Maximal violations of a Bell inequality by entangled spin-coherent states

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We study the violation of the Bell-Clauser-Horne-Shimony-Holt (Bell-CHSH) inequality for two-spin systems, of arbitrary spins j_1 and j_2, prepared in an entanglement of spin-coherent states of each of the spins. We show that the Bell-CHSH inequality is quite robustly violated for a wide range of values of the parameters that specify the spin-coherent states, and, for a particular choice of these parameters, maximal violations are obtained. That is, the violations can reach the Tsirelson bound, $2\sqrt{2}$, for any choices of the spins.

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Over the past 30 years or more, there has been interest in the question of whether or not classical realism, as embodied in various versions of Bell's theorem, is recovered in the limit of large quantum numbers, generally assumed to be the classical limit as per the correspondence principle. Some years ago, Garg and Mermin [1] studied Bell's inequality for two-spin j particles in a singlet state, given as

$$|SS\rangle = (2j+1)^{-1/2}\sum_{m=-j}^{j}(-1)^{j-m}|j,m\rangle_1 \otimes |j,-m\rangle_2, \quad (1)$$

and found violations for measurements along almost all pairs of directions. However, the magnitude of the violation decreased with increasing j, the decrease being the result of the author's use of a slowly varying function of the spin components which made their method insensitive to the rapidly varying part of the correlations. Later, Peres [2] pointed out that the Stern-Gerlach experiment, for arbitrary angular momentum, measures the dichotomic operator

$$\hat{\prod}^{(j)} = \sum_{m=-j}^{j}|j,m\rangle(-1)^{j-m}\langle j,m| = e^{i\pi(j-\hat{J}_z)} \quad (2)$$

if the detectors are positioned such that the $2j+1$ beams are well separated. The operator $\hat{\Pi}^{(j)}$ is just the parity operator for the $D^{(j)}$ unitary irreducible representation of SU(2). Using this observable, Peres found that the violations of Bell's theorem, as given in the form of the Clauser, Horne, Shimony, and Holt (Bell-CHSH) inequality [3], remains constant in the limit $j\to\infty$. Gerry and Albert [4] proposed an optical scheme for generating states that are very close to the spin-singlet states. These were two-mode field states that could be produced by the action of a 50:50 beam splitter on an input state containing N photons in one port and the vacuum at the other. The identification of the output state with a spin-j singletlike state, with $j=N/2$, comes about by introducing a Holstein-Primakoff [5] realization of the SU(2) algebra for each of the photon modes. The corresponding violations of the Bell-CHSH inequality, using displacement operations performed with beam splitters of low transmittance and a strong classical-like field at the other input port, followed by photon parity measurements, were discussed in Ref. [4].

Mermin and Schwartz [6] pointed out that there really is no reason to expect classical behavior to be approached in the limit of large spin, as the measurements that discriminate between the $2j+1$ values of M, no matter how large j, have a nonclassical character. Of course, what really matters is the degree to which the two particles in the singlet state of Eq. (1) remain entangled as $j\to\infty$. The linear entropy for particle 1, given as $S_1=1-\mathrm{Tr}_1\,\hat{\rho}_1^2$, where $\hat{\rho}_1=\mathrm{Tr}_2[\hat{\rho}_{12}]$, and where $\hat{\rho}_{12}=|SS\rangle\langle SS|$, is easily found to be $S_1=1-(2j+1)^{-2}$ which goes to unity as $j\to\infty$. Thus entanglement persists in the spin-singlet state, and in fact, becomes maximal, according to this measure, in the limit of large spin. However, the violation of the Bell-CHSH inequality obtained by Peres for the spin-singlet state is not the maximally allowed violation, $2\sqrt{2}$, the Tsirelson bound [7].

In this paper we study the case of two particles of arbitrary spins prepared in an entanglement of spin-coherent states and show that, under certain conditions, violations of the Bell-CHSH inequality up to the Tsirelson bound are possible. Spin-coherent states [8], also known as atomic coherent states or angular momentum coherent states [9] depending on context, are analogous to the ordinary coherent states of a harmonic oscillator in that they both may be considered as pure, near-classical states of their corresponding systems. An entanglement of spin-coherent states [as in Eq. (3) below] has a different character than the spin-singlet states mentioned above. In the latter, there is always a close pairwise correlation (actually an anticorrelation) between the z components of the spins of the two particles, i.e., the spin-singlet state is a superposition of the product states of the form $|j,m\rangle_1\otimes|j,-m\rangle_2$ and in this sense we could say that the entanglement embodied in spin-singlet states is microscopic. Furthermore, the angular momentum states, $|j,m\rangle$, of each of the particles are nonclassical states regardless of the size of the spin j. In contrast, spin-coherent states are classical-like and become more so in the limit of large spin. Yet an entanglement of spin-coherent states, especially of distinguishable spin-coherent states, would be expected to have strong nonclassical properties, including nonlocality.

Recall that starting with Bohm's [10] formulation of the argument of Einstein, Podolsky, and Rosen [11] (EPR), Bell [12] showed that the quantum mechanical spin-singlet states for [$j=1/2$ in Eq. (1)] lead to a conflict with local realistic hidden variable theories as embodied by violations of certain

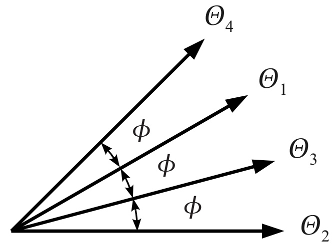

FIG. 1. Diagram showing the arrangements of the auxiliary angles Θ_1, Θ_2, Θ_3, and Θ_4. We make the choices $\Theta_1-\Theta_3=\phi$, $\Theta_1-\Theta_4=-\phi$, $\Theta_2-\Theta_3=-\phi$, such that $\Theta_2-\Theta_4=-3\phi$.

inequality. In the discussions of EPR, and of Bohm and Bell, the "elements of reality" were to be attached to microscopic particles. Bohr [13] argued that the elements of reality for such particles do not have an existence independent of the measuring apparatus. Certainly, for the spin-singlet state with $j=1/2$ we are dealing with microscopic particles. But for an entanglement of spin-coherent states of large angular momentum, the elements of reality are not attached to microscopic particles. This raises the question of the existence of elements of reality independent of the measuring apparatus when the particles have macroscopic spins. As recently emphasized by Laloë and Mullin [14], this is an important difference between demonstrations of violations of Bell-type inequalities involving the polarization states of two photons (equivalent to two spin-$\frac{1}{2}$ particles) and the system under discussion here.

We first examine entangled spin-coherent states of the form

$$|\Psi\rangle=\mathcal{N}[|\zeta_1,j_1\rangle_1\otimes|-\zeta_2,j_2\rangle_2+e^{i\Phi}|-\zeta_1,j_1\rangle_1\otimes|\zeta_2,j_2\rangle_2], \tag{3}$$

where the spin-coherent states are given by [8,9]

$$|\zeta,j\rangle=\exp\left(\frac{1}{2}\theta e^{-i\varphi}\hat{J}_+-\frac{1}{2}\theta e^{i\varphi}\hat{J}_-\right)|j,-j\rangle$$
$$=(1+|\zeta|^2)^{-j}\sum_{m=-j}^{j}\binom{2j}{j+m}^{1/2}\zeta^{j+m}|j,m\rangle, \tag{4}$$

where the complex parameter ζ is given by $\zeta=e^{-i\varphi}\tan(\theta/2)$. The normalization factor $\mathcal{N}$ is

$$\mathcal{N}=\frac{1}{\sqrt{2}}\left[1+\cos\Phi\left(\frac{(1-|\zeta_1|^2)}{(1+|\zeta_1|^2)}\right)^{2j_1}\left(\frac{(1-|\zeta_2|^2)}{(1+|\zeta_2|^2)}\right)^{2j_2}\right]^{-1/2}. \tag{5}$$

Our state $|\Psi\rangle$ of Eq. (3) represents an entanglement of two particles where particle 1 has spin j_1 and particle 2 has spin j_2. We need not have $j_1=j_2$. Because of entanglement, the spin-coherent state of each particle is indeterminate even though the spins of each particle can be large; that is, the spin-coherent states individually may represent macroscopic classical-like spin states. Furthermore, for the state $|\zeta,j\rangle$ the average of the spin vector has the components $\langle\mathbf{J}\rangle_{\zeta,j}=j(\sin\theta\cos\varphi,\sin\theta\sin\varphi,-\cos\theta)$ and for $|-\zeta,j\rangle$ we have $\langle\mathbf{J}\rangle_{-\zeta,j}=-j(\sin\theta\cos\varphi,\sin\theta\sin\varphi,\cos\theta)$ such that $\langle\mathbf{J}\rangle_{\zeta,j},\langle\mathbf{J}\rangle_{-\zeta,j}=j^2\cos(2\theta)$. For $\theta=\pi/2$, $\langle\mathbf{J}\rangle_{\zeta,j},\langle\mathbf{J}\rangle_{-\zeta,j}=-j^2$, the directions of the average spin vectors differ maximally, 180°. Also we have that $|\langle-\zeta,j|\zeta,j\rangle|^2=\cos^{4j}\theta$ indicating that the states have no overlap for $\theta=\pi/2$.

In terms of the angular momentum states $|j,m\rangle$ we can write

$$|\Psi\rangle=\sum_{m_1=-j_1}^{j_1}\sum_{m_2=-j_2}^{j_2}A_{m_1m_2}^{j_1j_2}|j_1,m_1\rangle_1\otimes|j_2,m_2\rangle_2, \tag{6}$$

where

$$A_{m_1m_2}^{j_1j_2}=\mathcal{N}(1+|\zeta_1|^2)^{-j_1}(1+|\zeta_2|^2)^{-j_2}\left[\binom{2j_1}{j_1+m_1}\binom{2j_2}{j_2+m_2}\right]^{1/2}$$
$$\times\zeta_1^{j_1+m_1}\zeta_2^{j_2+m_2}[(-1)^{j_2+m_2}+e^{i\Phi}(-1)^{j_1+m_1}]. \tag{7}$$

(Note the lack of pairing between the z components of the spins, as expected for our states.) We define our rotated-magnet state as $|\Psi'\rangle=e^{-i\theta_1J_{1x}}e^{-i\theta_2J_{2x}}|\Psi\rangle$, and using the same observables as Peres [2], a dichotomic operator, namely the SU(2) parity operators of Eq. (2), our correlation function is now

$$C(\theta_1,\theta_2)=\langle\Psi'|\hat{\prod}_1^{(j_1)}\otimes\hat{\prod}_2^{(j_2)}|\Psi'\rangle$$
$$=e^{i\pi(j_1+j_2)}\langle\Psi|e^{-i\pi(\hat{J}_{1z}\cos\theta_1+J_{1y}\sin\theta_1)}$$
$$\times e^{-i\pi(\hat{J}_{2z}\cos\theta_2+\hat{J}_{2y}\sin\theta_2)}|\Psi\rangle. \tag{8}$$

Using the identity $e^{-i\pi(\hat{J}_z\cos\theta+\hat{J}_y\sin\theta)}=e^{-i\pi\hat{J}_z/2}e^{-i(2\theta)\hat{J}_y}e^{-i\pi\hat{J}_z/2}$, the correlation function may be written as

$$C(\theta_1,\theta_2)=e^{i\pi(j_1+j_2)}\sum_{n_1=-j_1}^{j_1}\sum_{n_2=-j_2}^{j_2}\sum_{m_1=-j_1}^{j_1}\sum_{m_2=-j_2}^{j_2}[A_{n_1n_2}^{j_1j_2}]^*[A_{m_1m_2}^{j_1j_2}]$$
$$\times e^{-i(\pi/2)(n_1+n_2+m_1+m_2)}d_{n_1,m_1}^{(j_1)}(2\theta_1)d_{n_2,m_2}^{(j_2)}(2\theta_2), \tag{9}$$

where the $d_{m',m}^{(j)}(\beta)=\langle j,m'|e^{-i\beta\hat{J}_y}|j,m\rangle$. This is the result for the general case.

For the special case where $\zeta_1=\zeta_2=1$, we can analytically determine a closed form for the correlation function. To see this, we start, using Eq. (4), with the corresponding spin-coherent states for the cases $\zeta=\pm1$,

$$|\pm1,j\rangle\equiv\frac{1}{2^j}\sum_{m=-j}^{j}\binom{2j}{j+m}^{1/2}(\pm1)^{j+m}|j,m\rangle. \tag{10}$$

These states are orthogonal, $\langle\mp1,j|\pm1,j\rangle=0$, and satisfy the relation $\hat{\Pi}^{(j)}|\pm1,j\rangle=(-1)^{2j}|\mp1,j\rangle$. They can be shown, after a bit of algebra, to be eigenstates of the angular momentum operator J_x with eigenvalues $\pm j$: $J_x|\pm1,j\rangle=\pm j|\pm1,j\rangle$. The corresponding entangled spin-coherent states of Eq. (3) are

$$|\Psi\rangle=\frac{1}{\sqrt{2}}(|+1,j_1\rangle_1|-1,j_2\rangle_2+e^{i\Phi}|-1,j_1\rangle_1|+1,j_2\rangle_2). \tag{11}$$

Defining $|\Psi'\rangle=e^{-i\theta_1J_{1x}}e^{-i\theta_2J_{2x}}|\Psi\rangle$, we have

TABLE I. Results for S_{max} for the case with $j_1=3=j_2$ and the corresponding angles for various values of ζ_1 and ζ_2. We have used a search algorithm to obtain the angles which yield the maximal violations of the Bell-CHSH inequality.

ζ_1	ζ_2	θ_1	θ_2	θ_3	θ_4	S_{max}
1	1.55	−1.0470	1.8330	3.0030	3.2800	2.8000
1	3.35	−1.0470	1.8330	2.9690	3.3140	2.5960
0.85	3.45	−1.0499	0.2657	0.1822	2.9641	2.4891
0.85	3.65	−1.0500	0.2657	0.1839	2.9623	2.4660
1.15	1.55	−1.0424	0.2715	0.1508	3.0078	2.7336
1.15	3.35	−1.0492	0.2648	0.1791	2.9662	2.5251
0.65	0.65	−1.2966	0.0006	−0.0848	2.6961	2.2293
0.85	0.85	−1.3027	0.0039	−0.1193	2.7463	2.7129
1.15	1.15	−1.3048	0.0023	−0.1228	2.7465	2.7415
1.55	1.55	−1.2961	0.0006	−0.0835	2.6941	2.2154
1.75	1.75	−1.2864	0.0015	−0.0606	2.6574	2.0089
2.55	2.55	−1.2048	0.1026	0.0729	2.6023	1.5931

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}(e^{-i\theta_1 j_1}e^{i\theta_2 j_2}|+1,j_1\rangle_1|-1,j_2\rangle_2 + e^{i\Phi}e^{i\theta_1 j_1}e^{-i\theta_2 j_2}|-1,j_1\rangle_1|+1,j_2\rangle_2). \qquad (12)$$

It follows that

$$\hat{\prod}^{(j_1)}\hat{\prod}^{(j_2)}|\Psi'\rangle = \frac{(-1)^{2j_1+2j_2}}{\sqrt{2}}(e^{-i\theta_1 j_1}e^{i\theta_2 j_2}|-1,j_1\rangle_1|+1,j_2\rangle_2 + e^{i\Phi}e^{i\theta_1 j_1}e^{-i\theta_2 j_2}|+1,j_1\rangle_1|-1,j_2\rangle_2). \qquad (13)$$

Finally, we have as our correlation function for this special case

$$C(\theta_1,\theta_2) = \langle\Psi'|\hat{\prod}^{(j_1)}\hat{\prod}^{(j_2)}|\Psi'\rangle = (-1)^{2j_1+2j_2}\cos(2j_1\theta_1 - 2j_2\theta_2 + \Phi). \qquad (14)$$

The Clauser, Horne, Shimony, and Holt form of Bell's theorem is given as follows [3]: If Alice performs measurements with her detector set at angles θ_1 and θ_2 while Bob sets his detector at θ_3 and θ_4, then for

$$S = C(\theta_1,\theta_3) + C(\theta_1,\theta_4) + C(\theta_2,\theta_3) - C(\theta_2,\theta_4) \qquad (15)$$

we have $|S|\leq 2$ for local realistic hidden variable theories whereas quantum mechanically we can have $|S|\leq 2\sqrt{2}$, the former condition being the Bell-CHSH inequality, the latter being the Tsirelson inequality. Experimental values of $|S|$ in the range $2<|S|\leq 2\sqrt{2}$ falsify local realistic hidden variable theories and thus support quantum mechanics. In cases where $\Phi=0$ we obtain

$$S = (-1)^{2j_1+2j_2}[\cos(\Theta_1-\Theta_3) + \cos(\Theta_1-\Theta_4) + \cos(\Theta_2-\Theta_3) - \cos(\Theta_2-\Theta_4)], \qquad (16)$$

where $\Theta_1=2j_1\theta_1$, $\Theta_2=2j_1\theta_2$, $\Theta_3=2j_2\theta_3$, and $\Theta_4=2j_2\theta_4$ are a set of auxiliary angles, the angles θ_i being the actual angles set by Alice and Bob. For the choices $\Theta_1-\Theta_3=\phi$, $\Theta_1-\Theta_4=-\phi$, $\Theta_2-\Theta_3=-\phi$, and $\Theta_2-\Theta_4=-3\phi$, as indicated in Fig. 1, we obtain $S=3\cos\phi-\cos(3\phi)$ which obtains its maximum value $2\sqrt{2}$ for $\phi=\pi/4$.

Note that we obtain maximal violations of the Bell-CHSH inequality (as long as $\zeta_1=\zeta_2=1$) for all nonzero values of the spins j_1 and j_2 and that we need not have the spins equal. On the other hand, the measured angles θ_i must necessarily be smaller as the spins increase. The presence of the relative

TABLE II. Same as Table I now for the state of Eq. (17) but for a selection of values of ζ_1 and ζ_2.

ζ_1	ζ_2	θ_1	θ_2	θ_3	θ_4	S_{max}
1.15	1.15	−1.3139	0.0016	−0.9236	2.4888	2.6750
0.85	1.15	−1.3145	0.0014	−0.9253	2.4904	2.6511
1.3	1.3	−1.8358	0.0025	−0.1142	2.7283	2.5502
0.85	0.65	−1.8536	0.0028	−0.1217	2.7317	2.6129
1.6	2.35	−2.0470	−0.2554	0.1742	2.9120	2.1264
3	3	−1.1490	−0.0754	0.0399	2.5380	1.4204
1.15	1.85	−2.0591	−0.6124	−1.3205	3.1720	2.0629
2.15	3.55	−1.9728	−0.5660	−0.8390	3.0674	0.9689

phases Φ other than zero merely shifts the maximum to other angular differences; the maximal violation is, of course, still the Tsirelson bound $|S|_{max}=2\sqrt{2}$.

The Bell-CHSH inequality is violated, though generally not maximally, for cases where the condition $\zeta_1=\zeta_2=1$ does not hold. In Table I we list some results obtained numerically using the general form of the correlation function as given in Eq. (9) for the spins $j_1=3=j_2$. We have used a search algorithm for the angles which yield the maximal violations of the Bell-CHSH inequality (we have excluded relative maxima). We find strong violations as long as one of the parameters, ζ_1 in our case, is near unity (actually in the range we consider $\zeta_1=1.00\pm0.15$) while ζ_2 differs significantly from unity. When both ζ_1 and ζ_2 differ significantly from unity the violations become weaker and nonexistent for large enough deviations from unity.

Another entangled spin-coherent state we could consider is

$$|\Psi\rangle = \mathcal{N}[|\zeta_1,j_1\rangle_1 \otimes |\zeta_2,j_2\rangle_2 + e^{i\Phi}|-\zeta_1,j_1\rangle_1 \otimes |-\zeta_2,j_2\rangle_2], \qquad (17)$$

where the normalization factor is the same as before. In the special case where $\zeta_1=\zeta_2=1$, we find, using the same methods as above, the correlation function to be $C(\theta_1,\theta_2)=(-1)^{2j_1+2j_2}\cos(2j_1\theta_1+2j_2\theta_2+\Phi)$. For $\Phi=0$, the parameter S now reads as

$$S=(-1)^{2j_1+2j_2}[\cos(\Theta_1+\Theta_3)+\cos(\Theta_1+\Theta_4) + \cos(\Theta_2+\Theta_3)-\cos(\Theta_2+\Theta_4)], \qquad (18)$$

and thus with the choices $\Theta_1+\Theta_3=\phi$, $\Theta_1+\Theta_4=-\phi$, $\Theta_2+\Theta_3=-\phi$, and $\Theta_2+\Theta_4=-3\phi$, one solution of which is for $\Theta_{1,3}=\phi/2$ and $\Theta_{2,4}=-3\phi/2$, we again find for $\phi=\pi/4$ that $S=2\sqrt{2}$.

For the cases where the condition $\zeta_1=\zeta_2=1$ does not hold, we need to decompose Eq. (17) into the angular momentum basis as in Eq. (6). The expansion coefficients are now given by

$$A^{j_1j_2}_{m_1m_2}=\mathcal{N}(1+|\zeta_1|^2)^{-j_1}(1+|\zeta_2|^2)^{-j_2}\left[\binom{2j_1}{j_1+m_1}\binom{2j_2}{j_2+m_2}\right]^{1/2} \times \zeta_1^{j_1+m_1}\zeta_2^{j_2+m_2}[1+e^{i\Phi}(-1)^{j_1+j_2+m_1+m_2}], \qquad (19)$$

where the normalization factor is the same as Eq. (5). Some numerical results obtained by following the same procedure as before are tabulated in Table II. Again the Bell-CHSH inequality is violated over a wide range of parameters.

Finally, we point out that for the special cases where $\zeta=1$, the states $|1,j\rangle$ and $|-1,j\rangle$ are orthogonal, i.e., $\langle -1,j|1,j\rangle=0$. Then from Eqs. (3) and (17) with the choices of the phase $\Phi=0$ or π, we have a set of mutually orthogonal Bell states of the form

$$|\Psi_1\rangle=\frac{1}{\sqrt{2}}[|1,j_1\rangle_1\otimes|-1,j_2\rangle_2+|-1,j_1\rangle_1\otimes|1,j_2\rangle_2],$$

$$|\Psi_2\rangle=\frac{1}{\sqrt{2}}[|1,j_1\rangle_1\otimes|-1,j_2\rangle_2-|-1,j_1\rangle_1\otimes|1,j_2\rangle_2],$$

$$|\Psi_3\rangle=\frac{1}{\sqrt{2}}[|1,j_1\rangle_1\otimes|1,j_2\rangle_2+|-1,j_1\rangle_1\otimes|-1,j_2\rangle_2],$$

$$|\Psi_4\rangle=\frac{1}{\sqrt{2}}[|1,j_1\rangle_1\otimes|1,j_2\rangle_2-|-1,j_1\rangle_1\otimes|-1,j_2\rangle_2], \qquad (20)$$

in complete analogy with the standard set of Bell states. The difference here, of course, is that each of the spin-coherent states consists of $2j_{1,2}+1$ states in the angular momentum basis, and j_1 need not equal j_2.

In summary, we have shown that it is possible to obtain maximal violations of the Bell-CHSH inequality for an entanglement of distinguishable spin-coherent states, each of which could be macroscopic. Possible realizations of entangled spin-coherent states of the form of Eq. (3) or of Eq. (17) in the context of Bose-Einstein condensates and of optical fields will be discussed elsewhere.

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