Homogeneous and genuine Bell inequalities

F. De Zela*

Departamento de Ciencias, Sección Física, Pontificia Universidad Católica del Perú, Apartado 1761, Lima, Peru

(Received 19 November 2008; revised manuscript received 29 December 2008; published 4 February 2009)

We discuss homogeneous and inhomogeneous Bell inequalities, following Santos's classification. According to it, homogeneous inequalities entail only coincidence probabilities, whereas inhomogeneous inequalities entail coincidence probabilities together with single probabilities or with numbers. Because of technical limitations, all performed tests of Bell inequalities have been based on homogeneous inequalities whose derivation required additional assumptions besides realism and locality, thereby losing their genuine character. Here we derive, starting from the Clauser-Horne inequality, a homogeneous inequality that was at the basis of an experimental test performed some years ago by Torgerson *et al.* [Phys. Rev. A **51**, 4400 (1995)]. We show that its derivation does not require anything but realism and locality, contrary to what has been previously assumed. It can thus be considered a genuine Bell inequality, appropriate for testing local realism. Similar, homogeneous inequalities can be analogously derived. They constitute a promising family that is likely to serve as a basis for loophole-free tests of local realism. The existence of such a family proves false the assertion that all genuine Bell inequalities must be inhomogeneous.

DOI: 10.1103/PhysRevA.79.022102 PACS number(s): 03.65.Ud

I. INTRODUCTION

There is yet no conclusive experiment that favors quantum mechanics against local realism. Several tests based on Bell inequalities have been performed, but none of them is a completely loophole free. Although there is a general consensus about this last point, not all researches in the field would attribute to it equal importance. Thus, according to widespread opinion, local realism has been experimentally disproved (see, e.g., [1]) modulo some loopholes stemming from technical shortcomings that will be surely overcome, sooner or later. To this end, efforts are continuously being made either to close the remaining loopholes with the help of improved experimental techniques, or else to conceive new tests that are technically less demanding. On the other hand, there are researches for whom the absence of loophole-free tests would rather hint at a possible compatibility of quantum mechanics (QM) with local realism [2]. The fact is that all performed tests suffer from one or the other loophole, thereby allowing the construction of local-realistic models which are capable of explaining the reported results. Much attention has been devoted to close two important loopholes: the one stemming from low detector efficiency [3,4], and the other stemming from the difficulty of realizing a timelike interval between detection events [5–9]. While for closing the locality loophole there is a one-track approach, namely the improvement of experimental techniques, for closing the detection loophole there are at least two options: either to use almost perfect detectors [10], or to conceive new tests that lower the required detection efficiency [11–16]. However, all these efforts will remain useless, as long as the experimental data, viz. the counting rates, are used to construct quantities that are incorrectly identified with probabilities. This important point was raised by Santos [17,18], who emphasized that true probabilities correspond to ratios of counting rates to preparation rates. In all reported experiments, however, what people do is to construct some ratios of two counting rates, and these are incorrectly interpreted as probabilities instead of what they are: ratios of probabilities. The following situation then arises: in order to experimentally test Bell inequalities we need to properly measure some probabilities entering these inequalities. Such probabilities can be approximated by ratios of counting rates to preparation rates. As the last ones are usually unmeasurable, the corresponding test cannot be made. Alternatively, we can derive inequalities involving only ratios of probabilities, i.e., quantities that can be approximated by ratios of measurable counting rates. The test becomes then realizable; but at the same time it becomes useless. Indeed, it turns out that for this kind of test it is possible to construct a local realistic model which accounts for all experimental results, as Santos has proved [17,18].

Santos claimed that all performed tests are invalid. His claim rests upon the distinction between homogeneous and inhomogeneous Bell inequalities, followed by the observation that all performed experimental tests have been concerned with homogeneous inequalities. These are, allegedly, not genuine Bell inequalities, because either their derivation and/or the corresponding test would require additional assumptions besides realism and locality. According to Santos, all genuine Bell inequalities are inhomogeneous. They entail coincidence probabilities together with single probabilities or numbers. Homogeneous inequalities entail only coincidence probabilities. Santos maintains that it is pointless to test homogeneous inequalities, for whenever such an inequality is violated by QM, it is also violated by a local realistic model. This assertion constitutes a theorem and Santos proves it by explicit construction of a local hidden-variable (LHV) model [17,18]. It is important to emphasize that Santos's model does *not* exploit the detection loophole, as other authors did (see, e.g., [11,19]). Santos's LHV model works even for perfect detectors.

We are thus confronted with the following state of affairs: According to Santos's theorem, any experiment that is designed to test a homogeneous Bell inequality is, in principle,

*fdezela@pucp.edu.pe

incapable of discriminating between QM and LHV theories that are based only upon realism and locality. On the other hand, Bell's theorem states that it is possible to discriminate between QM and local realistic theories. This is so because these competing theories predict different results for some tests. But none of these tests can be based on a homogeneous Bell inequality. Otherwise, only one of the two theorems would hold true. We conclude that tests based on homogeneous Bell inequalities must entail some additional assumptions besides realism and locality. Therefore they are not "genuine" tests.

The validity of Santos's theorem makes Bell's theorem almost useless for the purpose of disproving local realism, because in order to show the violation of a genuine—and hence inhomogeneous—Bell inequality we should be able to measure probabilities that are, in fact, experimentally unaccessible [20]. Such a situation provides strong support to Santos's conjecture that realism and QM might be compatible. Is it then pointless to conceive new tests of local realism? Are inhomogeneous, experimentally unaccessible, inequalities the only genuine ones? We will answer these questions in the negative, by deriving homogeneous Bell inequalities that can be considered genuine, in the sense of being appropriate to disprove local realism. The point of departure from Santos's approach resides in some requirements that must be imposed on the probabilities entering LHV models. The sole requirement that Santos imposed on these probabilities was that they lie between zero and one. As we shall see, the probabilities entering our inequalities must be subjected to additional constraints, as long as they refer to a specified class of physical events.

II. SANTOS'S THEOREM

Santos's distinction between homogeneous and inhomogeneous inequalities is based upon Bell's formulation of LHV models. These consist of a set of hidden variables λ that serve to specify an otherwise "incomplete" description of a quantum state, plus a normalized distribution function $\rho(\lambda)\geq 0$, $\int\rho(\lambda)d\lambda=1$, and probability functions $0\leq p_i(\lambda,a)$, $p_j(\lambda,b)\leq 1$, with i,j each having the values $\pm$ that correspond to dichotomic observables. Labels a and b denote here—for the sake of concreteness—the angles to which two polarization analyzers are set before detectors D_a and D_b, respectively. These quantities are connected with the—in principle—measurable coincidence probabilities $P_{ij}(a,b)$ and single probabilities $P_i(a)$, $P_j(b)$, through the following relationships:

$$P_{ij}(a,b) = \int \rho(\lambda)p_i(\lambda,a)p_j(\lambda,b)d\lambda, \quad (1)$$

$$P_i(a) = \int \rho(\lambda)p_i(\lambda,a)d\lambda, \quad (2)$$

$$P_j(b) = \int \rho(\lambda)p_j(\lambda,b)d\lambda. \quad (3)$$

It is within this framework that Santos defines homogeneous Bell inequalities as those involving only the $P_{ij}(a,b)$, that is, as those comparing coincidence probabilities among themselves. Inhomogeneous inequalities are instead inequalities that contain the $P_{ij}(a,b)$ together with the $P_i(a)$ and $P_j(b)$, or together with numbers. According to Santos [17], all performed experiments have tested homogeneous inequalities of the form

$$\sum_{i,j} c_{ij}P(a_i,b_j) \geq 0, \quad (4)$$

with the c_{ij} being real numbers. Furthermore, Santos proved the following theorem [17]: If inequality (4) is violated by QM, then it is also violated by a LHV model. The proof of this theorem was a constructive one, in which a LHV model reproducing the predictions of QM for the $P_{ij}(a,b)$ was given. Hence we should conclude that none of the performed experiments has disproved local realism, unless they have also disproved QM. Santos argues that the performed experiments in fact did not rule out local realism because all these experiments tested inequalities like Eq. (4). The homogeneous character of these inequalities allowed people to test them by using only coincidence counting rates, thereby avoiding the necessity of measuring the $P(a_i,b_j)$ as ratios between coincidence rates to (unmeasurable) production rates. However, by so doing people chose to make one of two things. Either they introduced auxiliary assumptions besides realism and locality, or else they "renormalized" probabilities by dividing the $P(a_i,b_j)$ through a common factor like $\Sigma_{i,j}P(a_i,b_j)$. The second option was the choice taken by those who tested the celebrated Clauser-Horne-Shimony-Holt (CHSH) inequality [21], which is an inhomogeneous Bell inequality of the form $\Sigma_{i,j}c_{ij}P(a_i,b_j)\leq 2$. What people did in all these tests of the CHSH inequality was to replace the true correlations entering it by "renormalized" correlations. However, the resulting inequality cannot be derived from local realism alone. This has been the case even with the celebrated experiments of Aspect and co-workers [6] as well as with other, more recent ones [3]. In spite of these strong objections raised by Santos it seems that a great part of the physics community preferred to ignore them, or at least opted for not taking them as strong as they are.

It appears then that homogeneous inequalities are unsuitable for disproving local realism in favor of QM. Accordingly, the only usefulness such inequalities might have is that they could serve to disprove restricted classes of LHV models; that is, models incorporating additional assumptions besides realism and locality. This would be the case with some variants of two well known inhomogeneous Bell inequalities: the Clauser-Horne (CH) [22] and the CHSH [21] inequalities. Besides the already mentioned tests of the CHSH inequality, there have been other tests based on it. These tests have used homogeneous versions of the CH and of the CHSH inequalities, which were obtained by invoking supplementary assumptions that restricted the class of LHV models under test. These assumptions are violated by Santos's model. The models in the restricted LHV class satisfy inequalities that QM violates, and can thus be ruled out by the corresponding experiments. But these experiments do not rule out the unrestricted set of LHV models that could represent a "completion" of QM, in the sense of Einstein.

Henceforth, we will focus our attention on two tests of the homogeneous version of the CH inequality that were reported some years ago, showing its clear violation: a test by Ou and Mandel [23], and a test by Torgerson *et al.* [24]. Whereas the former has been often cited in the literature, the last one is not. This might be somewhat surprising, because the test by Torgerson *et al.* claimed to show a violation of the CH inequality by about 40 standard deviations at each data point. Moreover, as compared to the test of Ou and Mandel, the test of Torgerson *et al.* seems to address a broader class of hidden-variable models. Indeed, while Ou and Mandel invoked the no-enhancement assumption [22], Torgerson *et al.* did not. It seems that the test reported by Torgerson *et al.* has aroused more skepticism than other, often cited tests. As we shall see, this skepticism is not totally surprising, in view of an assumption made by Torgerson *et al.* in relation to single probabilities. This assumption is highly questionable and might be the reason why the test of Torgerson *et al.* has been only rarely cited. Our purpose here is to analyze the class of hidden-variable theories that the test of Torgerson *et al.* does rule out. By so doing, we will endeavor to discuss the tests of Bell inequalities in a way that transcends the particular case reported by Torgerson *et al.*, making it our battle horse for reaching our principal aim: to prove that it is possible to derive homogeneous Bell inequalities that are at the same time genuine.

To talk about a homogeneous and genuine Bell inequality seems to be a contradiction in terms, in view of the aforementioned LHV model provided by Santos. The contradiction is removed by strengthening and thereby modifying somewhat the notion of a genuine Bell inequality: we require from it to be derivable not only from realism—in a broad sense—and from locality. Thus there is a further restriction that we should impose, and this is that the underlying LHV theories be theories of physical reality. The probabilities $p_i(\lambda,a)$, $p_j(\lambda,b)$ must then be constrained by whatever conditions physical reality might impose upon them. Santos's proof of his theorem required from these probabilities to fulfill only the conditions $0\leq p_i(\lambda,a)$, $p_j(\lambda,b)\leq 1$ that every probability should satisfy, whenever it is assigned to the elements—termed "events"—of an abstract set. As soon as we require from $p_i(\lambda,a)$ and $p_j(\lambda,b)$ to be probabilities of physical events—in contrast to baldly assigning them to events of an abstract set—further restrictions can be called for. We stress that we hereby refer to conditions that are unavoidable, i.e., they should hold by necessity, as a consequence of some general features that we ascribe to physical reality. This should be distinguished from the form in which different authors have restricted the class of LHV models under study, namely by making supplementary assumptions on the $p_i(\lambda,a)$, $p_j(\lambda,b)$ that rest on plausibility arguments. In our case, we are referring to a self-consistent picture of the real world, a picture that nonetheless might very well entail some idealizations. For example, consider an ideal polarization analyzer. Let $p_+(\lambda,a)$ be the probability that a photon—characterized by a hidden variable λ—passes this analyzer when it has been set to the angle a, and let λ be the polarization angle for the photon. Then, we may safely require that $p_+(\lambda=a,a)=1$, without considering this equation to be an additional assumption alongside realism and locality. It is an assumption that we should rather impose on any LHV model that could be accepted as a self-consistent description of physical reality. And the word "photon" does not need here to be embodied with any naive meaning of a corpuscular entity; what is essential is the very concept of polarization as a dichotomic quantity. It is in this abstract sense that we refer to a self-consistent picture of physical reality, although we will often use the word "photon" and the like, for economy of language.

III. THE TESTS BY OU AND MANDEL AND BY TORGERSON *et al.*

Ou and Mandel and Torgerson *et al.* tested, respectively, two special forms of the CH inequality that were derived from the original, inhomogeneous version of it. In order to discuss the relationship among these different versions, let us briefly recall how the CH inequality can be obtained: Consider six real numbers, x, x', y, y', X, and Y, such that $0\leq x,x'\leq X$ and $0\leq y,y'\leq Y$. Then, it follows that [22]

$$xy - xy' + x'y + x'y' - x'Y - yX \leq 0. \tag{5}$$

Let us now take $i=j=+$ in Eqs. (1)–(3) and set $x=p_+(\lambda,a)\equiv p(\lambda,a)$, $y=p(\lambda,b)$, $x'=p(\lambda,a')$, $y'=p(\lambda,b')$, $X=Y=1$ in Eq. (5). After multiplying the resulting inequality by $\rho(\lambda)$ and integrating over λ, one obtains the CH inequality

$$P(a,b) - P(a,b') + P(a',b') + P(a',b) - P(a') - P(b) \leq 0. \tag{6}$$

Following Santos [17], we call Eq. (6) a "genuine" Bell inequality, viz. one which does not involve supplementary assumptions beyond realism and locality. We note that Eq. (6) is inhomogeneous; it compares coincidence probabilities against single probabilities. Another well-known inhomogeneous Bell inequality is the CHSH inequality [21], in which coincidence probabilities are compared against a pure number. It can be derived [22] as a corollary of Eq. (6). As already said, Santos pointed out [2,17] that so far only homogeneous Bell inequalities have been experimentally tested. This has been so because in a proper test all probabilities should have been interpreted as ratios of counting rates to emission rates. However, emission rates are usually unknown and hence should drop from the resulting expression. This can be achieved by writing down all rates in terms of the emission rate and other parameters like angular correlations and detector efficiencies. But proceeding in this way, these parameters enter with different weights if the inequality in question is an inhomogeneous one, like Eq. (6). In such a case, the resulting inequality entails one or more detector efficiencies, so that for the inequality to be violated there is some minimum efficiency that must be required. Several authors have analyzed both the CH and the CHSH inequalities including detector efficiencies [12–16,22,25]. They concluded that there is a threshold efficiency that should be reached for a test to be loophole free. Of course, reaching a threshold efficiency would be a necessary but not sufficient condition, as other loopholes might persist which exploit locality, background noise, depolarization, and so on. Experiments based on optical photons are among the best candidates to close all loopholes, but an overall detection efficiency $\eta > \eta_{thresh}=0.67$ is required for a two-photon test of the CH inequality [12,14]. Although such an η_{thresh} does represent an improvement with respect to the $\eta_{thresh}=0.83$ that would be required for a loophole-free test of the CHSH inequality [25], it is still beyond reach for present day technology: the overall efficiency for experiments with optical photons lies around 0.30 [15]. It is thus important, even for reasons besides our main concern, to delimit the class of local realistic theories that has been ruled out in experiments like the ones reported by Ou and Mandel and by Torgerson *et al.*

Let us first consider the experiments of Ou and Mandel [23]. They tested the CH inequality transformed to the following, homogeneous form:

$$P_{12}(\theta_1,\theta_2) - P_{12}(\theta_1,\theta_2') + P_{12}(\theta_1',\theta_2') + P_{12}(\theta_1',\theta_2) - P_{12}(\theta_1',\infty) - P_{12}(\infty,\theta_2) \leq 0. \tag{7}$$

Here, $P_{12}(\theta_1,\theta_2)$ means the joint probability of detecting a particle at each one of two detectors, D_1 and D_2, with these detectors having polarization analyzers (polarizers, for short) set in front of them to angles θ_1 and θ_2, respectively. $P_{12}(\theta_1,\infty)$ and $P_{12}(\infty,\theta_2)$ are probabilities with one or the other polarizer removed. Inequality (7) was proposed by Clauser and Horne [22] to make only joint probabilities appear, thereby avoiding the appearance of single probabilities. Now, in order to obtain inequality (7) the "no-enhancement assumption" must be invoked. This assumption states that the classical, single probabilities $p_{i=1,2}(\lambda,\theta)$ are bounded by some other probabilities $p_i(\lambda,\infty)$. Here, $p_i(\lambda,\theta)$ means the probability to register a particle at detector D_i when the polarizer is set to an angle θ, the registered particle being emitted in a state characterized by λ. It is required that $p_i(\lambda,\theta)$ satisfies the conditions $0 \leq p_i(\lambda,\theta) \leq p_i(\lambda,\infty) \leq 1$, with $p_i(\lambda,\infty)$ meaning the probability of counting a photon at D_i when the polarizer in front of it is absent. One can then start from Eq. (5), setting $x=p_1(\lambda,\theta_1)$, $y=p_2(\lambda,\theta_2)$, $x'=p_1(\lambda,\theta_1')$, $y'=p_2(\lambda,\theta_2')$, $X=p_1(\lambda,\infty)$, $Y=p_2(\lambda,\infty)$, and after multiplying the resulting inequality by $\rho(\lambda)$ and integrating over λ one obtains the CH inequality as given in Eq. (7). Although the above bounds assumed for $p_{i=1,2}(\lambda,\theta)$ are plausible, they represent an additional assumption that further restricts the class of hidden-variable models under test.

On the other hand, Torgerson *et al.* did not invoke the no-enhancement assumption because they tested the following inequality:

$$P_{12}(\theta_1,\theta_2) - P_{12}(\theta_1,\theta_2') - P_{12}(\theta_1'+\pi/2,\theta_2) - P_{12}(\theta_1',\theta_2'+\pi/2) \leq 0. \tag{8}$$

According to Torgerson *et al.*, Eq. (8) follows from the original form of the Clauser-Horne inequality:

$$P_{12}(\theta_1,\theta_2) - P_{12}(\theta_1,\theta_2') + P_{12}(\theta_1',\theta_2') + P_{12}(\theta_1',\theta_2) - P_1(\theta_1') - P_2(\theta_2) \leq 0, \tag{9}$$

"whenever the single channel probabilities $P_1(\theta_1)$ and $P_2(\theta_2)$ are equivalent to the joint probabilities $P_{12}(\theta_1,-)$ and $P_{12}(-,\theta_2)$, respectively." Here,

$$P_{12}(\theta_1,-) = P_{12}(\theta_1,\theta) + P_{12}(\theta_1,\theta+\pi/2)$$

and

$$P_{12}(-,\theta_2) = P_{12}(\theta,\theta_2) + P_{12}(\theta+\pi/2,\theta_2).$$

Indeed, one can easily check that by making the replacements $P_1(\theta_1') \to P_{12}(\theta_1',-)$ and $P_2(\theta_2) \to P_{12}(-,\theta_2)$ in Eq. (9), inequality (8) follows. That is, Torgerson *et al.* took for granted that one can set

$$P_1(\theta_1) = P_{12}(\theta_1,\theta) + P_{12}(\theta_1,\theta+\pi/2), \tag{10}$$

$$P_2(\theta_2) = P_{12}(\theta,\theta_2) + P_{12}(\theta+\pi/2,\theta_2). \tag{11}$$

They claimed that "this is certainly true for perfect detectors and it is true more generally if the fair sampling assumption is valid." Now, excepting the ideal case of perfect detectors, one should rather expect that Eqs. (10) and (11) are generally not fulfilled. This is because the left hand sides of Eqs. (10) and (11) generally depend on the efficiency of a single detector, whereas the right hand sides generally depend on the efficiencies of both detectors. How could the probability of a count at one detector depend on the efficiency of the other detector? Take, e.g., the case of D_1 having almost 100% efficiency and D_2 having almost null efficiency. Then, Eq. (10) would imply that the probability to detect a particle at D_1 is almost zero. And the fair sampling assumption does not help out of this contradiction. That is, although Torgerson *et al.* did not invoke the no-enhancement assumption, they made instead the very strong and restrictive assumption that is expressed through Eqs. (10) and (11). To invoke conditions (10) and (11) seems to make things far more restrictive than to invoke the no-enhancement assumption. Hence at first sight the test of Torgerson *et al.* seems to address a very limited class of hidden-variable models, if any at all. In fact, because Eqs. (10) and (11) seem to be meaningless for any but ideal detectors, one should expect inequality (8) to be violated in almost every measurement. From this viewpoint, it is not surprising that the results reported by Torgerson *et al.* do confirm this expectation.

Whenever Eq. (8) is violated, at least one of the assumptions used to derive it must be false. Togerson *et al.* based their derivation of Eq. (8) on realism, locality, and conditions (10) and (11) for the probabilities. Of course, the best candidates for being taken as false are the last ones, thereby making the violation of Eq. (8) useless for the scope of disproving local realism. However, as we shall see next, conditions (10) and (11) are in fact unnecessary to derive Eq. (8). We can derive it from realism, locality, and the following conditions for the $p_i(\lambda,\theta_i), i=1,2$:

$$p_i(\lambda,\theta_i) \leq p_i(\lambda,\theta) + p_i(\lambda,\theta+\pi/2), \ \forall \ \theta. \tag{12}$$

Assuming the validity of Eq. (12) we can derive Eq. (8) from Eq. (5) by setting $X=p_1(\lambda,\theta_x)+p_1(\lambda,\theta_x+\pi/2)$, $Y=p_2(\lambda,\theta_y)+p_2(\lambda,\theta_y+\pi/2)$, $x=p_1(\lambda,\theta_1)$, $y=p_2(\lambda,\theta_2)$, $x'=p_1(\lambda,\theta_1')$, $y'=p_2(\lambda,\theta_2')$. Multiplying the resulting inequality by $\rho(\lambda)$ and integrating over λ we obtain

$$P_{12}(\theta_1,\theta_2) - P_{12}(\theta_1,\theta_2') + P_{12}(\theta_1',\theta_2) + P_{12}(\theta_1',\theta_2') - P_{12}(\theta_1',\theta_y) - P_{12}(\theta_1',\theta_y+\pi/2) - P_{12}(\theta_x,\theta_2) - P_{12}(\theta_x+\pi/2,\theta_2) \leqslant 0. \quad (13)$$

Choosing $\theta_x=\theta_1'$, $\theta_y=\theta_2'$ we recover inequality (8). Similar forms could be derived by choosing θ_x and θ_y conveniently.

Having shown that inequality (8) is a particular case of Eq. (13) and thus follows from realism and locality, together with conditions (12), we pass now to analyze the restrictions that these last conditions might imply for the class of hidden-variable theories we can test through inequality (13). Note first that conditions (12) are fulfilled by necessity whenever $p_i(\lambda,\theta)+p_i(\lambda,\theta+\pi/2)=1$, $i=1,2$. So, let us analyze in which cases this last condition is satisfied. It is certainly satisfied if we assume that for each photon the following holds true: Each photon of the type λ would be detected at detector D_i in one of two cases, namely, when a polarizer is set to the angle θ before the detector, or when the polarizer is set to $\theta+\pi/2$. We expect this to be true for the case of ideal polarizers and perfect detectors. However, even assuming this ideal case we cannot directly conclude that $p_i(\lambda,\theta)+p_i(\lambda,\theta+\pi/2)=1$, $i=1,2$, because this would imply a counterfactual argument. Indeed, it would mean that if D_i did not detect the photon when the polarizer in front of it was set to θ, then it would have detected it in case the polarizer would have been set to $\theta+\pi/2$. In order to avoid counterfactual assertions we consider instead the complementary probabilities $\bar{p}_{i=1,2}(\lambda,\theta)$ that refer to the lack of detection at D_i when the polarizer in front of it has been set to θ. Then, we can safely require that $\bar{p}_i(\lambda,\theta)+\bar{p}_i(\lambda,\theta+\pi/2)=1$, $i=1,2$, without resorting to counterfactual arguments. The last equation follows from assuming that a photon must be absorbed whenever it goes through two consecutive, ideal polarizers, one being set to θ and the other to $\theta+\pi/2$. Note that we are consistently assuming—by invoking noncontextuality—that the value assigned to $\bar{p}_i(\lambda,\theta)$ does not depend upon the setting (to angle $\theta+\pi/2$) of a second polarizer. Because $p_i(\lambda,\theta)+\bar{p}_i(\lambda,\theta)=1$, it follows that $p_i(\lambda,\theta)+p_i(\lambda,\theta+\pi/2)=1$ must hold as well. Thus, up to this point, the validity of conditions (12) could be limited only by the efficiency of the polarizers, which we consider next.

Nonideal polarizers can have efficiencies larger than 99%. Thus, in any case, restrictions derived from the inefficiency of the polarizers are much less demanding than those stemming from detector inefficiencies. Let us consider once more a photon that is characterized by λ and sent through two consecutive, nonideal polarizers set to angles perpendicular to each other, θ and $\theta+\pi/2$. Let us denote by $\epsilon_i(\lambda,\theta)$ the (small) probability that the photon passes the two polarizers and is eventually detected at detector D_i. Then, it follows that $\bar{p}_i(\lambda,\theta)+\bar{p}_i(\lambda,\theta+\pi/2)=1-\epsilon_i(\lambda,\theta)$, $i=1,2$. Now, by replacing $p_i(\lambda,\theta)+\bar{p}_i(\lambda,\theta)=1$ in the last equation we conclude that $p_i(\lambda,\theta)+p_i(\lambda,\theta+\pi/2)=1+\epsilon_i(\lambda,\theta)$. This equation expresses the fact that with inefficient polarizers some of the photons that are polarized along θ could nevertheless pass a polarizer set to $\theta+\pi/2$ (or the other way around) so that $p_i(\lambda,\theta)+p_i(\lambda,\theta+\pi/2)\geqslant 1$. Anyhow, we conclude that conditions (12) are fulfilled, either with ideal or imperfect polarizers. As a consequence, the homogeneous inequality (13) follows without further assumptions than realism, locality, and the requirement that the $p_i(\lambda,\theta)$ be in accordance with a self-consistent description of physical reality. It can thus be considered a genuine Bell inequality. Finally, let us remark that while Eq. (13) is of the form (4), the theorem proved by Santos [17] does not apply to it, because its demonstration is based upon a class of probability functions which do not fulfill the conditions given in Eq. (12).

IV. CONCLUSIONS

We have derived inequality (13) without invoking additional assumptions beyond realism and locality. We are thus in possession of an inequality that, in spite of being homogeneous, may be considered a genuine Bell inequality. The class of local realistic theories that should fulfill it is a very broad class. Note that the underlying probabilities $p_i(\lambda,\theta)$ may incorporate inefficiencies of whatever sort. Hence inequality (13) should hold for actual experiments and its violation implies that either realism or locality, or both, must be rejected. The problem with similar, homogeneous inequalities that have been tested in the past was that they required to invoke additional assumptions, like the no-enhancement assumption, the fair sampling assumption, the free-will assumption, and so on.

Turning to the violation of Eq. (8) reported by Torgerson *et al.* we observe that these authors equated the probabilities $P_{12}(\theta_1,\theta_2)$ to coincidence ratios $R_{12}(\theta_1,\theta_2)/R(-,-)$, with

$$R(-,-) = R_{12}(\theta_1,\theta_2) + R_{12}(\theta_1,\bar{\theta}_2) + R_{12}(\bar{\theta}_1,\theta_2) + R_{12}(\bar{\theta}_1,\bar{\theta}_2),$$

$$\text{and } \bar{\theta} \equiv \theta + \pi/2.$$

While setting $P_{12}(\theta_1,\theta_2)=R_{12}(\theta_1,\theta_2)/R(-,-)$ could be questionable, this fact by itself does not invalidate the test. Indeed the test did show that inequality (8) was violated: its left-hand side was measured to be a positive number, and this is all that matters. The unknown rate of emitted photons, which should be the common denominator in all the $P_{12}(\theta_1,\theta_2)$, drops from Eq. (8) anyway, as it also occurs with the detector efficiencies. We conclude that the experiment reported by Torgerson *et al.* may be taken as a valid test of local realism. It thus represents an improvement with respect to the former test of Ou and Mandel, in which the no-enhancement assumption had to be invoked.

Strictly speaking, the class of LHV models that the test of Torgerson *et al.* rules out is a class entailing models that are realistic, local, and noncontextual. Noncontextuality has been required when treating the $\bar{p}_i(\lambda,\theta)$ as being fixed by λ and θ alone, without regard to the context. This does not follow from locality, because locality is compatible with both contextuality and noncontextuality. It should be worth mentioning in this context that a test of realistic and noncontextual models has been recently reported [26]. In a way, it is complementary to the test of Torgerson *et al.*, as it is also based on the CH inequality. However, it does not address locality, because it was performed with single photons.

Finally, we have seen that the Clauser-Horne inequality (6) can be brought without further assumptions into different homogeneous forms that are embodied in Eq. (13)—and of which Eq. (8) is just an example. Alternative forms could be better suited to perform other tests, particularly those aiming at closing the locality loophole.

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