Tunneling limit of strong-field photoionization: Dependence on ellipticity

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Some results from a recent work of Reiss [Phys. Rev. Lett. **101**, 043002 (2008); **101**, 159901(E) (2008)] are generalized for the case of elliptical polarization of a laser field. We also discuss the tunneling limit of strong-field photoionization.

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In his recent work [1] Reiss considered (among other things) the physical significance of the so-called tunneling limit of ionization by lasers: $\gamma \rightarrow 0$ [γ is the Keldysh adiabaticity parameter, introduced in Eq. (17) from Ref. [2], and generalized below in Eq. (3)]. According to Reiss, the limit $\gamma \rightarrow 0$ applies *only* to ionization by quasistatic electric fields. In the present work we would like to state this more precisely, because in fact Ref. [1] concerns only *linear polarization* (LP). The latter one was never mentioned once in Reiss' work; hence, it might suggest that the obtained results are valid for any polarization. [In fact, only the remark above Eq. (5) about a figure-8 pattern bespeaks LP in Ref. [1].] A freely strong plane-wave electromagnetic field is completely described by its frequency ω, intensity I, and polarization. But only two (ω, I) of these three quantities were evidently taken into account in Ref. [1]. The third important parameter, omitted in the analysis done by Reiss, is the laser field ellipticity. The main aim of the present work is to show that some conclusions from Ref. [1] (those regarding the limit $\gamma \rightarrow 0$) are valid only for *linear polarization* (LP). It appears that one can easily generalize them for any elliptical polarization. We also show that the case of *circular polarization* (CP) is qualitatively different from that of LP. In particular, one of the main conclusions from Ref. [1] about the onset of magnetic-field effects in the limit $\gamma \rightarrow 0$ does not apply for CP. In what follows we use atomic units: $\hbar = e = m_e = 1$ (substituting explicitly -1 for the electronic charge), and the same notation as in Ref. [1].

Reference [1] is based on a classical relativistic dynamics describing a free point charge (an electron) moving in the monochromatic plane-wave laser field. On the other hand, the well-known early tunneling theories [2–8] assumed the nonrelativistic and dipole (or long wavelength) approximation in the description of an outgoing electron. In the present work, unlike in Ref. [1], we assume that a tunneling theory should give the ionization or detachment rate in a laser field, when the outgoing electron tunnels through the Coulomb barrier (i.e., when tunneling is a dominant mechanism of ionization or detachment). In our opinion, a tunneling theory does not have to neglect the magnetic-field component of the laser [9], as it is assumed in Ref. [1]. There are numerous examples confirming our opinion in the recent review article of Popov [10]. Of course, for extremely strong fields relativistic treatment of tunneling [10–14] is necessary for any kind of polarization. In Fig. 1 from Ref. [12] there are specified three important regions of the laser field parameters (ω, I), where the ionization or detachment can take place. In the intermediate range of these parameters one can keep the nonrelativistic theory, but one has to take into account the magnetic-field component of the laser. Then the electric field $\vec{E}$ and the magnetic field $\vec{B}$ depend only on time and obey the condition $\vec{B} = \hat{n} \times \vec{E}$ ($\hat{n}$ is a unit vector in the propagation direction). In the present work we reconsider the area marked as "magnetic field important" in Fig. 1 from Ref. [12].

Let us assume that the field propagates along the x axis and its wave vector is given by $\vec{k} = k\hat{e}_x = (k, 0, 0)$ [$\hat{e}_x, \hat{e}_y, \hat{e}_z$ are real unit vectors; $k = \omega / c$; $\vec{r} = (x, y, z)$ in the laboratory frame]. The field of an arbitrary polarization can be described by the following vector potential (and the scalar potential equal to zero):

$$\vec{A}(\vec{r},t) = a[\hat{e}_z \cos(\delta/2)\cos(\omega t - kx) \pm \hat{e}_y \sin(\delta/2)\sin(\omega t - kx)], \tag{1}$$

where δ is the ellipticity parameter ($\delta \in [0, \pi/2]$, and the signs $\pm$ correspond to two different helicities). From Eq. (1) one can find the electric field vector ($\vec{E} = -c^{-1}\partial\vec{A}/\partial t$) of the electromagnetic plane wave and its amplitude,

$$E_0 = \frac{a\omega}{c}\sqrt{\frac{1 + \cos\delta}{2}} = \frac{a\omega}{c}\cos(\delta/2). \tag{2}$$

Keldysh, in his pioneering work, defined the parameter γ only for LP. If E_B denotes the binding energy (or the ionization potential) of an atom or ion, Eq. (17) from Ref. [2] in our notation is

$$\gamma = \frac{\omega\sqrt{2E_B}}{E_0} = \frac{c\sqrt{2E_B}}{a\cos(\delta/2)}, \tag{3}$$

where we generalize Keldysh's definition for an arbitrary polarization, using also Eq. (2). The motion of the charge in the field given by Eq. (1) can be found exactly for any δ. For LP ($\delta = 0$) and for CP ($\delta = \pi/2$) there are solutions to this problem in Sec. 48 (p. 134) of Ref. [15]. We generalize these solutions for any polarization here, and in the simplest frame of reference we get the following result:

$$x = \frac{a^2}{8c\omega\varepsilon^2}\cos\delta\sin 2(\omega t - kx), \tag{4a}$$

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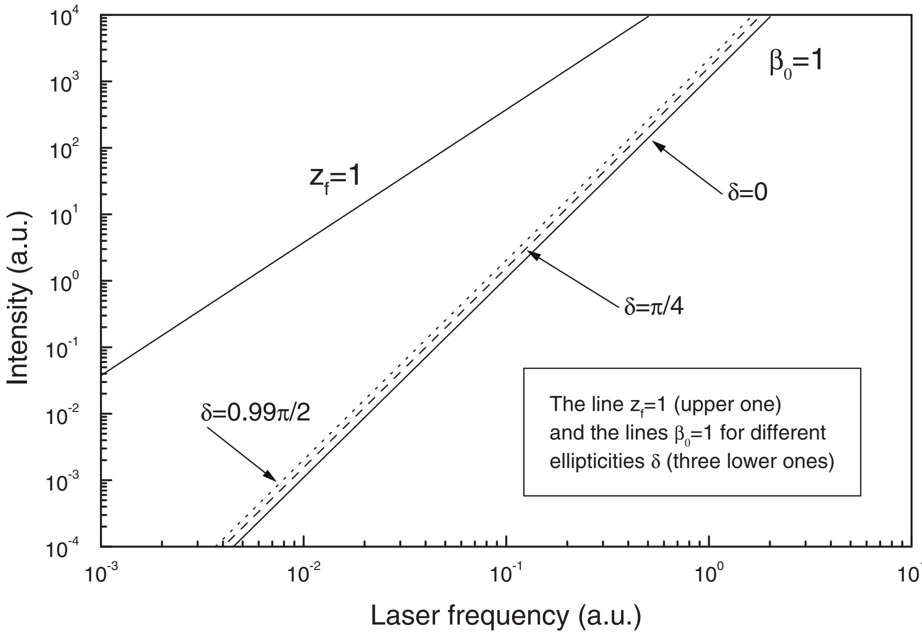

FIG. 1. The slanted lines indicate the intensity as a function of frequency, as a result of fixing either of two parameters. The line $z_f = 1$ corresponds to any ellipticity parameter δ. The lines $\beta_0 = 1$ correspond to $\delta = 0$ (linear polarization), $\delta = \pi/4$ (elliptical polarization), and $\delta = 0.99\pi/2$ (almost circular polarization). (See the text for more details.)

$$y = \mp \frac{a}{\omega\varepsilon}\sin(\delta/2)\cos(\omega t - kx), \tag{4b}$$

$$z = \frac{a}{\omega\varepsilon}\cos(\delta/2)\sin(\omega t - kx), \tag{4c}$$

where $\varepsilon = \sqrt{c^2 + a^2/2c^2}$. (Instead of the relativistic γ, as in Ref. [15], we use ε here. The simplest frame of reference is the one in which the charge is at rest on the average.) The amplitude of motion in the propagation direction, in Eq. (4a), may be treated as a direct measure of the effect of the magnetic field. In this way we generalize here Eq. (5) from Ref. [1] (see the Erratum), obtaining the following equation:

$$\beta_0 = \frac{a^2}{8c\omega\varepsilon^2}\cos\delta = \frac{z}{2c(1+z_f)}\cos\delta \approx \frac{z}{2c}\cos\delta, \tag{5}$$

where $z_f \equiv 2U_P/c^2 \equiv 2z\omega/c^2 \ll 1$ (z and z_f are the intensity parameters, and U_P is the ponderomotive potential—the time-averaged kinetic energy of a classical free charge oscillating in an electromagnetic plane-wave field). In the nonrelativistic approximation $U_P = (a/2c)^2 = [E_0/2\omega\cos(\delta/2)]^2$. Equation (5) comes from the fact that the speed of the ionized electron is much less than the speed of light. In Eq. (5) we used expressions for the laser field intensity in atomic units $[I = (a\omega/c)^2 = 4z\omega^3$, which are the same for all $\delta]$ and Eqs. (3) and (6) from Ref. [1]. Equations (4) and (5) are the main result of the present paper. For LP Eq. (5) reduces to Eq. (5) from Ref. [1], and for CP one obtains $\beta_0 = 0$ from Eq. (5). Therefore Eq. (8) from Ref. [1] is generalized here to

$$\beta_0 = 1 \Rightarrow I = 8c\omega^3/\cos\delta, \tag{6}$$

for $0 \leqslant \delta < \pi/2$. Figure 1 (which is similar to Fig. 2 from Ref. [1]) shows the line $z_f = 1$ and the lines $\beta_0 = 1$ from Eq. (5) for different ellipticities. For $\delta \neq 0$ the line $\beta_0 = 1$ moves toward the line $z_f = 1$ with increasing δ. However, due to logarithmic scales in Fig. 1, for typical ellipticities different from LP and CP ($0 < \delta < \pi/2$), the area where magnetic-field effects become important diminishes only very slightly with increasing δ. For CP ($\delta = \pi/2$) the intermediate area of the parameters ω, I disappears. This means that one should use a qualitatively different description (the fully relativistic one) of strong-field ionization beginning in the vicinity of the line $z_f = 1$ for CP. The plane-wave laser field is a transverse field. For CP the magnetic-field component of the Lorentz force acting on the charge is equal to zero because this component is always parallel to the velocity of the charge. Although the magnetic field in the circularly polarized plane wave may be freely strong, it cannot force the charge to move along the propagation direction. For CP, even in the fully relativistic regime, the motion takes place along a circle lying in the polarization plane. Equations (4) show that for $0 < \delta < \pi/2$ the relativistic charge moves in three spatial dimensions (in the simplest frame of reference), but for $\delta = 0$ or $\delta = \pi/2$—only in two dimensions. For example, for CP one obtains from Eqs. (4) that for $\omega = \text{const}$ and $I \to \infty$ the radius of the above mentioned circle approaches a finite limit c/ω instead of infinity predicted by the nonrelativistic dynamics. Therefore for any δ, the limit $\gamma \to 0$ for extremely strong laser fields requires relativistic treatment.

By the way, let us note that the nonrelativistic limit $\gamma \to 0$, when only the electric field vector (in the dipole approximation) is present, may have a very well-defined physical meaning. For example, the effect of a slowly rotating (with the frequency ω) electric field vector (of a constant length) on the bound system in the limit $\omega \to 0$ should be the same as for the static electric field. Therefore the ionization rates for CP and $I = \text{const}$, calculated in the nonrelativistic and dipole approximations [hence $\vec{E} = \vec{E}(t)$ and $\vec{B} = \vec{0}$], should coincide with the static field ionization rates. By analogy, the ionization rates for LP and $I = \text{const}$, in the same approximations, should coincide with the static field ionization rates averaged over a field period [16]. We have recently studied numerically the S-matrix theory of strong-field photoionization in this context for the $H(1s)$ atom and both polarizations [17,18]. Furthermore, recently Vanne and Saenz [19] have shown analytically that in the so-called velocity gauge in the low-frequency limit the ionization rate for LP is proportional to the laser frequency. As a result, I=const and $\omega \rightarrow 0$ lead to nulling of the ionization rate. In our opinion, this clearly *unphysical* result is a consequence of the lack of gauge invariance of the ionization probability amplitude in this case [20]. In the limit I=const and $\omega \rightarrow 0$ (then also $\gamma \rightarrow 0$) the ratio U_P/E_B goes to infinity. The latter fact should be the sufficient applicability condition for the nonrelativistic S-matrix theory which assumes that $\vec{E}=\vec{E}(t)$ and $\vec{B}=\vec{0}$. When the approximate, but gauge-invariant, Keldysh's theory [2] is used, one obtains at least an order of magnitude (or better) agreement with the exact static field results of Scrinzi [21] for the $H(1s)$ atom. As one should expect, the agreement usually improves with increasing the ratio U_P/E_B (see Fig. 5 in Ref. [17] and Figs. 1 and 2 in Ref. [20]).

As a side remark, we note that even for LP the magnetic field effects do not have to be strong in the intermediate regime of the parameters ω and I (the area between the line $z_f=1$ and the line $\beta_0=1$ and $\delta=0$ in Fig. 1). In a recent experiment [22] with argon and pulsed-laser 800-nm radiation at an intensity of up to 10^{19} W/cm^2 (cf. also Fig. 2 from Ref. [1]) the average Keldysh parameter γ was equal to 0.03. The authors of Ref. [22] confirmed the validity of the well-known Ammosov-Delone-Krainov–Wentzel-Kramers-Brillouin (ADK–WKB) tunneling model [3–8] (which is the nonrelativistic and dipole approximation theory) for the above parameters of the strong laser field. According to authors of Ref. [22], the results of their experiment may be interpreted within a two-step model, where the initial tunneling ionization process is dominated by the nonrelativistic effects, while the photoelectron continuum dynamics are strongly relativistic.

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