Strange behavior of the relativistic Einstein-Podolsky-Rosen correlations

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We show that configurations exist in which the correlation functions and the degree of violation of Bell-type inequalities in the relativistic Einstein-Podolsky-Rosen (EPR) experiment have local extrema for some values of the velocities of the EPR particles. Moreover, this strange behavior can be observed for both discussed relativistic spin operators and for spin-1/2 as well as spin-1 particles.

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Until recently, almost all papers concerning quantum-information processing were based on the nonrelativistic quantum mechanics. On the other hand, the present technological possibilities speed up the investigation of the relativistic aspects of the quantum Einstein-Podolsky-Rosen (EPR) correlations.

The aim of this paper is to report some strange behavior of the relativistic EPR correlation functions. We show that the correlation function, which in the relativistic case depends on the particle momenta, for some fixed configurations has local extrema. Such a behavior has not been reported in the previous works [1,2]. Such extrema can be observed for both spin-1/2 and spin-1 particles and for two different choices of the relativistic spin operator. This suggests that the discussed effect is a general property of the relativistic correlation functions. We also show that relativistic quantum correlations are stronger than nonrelativistic ones for a variety of configurations. Consequently, in such configurations Bell inequalities are more strongly violated by relativistic correlations than by nonrelativistic ones.

An appropriate treatment of the EPR experiment is hindered by very serious theoretical and interpretational difficulties concerning the relativistic quantum mechanics. One of the most frustrating problems is the lack of the Lorentz-covariant notion of localizability in the relativistic quantum mechanics. The position operator is needed not only to take into consideration the finite size of the detectors but also it is directly related with the definition and form of spin operator.

The most familiar choice of the position operator for a massive particle is the Newton-Wigner operator [3]

$$\hat{\mathbf{Q}}_{\mathrm{NW}} = -\frac{1}{2}\left[\frac{1}{\hat{P}^0}\hat{\mathbf{K}} + \hat{\mathbf{K}}\frac{1}{\hat{P}^0}\right] - \frac{\hat{\mathbf{P}}\times\hat{\mathbf{W}}}{m\hat{P}^0(m+\hat{P}^0)}, \tag{1}$$

where $\hat{P}^0$, $\hat{\mathbf{P}}$ are the four-momentum operators, m the particle mass, $\hat{\mathbf{K}}$ is the Lorentz boosts generator, and $\hat{\mathbf{W}}=\hat{P}^0\hat{\mathbf{J}}+\hat{\mathbf{P}}\times\hat{\mathbf{K}}$ is the space part of the Pauli-Lubanski four-vector $\hat{W}^\mu = \frac{1}{2}\epsilon^{\nu\gamma\delta\mu}\hat{P}_\nu\hat{J}_{\gamma\delta}$, $\hat{\mathbf{J}}$ is the total angular momentum operator, and $\hat{K}^i=\hat{J}^{0i}$, $\hat{J}^i=\varepsilon^{ijk}\hat{J}^{jk}$. The Newton-Wigner operator forms a vector with commuting, self-adjoint components, and is defined for arbitrary spin. Another popular choice of the position operator is [4]

$$\hat{\mathbf{Q}}_{\mathrm{c.m.}} = -\frac{1}{2}\left[\frac{1}{\hat{P}^0}\hat{\mathbf{K}} + \hat{\mathbf{K}}\frac{1}{\hat{P}^0}\right] \tag{2}$$

interpreted also as the center-of-mass position operator. For spinning particles components of this operator do not commute. Unfortunately, both operators do not form any autonomous geometrical object with a covariant transformation law.

Now, for observers in the same inertial frame spin is defined as a difference between total angular momentum (which is well defined as the generator of the rotations) and the orbital angular momentum $\hat{\mathbf{L}}=\hat{\mathbf{Q}}\times\hat{\mathbf{P}}$:

$$\hat{\mathbf{S}} = \hat{\mathbf{J}} - \hat{\mathbf{Q}}\times\hat{\mathbf{P}}. \tag{3}$$

However, to define the orbital angular momentum $\hat{\mathbf{L}}$ we should know the relativistic position operator $\hat{\mathbf{Q}}$. The lack of a generally accepted position operator results in ambiguities in the definition of relativistic spin operator. In particular, Newton-Wigner operator $\hat{\mathbf{Q}}_{\mathrm{NW}}$ leads to the following spin observable:

$$\hat{\mathbf{S}}_{\mathrm{NW}} = \frac{1}{m}\left(\hat{\mathbf{W}} - \hat{W}^0\frac{\hat{\mathbf{P}}}{\hat{P}^0+m}\right), \tag{4}$$

which satisfies usual spin algebra [su(2) Lie algebra]. $\hat{\mathbf{S}}_{\mathrm{NW}}$ is the only axial-vector operator being linear function of the Pauli-Lubanski four vector [5]. However, such spin operator is neither an autonomous geometrical object under Lorentz transformations nor even a part of an irreducible object.

For the position operator $\hat{\mathbf{Q}}_{\mathrm{c.m.}}$ the corresponding spin observable takes the form

$$\hat{\mathbf{S}}_{\mathrm{c.m.}} = \frac{\hat{\mathbf{W}}}{\hat{P}^0}. \tag{5}$$

Unfortunately, components of this operator do not form the spin algebra. Moreover their eigenvalues λ_i are momentum-dependent, i.e., $\lambda_i=\lambda\sqrt{[m^2+(k^i)^2]}/k^0$, $\lambda=-s,-s+1,\ldots,s$. Furthermore, in contrast to the operator $\hat{\mathbf{S}}^2_{\mathrm{NW}}$, the operator $\hat{\mathbf{S}}^2_{\mathrm{c.m.}}$ does not reduce to the relativistic spin-square operator $-W^\mu W_\mu/m^2$ equal to $s(s+1)\mathbb{1}$ in a unitary irreducible repre-

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sentation of the Poincaré group. Therefore the operator (5) cannot be treated as a proper spin observable. For this reason Czachor [6], and following him a number of authors (see, e.g., Refs. [1,7,8]), used the normalized operator (5). One can easily show that the spin observable used in Ref. [6] can be cast in the following form:

$$\hat{\mathbf{S}}(\mathbf{a}) = \frac{\mathbf{a}\cdot\hat{\mathbf{W}}}{\sqrt{m^2 + (\mathbf{a}\cdot\hat{\mathbf{P}})^2}}. \tag{6}$$

This operator has a proper spectrum; however, it cannot be treated as a projection of any spin observable on the direction $\mathbf{a}$, $|\mathbf{a}|=1$, because it is a nonlinear function of $\mathbf{a}$. In the following we will compare EPR correlations and Bell-type inequalities obtained with help of the spin operator (4) and the operator (6).

Let us consider two distant observers Alice and Bob in the same inertial frame, sharing a pair of particles with sharp momenta in a two-particle state $|\Psi\rangle$. We take into account only such measurements in which Alice and Bob register one particle each. Without loss of generality we can assume that particles are distinguishable and Alice registers the particle with momentum equal to $\mathbf{k}$ and Bob the particle with momentum equal to $\mathbf{p}$. Now let Alice measure spin component of her particle in direction $\mathbf{a}$ and Bob spin component of his particle in direction $\mathbf{b}$, where $|\mathbf{a}|=|\mathbf{b}|=1$. Their observables are $(\mathbf{a}\cdot\hat{\mathbf{S}}_{\mathrm{NW}})\otimes\mathbb{1}$ and $\mathbb{1}\otimes(\mathbf{b}\cdot\hat{\mathbf{S}}_{\mathrm{NW}})$, when one uses the spin operator $\hat{\mathbf{S}}_{\mathrm{NW}}$ defined in Eq. (4), or $\hat{\mathbf{S}}(\mathbf{a})\otimes\mathbb{1}$ and $\mathbb{1}\otimes\hat{\mathbf{S}}(\mathbf{b})$ when one uses the operator (6). Consequently, the normalized correlation function in the EPR-type experiment has the form

$$\mathcal{C}^{\Psi}(\mathbf{a},\mathbf{b}) = \frac{\langle\Psi|(\mathbf{a}\cdot\hat{\mathbf{S}}_{\mathrm{NW}})\otimes(\mathbf{b}\cdot\hat{\mathbf{S}}_{\mathrm{NW}})|\Psi\rangle}{s^2\langle\Psi|\Psi\rangle}, \tag{7}$$

for the spin operator $\hat{\mathbf{S}}_{\mathrm{NW}}$ or

$$\mathcal{C}^{\Psi}_{\mathrm{Cz}}(\mathbf{a},\mathbf{b}) = \frac{\langle\Psi|\hat{\mathbf{S}}(\mathbf{a})\otimes\hat{\mathbf{S}}(\mathbf{b})|\Psi\rangle}{s^2\langle\Psi|\Psi\rangle}, \tag{8}$$

for the operator (6) proposed by Czachor.

In this paper we will discuss EPR correlations in two-particle states which are singlets of the Lorentz group. Some of the formulas presented in this paper have been obtained in our previous works but we include them to the present paper to make it self-consistent. Let us remind first the notation concerning one-particle states. For the particle with mass m and spin s space of states is spanned by the four-momentum eigenvectors $|k,m,s,\sigma\rangle$. These vectors are normalized covariantly. The action of the Lorentz transformation Λ on the vector $|k,m,s,\sigma\rangle$ is of the form

$$U(\Lambda)|k,m,s,\sigma\rangle = \mathcal{D}^s_{\lambda\sigma}(R(\Lambda,k))|\Lambda k,m,s,\lambda\rangle, \tag{9}$$

where $\mathcal{D}^s$ is the matrix spin s representation of the SO(3) group, $R(\Lambda,k)=L^{-1}_{\Lambda k}\Lambda L_k$ is the Wigner rotation, and L_k designates the standard Lorentz boost defined by the relations $L_k\tilde{k}=k$, $L_{\tilde{k}}=I$, $\tilde{k}=(m,\mathbf{0})$. Throughout all the paper we will assume that both EPR particles have mass m. Moreover, for fixed values of the spin we will use the notation $|k,\sigma\rangle \equiv |k,m,s,\sigma\rangle$.

For $s=1/2$ pseudoscalar state of two particles with sharp momenta was discussed in Refs. [9,10]. It has the following form:

$$|\varphi(k,p)\rangle = \frac{-i}{\sqrt{2}\sqrt{\left(1+\frac{k^0}{m}\right)\left(1+\frac{p^0}{m}\right)}}\left\{\left[\mathbb{1}\left(1+\frac{k^0+p^0}{m}+\frac{kp}{m^2}\right) - \frac{i(\mathbf{k}\times\mathbf{p})\cdot\sigma}{m^2}\right]\sigma_2\right\}_{\sigma\lambda}|k,\sigma\rangle\otimes|p,\lambda\rangle, \tag{10}$$

where $\sigma,\lambda=\pm\frac{1}{2}$, $\boldsymbol{\sigma}=(\sigma_1,\sigma_2,\sigma_3)$, and σ_i are standard Pauli matrices. In the center-of-mass (c.m.) frame $[p=k^{\pi}\equiv(k^0,-\mathbf{k})]$ the state (10) is an ordinary singlet state.

Correlation function (7) for spin-1/2 particles in the state (10) was calculated in Ref. [9] and it reads

$$\mathcal{C}^{\varphi(k,p)}(\mathbf{a},\mathbf{b}) = -\mathbf{a}\cdot\mathbf{b} + \frac{(\mathbf{k}\times\mathbf{p})}{m^2+kp}\times\left((\mathbf{a}\times\mathbf{b}) + \frac{(\mathbf{a}\cdot\mathbf{k})(\mathbf{b}\times\mathbf{p})-(\mathbf{b}\cdot\mathbf{p})(\mathbf{a}\times\mathbf{k})}{(k^0+m)(p^0+m)}\right). \tag{11}$$

Notice that in the c.m. frame the above correlation function is the same as in the nonrelativistic case $\mathcal{C}^{\varphi(k,k^{\pi})}(\mathbf{a},\mathbf{b}) = -\mathbf{a}\cdot\mathbf{b}$.

On the other hand, one can check that correlation function (8) in the state (10) has the form

$$\mathcal{C}^{\varphi(k,p)}_{\mathrm{Cz}}(\mathbf{a},\mathbf{b}) = \frac{m^2}{\sqrt{m^2+(\mathbf{a}\cdot\mathbf{k})^2}\sqrt{m^2+(\mathbf{b}\cdot\mathbf{p})^2}}\times\left\{-\mathbf{a}\cdot\mathbf{b}+\frac{(\mathbf{a}\cdot\mathbf{k})(\mathbf{b}\cdot\mathbf{p})}{m^2} - \frac{[\mathbf{a}\cdot(\mathbf{k}+\mathbf{p})][\mathbf{b}\cdot(\mathbf{k}+\mathbf{p})]}{m^2+kp}\right\}. \tag{12}$$

In the c.m. frame it takes the form obtained by Czachor [6].

The unexpected behavior of the correlation functions (11) and (12) can be observed in the cases when observers are not in the c.m. frame of the pair of EPR particles. As an example, let us consider the situation when in Alice's and Bob's inertial frame

$$k^{\mu} = m(\sqrt{4x+1},\sqrt{x},0,-\sqrt{3x}), \tag{13a}$$

$$p^{\mu} = m(\sqrt{4x+1},-\sqrt{x},0,-\sqrt{3x}), \tag{13b}$$

where

$$x = \frac{\mathsf{W}^2}{4m^2} - 1. \tag{14}$$

Here W denotes invariant total energy of the two-particle system in the c.m. frame. In this case we can find such configurations in which both correlation functions (11) and (12) posses local extrema—see Fig. 1.

This behavior of the correlation function has interesting

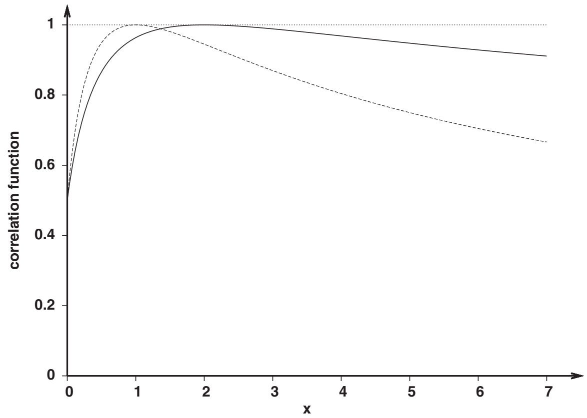

FIG. 1. The plot shows dependence of correlation functions $\mathcal{C}^{\varphi(k,p)}(\mathbf{a},\mathbf{b})$ (solid line) and $\mathcal{C}_{\mathrm{Cz}}^{\varphi(k,p)}(\mathbf{a},\mathbf{b})$ (dashed line) on x for k and p given in Eqs. (13) and $\mathbf{a}=(0,0,1)$, $\mathbf{b}=(\frac{\sqrt{3}}{2},0,-\frac{1}{2})$.

physical consequences. The violation of the Clauer-Horne-Shimony-Holt (CHSH) inequality [11]

$$(\mathrm{CHSH}) = |\mathcal{C}(\mathbf{a},\mathbf{b}) - \mathcal{C}(\mathbf{a},\mathbf{d}) + \mathcal{C}(\mathbf{c},\mathbf{b}) + \mathcal{C}(\mathbf{c},\mathbf{d})| \leqslant 2 \qquad (15)$$

depends on the particle momenta and on the chosen spin operator. There are configurations in which the quantity CHSH possesses local maximum and exceeds 2 for both correlation functions $\mathcal{C}^{\varphi(k,p)}(\mathbf{a},\mathbf{b})$ and $\mathcal{C}_{\mathrm{Cz}}^{\varphi(k,p)}(\mathbf{a},\mathbf{b})$—see Fig. 2. There are also such configurations in which the function $\mathcal{C}^{\varphi(k,p)}(\mathbf{a},\mathbf{b})$ violates the CHSH inequality while the function $\mathcal{C}_{\mathrm{Cz}}^{\varphi(k,p)}(\mathbf{a},\mathbf{b})$ does not (Fig. 3) and vice versa. Notice that $x=0$ corresponds to the nonrelativistic case. Therefore, in all configurations depicted in Figs. 2 and 3 the CHSH inequality is not violated in the nonrelativistic case.

The situation is more surprising for spin-1 particles. In this case we can observe similar phenomena even in the c.m. frame. To see this let us consider the scalar state of two spin-1 particles [12]

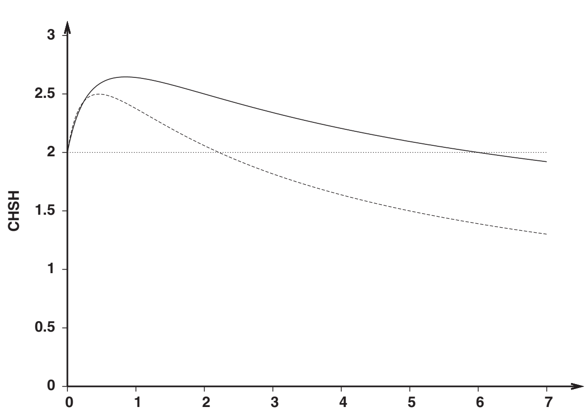

FIG. 2. The plot shows dependence of the left hand side of the CHSH inequality for $\mathcal{C}^{\varphi(k,p)}(\mathbf{a},\mathbf{b})$ (solid line) and for $\mathcal{C}_{\mathrm{Cz}}^{\varphi(k,p)}(\mathbf{a},\mathbf{b})$ (dashed line) on x for k and p given in Eq. (13), $\mathbf{a}=(0,0,1)$, $\mathbf{b}=(0,0,1)$, $\mathbf{c}=(\frac{\sqrt{3}}{2},0,\frac{1}{2})$, $\mathbf{d}=(\frac{\sqrt{3}}{2},0,\frac{1}{2})$.

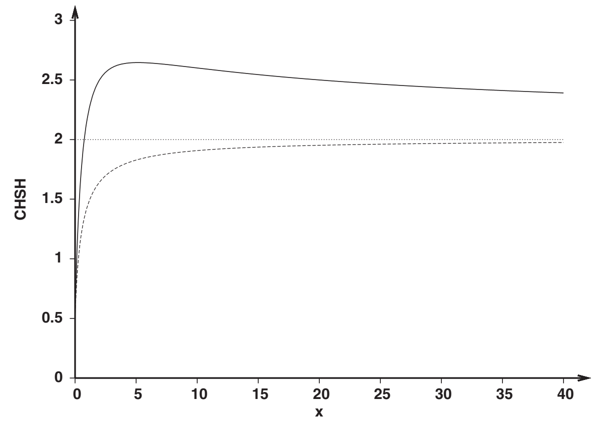

FIG. 3. The plot shows dependence of the left-hand side of the CHSH inequality for $\mathcal{C}^{\varphi(k,p)}(\mathbf{a},\mathbf{b})$ (solid line) and for $\mathcal{C}_{\mathrm{Cz}}^{\varphi(k,p)}(\mathbf{a},\mathbf{b})$ (dashed line) on x for k and p given in Eqs. (13), $\mathbf{a}=(0,0,1)$, $\mathbf{b}=(0,0,1)$, $\mathbf{c}=(\frac{\sqrt{3}}{2},0,-\frac{1}{2})$, $\mathbf{d}=(\frac{\sqrt{3}}{2},0,\frac{1}{2})$.

$$|\psi(k,p)\rangle = \sum_{\sigma,\lambda=0,\pm 1} e^{\mu}{}_{\sigma}(k) e_{\mu\lambda}(p) |k,\sigma\rangle \otimes |p,\lambda\rangle, \qquad (16)$$

where the explicit form of the amplitudes $e^{\mu}{}_{\sigma}(k)$ can be found in Ref. [12].

The correlation function $\mathcal{C}^{\Psi}(\mathbf{a},\mathbf{b})$ [Eq. (7)] in the state (16) was calculated in Ref. [12]. In the c.m. frame this correlation function takes the form

$$\mathcal{C}^{\psi(k,k^{\pi})}(\mathbf{a},\mathbf{b}) = \frac{2}{2+(1+2x)^2}[-(1+2x)(\mathbf{a}\cdot\mathbf{b}) + 2x(\mathbf{a}\cdot\mathbf{n})(\mathbf{b}\cdot\mathbf{n})], \qquad (17)$$

where $\mathbf{n}=\frac{\mathbf{k}}{|\mathbf{k}|}$ and x is defined in Eq. (14). In the c.m. frame x is connected to the velocity of the particle via the relation $(v/c)^2=x/(x+1)$. Therefore the correlation function (17) depends only on the velocity of the particles, not on its mass.

As one can check, the correlation function $\mathcal{C}_{\mathrm{Cz}}^{\Psi}(\mathbf{a},\mathbf{b})$ [Eq. (8)] in the state (16) has the following form:

$$\mathcal{C}_{\mathrm{Cz}}^{\psi(k,p)}(\mathbf{a},\mathbf{b}) = 2\frac{-\mathbf{a}\cdot\mathbf{b}(kp) - (\mathbf{a}\cdot\mathbf{p})(\mathbf{b}\cdot\mathbf{k})}{\left(2+\frac{(kp)^2}{m^4}\right)\sqrt{m^2+(\mathbf{a}\cdot\mathbf{k})^2}\sqrt{m^2+(\mathbf{b}\cdot\mathbf{p})^2}}. \qquad (18)$$

In the c.m. frame it reduces to

$$\mathcal{C}_{\mathrm{Cz}}^{\psi(k,k^{\pi})}(\mathbf{a},\mathbf{b}) = 2\frac{-\mathbf{a}\cdot\mathbf{b}(1+2x) + x(\mathbf{a}\cdot\mathbf{n})(\mathbf{b}\cdot\mathbf{n})}{[2+(1+2x)^2]\sqrt{1+(\mathbf{a}\cdot\mathbf{n})^2x}\sqrt{1+(\mathbf{b}\cdot\mathbf{n})^2x}}. \qquad (19)$$

In Fig. 4 we have plotted the functions (17) and (19) for the fixed configuration. We again observe local maxima for both functions.

In this case physical consequences are stronger than for $s=1/2$ particles. According to the Mermin's paper [13], in the EPR-type experiments with the pair of spin 1 particles in the singlet state the following inequality has to be satisfied:

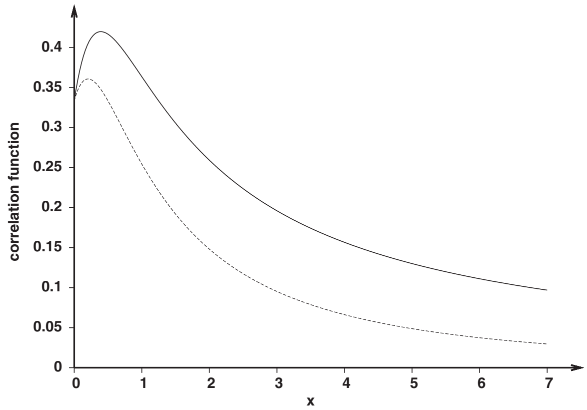

FIG. 4. The plot shows dependence of correlation functions $\mathcal{C}^{\psi(k,k^{\pi})}(\mathbf{a},\mathbf{b})$ (solid line) and $\mathcal{C}_{\mathrm{Cz}}^{\psi(k,k^{\pi})}(\mathbf{a},\mathbf{b})$ (dashed line) in the c.m. frame on x for $\mathbf{a}\cdot\mathbf{b}=-1/2$, $\mathbf{a}\cdot\mathbf{n}=\mathbf{b}\cdot\mathbf{n}=1/2$.

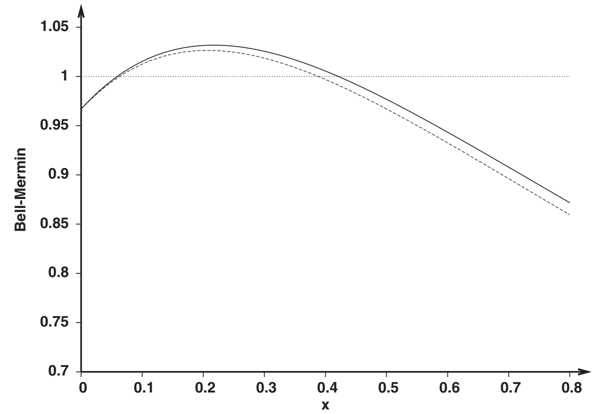

FIG. 5. The plot shows dependence of the left-hand side of the Bell-Mermin inequality in the c.m. frame for $\mathcal{C}^{\psi(k,k^{\pi})}(\mathbf{a},\mathbf{b})$ (solid line) and $\mathcal{C}_{\mathrm{Cz}}^{\psi(k,k^{\pi})}(\mathbf{a},\mathbf{b})$ (dashed line) on x for $\mathbf{a}=(0.995004,0,0.0998334)$, $\mathbf{b}=(-0.40899,0.907061,0.0998334)$, $\mathbf{c}=(-0.581043,-0.807727,0.0998334)$, and $\mathbf{n}=(0,0,1)$.

$$(\text{Bell-Mermin}) = C_{\mathbf{a}\cdot\mathbf{b}} + C_{\mathbf{b}\cdot\mathbf{c}} + C_{\mathbf{c}\cdot\mathbf{a}} \leqslant 1, \tag{20}$$

in the theory which fulfills the assumptions of local realism. One can show [13] that in the nonrelativistic case this inequality is satisfied for each configuration. However, both relativistic correlation functions (17) and (19) can violate the inequality (20). We have depicted such a situation in Fig. 5.

It should be stressed that previous works suggest that for fixed measurements directions the degree of violation of Bell-type inequalities by a pair of spin-1/2 particles monotonically decreases with increasing velocity of the particles [1,2]. Our results show that such a statement is false, in general, also for spin-1 particles. We have shown that, at least for certain states, there exist configurations in which the correlation functions and the degree of violation of Bell-type inequalities have local extrema for some values of the velocities of the EPR particles. Moreover, this strange behavior can be observed for both discussed spin operators and for spin-1/2 as well as spin-1 particles. The most surprising fact is that EPR experiment in a fixed configuration can distinguish the values of the velocity of the particles corresponding to local extrema. This observation is supported by the recent results obtained in Ref. [14], where the helicity and linear polarization correlations of spin-1 particles were analyzed.

We have shown also that relativistic quantum correlations are stronger than nonrelativistic ones for a variety of configurations. Consequently, in such configurations Bell inequalities are violated stronger by relativistic correlations than by nonrelativistic ones.

Let us notice also that in some configurations the correlation function and the degree of violation of CHSH inequality strongly depend on the relativistic spin operator used in calculations (compare, e.g., Figs. 2 and 3). This observation could help us to determine experimentally which of the discussed spin operators is a proper one. In the recent experiments with protons [15] the particles were too slow to distinguish different spin operators. The main result of our paper is the observation that the discussed strange behavior of the correlation functions seems to be a general property of the relativistic quantum mechanics independent of the chosen relativistic spin operator.

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[1] D. Ahn, H. J. Lee, Y. H. Moon, and S. W. Hwang, Phys. Rev. A **67**, 012103 (2003).
[2] H. Terashima and M. Ueda, Int. J. Quantum Inf. **1**, 93 (2003).
[3] T. D. Newton and E. P. Wigner, Rev. Mod. Phys. **21**, 400 (1949).
[4] H. Bacry, *Localizability and Space in Quantum Physics*, Vol. 308 of *Lecture Notes in Physics* (Springer-Verlag, Berlin, 1988).
[5] N. N. Bogolubov, A. A. Logunov, and I. T. Todorov, *Introduction to Axiomatic Quantum Field Theory* (Benjamin, Reading, MA, 1975).
[6] M. Czachor, Phys. Rev. A **55**, 72 (1997).
[7] D. Lee and E. Chang-Young, New J. Phys. **6**, 67 (2004).
[8] S. Moradi, Phys. Rev. A **77**, 024101 (2008).
[9] P. Caban and J. Rembieliński, Phys. Rev. A **72**, 012103 (2005).
[10] P. Caban and J. Rembieliński, Phys. Rev. A **74**, 042103 (2006).
[11] L. E. Ballentine, *Quantum Mechanics: A Modern Development* (World Scientific, Singapore, 1998).
[12] P. Caban, J. Rembieliński, and M. Włodarczyk, Phys. Rev. A **77**, 012103 (2008).
[13] N. D. Mermin, Phys. Rev. D **22**, 356 (1980).
[14] P. Caban, Phys. Rev. A **77**, 062101 (2008).
[15] H. Sakai *et al.*, Phys. Rev. Lett. **97**, 150405 (2006).