this case we can treat the CP potential as the energy shift of the ground level of the atom  $U = \Delta \epsilon_A$  [9]. Later we will generalize the result to an arbitrary initial state of atom  $A$  and arbitrary number of photons at the transition frequency of atom  $A$  using symmetry properties of the potential.

The Hamiltonian of the atom–A–EM-field–medium system reads ( $\hbar = 1$ ,  $c = 1$ )  $H = H_0 + H'$ , where the Hamiltonian of free system is  $H_0 = \sum_i \epsilon_i b_i^\dagger b_i + \sum_{\mathbf{k}\lambda} |\mathbf{k}| (\alpha_{\mathbf{k}\lambda}^\dagger \alpha_{\mathbf{k}\lambda} + 1/2) + H_{\text{med}}^0$ . The interaction Hamiltonian is  $H = H'_{\text{med}} - \int \psi^\dagger(X) \mathbf{dE}(X) \psi(X) \mathbf{dr}$ , where  $\alpha_{\mathbf{k}\lambda}$  is the annihilation opera-

tor of the state  $\{\mathbf{k}, \lambda\}$  of the EM field,  $\epsilon_i$  is the bare energy of atom  $A$  in the state  $i$ ,  $b_i$  is the annihilation operator of this state,  $\psi(X) = \sum_i \phi_i(\mathbf{r}, \mathbf{R}_A) b_i$  with  $\phi$  as the wave function of atom  $A$ ,  $X = \{\mathbf{r}, t\}$ , and  $\mathbf{E}$  is the operator of EM field.  $\mathbf{d}$  is the dipole moment of the atom,  $H_{\text{med}}^0$  is the Hamiltonian of the medium, provided it does not interact with the EM field,  $H'_{\text{med}}$  is the part of the Hamiltonian of the medium describing the interaction with the EM field.

The Keldysh Green's functions of atom  $A$  and EM field in the interaction representation are [7,8]

$$G_{ll'}(X, X') = -i \langle T_c \psi_l(X) \psi_{l'}^\dagger(X') S_c \rangle, \\ D_{ll'}^{\nu\nu'}(X, X') = -i \langle T_c E_l^\nu(X) E_{l'}^{\nu'}(X') S_c \rangle, \quad (2)$$

where  $S_c$  is the scattering matrix  $S_c = T_c \exp[\sum_{l=1,2} (-1)^l i \int H_l' dt]$  given on the Keldysh time contour, which goes in time from  $-\infty$  to  $\infty$  ( $l=1$ ) and then from  $\infty$  to  $-\infty$  ( $l=2$ ),  $T_c$  is the time-ordering operator on the Keldysh contour.

For our purpose we need only the Green's function  $G_{12}$  defined in the Heisenberg representation (the operators are marked with tilde) as  $G_{12}(X, X') = -i \langle \tilde{\psi}^\dagger(X') \tilde{\psi}(X) \rangle$  [8]. It is proportional to the density matrix of atom  $A$ . The Green's function  $G_{12}$  obeys an integral equation, which for the quasiclassical case is the standard kinetic equation [8].

The solution of the integral equation for  $G_{12}$  for the case of single atom  $A$  interacting with EM field can be found with the help of the method proposed in Refs. [6,10]. Treating the high-order correlation functions of atomic operators, which emerge in perturbation series, we implement exact Wick's theorem [11] representing the atomic time-ordered operators as

$$T_c \psi_l(X) \psi_{l'}^\dagger(X') =: \psi_l(X) \psi_{l'}^\dagger(X') : + i g_{ll'}^0(X, X'), \quad (3)$$

where  $: \dots :$  means normal ordering,

$$g_{ll'}^0(X, X') = -i \langle T_c \psi_l(X) \psi_{l'}^\dagger(X') \rangle_{\text{vac}} \quad (4)$$

is the free atomic propagator,  $\langle \dots \rangle_{\text{vac}}$  means the averaging over vacuum electronic state. The average of normal products over the initial state occupied by a single atom  $\langle : \psi_l(X) \psi_{l'}^\dagger(X') \psi_{l_1}^\dagger(X_1) \psi_{l_2}^\dagger(X_2) \dots \rangle = 0$  for all the orders but the first one. For the first order it is proportional to the Green's function  $G_{12}^0$  for a free atom.

Expanding  $S_c$  in the perturbation series and summing up all corresponding diagrams, we find [6,10]

$$G_{12}(X, X') = G_{12}^0(X, X') + \int G_{12}^0(X, X_1) M_{22}(X_1, X_2) \\ \times g_{22}(X_2, X') dX_1 dX_2 + \int g_{11} M_{11} G_{12}^0 dX_1 dX_2 \\ + \int g_{11} M_{11} G_{12}^0 M_{22} g_{22} dX_1 \dots dX_4 \\ - \int g_{11} M_{12} g_{22} dX_1 dX_2 \quad (5)$$

where  $g$  is the atomic propagator obeying the Dyson equation in the energy representation

$$g_{ll'}(E, \mathbf{r}, \mathbf{r}') = g_{ll'}^0(E, \mathbf{r}, \mathbf{r}') + \int g_{ll_1}^0(E, \mathbf{r}, \mathbf{r}_1) M_{l_1 l_2}(E, \mathbf{r}_1, \mathbf{r}_2) \\ \times g_{l_2 l'}(E, \mathbf{r}_2, \mathbf{r}') d\mathbf{r}_1 d\mathbf{r}_2.$$

$M_{ll'}(X, X') = i(-1)^{l+l'} d^\nu d^{\nu'} g_{ll'}(X, X') D_{l_1 l_1}^{\nu \nu'}(X', X)$  is the mass operator. We assume Einstein summation assumption throughout the paper. Under the assumption that atom  $A$  does not change its initial state, the last term of Eq. (5) can be omitted [6]. We should stress that Eq. (5) is not an integral equation like the standard Keldysh one, but represents the solution for  $G_{12}$ . We suppose that at the initial moment of time  $t_0$ , atom  $A$  was in its initial unperturbed state 0. Then  $G_{12}^0 = -i \phi_0(\mathbf{r}) \phi_0^*(\mathbf{r}') \exp[-i\epsilon_0(t-t')] \theta(t-t_0) \theta(t'-t_0)$ .

Direct calculation yields

$$G_{12} = -i \phi_0(\mathbf{r}) \phi_0^*(\mathbf{r}') \exp[-i\epsilon_0(t-t')] \exp[iM_{11}^0(t-t_0)] \\ \times \exp[-iM_{22}^0(t'-t_0)] \theta(t-t_0) \theta(t'-t_0), \quad (6)$$

with  $M_{ll'}^0 = \langle \phi_0 M_{ll'} \phi_0 \rangle$ . Thus, the energy shift of the ground level of atom  $A$  is  $U = \Delta \epsilon_A = \text{Re } M_{11}^0$ .  $\text{Im } M$  describes the decay of the level 0 as a result of excitations. After some simple algebra, and using explicit expression for free atomic propagator  $g_{11}^0(E, \mathbf{r}, \mathbf{r}') = \sum_i \frac{\phi_i(\mathbf{r}) \phi_i^*(\mathbf{r}')}{E - \epsilon_i - i0}$  we obtain

$$U = \text{Re } M_{11}^0 = -\text{Re} \frac{i}{2\pi} \int_0^\infty \alpha_A^{\nu \nu'}(\omega) D_{11}^{\nu \nu'}(\omega, \mathbf{R}_A, \mathbf{R}_A) d\omega. \quad (7)$$

This formula generalizes the one obtained in [6] to the case of external EM field. The polarizability of the atom at the initial state 0 is given by the standard formula

$$\alpha_A^{\nu \nu'}(\omega) = \left( \frac{d^\nu d^{\nu'}}{\omega_A - \omega - i0} + \frac{d^{\nu'} d^\nu}{\omega_A + \omega + i0} \right), \quad (8)$$

with  $\omega_A$  as the transition frequency of the atom from the ground state 0 to the excited state,  $d^\nu$  is the  $\nu$ th projection of the electric-dipole matrix element between the ground state and the excited one,  $i0$  describes the analytical properties of the polarizability.

The Green's tensor  $D_{11}$  is the sum of the retarded tensor  $D_R$  and  $D_{12}$  [7,8], the latter is proportional to the density matrix of photons  $D_{12}^{\nu \nu'}(X, X') = -i \langle \tilde{E}^{\nu'}(X') \tilde{E}^\nu(X) \rangle$ . Thus

$$U = -\text{Re} \frac{i}{2\pi} \int_0^\infty \alpha_A^{\nu \nu'}(\omega) D_R^{\nu \nu'}(\omega, \mathbf{R}_A, \mathbf{R}_A) d\omega \\ - \text{Re} \frac{i}{2\pi} \int_0^\infty \alpha_A^{\nu \nu'}(\omega) D_{12}^{\nu \nu'}(\omega, \mathbf{R}_A, \mathbf{R}_A) d\omega. \quad (9)$$

The first term of Eq. (9) is the standard potential for the ground-state atom interacting with the vacuum EM field in the presence of an arbitrary dielectric body [12]. The second term describes the interaction of the atom with the external EM field.

As an example we consider a ground-state atom embedded in the system EM field—dielectric medium, provided the medium and the field are at thermal equilibrium, but the atom is not. We suppose that the atom does not change its initial state, which means that the time scales are short compared to the inverse ground-state heating rates of the atom  $\Gamma^{-1}$ . This situation has been recently considered by Buhmann and Scheel [3] (see also [4]). Substituting the Green's tensors of equilibrium EM field  $D_{12}(\omega, \mathbf{R}_A, \mathbf{R}_A) = \text{Re } D_R(\omega, \mathbf{R}_A, \mathbf{R}_A) + i \text{Im } D_R(\omega, \mathbf{R}_A, \mathbf{R}_A) \coth \omega/2T$  [5] we find

$$U = T \sum_{m=0}^{\infty} \left( 1 - \frac{1}{2} \delta_{m0} \right) \alpha_A^{\nu' \nu} (i\xi_m) D_r^{\nu \nu'} (i\xi_m) - 1/3 \sum_j N(\omega_{jk}) |d_{kj}|^2 \text{Re } D_r(\omega_{jk}) \theta(\omega_{jk}). \quad (10)$$

This result coincides with Eq. (25) of [3].

### III. INTERACTION BETWEEN TWO ATOMS IN ELECTROMAGNETIC FIELD

Now we consider the dispersion interaction between two atoms in EM field. Let a two-level atom  $B$  be at a position  $\mathbf{R}_B$ . It can be either excited with probability  $p_e$  or in its ground state with probability  $p_g$ . For simplicity we suppose that the atoms are exposed to isotropic unpolarized EM field, i.e.,  $N_{\mathbf{k}\lambda}$  depends only on  $|\mathbf{k}| = \omega$ ,  $N_{\mathbf{k}\lambda} = N(\omega)$ . We will attack the problem perturbatively. The Green's tensor  $D_{12}$  can be calculated with the help of Keldysh technique [7,8]. For positive frequencies we get

$$D_{12}(\omega, \mathbf{R}_A, \mathbf{R}_A) = D_{12}^0(\omega, \mathbf{R}_A, \mathbf{R}_A) - 2N(\omega) D_R^0(\omega, \mathbf{R}_A, \mathbf{R}_B) \alpha_B(\omega) D_R^0(\omega, \mathbf{R}_B, \mathbf{R}_A) + N(\omega) 2\pi i d_B d_B \delta(\omega - \omega_B) p_g |D_R^0(\omega, \mathbf{R}_A, \mathbf{R}_B)|^2 - [N(\omega) + 1] 2\pi i d_B d_B \delta(\omega - \omega_B) p_e |D_R^0(\omega, \mathbf{R}_A, \mathbf{R}_B)|^2. \quad (11)$$

Here we suppressed the tensor indices  $\nu$ .  $D^0$  is the free photon Green's tensor. The first term of Eq. (11) describes the free EM field and results in optical Stark shift of the level of atom  $A$ . We will omit this term. The second term is proportional to the scattering part of the EM Green's tensor and the number of photons. It describes the scattering of real photons on atom  $B$ . The third term is responsible for the absorption of a real photon by ground-state atom  $B$ . The last term is due to spontaneous or stimulated emission of a photon by excited atom  $B$ . To support this interpretation, we use the explicit expression for the Green's tensor  $D_R^0$  [5],

$$D_R^{\nu \nu'}(\omega, \mathbf{r}, \mathbf{r}') = \omega^2 \left[ \delta_{\nu \nu'} \left( 1 + \frac{i}{\omega R} - \frac{1}{(\omega R)^2} \right) + s^\nu s^{\nu'} \left( \frac{3}{(\omega R)^2} - \frac{3i}{\omega R} - 1 \right) \right] \frac{\exp(i\omega R)}{R}, \quad (12)$$

where  $s^\nu = (r - r')^\nu / R$ ,  $R = |r - r'|$ , and notice that

$$|D_R^0|^2 = \frac{2\omega^4}{R^2} [1 + 1/(\omega R)^2 + 3/(\omega R)^4], \quad (13)$$

which is proportional to the correlation function of the field created by a classical dipole [13].

The interaction potential thus consists of the sum of two independent terms—the equilibrium potential,  $U_{\text{eq}}$  [Eq. (1)], and the nonequilibrium one,  $U_{\text{neq}}$ ,

$$U = U_{\text{eq}} + U_{\text{neq}},$$

$$U_{\text{eq}} = \text{Re} \frac{i}{\pi} \int_0^\infty d\omega [N(\omega) + 1/2] \alpha_A(\omega) \alpha_B(\omega) \times [D_R^0(\omega, \mathbf{R}_A, \mathbf{R}_B)]^2,$$

$$U_{\text{neq}} = \frac{1}{3} \text{Re} |d_B|^2 \alpha_A(\omega_B) |D_R^0(\omega_B, \mathbf{R}_A, \mathbf{R}_B)|^2 \times \{p_g N(\omega_B) - p_e [N(\omega_B) + 1]\}. \quad (14)$$

The equilibrium potential,  $U_{\text{eq}}$ , coincides with the one obtained with the help of Lifshitz formula [14] or Milonni-Smith formula [15]. The nonequilibrium potential,  $U_{\text{neq}}$ , describes the absorption of a photon by atom  $B$  (first term in the square brackets) as well as the spontaneous and stimulated emission by atom  $B$  (the second term in the curly brackets). For the thermal equilibrium, the probability to find an atom in its  $n$ th state  $p_n$  is given by the Boltzmann distribution  $p_e = p_g e^{-\omega_B/T}$ , while the number of photons obeys the Bose statistics  $N(\omega, T) = (e^{\omega/T} - 1)^{-1}$ . Thus,  $U_{\text{neq}} = 0$  due to the detailed balance, and we find that Eq. (14) coincides with the Lifshitz formula [14].

The results obtained above are valid only if atom  $A$  is in its ground state during the interaction process. This means that the number of photons at the transition energies of atom  $A$  is negligible [ $N(\omega_A) \approx 0$ ], while the number of photons at the frequency of atom  $B$  is sufficiently large  $N(\omega_B) \gg N(\omega_A)$ . Our formalism does not allow us to calculate analytically the interaction potential for the case when atom  $A$  can be excited by the electromagnetic field. But to extend our result to the case free from the above mentioned assumption, we can use the symmetry properties of the potential: (i) the potential is obviously symmetric with respect to atomic indices; (ii) if both atoms are thermalized  $U_{\text{neq}} = 0$ . Using these properties, we calculate the potential in general case,

$$U_{\text{neq}} = \frac{2}{9} \frac{|d_B|^2 |d_A|^2 \omega_A}{\omega_A^2 - \omega_B^2} |D_R^0(\omega_B, \mathbf{R}_A, \mathbf{R}_B)|^2 \times \{p_g^B N(\omega_B) - p_e^B [N(\omega_B) + 1]\} (p_g^A - p_e^A) + \frac{2}{9} \frac{|d_B|^2 |d_A|^2 \omega_B}{\omega_B^2 - \omega_A^2} |D_R^0(\omega_A, \mathbf{R}_A, \mathbf{R}_B)|^2 \times \{p_g^A N(\omega_A) - p_e^A [N(\omega_A) + 1]\} (p_g^B - p_e^B) \quad (15)$$

To obtain the quantitative result we substitute the Green's tensor Eq. (13) into Eq. (15),

$$\begin{aligned}
U_{\text{neq}} = & \frac{4}{9R^2} \frac{|d_B|^2 |d_A|^2 \omega_A \omega_B}{\omega_A^2 - \omega_B^2} \left[ \omega_B^3 \left( 1 + \frac{1}{(\omega_B R)^2} + \frac{3}{(\omega_B R)^4} \right) \right. \\
& \times \{p_g^B N(\omega_B) - p_e^B [N(\omega_B) + 1]\} (p_g^A - p_e^A) \\
& - \omega_A^3 \left( 1 + \frac{1}{(\omega_A R)^2} + \frac{3}{(\omega_A R)^4} \right) \\
& \left. \times \{p_g^A N(\omega_A) - p_e^A [N(\omega_A) + 1]\} (p_g^B - p_e^B) \right] \quad (16)
\end{aligned}$$

The distance dependence of the nonequilibrium potential is determined only by the characteristic wavelength of the atomic transitions and does not depend on the initial states of the atoms or the state of the EM field. The only exception is the system of two ground-state atoms in a EM vacuum (for this case  $U_{\text{neq}}=0$ ). For the nonretarded regime  $R \ll \lambda$ , the nonequilibrium potential is  $U_{\text{neq}}=C_6/R^6$ , which is the same as for the equilibrium potential. For the retarded regime  $R \gg \lambda$ ,  $U_{\text{neq}}=C_2/R^2$ . This means that the nonequilibrium potential decays with the distance much more slowly than the equilibrium one, which drops like  $R^{-7}$  for low temperatures  $T$  ( $R \ll \lambda_T=c/T$ ) and  $R^{-6}$  for large temperatures ( $R \gg \lambda_T$ ). The coefficients  $C_6$  and  $C_2$  depend on the initial states of the atoms and the state of EM field. They can be positive or negative or even resonant.

As an example we consider two two-level atoms embedded in the thermal EM field at temperature  $T$ .

(1) Let both atoms be in their ground states.

If the temperature is low compared to the transition frequencies ( $T \ll \omega_{A,B}$ ), but high compared to the frequency difference ( $T \gg |\omega_A - \omega_B|$ ), we find

$$\begin{aligned}
U_{\text{neq}} = & \frac{4}{9R^2} \frac{\exp(-\omega_A/T) |d_A|^2 |d_B|^2 \omega_A \omega_B}{\omega_A + \omega_B} \\
& \times \left[ -(\omega_B^2 + \omega_A \omega_B + \omega_A^2) - \frac{1}{R^2} + \frac{3}{\omega_A \omega_B R^4} \right]. \quad (17)
\end{aligned}$$

This result is not resonant and exponentially small compared to the equilibrium contribution, but the asymptotic behavior of Eq. (17) for large distances ( $R \gg \lambda$ ) is  $1/R^2$ , while the asymptotics of the equilibrium contribution ( $R \gg 1/T$ ) is  $1/R^6$  [5].

For  $T \ll |\omega_A - \omega_B|$ ,

$$\begin{aligned}
U_{\text{neq}} = & \frac{4}{9R^2} \frac{\exp(-\omega_B/T) |d_A|^2 |d_B|^2 \omega_A \omega_B^4}{\omega_A^2 - \omega_B^2} \\
& \times \left( 1 + \frac{1}{(\omega_B R)^2} + \frac{3}{(\omega_B R)^4} \right), \quad (\omega_A > \omega_B), \quad (18)
\end{aligned}$$

$$\begin{aligned}
U_{\text{neq}} = & -\frac{4}{9R^2} \frac{\exp(-\omega_A/T) |d_A|^2 |d_B|^2 \omega_B \omega_A^4}{\omega_A^2 - \omega_B^2} \\
& \times \left( 1 + \frac{1}{(\omega_A R)^2} + \frac{3}{(\omega_A R)^4} \right), \quad (\omega_A < \omega_B). \quad (19)
\end{aligned}$$

The contributions of the nonequilibrium potential are resonant.

For a high-temperature case ( $T \gg \omega_{A,B}$ ) we obtain the universal expression, which does not depend on the initial states of the atoms

$$U_{\text{neq}} = \frac{4}{9R^2} T |d_A|^2 |d_B|^2 \omega_A \omega_B \left( -1 + \frac{3}{(\omega_B \omega_A)^2 R^4} \right). \quad (20)$$

The equilibrium contribution at high temperatures ( $T \gg 1/R$ ) is [5]

$$U_{\text{eq}} = -\frac{4}{3R^6} T |d_A|^2 |d_B|^2 \frac{3}{\omega_B \omega_A R^6}. \quad (21)$$

The total potential is

$$U = U_{\text{eq}} + U_{\text{neq}} = \frac{4}{9R^2} T |d_A|^2 |d_B|^2 \omega_A \omega_B. \quad (22)$$

For  $T \gg \omega_{A,B}$ ,  $T \gg 1/R$  the total interaction potential drops like  $1/R^2$  with the distance  $R$ , even though  $R \ll \lambda$ .

(2) Two excited atoms.

If  $T \ll \omega_{A,B}$ ,

$$\begin{aligned}
U_{\text{neq}} = & \frac{4}{9R^2} \frac{|d_A|^2 |d_B|^2 \omega_A \omega_B}{\omega_A + \omega_B} \\
& \times \left[ -(\omega_B^2 + \omega_A \omega_B + \omega_A^2) - \frac{1}{R^2} + \frac{3}{\omega_A \omega_B R^4} \right]. \quad (23)
\end{aligned}$$

This result is no more exponentially small (although is not resonant) compared to the equilibrium potential and could be greater than the latter. For large distances we once again obtain the  $R^{-2}$  dependence of the potential. If  $T \gg \omega_{A,B}$ , we obtain Eq. (22).

(3) Atom  $A$  is in its ground state, atom  $B$  is excited.

$T \ll \omega_{A,B}$ ,

$$U_{\text{neq}} = \frac{4}{9R^2} \frac{|d_A|^2 |d_B|^2 \omega_A \omega_B^4}{\omega_A^2 - \omega_B^2} \left( 1 + \frac{1}{(\omega_B R)^2} + \frac{3}{(\omega_B R)^4} \right). \quad (24)$$

This nonequilibrium contribution is resonant and exceeds the equilibrium contribution.

For  $T \gg \omega_{A,B}$  we once again obtain the result [Eq. (22)].

In Fig. 1 we show the potential for two atoms embedded in a thermal electromagnetic field as a function of the distance between the atoms, if the temperature is of the order of the transition frequencies. The contribution of the nonequilibrium term is significant for two ground-state atoms even at small distances ( $R \ll \lambda$ ), although the  $R$  dependence of the equilibrium potential and the nonequilibrium one is the same ( $R^{-6}$ ). For the retarded case  $R \gg \lambda$ , one can see the slow down of the interaction potential in comparison with the equilibrium situation. In the inset we show the spectral dependence of the nonequilibrium potential. The interaction

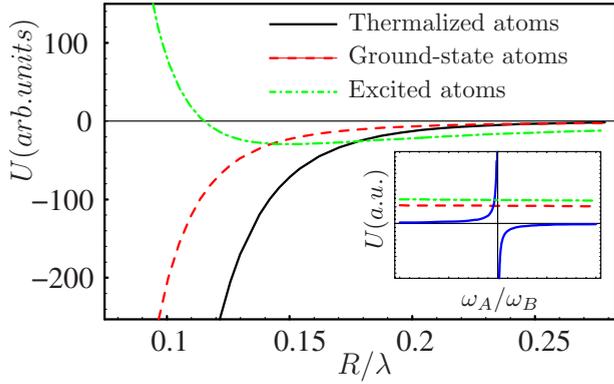


FIG. 1. (Color online) Interaction between two atoms in thermal EM field. Main figure: the interaction potential  $U$  (arbitrary units) vs the distance  $R/\lambda$  between two two-level atoms ( $\omega_B=1.05\omega_A$ ) embedded in the thermal EM field at temperature  $T=\omega_A$ . Thermalized atoms—solid line, ground-state atoms—dashed line, and excited atoms—dashed-dotted line. Inset: nonequilibrium potential between two atoms in thermal EM field vs  $\omega_A/\omega_B$ .  $R=0.12\lambda$ ,  $T=\omega_A$ .  $A$  is ground-state,  $B$  is excited—solid line, ground-state atoms—dashed line, and excited atoms—dashed-dotted line.

between the ground-state atom and the excited one is resonant, while the interaction between two ground-state atoms or two excited ones is not.

Now we consider the interaction potential between two atoms in EM vacuum [ $N(\omega_A)=N(\omega_B)=0$ ],

$$U_{\text{neq}} = -\frac{4}{9R^2} \frac{|d_B|^2 |d_A|^2 \omega_A \omega_B}{\omega_A^2 - \omega_B^2} \times \left[ \omega_B^3 \left( 1 + \frac{1}{(\omega_B R)^2} + \frac{3}{(\omega_B R)^4} \right) p_e^B (p_g^A - p_e^A) - \omega_A^3 \left( 1 + \frac{1}{(\omega_A R)^2} + \frac{3}{(\omega_A R)^4} \right) p_e^A (p_g^B - p_e^B) \right]. \quad (25)$$

This result coincides with the one obtained by Power and Thirunamachandran [16,17].

The approach we developed in this presentation is valid for the initial stage of interaction provided the atoms and the EM field are not in the equilibrium. Obviously, after the equilibration the nonequilibrium term is zero and we obtain the standard equilibrium potential.

In [10] we considered dispersion interaction between two media of excited atoms. A significant discrepancy between the results of [10] and the Lifshits formula were found, provided the EM field was in its vacuum state. According to the findings of the present paper, the results obtained using the approach of [10] and the Lifshitz formula coincide, provided the equilibrium EM field is taken into account.

#### IV. CONCLUSIONS

In conclusion, we considered the atom-atom dispersion interaction in EM fields. We showed that the absorption and emission of photons results in the strong modification of the interaction potential at the initial stage of the nonequilibrium dynamics.

[1] M. Antezza, L. P. Pitaevskii, and S. Stringari, Phys. Rev. Lett. **95**, 113202 (2005).  
 [2] J. M. Obrecht, R. J. Wild, M. Antezza, L. P. Pitaevskii, S. Stringari, and E. A. Cornell, Phys. Rev. Lett. **98**, 063201 (2007).  
 [3] S. Y. Buhmann and S. Scheel, Phys. Rev. Lett. **100**, 253201 (2008).  
 [4] M.-P. Gorza and M. Ducloy, Eur. Phys. J. D **40**, 343 (2006).  
 [5] E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics, Part 2, Course on Theoretical Physics* (Pergamon, Oxford, 1980), Vol. 9.  
 [6] Y. Sherkunov, Phys. Rev. A **75**, 012705 (2007).  
 [7] L. Keldysh, Zh. Eksp. Teor. Fiz. **47**, 1515 (1964).  
 [8] E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics, Course on Theoretical Physics* (Pergamon, Oxford, 1981), Vol. 10.

[9] H. B. G. Casimir and D. Polder, Phys. Rev. **73**, 360 (1948).  
 [10] Y. Sherkunov, Phys. Rev. A **72**, 052703 (2005).  
 [11] V. Berestetskii, E. Lifshitz, and L. Pitaevskii, *Relativistic Quantum Theory, Part 1, Course on Theoretical Physics* (Pergamon, Oxford, 1971), Vol. 4.  
 [12] J. M. Wylie and J. E. Sipe, Phys. Rev. A **32**, 2030 (1985).  
 [13] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields, Course on Theoretical Physics* (Pergamon, Oxford, 1971), Vol. 2.  
 [14] E. M. Lifshitz, Zh. Eksp. Teor. Fiz. **2**, 73 (1956).  
 [15] P. W. Milonni and A. Smith, Phys. Rev. A **53**, 3484 (1996).  
 [16] E. A. Power and T. Thirunamachandran, Phys. Rev. A **47**, 2539 (1993).  
 [17] E. A. Power and T. Thirunamachandran, Phys. Rev. A **51**, 3660 (1995).