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Complete state reconstruction of a two-mode Gaussian state via local operations and classical communication

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We propose a strictly local protocol that is completely equivalent to global quantum state reconstruction for a bipartite system. We show that the joint density matrix of an arbitrary two-mode Gaussian state, entangled or not, is obtained via local operations and classical communication only. In contrast to previous proposals, simultaneous homodyne measurements on both modes are replaced with local homodyne detections and a set of local projective measurements.

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The feasibility of a quantum information task is related to the reduced or absence of nonlocal resources needed to its implementation and is an important asset for quantum communication purposes [1], setting the limit for its widespread use. However, quantum state tomography (QST) [2,3], a key tool in quantum information, is performed mostly through nonlocal operations. QST is a complete state reconstruction scheme implemented through a set of measurements over an ensemble of identical quantum systems. For qubit systems it corresponds to the determination of all the Stokes parameters [4]. For Gaussian continuous variable (CV) systems, as given by quantized electromagnetic field modes, it stands on a set of joint quadrature measurements, from which the joint density matrix ρ is reconstructed. Thus for Gaussian states, QST is equivalent to the measurement of global covariance matrices of the modes. For a two-mode Gaussian state most QST protocols to date either require simultaneous homodyne measurements (HMs) on both modes [5–7], with an exquisite control of both local oscillator (LO) phases, or require previous nonlocal operations on the modes to achieve a complete state reconstruction [8]. It is desirable, therefore, the construction of a QST protocol not requiring any nonlocal operation and no phase locking. In other words, a process operationally equivalent to QST, but without unnecessary nonlocal resources to its implementation.

In this Rapid Communication we show how one can reconstruct the whole density matrix ρ_{12} of an arbitrary two-mode Gaussian state via local operations and classical communication (LOCC) only. Since simultaneous HMs of the two modes [5,6] are not required, there is no need for constrained control of the LO phases, thus increasing the overall efficiency of the protocol and also reducing the computational postprocessing of data (see Ref. [9] for an interesting single homodyning alternative scheme). Instead, a set of local parity and vacuum projections plus local squeezing are required. Our protocol is built basically on three premises: (i) Alice and Bob can implement independent single mode local QST, certifying that they have a Gaussian state. Actually, after confirming (or being informed previously) that one deals with a Gaussian state, only HMs of the variances of the modes will suffice. (ii) Both Alice and Bob are able to implement local squeezing and a local rotation on the quadratures of their modes. (iii) Bob (Alice) can make two types of local measurements: even (odd) parity projections and vacuum projections of his (her) mode.

A bipartite two-mode Gaussian state ρ_{12} is completely described [10,11] by its Gaussian characteristic function $C(\mathbf{z})=\mathrm{Tr}[D(z)\rho_{12}]=e^{-(1/2)\mathbf{z}^\dagger\mathbf{V}\mathbf{z}}$, where $\mathbf{z}^\dagger=(z_1^*,z_1,z_2^*,z_2)$ are complex numbers, $D(z)=e^{-\mathbf{z}^\dagger\mathbf{E}\mathbf{v}}$ is the displacement operator, with $\mathbf{E}=\mathrm{diag}(\mathbf{Z},\mathbf{Z})$, $\mathbf{Z}=\mathrm{diag}(1,-1)$, and $\mathbf{v}=(v_1,v_2,v_3,v_4)^T=(a_1,a_1^\dagger,a_2,a_2^\dagger)^T$ as the annihilation and creation operators of modes 1 and 2, respectively. T is the transposition, so that $\mathbf{v}$ is a column vector, and we have assumed all the first-order moments to be null [12]. The covariance matrix $\mathbf{V}$ describing all the second-order moments $V_{ij}=(-1)^{i+j}\langle v_i v_j^\dagger + v_j^\dagger v_i\rangle/2$ is given by

$$\mathbf{V}=\begin{pmatrix}\mathbf{V}_1 & \mathbf{C}\\ \mathbf{C}^\dagger & \mathbf{V}_2\end{pmatrix}=\begin{pmatrix} n_1 & m_1 & m_s & m_c\\ m_1^* & n_1 & m_c^* & m_s^*\\ m_s^* & m_c & n_2 & m_2\\ m_c^* & m_s & m_2^* & n_2\end{pmatrix}. \quad (1)$$

Here $\mathbf{V}_1$ and $\mathbf{V}_2$ are the local covariance matrices of modes 1 and 2, respectively, giving the local properties of the two modes while $\mathbf{C}$ is the correlation between them. Finally, in addition to being positive semidefinite, $\mathbf{V}\geq\mathbf{0}$, a physical Gaussian state must satisfy the generalized uncertainty principle, $\mathbf{V}+\frac{1}{2}\mathbf{E}\geq\mathbf{0}$ [10].

The main goal of Alice and Bob is to obtain via LOCC the matrix $\mathbf{V}$. Therefore, the first logical and trivial step consists in the measurement of $\mathbf{V}_1$ and $\mathbf{V}_2$ by Alice and Bob, respectively. These two covariance matrices are locally obtained via any standard single-mode HM technique (or local QST). Up to now no classical communication is needed and only after finishing this task Bob (Alice) informs Alice (Bob) of his (her) result. It is worth noting that we assume Alice and Bob have at their disposal a trustful source, in the sense that it produces as many as needed identical copies of the two-mode Gaussian state.

The next nontrivial step is the determination of $\mathbf{C}$. To achieve such a goal, Alice and Bob need to work collaboratively [13]. First, on a subensemble of the copies, Bob implements parity measurements on his mode and informs Alice

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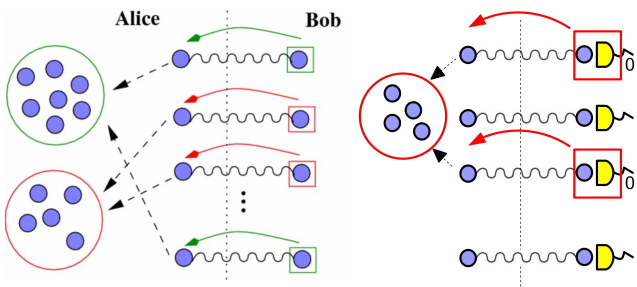

FIG. 1. (Color online) Left: Alice separates her copies in two groups conditioned on an even (green, top) or odd (red, bottom) parity result obtained by Bob. Right: Alice selects copies corresponding to Bob's no-photon results.

the respective outcomes for each copy, i.e., even parity (even number of photons) or odd parity (odd number of photons). With this information Alice separates her copies in two distinct groups, the even and the odd ones [13], as depicted in Fig. 1. Alice's even group can be described by the non-normalized density matrix $\rho_1^e=\mathrm{Tr}_2\{P_2^e\rho_{12}P_2^e\}=\Sigma_{n=0}^{\infty}\,{}_2\langle 2n|\rho_{12}|2n\rangle_2$, where $P_2^e=\mathcal{I}_1\otimes\Sigma_{n=0}^{\infty}|2n\rangle_2\,{}_2\langle 2n|$, $\mathcal{I}_1$ is the identity operator, and $|n\rangle_2$ is the nth Fock state for mode 2. Using a similar notation, Alice's odd group is given as $\rho_1^o=\mathrm{Tr}_2\{P_2^o\rho_{12}P_2^o\}=\Sigma_{n=0}^{\infty}\,{}_2\langle 2n+1|\rho_{12}|2n+1\rangle_2$. But one can show that [14] $\sigma_1=2\sqrt{\det(\mathbf{V}_2)}(\rho_1^e-\rho_1^o)=\int d\mathbf{z}_1\, e^{\mathbf{z}_1^\dagger\mathbf{Z}\mathbf{a}_1}e^{-(1/2)\mathbf{z}_1^\dagger\mathbf{\Gamma}_1\mathbf{z}_1}$, where $d\mathbf{z}_1=(1/\pi)d\,\mathrm{Re}(z_1)d\,\mathrm{Im}(z_1)$ and $\mathbf{\Gamma}_1$ is the Schur complement [15] of $\mathbf{V}_2$,

$$\Gamma_1=\mathbf{V}_1-\mathbf{C}\mathbf{V}_2^{-1}\mathbf{C}^\dagger=\begin{pmatrix}\eta_1 & \mu_1\\ \mu_1^* & \eta_1\end{pmatrix}. \tag{2}$$

However, any one-mode Gaussian operator can be written as $\int d\mathbf{z}_1\, e^{\mathbf{z}_1^\dagger\mathbf{Z}\mathbf{a}_1}e^{-(1/2)\mathbf{z}_1^\dagger\mathbf{\Gamma}_1\mathbf{z}_1}$, with $\mathbf{\Gamma}_1$ being its covariance matrix [10]. Therefore, σ_1 is a Gaussian operator whose covariance matrix elements are $\eta_1=2\sqrt{\det(\mathbf{V}_2)}(\langle a_1^\dagger a_1\rangle_e-\langle a_1^\dagger a_1\rangle_o)$ and $\mu_1=2\sqrt{\det(\mathbf{V}_2)}(\langle a_1^2\rangle_e-\langle a_1^2\rangle_o)$, where $\langle\cdot\rangle_e=\mathrm{Tr}(\cdot\rho_1^e)$ and $\langle\cdot\rangle_o=\mathrm{Tr}(\cdot\rho_1^o)$. Summing up, $\mathbf{\Gamma}_1$ can be obtained with the knowledge of $\det(\mathbf{V}_2)$ and the second moments of ρ_1^e and ρ_1^o, all of which determined via LOCC [16]. Defining $\gamma=(n_1-\eta_1)(n_2^2-|m_2|^2)$ and $\delta=(m_1-\mu_1)(n_2^2-|m_2|^2)$, Eq. (2) gives two independent equations, which alone cannot give m_s and m_c unequivocally,

$$\gamma=n_2(|m_c|^2+|m_s|^2)-2\,\mathrm{Re}(m_2m_sm_c^*), \tag{3}$$

$$\delta=2n_2m_sm_c-m_2^*m_c^2-m_2m_s^2. \tag{4}$$

A unique solution, though, can be obtained if we consider an additional subensemble on which Bob performs another kind of projective measurement. The results of this measurement are communicated to Alice who builds a local covariance matrix that is related to the original one through the Schur complement structure, similar to Eq. (2). In the present case we consider the simplest choice, i.e., Bob is able to perform a vacuum state projection on his copies: photon-number measurements with no outcome. For each measurement, Bob informs Alice to which copies a no-photon result ($\rho_2\to|0\rangle_2\,{}_2\langle 0|$) occurred. Alice, then, proceeds in a similar fashion as before but considering only the vacuum-projected subensemble (Fig. 1, right), described by the density matrix $\rho_1^{\mathrm{vac}}=\mathrm{Tr}_2(|0\rangle_2\,{}_2\langle 0|\rho_{12})/\mathrm{Tr}_{12}(|0\rangle_2\,{}_2\langle 0|\rho_{12})$. One can show that [14] $\rho_1^{\mathrm{vac}}=\int d\mathbf{z}_1\, e^{\mathbf{z}_1^\dagger\mathbf{Z}\mathbf{a}_1}e^{-(1/2)\mathbf{z}_1^\dagger\mathbf{\Pi}_1\mathbf{z}_1}$, where

$$\mathbf{\Pi}_1=\mathbf{V}_1-\mathbf{C}\left(\mathbf{V}_2+\frac{1}{2}\mathbf{I}\right)^{-1}\mathbf{C}^\dagger=\begin{pmatrix}\xi_1 & \nu_1\\ \nu_1^* & \xi_1\end{pmatrix}, \tag{5}$$

with $\mathbf{I}$ as the identity matrix of dimension 2. Here, $\xi_1=\langle a_1^\dagger a_1\rangle_{\mathrm{vac}}$ and $\nu_1=\langle a_1^2\rangle_{\mathrm{vac}}$, where $\langle\cdot\rangle_{\mathrm{vac}}=\mathrm{Tr}(\cdot\rho_1^{\mathrm{vac}})$. Explicitly, Eq. (5) gives us two more equations:

$$\alpha=\left(n_2+\frac{1}{2}\right)(|m_c|^2+|m_s|^2)-2\,\mathrm{Re}(m_2m_sm_c^*), \tag{6}$$

$$\beta=2\left(n_2+\frac{1}{2}\right)m_sm_c-m_2^*m_c^2-m_2m_s^2, \tag{7}$$

in which $\alpha=(n_1-\xi_1)[(n_2+1/2)^2-|m_2|^2]$ and $\beta=(m_1-\nu_1)[(n_2+1/2)^2-|m_2|^2]$. It is worth noting that α and β, as well as γ and δ, are functions of parameters locally obtained by Alice and Bob. In order to solve Eqs. (3), (4), (6), and (7) for m_c and m_s we write $m_j=|m_j|e^{i\theta_j}$, where $j=1$, 2, c, s, and θ_j is real. In this notation, our task is to determine θ_c, θ_s, $|m_c|$, and $|m_s|$ with the aid of $\mathbf{\Gamma}_1$ and $\mathbf{\Pi}_1$ obtained by Alice via LOCC. The other quantities n_1, n_2, $|m_1|$, $|m_2|$, θ_1, and θ_2 are easily obtained via local HM of modes 1 and 2.

(i) *Determination of* θ_c *and* θ_s. Subtracting Eq. (4) from Eq. (7) we have $\beta-\delta=m_sm_c$, which gives

$$|m_sm_c|=|\beta-\delta|, \tag{8}$$

$$\theta_s+\theta_c=\mathrm{Arg}(\beta-\delta), \tag{9}$$

where $\mathrm{Arg}(z)$ is the phase of the complex number z. By the same token, subtracting Eq. (3) from Eq. (6) we have

$$|m_c|^2+|m_s|^2=2(\alpha-\gamma). \tag{10}$$

Inserting Eqs. (8) and (10) into Eq. (6) we get, $\alpha=(2n_2+1)(\alpha-\gamma)-2|m_2(\beta-\delta)|\cos(\theta_2+\theta_s-\theta_c)$. We could have used Eq. (3) as well. Solving, then, for $\theta_s-\theta_c$ we obtain

$$\theta_s-\theta_c=\cos^{-1}\{[\alpha n_2-\gamma(n_2+1/2)]/|m_2(\beta-\delta)|\}-\theta_2. \tag{11}$$

Equations (9) and (11) can be easily solved to give θ_c and θ_s, the phases of m_c and m_s. It is worth mentioning that Eq. (11) is only valid when $|(\beta-\delta)m_2|\neq 0$. Later we show how to overcome this limitation.

(ii) *Determination of* $|m_c|$ *and* $|m_s|$. From Eq. (10) we note that if we had $|m_c|^2-|m_s|^2$ the problem would be solved. Manipulating the real and imaginary parts of Eq. (7) we get

$$|m_c|^2-|m_s|^2=|\beta|\sin(\theta_\beta-\theta_c-\theta_s)/[|m_2|\sin(\theta_2-\theta_c+\theta_s)]. \tag{12}$$

Here θ_β is the phase of β. Equations (10) and (12) can be directly solved to give $|m_c|$ and $|m_s|$, the moduli of m_c and m_s. Equation (12) is only valid for $|m_2\sin(\theta_2-\theta_c+\theta_s)|\neq 0$. Thus, all the covariance matrix elements can be locally reconstructed with a set of appropriate measurements and classical

TABLE I. Overview of the general strategies. Here $j=2,s,c$.

	$m_j=0$	$\sin(\theta_2-\theta_c+\theta_s)=0$
Local squeezing	Yes	Yes
Local quadrature rotation	No	Yes

communication, establishing the following important connection to Gaussian QST: *global QST is completely equivalent to local covariance matrix HM, local parity and vacuum-state projections, and classical communication.* This is our central result and in the rest of this Rapid Communication we show how the necessary conditions $|(\beta-\delta)m_2|\neq 0$ and $|m_2\sin(\theta_2-\theta_c+\theta_s)|\neq 0$ can always be obtained by the addition of local squeezing [17].

(iii) *Overcoming* $|(\beta-\delta)m_2|=0$ *or* $|m_2\sin(\theta_2-\theta_c+\theta_s)|=0$. To properly solve these problems we must know which quantity is zero. The simplest check is implemented when Bob reconstructs $\mathbf{V}_2$, which allows him to know if $m_2=0$. Alice and Bob can also discover if $m_c=m_s=0$ (implying $\beta-\delta=m_sm_c=0$) by testing if $\mathbf{V}_1=\mathbf{\Gamma}_1=\mathbf{\Pi}_1$, since the absence of correlation ($\mathbf{C}=\mathbf{0}$) between the modes cannot change what the parties measure locally [see Eqs. (2) and (5)]. Also, if $\mathbf{V}_1\neq\mathbf{\Gamma}_1$ or $\mathbf{V}_1\neq\mathbf{\Pi}_1$ Alice and Bob are sure that $\mathbf{C}\neq\mathbf{0}$ and the first nontrivial check sets in. They must discover if either $m_c=0$ and $m_s\neq 0$ or $m_c\neq 0$ and $m_s=0$ or both $m_c\neq 0$ and $m_s\neq 0$. If either m_c or m_s is zero it is obvious that $|I_3|=|\det(\mathbf{C})|=||m_s|^2-|m_c|^2|=|m_s|^2+|m_c|^2$. But one can show [13] that $|I_3|=\sqrt{\det(\mathbf{V}_2)\det(\mathbf{V}_1-\mathbf{\Gamma}_1)}$ and using Eq. (10) we see that if $|I_3|=2(\alpha-\gamma)$ we know for sure that either m_s or m_c is zero. If we do not have an equality $m_c\neq 0$ and $m_s\neq 0$. For our purposes, as we explain below, we do not need to know which quantity, m_c or m_s, is zero [18]. Finally, to discover if $\sin(\theta_2-\theta_c+\theta_s)=0$ we use Eq. (11) and the phase of m_2. Of course, Eq. (11) is only valid if $|m_2(\beta-\delta)|\neq 0$. Therefore, if $|m_2(\beta-\delta)|=0$ we first need to solve this problem in order to test if $\sin(\theta_2-\theta_c+\theta_s)=0$. Since now we know which parameter is zero we are ready to show how Alice and Bob can overcome this situation allowing them to use Eqs. (9)–(12) to obtain $\mathbf{C}$. See Table I for an overview of the strategies to solve these problems.

If $m_2=0$ the most general solution [19] is achieved implementing a local symplectic transformation (local quadrature squeezing and rotation) on mode 2 [20], $\mathbf{S}=\mathrm{diag}(\mathbf{I}_1,\mathbf{S}_2)$, where $\mathbf{I}_1$ is a 2×2 identity matrix acting on system 1 and $\mathbf{S}_2$ is given as

$$\mathbf{S}_2=\begin{pmatrix} e^{-is_2}\cosh r_2 & \sinh r_2 \\ \sinh r_2 & e^{is_2}\cosh r_2 \end{pmatrix} \tag{13}$$

with s_2 and r_2 being real parameters. The correlation matrix $\tilde{\mathbf{V}}$ is connected to $\mathbf{V}$ by $\tilde{\mathbf{V}}=\mathbf{S}\mathbf{V}\mathbf{S}^\dagger$ [20] or, equivalently for $j=1,2$, $\tilde{\mathbf{V}}_j=\mathbf{S}_j\mathbf{V}_j\mathbf{S}_j^\dagger$ and $\tilde{\mathbf{C}}=\mathbf{S}_1\mathbf{C}\mathbf{S}_2^\dagger$. Applying $\mathbf{S}$ to Eq. (1), the off-diagonal term of $\tilde{V}_2$ is $\tilde{m}_2=e^{-2is_2}m_2\cosh^2 r_2+m_2^*\sinh^2 r_2+e^{-is_2}n_2\sinh(2r_2)$. Setting $s_2=0$ and using $m_2=0$ we have

$$\tilde{m}_2=n_2\sinh(2r_2), \tag{14}$$

i.e., a covariance matrix with $\tilde{m}_2\neq 0$. After this operation we can proceed with the original protocol to reconstruct $\tilde{\mathbf{V}}$, which can be transformed back to give $\mathbf{V}=\mathbf{S}^{-1}\tilde{\mathbf{V}}\mathbf{S}^{\dagger-1}$ with $\mathbf{S}^{-1}=\mathrm{diag}(\mathbf{I}_1,\mathbf{S}_2^{-1})$ and

$$\mathbf{S}_2^{-1}=\begin{pmatrix} e^{is_2}\cosh r_2 & -\sinh r_2 \\ -\sinh r_2 & e^{-is_2}\cosh r_2 \end{pmatrix}. \tag{15}$$

If either $m_s=0$ or $m_c=0$, or equivalently $\beta-\delta=0$, we can obtain another matrix $\tilde{\mathbf{C}}=\mathbf{S}_1\mathbf{C}\mathbf{S}_2^\dagger$, where both parameters are not zero via a local squeezing operation alone. This leads to

$$\tilde{m}_s=e^{is_2}m_s\cosh r_2+m_c\sinh r_2, \tag{16}$$

$$\tilde{m}_c=e^{-is_2}m_c\cosh r_2+m_s\sinh r_2. \tag{17}$$

Setting $s_2=0$ in Eqs. (16) and (17) we see that $\tilde{m}_s$ and $\tilde{m}_c$ are combinations of m_s and m_c. Therefore, if $m_s=0$ or $m_c=0$ the present coefficients are necessarily different from zero whenever we apply a local squeezing operation on mode 2. As anticipated, we do not need to know which quantity was originally zero. As before, after this local transformation we proceed with the original protocol obtaining $\tilde{\mathbf{V}}$ and then $\mathbf{V}$. It is worth noting that when the two situations occur simultaneously, i.e., $m_2=0$ and $m_s=0$ or $m_c=0$, the same local squeezing operation solves at once both problems, as can be seen in Eqs. (14), (16), and (17).

Lastly, after being sure that $|m_2m_cm_s|\neq 0$ we can proceed to test if $\sin(\theta_2-\theta_c+\theta_s)=0$ using Eq. (11) and the phase of m_2, which are all quantities locally determined. In case of a positive result, there exist three possible solutions. The first one is valid when $m_2\neq m_1$ and is achieved reversing the roles of Alice and Bob in the protocol, as discussed above. The remaining two possibilities, and more general, is to locally and unitary transform mode 1 or mode 2 before we implement the protocol, in the same fashion as before. Therefore, we need to show that there exists at least one local unitary operation acting on mode 1 or mode 2 that eliminates such a problem.

Let us begin with mode 2. Applying the symplectic local transformation $\mathbf{S}_2$ we get, after assuming that $\sin(\theta_2-\theta_c+\theta_s)=0$,

$$\tan(\tilde{\theta}_2+\tilde{\theta}_s-\tilde{\theta}_c)=2\mathcal{A}_\pm\sin(\theta_2-s_2)\sinh(2r_2)/\mathcal{B}_\pm, \tag{18}$$

$$\mathcal{A}_\pm=|m_2|(|m_c|^2+|m_s|^2)\mp 2n_2|m_cm_s|, \tag{19}$$

$$\begin{aligned}\mathcal{B}_\pm={}&\pm 3|m_2m_cm_s|-n_2(|m_c|^2+|m_s|^2)+[\pm|m_2m_cm_s|\\ &+n_2(|m_c|^2+|m_s|^2)]\cosh(4r_2)\pm 2|m_2m_cm_s|\cos(2\theta_2\\ &-2s_2)\sinh^2(2r_2)+\mathcal{A}_\mp\cos(\theta_2-s_2)\sinh(4r_2).\end{aligned} \tag{20}$$

Here $\mathcal{A}_\pm$ ($\mathcal{B}_\pm$) stand for the two possible values for the cosine, i.e., $\cos(\theta_2-\theta_c+\theta_s)=\pm 1$, respectively. From Eqs. (18) and (19) we see that a local squeezing alone ($r_2\neq 0$ and $s_2=0$) on mode 2 can make $\tan(\tilde{\theta}_2+\tilde{\theta}_s-\tilde{\theta}_c)\neq 0$ if m_2 is not real ($\theta_2\neq 0$). However, whenever m_2 is real a rotation on the quadratures ($s_2\neq 0$) is mandatory. There is one last loophole to fix, namely, the rare instances in which $\mathcal{A}_+=0$ (note that $\mathcal{A}_-$ is always different from zero). This is fixed by allowing the other party, in this case Alice, to implement a local squeezing on mode 1. As shown in Eqs. (16) and (17) this operation allows Alice to change at her will the phases of m_s and m_c without altering θ_2, solving completely the last problem. By the way, this is the other possible solution for the $\sin(\theta_2-\theta_c+\theta_s)=0$ case, i.e., a local squeezing directly on mode 1.

In summary, we have shown a strictly local protocol in which a two-mode Gaussian state is completely reconstructed without relying on simultaneous HMs or nonlocal resources. Actually, the only resources needed for this protocol are the ability to perform single-mode HM, local parity and vacuum projective measurements, and classical communication. We also showed the complete equivalence of this local protocol to QST for Gaussian states. This equivalence is important for quantum communication purposes since now we can achieve the same goals of QST without nonlocal resources and simultaneous HMs. The set of local parity measurements required here, however, may restrict the implementation of the protocol, apart from instances where this measurement can be, in principle, performed [22]. Finally, this present protocol raises several interesting problems yet to be solved. First, it is unknown if a similar local protocol can be devised for more than two modes and, second, if there exist other optimal sets of measurements, other than parity and vacuum projections, allowing the complete state reconstruction of a two-mode (or many-mode) Gaussian (or non-Gaussian) state in a simpler local way.

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