Cavity-enhanced detection of magnetic order in lattice spin models

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We develop a general scheme for detecting spin correlations inside a two-component lattice gas of bosonic atoms, stimulated by the recent theoretical and experimental advances on analogous systems for a single component quantum gas. Within a linearized theory for the transmission spectra of the cavity mode field, different magnetic phases of a two-component (spin 1/2) lattice bosons become clearly distinguishable. In the Mott-insulating (MI) state with unit filling for the two-component lattice bosons, three different phases: antiferromagnetic, ferromagnetic, and the *XY* phases are found to be associated with drastically different cavity photon numbers. Our suggested study can be straightforwardly implemented with current cold atom experiments.

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I. INTRODUCTION

Atomic quantum gases trapped in optical standing waves have become ideal systems for implementing lattice spin models after the pioneering theoretical proposal [1] and the experimental observation [2] of the superfluid (SF) to Mott insulator (MI) transition in the Bose-Hubbard model. When atoms of two-species or two-components are loaded into an optical lattice, a variety of more general effective spin models can be constructed [3–5], including the well-known anisotropic Heisenberg *XXZ* model. The development of noise spectroscopy [6–11] has provided an astounding breakthrough that overcomes several significant hurdles in detecting quantum correlations, or in measuring the second order spin moments for various magnetic phases of lattice models.

Cold atoms are usually probed with time of flight methods, which measures the atomic density or matter-wave interference patterns upon being released from traps and often after significant expansions. The near resonant imaging light generally destroys the atomic state. Several quantum limited detection schemes have since been suggested, capable of quantum nondemolition detections of strongly correlated states in atomic lattice models [12,13]. A very interesting approach relies on the enhanced detection sensitivity provided by an optical cavity, as was first proposed by Mekhov *et al.* [14,15]. The transmission spectra, calculated to first order, or within the linear response theory of the amplitude for the probe field, assumes the initial state of atoms to remain unchanged when expectation values are taken and carries unambiguous signatures of magnetic orders in an atomic Bose-Hubbard model.

Several experimental groups have recently succeeded in the difficult first step of coupling atomic condensates into high-Q optical cavities [16,17], highlighting the prospects for creating and detecting exotic quantum phases of lattice spin models [18]. A promising new direction worthy of theoretical investigation concerns the study of atomic lattice spin models coupled with optical cavities, generalizing the single component study [14,15]. Nonlocal quantum spin correlations of the various magnetic orders could analogously be reflected through the photon numbers and statistics.

This paper describes a scheme for detecting spin correlations in a two-species or two-component bosonic atom lattice [3,4,19,20]. Our study shows that atomic spin correlations are faithfully mapped onto the transmission spectra of the cavity probe field, making them easily diagnosed through cavity QED based techniques.

II. MODEL

Our model is based on the scattering of two Raman matched incident laser beams from a lattice of effective spin 1/2 bosonic atoms [21,22]. Similar to the original model [15] for single component bosons, we consider N atoms with two internal states identically trapped in an optical lattice with M sites formed by far off-resonant standing-wave laser beams. As schematically illustrated in Fig. 1, $K<M$ lattice sites are located within the overlapped region of the two fundamental modes of the cavities. We consider two nonde-

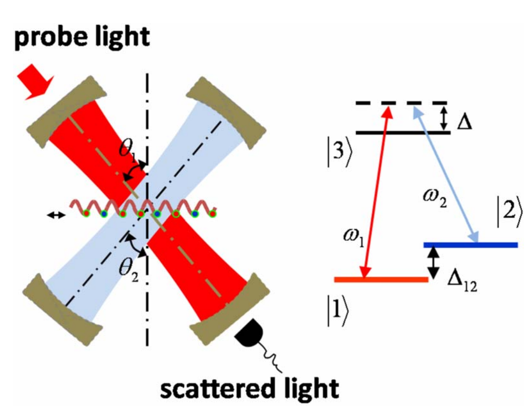

FIG. 1. (Color online) Schematic illustration of the proposed experimental setup and the level diagram for a bosonic atom with two states resonantly coupled to the two cavities.

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generate hyperfine states $|1\rangle$ and $|2\rangle$, the two stable ground states that are coupled to a common excited state $|3\rangle$ with a blue common detuning Δ and no differential detuning, forming a Raman coupled Λ-type atom model. The resonant cavity modes are denoted by matching labels with frequencies ω_l ($l=1,2$). For large detuning Δ, we adiabatically eliminate the excited state $|3\rangle$ [24,25] and end up with two-state atoms effectively coupled in the overlapped region of two optical cavities. For a single atom, the effective coupling is described by $\Omega a_1^\dagger a_2 b_1^\dagger b_2$+H.c. and the ac Stark shift becomes $\delta_l a_l^\dagger a_l b_{\sigma=l}^\dagger b_{\sigma=l}$ with $\delta_l = g_l^2/\Delta$ and $\Omega = g_1 g_2/\Delta$. The peak value for the dipole coupling with their respective cavity mode is denoted by g_σ for transition between $|\sigma\rangle \leftrightarrow |3\rangle$. $b_{\sigma=1,2}$ ($a_{l=1,2}$) denotes the corresponding annihilation operator for the atom (cavity mode photon).

Following the notations of Ref. [15], the Hamiltonian for effective spin 1/2 bosons in a lattice coupled to two optical cavities takes the form $H_B + H_I$, with

$$H_I = \sum_{l=1,2} \hbar\omega_l a_l^\dagger a_l - i\hbar\eta(a_1 e^{i\omega_{1p}t} - \text{H.c.}) + \hbar\delta_1 \sum_{i=1}^{K} |u_1|^2 n_{i1} a_1^\dagger a_1 + \hbar\delta_2 \sum_{i=1}^{K} |u_2|^2 n_{i2} a_2^\dagger a_2 + \hbar\Omega \sum_{i=1}^{K} (A_i a_1^\dagger a_2 b_{i1}^\dagger b_{i2} + \text{H.c.}), \quad (1)$$

where $n_{i\sigma} = b_{i\sigma}^\dagger b_{i\sigma}$ gives the number of atoms in state $|\sigma\rangle$ at site i and $u_{1,2}(\mathbf{r})$ is the mode function of the cavity with wave vector $\mathbf{k}_{1,2}$. The coefficients $A_i(\theta_1,\theta_2) = u_1^*(\mathbf{r}_i) u_2(\mathbf{r}_i)$ due to emission-absorption or absorption-emission cycle's are responsible for the geometric dependence of the effective coupling [15].

With atoms assumed to occupy only the lowest Bloch band, our model generalizes the familiar Bose-Hubbard for two components: H_B as in Eq. (1) of Ref. [19] for two species. Following the work of Ref. [15], we perform a linear calculation to the first order in cavity probe field, thus we leave out the dynamics of how various quantum phases of the atomic lattice are realized or dynamically created through the tuning of lattice parameters. This further justifies the neglect of atomic tunneling as well as the on-site intracomponent and intercomponent interactions. In addition to the coupling of each atomic component with its corresponding cavity mode, Raman matched two-photon processes can transfer atoms between the two effective spin states, unless the atoms are prepared in the so-called dark state $|\text{dark}\rangle \sim \langle a_2\rangle g_2 |1\rangle - \langle a_1\rangle g_1 |2\rangle$ corresponding to coherent population trapping (CPT) [23]. We also assumed large detuning between cavity and atoms, to keep the actual excitations low, or negligible; thus any Raman type population transfers only affect the initial state to higher orders than the linear response theory calculation we provide. The second term in Eq. (1) describes the coherent pumping of cavity 1 at frequency ω_{1p} with amplitude η.

III. SEMICLASSICAL THEORY

We first consider the simplest case with no external pumping on cavity 1, i.e., $\eta=0$, and assume cavity mode a_2 to be a classical field, or a c-number amplitude as in Ref. [15]. In the frame rotating with frequency ω_2, a_1 evolves in time according to the Heisenberg equation

$$\dot{a}_1 = -i\left(\Delta_{12} + \delta_1 \sum_i^K |u_1|^2 n_{i1}\right) a_1 - i\Omega\hat{D} a_2 - \kappa a_1, \quad (2)$$

where $\Delta_{12} = \omega_1 - \omega_2$ and κ denotes the cavity decay rate and is put in by hand. Its corresponding Langevin noise is neglected. We have defined the analogous operator $\hat{D} = \sum_{i=1}^K A_i S_i^-$, in terms of the effective lattice spin operators $S_i^- = b_{i1}^\dagger b_{i2}$ and $S_i^+ = (S_i^-)^\dagger$, which obey the standard commutation relation at the same site and commute with each other on different sites. Neglecting the presumably much smaller cavity field induced ac Stark shift in comparison to Δ_{12} or κ [14], a_1 and the photon number is easily obtained as

$$a_1 = C\hat{D}, \quad a_1^\dagger a_1 = |C|^2 \hat{D}^\dagger \hat{D}, \quad (3)$$

where $C = -i\Omega a_2/(i\Delta_{12} + \kappa)$. The photon number $\langle a_1^\dagger a_1\rangle$, clearly provides information about the spin correlation in the two-component bose lattice through the moments associated with the same site $\langle S_i^+ S_i^-\rangle$ and between the different sites $\langle S_i^+ S_j^-\rangle$. The angular dependence can become totally different due to the geometric coefficients $A_i(\theta_1,\theta_2)$. Within the linear response, the above averages are expectation values with respect to whatever initially prescribed atomic ground state.

The quantum phases for a two-component lattice bosons at commensurate fillings have attracted significant attention [19,20]. The phase diagram consists of (1) 2MI where both boson components are in the MI phase, (2) SF+MI where one is SF and the other is MI, and (3) 2SF where both components are SF. Deep inside the MI phase the ground state of the system may be characterized by filling the lattice site with even or odd numbers of atoms [20]. In addition to the usual even filling phase with $n_1 = n_2$, a particularly interesting phase arises when the total filling factor is odd, especially at unit filling, i.e., for $n_1 + n_2 = 1$. This exotic phase has been extensively studied [3,4,19,20] by adopting a trial wave function $|\Psi_{MI}\rangle = \Pi_{i\in A, j\in B} |\psi_A\rangle_i |\psi_B\rangle_j$, which is of a form composed of two sublattices A and B with $|\psi_{A,B}\rangle = \cos(\theta_{A,B}/2)|1,0\rangle + e^{i\phi_{A,B}} \sin(\theta_{A,B}/2)|0,1\rangle$. $|n_1,n_2\rangle_i$ denotes the state with n_1 (n_2) number of component-1 (-2) atoms at site i and θs and ϕs are variational parameters. Three types of spin exchange interactions are identified: (I) antiferromagnetic (AFM) phase with $\theta_A = 0(\pi)$ and $\theta_B = \pi(0)$; (II), ferromagnetic (FM) phase with $\theta_A = \theta_B = 0$; and (III) XY phase with $\theta_A = \theta_B \neq 0$. The 2SF phase, whose quantum state is $\Psi_{\text{SF}} \sim (\Sigma_i b_{i1}^\dagger)^{N_1} (\Sigma_j b_{j2}^\dagger)^{N_2} |0\rangle$ with $N_{1,2}$ the total number of component-1 (-2) atoms [26], will serve as a reference for presenting our results.

The scattered photons are explicitly tabulated in Table I. For a 1D optical lattice of a spatial period $d = \lambda/2$ and with atoms trapped at sites centered at $x_j = jd$, the mode functions are $u_{1,2}(\mathbf{r}_j) = \exp(ij|\mathbf{k}_{1,2}|d \sin\theta_{1,2})$ for a traveling wave and/or $u_{1,2}(\mathbf{r}_j) = \cos(ij|\mathbf{k}_{1,2}|d \sin\theta_{1,2})$ for a standing wave form. Atoms in the FM phase do not scatter because the two coupling paths to the excited state $|3\rangle$ destructively cancels as in the dark state. For the notation we use, the FM state corresponds to all atoms staying in state $|1\rangle$, then a semiclassical light

TABLE I. Cavity 1 photon number for the four quantum phases of the two-component Bose-Hubbard model at the diffraction maxima (minima) with $\theta_1=0$ ($\theta_1=\pi/2$) and $\theta_2=0$. For the XY phase $\theta_A=\theta_B=\pi/3$.

	$\langle a_1^\dagger a_1\rangle_{\theta_1=0}$	$\langle a_1^\dagger a_1\rangle_{\theta_1=\pi/2}$
AF	$K\|C\|^2/2$	$K\|C\|^2/2$
FM	0	0
XY	$(K+3K^2)\|C\|^2/16$	$K\|C\|^2/16$
SF	$n_2(n_1K+1)K\|C\|^2$	$n_2K\|C\|^2$

amplitude $\langle a_2\rangle\, g_2$ clearly will not be able to cause any scattering. While the initial atomic states of the AF and XY phases under the single excitation of a semiclassical light are not any more dark states, they will scatter. These features thus completely characterize the many-body spin correlations of the quantum phases for the two-component Bose-Hubbard model.

To map quantum fluctuations of lattice spins faithfully onto the probe cavity photon statistics, we define a noise function $R(\theta_1,\theta_2)=\langle\hat{D}^\dagger\hat{D}\rangle-\langle\hat{D}^\dagger\rangle\langle\hat{D}\rangle$, whose angular distribution is compared in Fig. 2 for all four quantum phases. The structure in the angular distribution comes from the summation of the geometric coefficients from different sites, reflecting both the on-site and off-site lattice spin correlations. In the SF phase with $n_1=n_2=1/2$, the respective noise functions are completely different for the two choices of cavity modes. For the traveling wave, the noise function is zero for the FM phase, but takes nonzero values and is isotropic for the XY and the AF phases. The angular dependence for the standing wave mode case is richer than that for the traveling waves. The structures in the angle dependence can be attributed to dependence on the summation of the geometric coefficients, and physically due to both on-site and off-site lattice spin correlations.

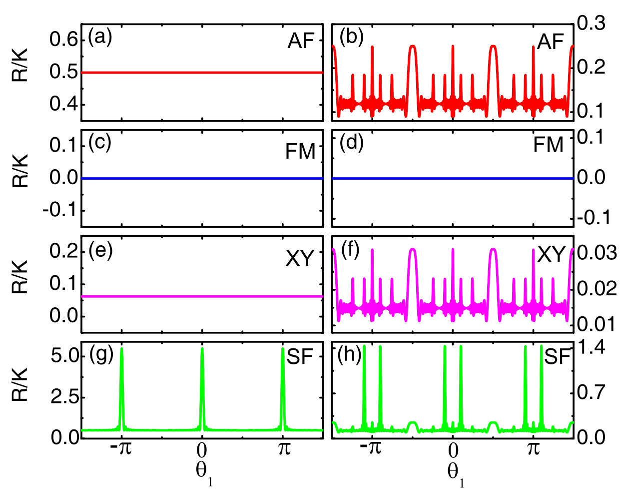

FIG. 2. (Color online) The angular distribution of $R(\theta_1,\theta_2)$ for the four quantum phases evaluated for different choices of cavity mode functions: the left (right) panels are for two traveling (standing) waves and for $\theta_2=0$ ($\theta_2=0.1\pi$). We have assumed $N=M=2K=40$ and in the SF phase $n_1=n_2=1/2$. For the XY phase $\theta_A=\theta_B=\pi/3$.

IV. QUANTIZED MODEL

We next consider the more general case with coherent pumping for cavity 1 at frequency ω_{1p} [15]. The dissipations for both cavities are assumed the same with the associated Langevin noise terms neglected in the Heisenberg operator equations. Within a linearized calculation, we decorrelate the atomic and field operators and replace in the Heisenberg equations for $a_{1,2}$ the atomic operators by their respective expectation values, which leads to $\langle a_l^\dagger\rangle\langle a_l\rangle=|\langle a_l\rangle|^2$. To simplify our result, we further assume $|u_{1,2}(\mathbf{r}_i)|^2=1$, which occurs for the diffraction maxima with $A_i=1$ at $\theta_1=0$ or the minima with $A_i=(-1)^i$ at $\theta_1=\pi/2$ when the 1D lattice is lined up at $\theta_2=0$. The cavity photons are found to be

$$\langle a_1^\dagger\rangle\langle a_1\rangle=\eta^2(\kappa^2+\zeta_2^2)/B,\quad \langle a_2^\dagger\rangle\langle a_2\rangle=\eta^2\alpha^*\alpha/B, \qquad (4)$$

where $B=\kappa^4+\kappa^2(\zeta_1^2+\zeta_2^2+2\alpha^*\alpha)+(\zeta_1\zeta_2-\alpha^*\alpha)^2$, $\alpha=\Omega\Sigma_i^K A_i\langle S_i^-\rangle$, and $\zeta_l=\Delta_{lp}+\delta_l\Sigma_i^K\langle n_{il}\rangle$. The detuning $\Delta_{lp}=\omega_l-\omega_{1p}$ are assumed the same for $l=1,2$ because $\Delta_{12}\ll\omega_{1,2}$. If the cavity coupling is assumed identical, we end up with $\delta_{1,2}=\Omega=\delta$. Equation (4) shows that probe photon numbers depend on the average values of on-site atom numbers $\langle n_{i\sigma}\rangle$ and the lattice spin operators $\langle S_i^\pm\rangle$. A crucial term for spin correlation $\alpha^*\alpha$ appears in the expression for $\langle a_2^\dagger\rangle\langle a_2\rangle$. At the diffraction minima or maxima $\alpha=0$ so that no photon will be detected from cavity 2 except for the XY phase. This then allows for simplified expressions of the scattered photon numbers $\langle a_1^\dagger\rangle\langle a_1\rangle$ from the AF and FM phases into $\langle a_1^\dagger\rangle\langle a_1\rangle=\eta^2/(\kappa^2+\zeta_1^2)$, which only depends on the detuning Δ_{1p} and atom numbers for component-1 in the overlapped K sites $N_1^K=\Sigma_i^K\langle n_{i1}\rangle$. When $\alpha\neq0$, however, $\langle a_1^\dagger\rangle\langle a_1\rangle$ for the XY phase at the diffraction maxima depends on two parameters ζ_1 and ζ_2 including the detunings Δ_{1p}, Δ_{2p}, and the number of atoms for both components in the overlapped region of K sites. Measuring photon numbers $\langle a_1^\dagger\rangle\langle a_1\rangle$ thus gives sufficient information to distinguish magnetic orders or quantum phases of the two-component Bose-Hubbard model.

An especially interesting property concerns the dependence of the probe photon numbers on the detuning Δ_{1p}, as is illustrated in Fig. 3 for the four quantum phases. For the FM and AF phases, we find Lorentzians with width κ and shifted by δN_1^K as in the classical result of a single component Bose-Hubbard model [15]. In contrast, for the SF phase the photon number distribution is an envelope of a comb for a good cavity ($\kappa=0.1\delta$) while a smooth broadened contour for a bad cavity ($\kappa=\delta$). In the SF case, individual atoms are completely delocalized over all sites causing significant number fluctuations over each site within the K-site region. The corresponding quantum state is a superposition of Fock states containing all possible distributions of N_1^K atoms for component-1 at K sites, which gives rise to scattering terms from all possible atomic distributions. For the XY phase, the double peaked feature provides evidence for different population of atoms in the two internal states, with the relative heights of the two peaks being controlled by the variational parameters $\theta_{A,B}$. This structure in the easy plane XY phase is essentially identified with the so-called superfluid counterflow (SCF) phase, which can be qualitatively understood as a paired superfluid vacuum (PSF) phase, a strongly correlated

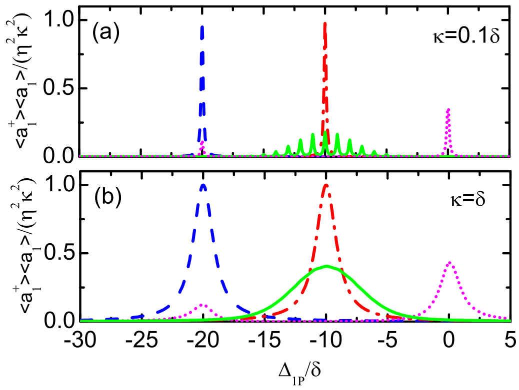

FIG. 3. (Color online) Cavity 1 photon numbers as a function of cavity-probe detuning for the four quantum phases: AF (red dashed dot), FM (blue dashed), *XY* (pink dotted), and SF (green solid). In our simulation we use $K=20$ for all phases and in the SF phase $n_1=n_2=1/2$. For the *XY* phase $\theta_A=\theta_B=0.6\pi$.

superfluid ground state already predicted from numerical simulations [4]. These distinct features of the transmission spectrum we discuss for the various quantum phases form the basis for easily detecting and differentiating the corresponding magnetic orders in the two-component Bose-Hubbard model.

As with the original cavity scheme of Mekhov *et al.* [14,15], the scheme we propose, is constructed to detect high order moments. The different phases (in the sense of quantum states of matter) of a two-component lattice bose gas are resolved from the statistics of scattered photons or pseudospins. In this sense, it is analogous to the so-called noise spectroscopy of quantum gases [6,28], albeit somewhat superior due to the enhanced collection efficiency aided by cavities. The Ramsey spectroscopy [27], as proposed by Kuklov, measures the first order moments of atomic pseudospins. The SCF state or the paired condensation phase is a special case, where the order parameters are simply field operators themselves. Thus their presence can be probed by the Ramsey spectroscopy measurement of the relative phase (in the sense of amplitude and phase).

V. CONCLUSIONS

In summary, we have generalized the model of a single component atomic lattice gas described by the Bose-Hubbard model coupled to near resonant optical cavities to the case of a two-component Bose-Hubbard model. We have shown conclusively through the probe cavity photon numbers and its spectra dependence on various system parameters that different quantum phases of the two-component Bose-Hubbard model can be easily distinguished and confirmed. Our results shine new light on atomic lattice gases coupled to cavity QED systems.

Note added in proof: Recently a similar scheme has been applied in the detection of the excitation spectrum of ultracold atoms in optical lattices and the universaility class of quantum phase transitions [29].

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