Few qubit atom-light interfaces with collective encoding

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Samples with a few hundred atoms within a few μm sized regions of space are large enough to provide efficient cooperative absorption and emission of light, and small enough to ensure strong dipole-dipole interactions when atoms are excited into high-lying Rydberg states. Based on a recently proposed collective encoding scheme, we propose to build few-qubit quantum registers in such samples. The registers can receive and emit quantum information in the form of single photons, and they can employ entanglement pumping protocols to perform ideally in networks for scalable quantum computing and long distance quantum communication.

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I. INTRODUCTION

Physicists are currently exploring the potential of quantum-information processing with particular efforts devoted to the construction of practical devices for quantum communication and quantum computing. The most challenging task for quantum communication is to reach long distances, and for quantum computing it is to achieve scalability toward a significant number of controllable quantum bits. A combination of stationary qubits in atoms and flying qubits in photons, transmitted along waveguides, through optical fibers or free space, is expected to be an important ingredient in the achievement of the ambitious long term goals for quantum information, and interfacing of stationary and flying qubits constitutes a very active field of research.

Following the proposal for quantum repeaters based on optically dense media [1], significant progress has been made [2,3] on the entanglement of collective qubit degrees of freedom in physically separated atomic samples. These experiments involve large atomic gases, and while errors and imperfections may be detected and "repeat-until-successful" strategies may be employed to secure the formation of entangled states [1,4], effective restoration of quantum information by error correction procedures is difficult to achieve in these systems. The latter problem would be solved by interfacing light with a small quantum processor, capable of performing one- and two-bit gates among its qubits. Interfacing light with a single atom or ion can be done with probabilistic protocols [5–8], and hence one can entangle remote quantum processors in a conditional manner [9]. Although progress has been made recently on the free space coupling of a single atom to a focused single photon field [10,11], light couples with a larger degree of directional selectivity to an atomic ensemble, and we shall show in the following that a deterministic scheme is within reach for the coupling of photonic qubits to a small atomic ensemble of a few hundred atoms.

In contrast to the macroscopic samples [2,3], we suggest to confine the atoms within a 10-μm-wide volume so that the Rydberg blockade mechanism [12] can be used for quantum gate operations on collectively encoded qubits [13,14] in different internal states in the atomic ensembles. The Rydberg blockade mechanism is due to the strong long range interaction between pairs of Rydberg excited atoms which causes a single excited atom to significantly shift the Rydberg energy level of its neighbors out of resonance and hence block the excitation of those atoms. The Rydberg blockade is well documented: A dramatic suppression of Rydberg state excitation in large atomic clouds has been observed and interpreted as evidence for the Rydberg blockade between atoms and their immediate neighbors [15–19]. In a smaller sample of atoms, the weaker interaction between low-lying Rydberg excited atoms has led to decoherence of Rabi oscillations of the sample [20], and recent experiments [21,22] have demonstrated an almost perfect blockade between a single pair of optically trapped atoms at 10 μm separation.

The paper is organized as follows. In Sec. II, we derive and solve the equations for the single photon field emitted by an atomic ensemble containing initially a single optical excitation. We note that absorption of a light pulse is the time reversed process of emission, and conclude that a small atomic ensemble can absorb a single photon pulse with near-unit efficiency. In Sec. III, we describe the entanglement pumping scheme that allows the production of high fidelity entangled states by subsequent photon absorptions and local operations on few-qubit registers. We explain how the few qubits required can be identified with the collective population in different internal states of the atoms, and we describe how a universal set of quantum gates can be carried out on the qubits by means of the collective Rydberg blockade. In Sec. IV, we conclude and present a few ideas on how to deal with errors and the gradual degradation of the purity and symmetry of the atomic ensemble due to losses and small asymmetries in the atom-light coupling.

II. COLLECTIVE EMISSION OF A SINGLE PHOTON

To determine the ability of a collection of atoms to absorb a single photon, we first study the time-reversed process of cooperative spontaneous emission [23]. Following [24], it was shown in [25] that even a fairly small cloud of Rydberg blocked atoms constitutes a directional source of single photons. Our analysis is based on a solution of the full time-dependent problem of light emission, as we aim to extract precise information about the spatiotemporal field mode coupled to our system.

We assume the initial collective atomic state

$$|\Psi_0\rangle = \frac{1}{\sqrt{N}}\sum_{j=1}^{N} e^{i\mathbf{k}_0\cdot\mathbf{r}_j}|e_j\rangle \otimes |0\rangle, \tag{1}$$

where $|e_j\rangle$ is shorthand for the state with atom j excited and the other atoms in their ground state $|g\rangle$, and $|0\rangle$ denotes the field state with no photons. This state may be prepared using laser fields with wave vectors $\mathbf{k}_{1,2}$ driving a resonant two-photon excitation into a Rydberg state, so that the blockade interaction prevents transfer of more than a single atom to the Rydberg excited state. A resonant π pulse with wave vector $\mathbf{k}_3$ hereafter drives the atomic excitation into the excited state $|e\rangle$, producing the state (1) with $\mathbf{k}_0=\mathbf{k}_1+\mathbf{k}_2-\mathbf{k}_3$.

To study the time dependence of the emission of light from the excited ensemble, we shall use the approach in [26]. For simplicity, effects of photon polarization are neglected, but may readily be incorporated in a more detailed analysis. The atoms and the quantized field modes are governed by the Hamiltonian $H_0=\Sigma_{j=1}^{N}\hbar\omega_0|e_j\rangle\langle e_j|+\Sigma_{\mathbf{k}}\hbar ck a_{\mathbf{k}}^{\dagger}a_{\mathbf{k}}$, and their interaction reads $V_I=\Sigma_{j=1}^{N}\Sigma_{\mathbf{k}}\hbar g_{\mathbf{k}}a_{\mathbf{k}}^{\dagger}|g\rangle\langle e_j|e^{-i\mathbf{k}\cdot\mathbf{r}_j}e^{i(ck-\omega_0)t}$, where $g_{\mathbf{k}}$ is the atom-photon coupling constant. The initial state Ψ_0 evolves into a state on the form

$$|\Psi(t)\rangle = \sum_{j=1}^{N}\alpha_j e^{-i\omega_0 t}|e_j\rangle \otimes |0\rangle + \sum_{\mathbf{k}}\kappa_{\mathbf{k}}e^{-ickt}|g\rangle \otimes |\mathbf{k}\rangle, \tag{2}$$

where $|g\rangle$ is shorthand for the collective state with all atoms in the ground state, and $|\mathbf{k}\rangle$ is the state with a single photon with wave number $\mathbf{k}$. The amplitudes α_j and $\kappa_{\mathbf{k}}$ are time dependent in the interaction picture, and from the formal solution of the Schrödinger equation for the photon state amplitudes $\kappa_{\mathbf{k}}$, we obtain the atomic amplitude equations

$$\dot{\alpha}_j = -\sum_{j=1}^{N}\sum_{\mathbf{k}}|g_{\mathbf{k}}|^2 e^{i\mathbf{k}\cdot(\mathbf{r}_j-\mathbf{r}_{j'})}\int_0^t e^{i(ck-\omega_0)(t'-t)}\alpha_j(t')dt'.$$

Following [26], we apply the Weisskopf-Wigner approximation and we discard a multiatom "Lamb-shift" term, which is expected to be at least an order of magnitude smaller than the terms retained in our derivation. Introducing new variables $\beta_j=e^{-i\mathbf{k}_0\cdot\mathbf{r}_j}\alpha_j$ we arrive, for ct larger than the sample size, at the linear set of equations

$$\dot{\beta}_j = -\gamma_1\sum_{j'=1}^{N}F(\mathbf{r}_j-\mathbf{r}_{j'})\beta_{j'}, \tag{3}$$

where $\gamma_1=\int\frac{d\Omega_{\mathbf{n}}}{4\pi}\pi|g_{\mathbf{k}}|^2\,\rho(ck)|_{\mathbf{k}=\mathbf{n}k_0}$, and $2\gamma_1$ is the usual single atom decay rate, and where

$$F(\mathbf{r}_j-\mathbf{r}_{j'}) = \frac{\sin(k_0|\mathbf{r}_j-\mathbf{r}_{j'}|)}{k_0|\mathbf{r}_j-\mathbf{r}_{j'}|}e^{-i\mathbf{k}_0\cdot(\mathbf{r}_j-\mathbf{r}_{j'})}.$$

The fully symmetric, super-radiant state (1) initially decays with the rate

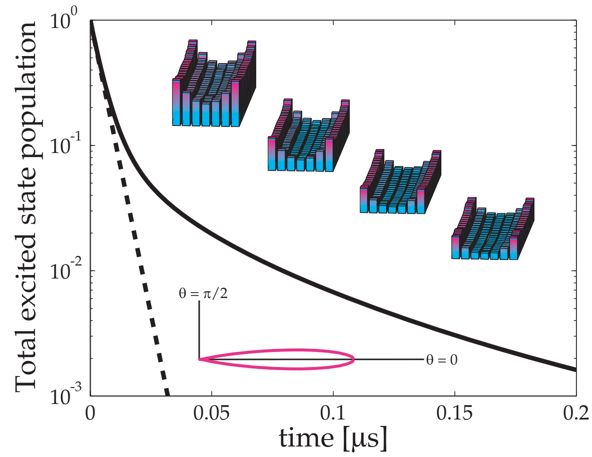

FIG. 1. (Color online) Excited state population in an ^{87}Rb sample with $7\times7\times20$ atoms (solid curve). The population departs from the exponential collective decay law (dashed line) around $t=10^{-8}$ s, where the excited state population on the individual atoms in the four top layers of the sample is shown in the upper part of the graph. The bottom inset shows the directional photon density at $t=10^{-7}$ s.

$$\gamma_{col} = \frac{\gamma_1}{N}\sum_{j=1}^{N}\sum_{j'=1}^{N}F(\mathbf{r}_j-\mathbf{r}_{j'}), \tag{4}$$

but it is not an exact eigenvector for Eq. (3), which is, however, easily solved for our system with only a few hundred atoms by diagonalization of the matrix F.

In our numerical simulations we have studied a cubic lattice with an elongated sample of $7\times7\times20$ (=980) atoms. With a lattice spacing of 0.37 μm, the maximum distance between any two atoms is 8.3 μm, short enough to achieve the Rydberg blockade. We use numbers characteristic for ^{87}Rb and the $5P_{1/2}$ excited state with a spontaneous emission rate of $2\gamma_1=37\ \mu s^{-1}$.

As demonstrated in Fig. 1, the excited state population initially decays as $\exp(-2\gamma_{col}t)$ (dashed line), where $\gamma_{col}=5.7\gamma_1$. The upper insets in Fig. 1 show the excited state population on the 7×20 atoms in each of the four upper layers of the ensemble at $t=10^{-8}$ s. At this time, the symmetry of the atomic excited state population in the sample is broken, explaining the slower decay of the remaining few percent of excitation in the system.

The Schrödinger equation for the field amplitudes $\kappa_{\mathbf{k}}$ are first order equations with the atomic amplitudes β_j as source terms,

$$i\dot{\kappa}_{\mathbf{k}} = \sum_{j=1}^{N}g_{\mathbf{k}}e^{i(\mathbf{k}_0-\mathbf{k})\cdot\mathbf{r}_j}e^{i(ck-\omega_0)t}\beta_j,$$

and the field emanating from the sample is given explicitly by the analytical evaluation of the integrals over time of the exponentially damped atomic eigenmodes of Eq. (3), weighted by the expansion coefficients of the initial atomic state on these eigenmodes. The eigenmodes and eigenvalues are known from the numerical diagonalization of F. In the super-radiant stage, light emission occurs predominantly

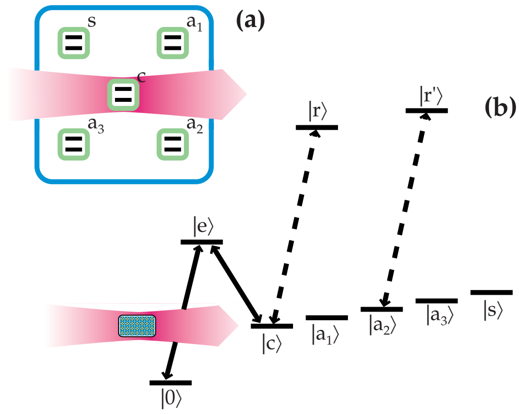

FIG. 2. (Color online) (a) A five-qubit register consisting of a communication qubit (c), a storage qubit (s), and three auxiliary qubits ($a_{1,2,3}$) [28]. (b) The collective encoding implementation, with a collective internal state transition interacting with the field mode, and long lived and Rydberg internal states used for encoding and coupling of the five qubits.

within a narrow emission cone. This is illustrated in the lower inset of Fig. 1, showing the total photon emission probability as a function of direction.

With more than 95% probability the photon is emitted in a direction within 0.3 rad off the axis of the sample. A time reversal argument ensures that a field with a spatial dependence which is the complex conjugate of the fields found above will travel in the opposite direction and become extinct by the atomic excitation. This offers the possibility to split a single photon by a beam splitter and direct it toward two separate ensembles in a deterministic protocol for entanglement generation with a fidelity of 95% or higher.

III. LIGHT-ATOM INTERFACE

A. Entanglement pumping

In the above analysis we have found that a few hundred atoms within a 10-μm-wide volume in space have a high coupling fidelity (in excess of 95%) to a single photonic qubit. Using the Rydberg blockade gates [12], it also offers efficient means for the ensuing entanglement pumping [27] via two-bit gates from the information receiving qubit toward other bits in the register, cf. Fig. 2.

The entanglement pumping protocol [28], involving auxiliary qubits, measurements, and multiple rounds of communication can raise a 90% transmission fidelity to arbitrarily high degrees of entanglement between two samples. The idea in [28] is to obtain multiple pairs of entangled qubits and successively transfer them to auxiliary bits. Local two-bit quantum operations between members of different pairs of entangled qubits followed by local measurements and a comparison of measurement results by classical communication provide a very efficient means to achieve a single near-perfect entangled pair of qubits.

Figure 2(a) illustrates the five-qubit register designs, proposed in [28]. Five separate physical systems take the role of a communication qubit, "c," three auxiliary qubits for temporary storage and entanglement pumping, "a_i, $i=1,2,3$," and a storage qubit for the perfected state, "s." A chain of trapped ions with a single ion residing in an optical cavity for communication or ^{13}C atoms in the proximity of an optically addressable nitrogen-vacancy center in diamond are proposed in [28] as candidates for these five physical qubits. We refer the reader to [28] for the algorithmic details and turn to the description of our physical implementation of local five-qubit registers needed for the entanglement pumping protocol.

B. Collective encoding of qubit registers

An ensemble of $K>N$ identical, collectively addressed particles with $(N+1)$ internal levels, $|i\rangle$, $i=0,\ldots,N$, can encode the qubits of an N-bit register [13,14]. The $|i=0\rangle$ state is our "reservoir" state, populated initially by all members of the ensemble, and we associate the computational register state $|b_1,b_2,\ldots,b_N\rangle$ $(b_i=0,1)$ with the *symmetric* state of the ensemble with b_i ensemble members populating the single atom states $|i\rangle$. The register state $|0_1,0_2,\ldots,0_N\rangle=\otimes_{j=1}^{K}|0\rangle_j$ is the starting point for our analysis.

Figure 2(b) illustrates the use of a generic single-atom level scheme for our collective encoding with a reservoir state, and five different long-lived states playing the same roles as the five physical qubits in Fig. 2(a). The figure also shows an optically excited state and two Rydberg excited states, needed for optical interfacing and one- and two-bit operations, respectively.

In a recent publication [29] we have suggested that up to 1000 bit registers may be built, using the collective encoding in holmium atoms, and using multiple individually addressable ensembles which are within the Rydberg interaction distance of each other. In the present work, our interest is in small registers with the capacity to store only five qubits. In the holmium ground state with hyperfine quantum numbers of $F=4,\ldots,11$, we have access to eight different field insensitive $M_F=0$ electronic ground states, which would make long coherence times of a five-bit register feasible, while a modified storage scheme with qubit values zero and unity encoded in the collective population of state pairs $\{|F,M_F\rangle,|F',-M_F\rangle\}$ provides an adequate number of (first order) field insensitive qubits [30] in the electronic ground state of, e.g., rubidium or cesium.

The absorption of a photon on the $|0\rangle-|e\rangle$ collective transition, sketched in Fig. 2, must be followed by an immediate transition between the excited state $|e\rangle$ and the long-lived "communication state" $|c\rangle$, so that the stationary communication qubit acquires the state of the incident photonic qubit with $\geqslant$95% fidelity. From here, Rydberg gates between the collective communication, auxiliary, and storage qubits are used to implement the algorithm proposed in [28]. Unlike Ref. [28] which assumes a probabilistic transfer, we achieve our 95% fidelity in a deterministic manner in each round of communication, and we are thus in the most favorable setting for efficient entanglement pumping and broadcasting of high fidelity entanglement over arbitrarily long distances and to large scale networks. In the collective encoding all qubits may interchangeably take the roles of communicating, auxiliary, and storage qubits, unless polarization and dipole selection rules make it advantageous to fix these roles from the beginning, e.g., to ensure that the decay of the excited state during photon emission puts the atomic excited state population into the reservoir state and not into any of the qubit encoding states.

C. Rydberg blockade gates on collective qubits

To encode and controllably manipulate an N-bit quantum register in a single mesoscopic ensemble of atoms, one uses the fact that, due to the blockade effect, coherent driving on the $|0\rangle \leftrightarrow |r\rangle$ transition drives a closed two-level transition between collective states with none and a single Rydberg excited atom in the multiatom sample. To carry out an arbitrary unitary operation on the ith qubit, the contents of the state $|i\rangle$ are transferred to the Rydberg excited state $|r\rangle$, followed by the desired unitary operation on the $|0\rangle \leftrightarrow |r\rangle$ two-state transition, and subsequent return of the $|r\rangle$ state component to the qubit state $|i\rangle$ [13]. A two-qubit phase gate involves the excitation of the control qubit internal state $|i\rangle$ into one Rydberg state $|r\rangle$, followed by a complete, 2π Rabi rotation between the target qubit state $|j\rangle$ and another Rydberg state $|r'\rangle$, and concluded by the return of the population from $|r\rangle$ to $|i\rangle$. If the control qubit is in its logical 1-state, the resulting unit occupancy of the $|r\rangle$ state blocks the Rabi cycle and nothing happens to the target qubit, while a control qubit logical value of 0 causes no blockade, and hence we obtain a controlled -1 factor on the $|j\rangle$ state amplitude due to the full Rabi cycle. Note that this two-qubit gate is very similar to the one proposed in [12], except that it does not require access to individual particles, and it makes use of two different Rydberg states in the atoms.

We note that a general quantum register state is a superposition of five-bit collective states $|b_1, b_2, \ldots, b_5\rangle$ with zero or unit collective occupancy of the atomic states c, a_1, a_2, a_3, and s, and all optical transitions occur, due to the linearity of quantum mechanics, on every component of that superposition. Since the number of atoms in the reservoir state $|0\rangle$ depends on the occupancy of all the qubit levels, it may attain values ranging from $K-5$ to K (when all qubits are equal to unity and zero, respectively), and laser coupling schemes which yield the precise $|0\rangle \leftrightarrow |r\rangle$ pulses, irrespective of such variations, must be employed [13,14].

IV. CONCLUSION

In conclusion, collective encoding of few qubits in small ensembles of atoms offers a promising approach to interfaces of flying and stationary qubits, and they hold the potential to provide scalable quantum computing and long distance quantum communication. We emphasize that the collective encoding both yields the efficient coupling to single photons and alleviates the need for addressing of individual atoms. One might worry that the loss or misplacement of a few atoms from the sample would cause a significant change in the field mode, and hence unrealistic demands on the ability to trap atoms would have to be met. We have tested this concern by removing up to tens of atoms from random locations in our lattice system. We have then computed the field mode emitted by the modified structure and determined the overlap of this field mode with the one emitted by the original sample. These overlaps are very robust and in excess of 99% in all our simulations. This also implies that the read out, needed in the entanglement pumping protocol, can be made by state selective ionization of qubit internal states, removing a single atom from the sample for each read out of a "1"-result without affecting the symmetric state of the remaining atoms. Another more serious concern is the gradual destruction of the symmetric collective state of the system, due to the nonperfect matching with the super-radiant mode. Entanglement pumping can correct some errors, but when the absorption fails we do not only have a $<5\%$ qubit error: the system may actually leave the computational subspace of symmetric states. For a sufficiently large sample, the system is robust against such errors for a limited amount of time, and methods exist to counter the errors [14]. We suggest, in addition, to frequently restore the symmetry of the sample by optically pumping the communication qubit content into the reservoir state. Another way to obtain a renewable communication qubit in the sample may be to apply a more elaborate architecture with individually addressable ensembles within the Rydberg blockade radius of each other or with a mixture of two different species, contained within the same volume, and where the Rydberg blockade may also apply between species. One species, used for communication, may then be optically pumped at any time to maintain the symmetry of the system, needed for the interaction with the optical field.

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