High-fidelity state transfer in binary-tree spin networks

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Quantum state propagation over binary-tree configurations is studied in the context of quantum spin networks. For a binary tree of order 2, a simple protocol is presented which allows one to achieve arbitrary high transfer fidelity. It does not require fine-tuning of local fields and two-node coupling of the intermediate spins. Instead it assumes simple local operations on the intended receiving node: their role is to brake the transverse symmetry of the network that induces an effective refocusing of the propagating signals. Some ideas on how to scale up these effects to a binary tree of arbitrary order are discussed.

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I. INTRODUCTION

The paradigmatic approach to quantum communication assumes the possibility of "loading" quantum information (i.e., qubits) into mobile physical systems which are then transmitted from the sender of the messages to their intended receiver. Such *flying qubit* architecture for quantum communication has found its natural implementation in optics where photons play the role of information carriers. In many respects this appears to be the most reasonable choice, especially when long distances are involved in the communication. However, the recent development of controllable quantum many-body systems, such as optical lattices [1], phonons in ion traps [2], Josephson arrays [3], and polaritons in optical cavities [4], makes it plausible to consider alternative quantum-communication scenarios such as the so-called *quantum wire* architectures [5]. Here the transfer of quantum information proceeds over an extended network of coupled quantum systems (e.g., spins) which are *at rest* with respect to the communicating parties. In this case the messages are encoded into the internal states of the spins, while the information flow proceeds by their mutual interactions, which, when properly tuned, induce a net transfer of messages from two separate regions of the network [6–8]. The quantum wire architecture is, of course, of limited application, since it assumes the sender and the receiver to have access to the same quantum network (in any real implementation the latter will always have a reduced size). However, these techniques may play an important role in the creation of clusters of otherwise independent quantum computational devices. Furthermore, the study of quantum network communication protocols is an ideal playground to test and devise new quantum communication protocols.

Perfect transfer among any two regions of a quantum network can always be achieved if one allows the communicating parties to have direct access on the individual nodes of the network (for instance, this can be done by swapping sequentially the information from one node to a subsequent one). These strategies are, however, extremely demanding in terms of control and, even in the absence of external noise, are arguably prone to error due to the large number of quantum gates that have to be applied to the system. A less demanding approach consists in fixing the interaction of the network once and for all and letting the Hamiltonian evolution of the system to convey the sender message to the receiver. In this context perfect transmission can be achieved either by engineering the spin couplings [9–12] or by choosing proper encoding and decoding protocols [13–17].

In this paper we discuss the propagation of quantum information over binary-tree (BT) quantum networks. Together with the star configuration, the BT configuration is arguably the most significant network topology in circuit design. The former is typically used as hubs to wire different computational devices (for an analysis of such a system in the context of spin network communication, see Ref. [11]). Star configurations have been also extensively studied for entanglement distribution [18] and cloning [19]. BT networks instead are employed to route toward external memory elements (i.e., database). The information flow on unmodulated and uncontrolled BT was first discussed in Ref. [20] while, more recently, BT quantum networks have been employed to design efficient quantum random access memory elements [21].

The paper is organized as follows. In Sec. II we start by analyzing the first nontrivial BT system, introducing the notation, and setting the problem. In Sec. III we then describe a transfer protocol that allows one to deliver a generic quantum message to any desired final edges of the second-order BT network by exploiting simple end-gate operations. In Sec. IV we discuss various techniques that allow us to scale up the protocol, adapting it to a BT of arbitrary order. The paper finally ends with the conclusions and discussion in Sec. V.

II. SYSTEM DESCRIPTION

First-order BT networks are just particular instances of star networks [11]. Consequently the simplest nontrivial example of BT networks is the second-order one shown in Fig. 1. In the following we will assume the lines connecting the nodes to represent *XY* (exchange) spin interactions (the results, however, can be generalized to include *XXZ* or Heisenberg couplings). The resulting Hamiltonian is thus

$$H = \frac{J_0}{2}\sum_{\langle i,j\rangle}(\sigma_x^i\sigma_x^j + \sigma_y^i\sigma_y^j) + \sum_j \frac{\omega_j}{2}(\sigma_z^j + 1), \qquad (1)$$

where the summation is performed over all couples i and j, which are connected through an edge, where $\sigma_{x,y,z}^j$ are the

Pauli matrices associated with the ith node and where the ω_i's appear in consequence of the interaction with local magnetic fields [in this expression the labels i and j stand for the joint indexes (a,b) of Fig. 1]. As usual [5], we assume that initially the system is in the ferromagnetic "all-spin-down" ground state $|\varnothing\rangle \equiv |0\cdots 0\rangle$. At time $t=0$ we then place an (unknown) qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ on the leftmost site (0,0) (for instance, by swapping it from an external memory). With this choice the global state of the network is now described by the vector

$$|\Psi_{\text{in}}\rangle = \alpha|\varnothing\rangle + \beta|1_{(0,0)}\rangle, \tag{2}$$

where $|1_{(a,b)}\rangle$ represents the network state where the node (a,b) is in the spin up state $|1\rangle$, while the remaining ones are in the down state $|0\rangle$—i.e.,

$$|1_{(a,b)}\rangle \equiv |0\cdots 0\ 1_{(a,b)}\ 0\cdots 0\rangle. \tag{3}$$

Knowing that the z component of the total spin is preserved by the Hamiltonian evolution of the system (i.e., $[H, S_z^{\text{tot}}] = 0$), we can conclude that the dynamics is costrained in the subspace of single flips: On this subspace H acts in a very simple way that can be inferred from the graphical structure of the network—i.e.,

$$H|1_{(a,b)}\rangle = \omega_{(a,b)}|1_{(a,b)}\rangle + J_0 \sum_{(c,d)} |1_{(c,d)}\rangle, \tag{4}$$

where the sum is taken over all sites (c,d) connected with (a,b). Our goal is to find a procedure that would allow us to transfer the qubit state $|\psi\rangle$ to the rightmost sites $(2,b)$ with $b \in \{1,2,3,4\}$ of our choice—i.e.,

$$|\Psi_{\text{in}}\rangle \to |\Psi_{\text{fin}}^{(b)}\rangle \equiv \alpha|\varnothing\rangle + \beta|1_{(2,b)}\rangle. \tag{5}$$

Following Refs. [10,11], one could try to solve this problem by fine-tuning the parameters J_0 and ω_j of H in such a way that the free Hamiltonian evolution of the system will be able to transform $|\Psi_{\text{in}}\rangle$ into $|\Psi_{\text{fin}}^{(b)}\rangle$ after some time interval τ.[1] This, however, is in general a quite complex calculation which entails solving an inverse eigenvalue problem. Moreover, if any, the solutions obtained using such a strategy will be arguably highly asymmetrical in the distribution of the local magnetic fields ω_j's. To avoid all this, here we will pursue a different approach by limiting the freedom one has in choosing the Hamiltonian parameters, but as in Refs. [14,16,17], allowing local manipulation on the receiving node of the network [i.e., $(2,b)$]. Under these conditions we can show that a simple protocol exists that realizes the transformation (3) with arbitrary accuracy. It assumes a homogeneous network structure where all the ratios ω_j/J_0 are chosen to be identical and equal to some fixed value, and it is composed of the following three steps.

(1) The system is allowed to evolve freely under the action of H for some time τ.

(2) At this point, on the receiving node $(2,b)$ is performed a fast (ideally instantaneous) local phase-shift transformation $\mathcal{S}_{(2,b)}$.

(3) The network is then allowed to evolve for an extra time interval 2τ.

During the first step, due to the homogeneity of the Hamiltonian, the information flows along the left-right axis of the network, while delocalizing along the south-north axis. The value of τ is approximately the time interval an excitation takes to travel from the leftmost node (0,0) to the rightmost column formed by the nodes (2,1), (2,2), (2,3), and (2,4). The role of the local phase-shift transformation $\mathcal{S}_{(2,b)}$ of step (2) is to brake the south-north symmetry of the resulting state by flipping the sign of a specific wave-vector component. The system is then allowed to evolve freely for a time interval which is twice the initial one: this is approximately the time it takes an excitation to leave the rightmost column, "bounce back" to the leftmost node, and return to the rightmost network column. Due to the symmetry brake introduced at the second step, however, the signal will now not diffuse over all the four sites (2,1), (2,2), (2,3), and (2,4), but instead it will focus on the intended receiving node $(2,b)$. A detailed description of the protocol will be presented in Sec. III.

A. Diagonalization of the Hamiltonian

To solve our problem we can exploit the fact that the ground state $|\varnothing\rangle$ of the network does not evolve to restrict ourselves to the case $\alpha=0$—i.e., $|\psi\rangle = |1\rangle$. We then simplify the structure of the Hamiltonian (1) assuming all ω_j's to be identical—i.e., $\omega_j \equiv \omega$. In dealing with magnetic spins this means that we are applying a homogeneous magnetic field of constant strength $B \propto -\omega$ all over the system. We could set $\omega=0$, since the energy is defined up to a constant, but we let it be nonzero to guarantee that $|\varnothing\rangle$ is the ground state of the system. We now choose the following basis for the single-excitation sector, which divides the Hamiltonian in invariant blocks:

$$B_1 \begin{cases} |v_0\rangle \equiv |1_{(0,0)}\rangle, \\ |v_1\rangle \equiv |1_{(0,1)}\rangle, \\ |v_2\rangle \equiv \dfrac{1}{\sqrt{2}}(|1_{(1,1)}\rangle + |1_{(1,2)}\rangle), \\ |v_3\rangle \equiv \dfrac{1}{2}(|1_{(2,1)}\rangle + |1_{(2,2)}\rangle + |1_{(2,3)}\rangle + |1_{(2,4)}\rangle), \end{cases}$$

$$B_2 \begin{cases} |v_4\rangle \equiv \dfrac{1}{\sqrt{2}}(|1_{(1,1)}\rangle - |1_{(1,2)}\rangle), \\ |v_5\rangle \equiv \dfrac{1}{2}(|1_{(2,1)}\rangle + |1_{(2,2)}\rangle - |1_{(2,3)}\rangle - |1_{(2,4)}\rangle), \end{cases}$$

[1] Along these lines, for instance, one could map the propagation over the network into a simpler problem by setting a subset of the ω_j's to a value Λ much greater than the remaining constants of the systems. This will induce an effective decoupling of the selected nodes from the remaining part of the network, simplifying the underlying topology.

$$B_3 \begin{cases} |v_6\rangle \equiv \frac{1}{2}(|1_{(2,1)}\rangle - |1_{(2,2)}\rangle + |1_{(2,3)}\rangle - |1_{(2,4)}\rangle), \\ |v_7\rangle \equiv \frac{1}{2}(|1_{(2,1)}\rangle - |1_{(2,2)}\rangle - |1_{(2,3)}\rangle + |1_{(2,4)}\rangle). \end{cases}$$

In this basis the matrix representing H is given by

$$\begin{pmatrix} \begin{pmatrix} \omega & J_0 & & \\ J_0 & \omega & J & \\ & J & \omega & J \\ & & J & \omega \end{pmatrix} & & & \\ & \begin{pmatrix} \omega & J \\ J & \omega \end{pmatrix} & & \\ & & \begin{pmatrix} \omega & 0 \\ 0 & \omega \end{pmatrix} & \end{pmatrix}, \tag{6}$$

with $J \equiv \sqrt{2}J_0$. This shows that the evolution of the network can be effectively described as three independent linear chains, the first composed of four nodes and the other of two elements each. The basic idea in deriving the above basis is that any state in the form $|1_{(a,b)}\rangle$ is decoupled from the "singlet" superposition $\frac{1}{\sqrt{2}}(|10\rangle - |01\rangle)$ of the two nearest-neighbor qubits on its right, e.g., the two states $\frac{1}{\sqrt{2}}(|1_{(2,b)}\rangle - |1_{(2,b+1)}\rangle)$ with $b=1,3$ are decoupled from the whole network and provide an alternative basis for the block B_3.

Of special interest for us is, of course, the block B_1, which is the only one to have an overlap with the input state (2). It is clear that the case $J_0=J$ would be much simpler to deal with (in this case, for instance, one could adapt the linear chain analysis of Refs. [6,10] to simplify the calculation). Such an option, however, is not possible if we assume the coupling strengths of the network to be fixed *a priori*. Anyway we can use a trick to obtain the same result without adjusting the coupling strength, which, as discussed in the final paragraph of the present section, allows us to improve also the controllability of the setup. We suppose thus to modify the system by adding an additional spin connected only to site (0, 1) with the usual XY coupling of strength J_0, as shown in the inset of Fig. 1. With this choice the Hamiltonian (1) is replaced by

$$H_{\text{new}} \equiv H + \frac{J_0}{2}(\sigma_{\text{aux}}^x \sigma_{0,1}^x + \sigma_{\text{aux}}^y \sigma_{0,1}^y) + \frac{\omega}{2}(\sigma_{\text{aux}}^z + 1). \tag{7}$$

Now that we enlarged the Hilbert space, we have to deal with the nine-dimensional space of single flips. However, since the singlet state $\frac{1}{\sqrt{2}}(|1_{(0,0)}\rangle - |1_{\text{aux}}\rangle)$ is decoupled from the rest, if we encode the "logic" state $|1\rangle$ on the sending end of the network as $|v_0^{\text{new}}\rangle \equiv \frac{1}{\sqrt{2}}(|1_{(0,0)}\rangle + |1_{\text{aux}}\rangle)$, instead of using $|1_{(0,0)}\rangle$, not only do we recover a dynamics costrained in an eight-dimensional space, but we obtain also an effective coupling of strength $J=\sqrt{2}J_0$ between $|v_0^{\text{new}}\rangle$ and $|v_1\rangle$ [here $|1_{\text{aux}}\rangle$ is the analog of the states (3) with the spin up localized on the auxiliary node]. The four-dimensional block of our effective Hamiltonian thus becomes

$$H_{(4)} \equiv \begin{pmatrix} \omega & J & & \\ J & \omega & J & \\ & J & \omega & J \\ & & J & \omega \end{pmatrix}. \tag{8}$$

Following Ref. [10], this can be easily put in diagonal form, obtaining the eigenvalues

$$E_1 \equiv \frac{2\omega - (\sqrt{5}+1)J}{2}, \quad E_2 \equiv \frac{2\omega - (\sqrt{5}-1)J}{2},$$

$$E_3 \equiv \frac{2\omega + (\sqrt{5}-1)J}{2}, \quad E_4 \equiv \frac{2\omega + (\sqrt{5}+1)J}{2}, \tag{9}$$

with the corresponding eigenstates described by the vectors

$$|e_1\rangle \equiv \frac{1}{\sqrt{5+\sqrt{5}}}\left(-1, \frac{1+\sqrt{5}}{2}, -\frac{1+\sqrt{5}}{2}, 1\right),$$

$$|e_2\rangle \equiv \frac{1}{\sqrt{5-\sqrt{5}}}\left(1, \frac{1-\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}, 1\right),$$

$$|e_3\rangle \equiv \frac{1}{\sqrt{5-\sqrt{5}}}\left(-1, \frac{1-\sqrt{5}}{2}, -\frac{1-\sqrt{5}}{2}, 1\right),$$

$$|e_4\rangle \equiv \frac{1}{\sqrt{5+\sqrt{5}}}\left(1, \frac{1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}, 1\right), \tag{10}$$

expressed in the basis $|v_0^{\text{new}}\rangle$, $|v_1\rangle$, $|v_2\rangle$, and $|v_3\rangle$.

Apart from simplifying the spectral properties of the Hamiltonian, the introduction of the site "aux" adds also an additional feature that substantially enhances our ability of controlling the system. We have already mentioned that the singlet state $\frac{1}{\sqrt{2}}(|1_{(0,0)}\rangle - |1_{\text{aux}}\rangle)$ is decoupled from all other vectors of the system. Therefore we can "entrap" our qubit of information at the leftmost end of the network for as much time as we like by encoding its logic-1 component in such a singlet state. When we want the transfer to start, we simply apply a local phase shift $\mathcal{S}_{\text{aux}}$ on the auxiliary spin that induces the mapping $|1_{\text{aux}}\rangle \to -|1_{\text{aux}}\rangle$. This will transform $\frac{1}{\sqrt{2}}(|1_{(0,0)}\rangle - |1_{\text{aux}}\rangle)$ into $|v_0^{\text{new}}\rangle$, bringing the encoded message into the four-dimensional subspace associated with the Hamiltonian (8) and allowing the first step of the above protocol to begin.

III. TRANSFER PROTOCOL

In this section we analyze in detail the performance of the protocol defined in Sec. II. Without loss of generality we consider the case in which the receiving node is (2, 1)—i.e., $b=1$. We recall that our aim is to obtain the transition $|v_0^{\text{new}}\rangle \to |1_{(2,1)}\rangle$, and we notice that

$$|1_{(2,1)}\rangle = \frac{1}{2}(|v_3\rangle + |v_5\rangle + |v_6\rangle + |v_7\rangle). \tag{11}$$

The protocol: step 1. In the first stage of the protocol the

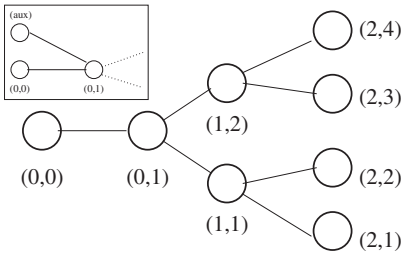

FIG. 1. Spin tree of second order. The nodes are connected through edges which describe the *XY* interactions defined by the Hamiltonian (1) and are identified by a double index (a,b), with (0,0) corresponding to the leftmost graph element. Inset: auxiliary spin added to the BT in order to homogenize the effective couplings in the block form representation (6). The additional spin provides also a controllable trigger to start the information transfer.

system is initialized into $|v_0^{\text{new}}\rangle$ and freely evolves for some time τ. The goal here is to find ω and τ such that this vector is mapped into $|v_3\rangle$, which represents a symmetric combination in which the input excitation is spread all over the rightmost nodes of the network. As already noticed, this process is formally equivalent to the information transfer along a linear chain of four spins coupled by uniform *XY* first-neighbor interactions. From Ref. [10] we know that such transferring cannot be exact. Nevertheless, the transfer fidelity can be made arbitrarily close to 1. Indeed, defining $U(\tau)=\exp[-iH\tau]$ and using Eq. (9), we have

$$\begin{aligned}\langle v_3|U(\tau)|v_0^{\text{new}}\rangle &= \sum_{k=1}^{4} e^{-iE_k t}\langle v_3|e_k\rangle\langle e_k|v_1\rangle \\ &= \frac{\sqrt{5}-5}{20}(e^{-iE_1\tau}-e^{-iE_4\tau}) \\ &\quad + \frac{\sqrt{5}+5}{20}(e^{-iE_2\tau}-e^{-iE_3\tau}). \end{aligned} \tag{12}$$

This will be exactly 1 if one could find τ such that $e^{-iE_1\tau}=e^{-iE_3\tau}=-1$ and $e^{-iE_2\tau}=e^{-iE_4\tau}=1$. Even though these conditions are impossible to be satisfied exactly [10], an approximate solution is obtained by choosing

$$\omega = \frac{7+\sqrt{5}}{2}J = \frac{7+\sqrt{5}}{\sqrt{2}}J_0 \tag{13}$$

and

$$\tau = \tau_n \equiv \frac{2n+1}{J}\pi, \tag{14}$$

with n integer. Under this assumption Eq. (12) yields

$$\langle v_3|U(\tau_n)|v_0^{\text{new}}\rangle = \frac{1}{2}(1+e^{-i\varphi_n}), \tag{15}$$

where $\varphi_n \equiv \sqrt{5}(2n+1)\pi$. The exponential in Eq. (15) never takes the value 1, but since $\sqrt{5}$ is an irrational number, it approaches it indefinitely. Therefore for any $\varepsilon>0$ we can choose n such that $|\langle v_3|U(\tau_n)|v_0^{\text{new}}\rangle| \geqslant 1-\varepsilon$. As a result, the state of the system, with high accuracy, is now described by the vector $|v_3\rangle$.

The protocol: step 2. As a second step, we act locally on the node (2, 1), applying the phase-shift unitary transformation $\mathcal{S}_{21}$, which changes the sign to the state $|1_{(2,1)}\rangle$, i.e.,

$$\mathcal{S}_{21}|1_{(2,1)}\rangle = -|1_{(2,1)}\rangle, \tag{16}$$

while preserving the remaining single-excitation states. This can be done, for instance, by acting with an intense magnetic field which acts locally on (2, 1) for a time interval shorter than the characteristic times of the Hamiltonian H. When acting on $|v_3\rangle$ the unitary $\mathcal{S}_{21}$ yields the transformation

$$\mathcal{S}_{21}|v_3\rangle = \frac{1}{2}(|v_3\rangle - |v_5\rangle - |v_6\rangle - |v_7\rangle). \tag{17}$$

This superposition contains the four states that compose the state $|1_{(2,1)}\rangle$, but the relative phases are wrong—see Eq. (11). Luckily, the third step fixes this issue with free evolution only.

The protocol: step 3. Finally, we just have to wait for a time $2\tau_n$ and the relative phases adjust themselves to give the state $|1_{(2,1)}\rangle$. In fact, by explicit calculations it can be shown that

$$\begin{aligned}\langle v_3|U(2\tau_n)|v_3\rangle &= \frac{1}{2}(1+e^{-i2\varphi_n}) \simeq 1, \\ \langle v_k|U(2\tau_n)|v_k\rangle &= -e^{-i\varphi_n} \simeq -1, \quad \text{for } k=5,6,7. \end{aligned} \tag{18}$$

Both expressions are justified by the fact that $e^{-i\varphi_n}\simeq 1$, and they imply that after the third step we have reached state $|1_{(2,1)}\rangle$ with as good an approximation as we like.

The above operations can be summarized with the application of the unitary operator

$$V_n \equiv U(2\tau_n)\mathcal{S}_{(2,1)}U(\tau_n). \tag{19}$$

Therefore the resulting transfer fidelity can be expressed as

$$\begin{aligned}F_n &\equiv |\langle 1_{(2,1)}|V_n|v_0^{\text{new}}\rangle|^2 \\ &= \left|\frac{1+e^{-i\varphi_n}}{8}\right|^2 \left|3e^{-i\varphi_n} + \frac{1+e^{-2i\varphi_n}}{2} + (1+e^{-i\varphi_n})^2\right|^2 \\ &= \frac{1}{4}\cos^2(\varphi_n/2)[3-\cos^2(\varphi_n/2)]^2, \end{aligned} \tag{20}$$

which approaches 1 if $e^{-i\varphi_n}\simeq 1$, giving us the desired result—see Fig. 2.

IV. SOME IDEAS TO SCALE UP THE SYSTEM

Unfortunately our protocol does not extend easily to higher-order trees (or at least we could not find a simple way of doing it). The idea we pursued in trying to scale up a second-order BT is to connect in some way the ends of a tree to the beginning of another. The resulting structure is no longer a tree of the type described above, but it is still a valid means to obtain a larger number of outputs. As an example, we could connect (say) two second-order trees to the ends of

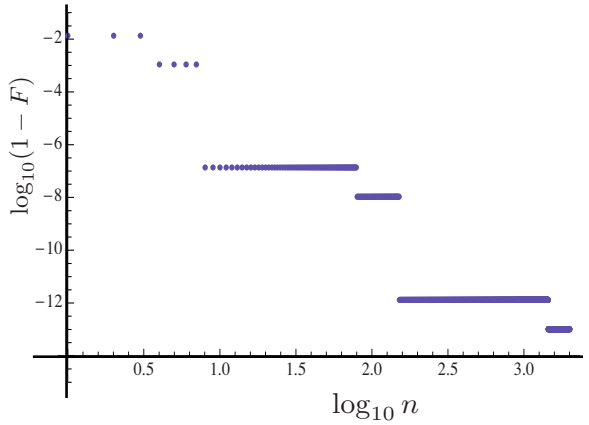

FIG. 2. (Color online) Minimum "infidelity" $(1-F_n)$ achievable according to Eq. (20) for $n\in[0,N]$ as a function of N—i.e., $(1-F)_{\min}\equiv\min_{n\leqslant N}(1-F_n)$. Notice that already for $n=8$ we get values of F greater than $1-10^{-6}$. From the plot we can infer an almost linear dependence of $(1-F)_{\min}$ with respect to N, yielding the (approximated) behavior $F_{\min}\sim 1-cN^{-\gamma}$, with $\gamma\sim 1$ and c being constant.

a first-order tree to obtain an eight-output quantum switch or four second-order trees to the ends of a fifth second-order tree to obtain a 16-output quantum switch—see Fig. 3. The former setup can be solved by properly merging the protocol of Ref. [11] with our second-order BT propagation scheme; this, however, will require us to employ nonuniform magnetic fields at least for the first spins and does not admit simple concatenation. We thus decided to focus on the second architecture, which instead can be trivially concatenated to form a larger setup. We found a relatively simple way to make the required connections, but at the expense of considering some coupling strength engineering and including antiferromagnetic interactions, which means that the "all-down" configuration is no longer the ground state, although still stationary. A combination of time evolution and a phase shift will do all the work. First of all, each receiving end of a tree must be accompanied by an auxiliary qubit, as done before for the sending end. This is similar to what happened when is $(0,0)_{\text{aux}}$ is introduced. It is easy to see that the rightmost singlets

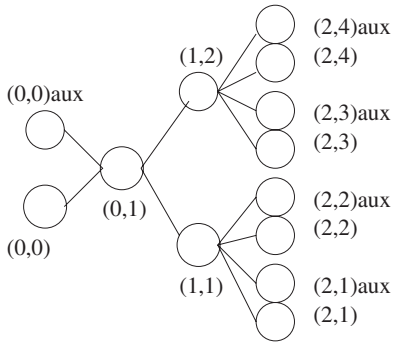

FIG. 3. Modified tree for the purpose of scaling up. Each rightmost qubit is accompanied now by an auxiliary; moreover, all rightmost couplings are adjusted by a factor $1/\sqrt{2}$.

$$|s_{2b}\rangle\equiv\frac{1}{\sqrt{2}}(|1_{(2,b)}\rangle-|1^{\text{aux}}_{(2,b)}\rangle)$$

are isolated, while the corresponding triplets

$$|t_{2b}\rangle\equiv\frac{1}{\sqrt{2}}(|1_{(2,b)}\rangle+|1^{\text{aux}}_{(2,b)}\rangle)$$

interact with the network and evolve, with an effective coupling strength $\sqrt{2}$ times the original one. In order for our protocol to be still valid, we need to modify the coupling strengths of the rightmost branches so that matrix (8) (now with $J_0=J$) remains unchanged. Moreover, the local operation on the receiving end $(2,b)$ must now be performed simultaneously on $(2,b)$ and $(2,b)_{\text{aux}}$, i.e., we must now apply $\mathcal{S}_{2b}\otimes\mathcal{S}^{\text{aux}}_{2b}$. In this way, once the excitation reaches one of the end triplets of the tree, it can be trapped there with a phase shift on the auxiliary qubit of that site, storing information in the relative singlet. To clarify this, we outline that in this new configuration our protocol is capable of achieving the (approximate) transfer:

$$\alpha|\varnothing\rangle+\beta|1_{(0,0)}\rangle\rightarrow\alpha|\varnothing\rangle+\beta|t_{2b}\rangle. \tag{21}$$

Now by applying a local phase shift $\mathcal{S}^{\text{aux}}_{2b}$ the state is transformed into

$$\alpha|\varnothing\rangle+\beta|s_{2b}\rangle, \tag{22}$$

which is decoupled from the rest.

If by some means we could transfer this state to the singlet at the beginning of the next tree (denoted by primed indexes), i.e., obtain the state

$$\alpha|\varnothing\rangle+\beta\frac{1}{\sqrt{2}}(|1_{(0',0')}\rangle-|1^{\text{aux}}_{(0',0')}\rangle)\equiv\alpha|\varnothing\rangle+\beta|s_{0'0'}\rangle, \tag{23}$$

we could then perform the local operation $\mathcal{S}^{\text{aux}}_{(0',0')}$ to obtain the corresponding triplet state

$$\alpha|\varnothing\rangle+\beta|t_{0'0'}\rangle,$$

which can be transferred along the new tree with the usual protocol. The structure shown in Fig. 4 (which we will call a "singlet link") achieves perfect transfer between two singlets, since in the subspace $\{|s_{21}\rangle,|1_0\rangle,|s_{0'0'}\rangle\}$ it is equivalent to a chain of length 3 with constant couplings [10]; moreover, the evolution of $|t_{21}\rangle$ is decoupled from that of $|t_{0'0'}\rangle$ thanks to the opposite signs of the couplings along the vertical axis. The lines stand for XY interaction of strength J(ferromagnetic) and $-J$(antiferromagnetic), respectively. As an example, we have considered site (2, 1) of a second-order tree plus its auxiliary qubit, connected with site $(0',0')$ plus its auxiliary of another tree. We outline again that we are working in the subspace of single flips, as our Hamiltonian still conserves S^{tot}_z. In the considered example we have

$$H|s_{21}\rangle=\omega|s_{21}\rangle+\sqrt{2}J|1_0\rangle,$$

$$H|t_{21}\rangle=\omega|t_{21}\rangle+J|1_{(1,1)}\rangle,$$

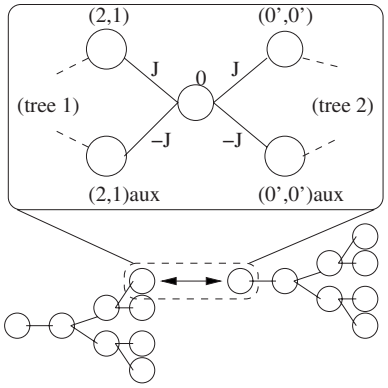

FIG. 4. The "singlet link" used to connect the outputs of the first tree to the inputs of the next column of trees (only the first one is represented here). The auxiliary qubits are not represented for simplicity.

$$H|s_{0'0'}\rangle = \omega|s_{0'0'}\rangle + \sqrt{2}J|1_0\rangle,$$

$$H|t_{0'0'}\rangle = \omega|t_{0'0'}\rangle + \sqrt{2}|1_{(0',1')}\rangle,$$

where $|t_{0'0'}\rangle \equiv \frac{1}{\sqrt{2}}(|1_{(0',0')}\rangle + |1^{\text{aux}}_{(0',0')}\rangle)$.

We can see from the above equations that once the information enters a tree through a triplet state, it does not come out of it until we make a phase shift on the desired end (and at the right time). At this point the information goes to the singlet and propagates to the starting singlet of another tree, thanks to the singlet link; then, it is transferred to the corresponding triplet with a local phase shift and propagation begins on the next tree. We shall repeat this procedure until information reaches the desired end on the last array of trees. Of course, we must control *a priori* the total error due to the presence of second-order trees. Fixing a "single-tree time" $\bar{\tau}$, will give a satisfactory overall transfer fidelity.

V. CONCLUSIONS

In this paper we have presented a protocol for quantum state transfer on BT spin networks of order 2. As in Refs. [14,16,17], it is based on local operations that must be performed on the receiving nodes. Differently from [16,17], however, it does not involve a swapping operation between the receiving nodes and external memories and arbitrarily high fidelity can be obtained in just three operational steps. Generalization of this technique to higher-order BT is currently under investigation: arguably this will involve more complex end-gates operations possibly on more than one of the rightmost nodes. We have, however, provided a simple way to scale up the problem by concatenating smaller BT networks through connecting gates which can be turned on and off by simple local phase gate transformations.

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