Fidelity of asymmetric dephasing channels

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We study the properties of the fidelity of one-qubit operations in a noisy channel and reveal its properties in dependence on coupling to the outer environment. We show that for an asymmetric qubit-environment coupling, the fidelity can be improved by a tuning of the external parameters acting on the qubit energy splitting. In particular, for the case of a spin qubit, the fidelity can be improved by an appropriate tuning of the external magnetic field. We observe that within tailored parameter regimes, the fidelity (typically being oscillatory) evolves monotonically and remains significant in the long-time regime, for both an environment prepared in vacuum and coherent states. This result holds true also for the entanglement fidelity of the two-qubit system.

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I. INTRODUCTION

Quantum information science is a rapidly developing research area which has the potential to directly impact future technologies. The main challenge consists in designing and building of quantum computers. One of the principal issues of quantum information engineering is to develop methods of controlling the quantum operations, in particular quantum computing gates, teleportation, coding, quantum state storage, transmission of information, etc. These operations are analyzed in terms of well-defined quantifiers and measures. There exist various quantities measuring the imperfection of a quantum communication [1,2]. The input-output fidelity is one of such quantifiers characterizing the quality of information transmission through quantum channels, T, which is defined as a linear, completely positive, trace-preserving map T of a quantum state represented by a density operator ρ. If the state ρ is the input state, then $T(\rho)$ is the output state and we say that the state ρ is transmitted through the channel T. If T is known then the input-output fidelity is defined as [3]

$$F(T(\rho),\rho)=\{\mathrm{Tr}[\sqrt{T(\rho)}\rho\sqrt{T(\rho)}]^{1/2}\}^2. \quad (1)$$

It measures the stability of quantum evolution, estimates how close are two states and their separation can be measured by, e.g., the Bures metric $\|\rho_1-\rho_2\|^2=2[1-\sqrt{F(\rho_1,\rho_2)}]$ [1].

For an ideal, undisturbed, transmission, we expect $F=1$. Practically, almost all channels are noisy; i.e., the system is in contact with its (infinite) environment. The influence of the environment results in decoherence, an uncontrollable and irreversible process. Noisy quantum channels have been studied in very different contexts—e.g., for systems in the presence of stochastic decoherence [4], for infinite Gaussian channels [5], or in the problem of holonomic quantum gates in semiconductor quantum dots [6]. The natural question—to what extent is such a channel able faithfully to transmit qubits—can be answered if its reduced, with respect to the environment, dynamics is known. In other words, the reduced density operator $\rho(t)$ has to be determined. Then the transmission process is evaluated by the input-output fidelity $F(\rho(t),\rho(0))$, where now the noisy channel T is defined by the evolution map $T:\rho(0)\rightarrow\rho(t)$. Because of the decoherence process, the fidelity is smaller than 1 and frequently decays to zero in the long-time regime [7]. The problem is to minimize the influence of the environment and to optimize the fidelity. However, in a general case, it is impossible to obtain an exact form of $\rho(t)$. Approximate methods, especially when are mathematically uncontrollable, can lead to unjustified conclusions. The method of quantum dynamical semigroups is particularly convenient in the widely used Markovian domain which works well in the weak-coupling regime or in the singular coupling limit [8]. It has been used for studying, e.g., entanglement dynamics [9] and fidelity of quantum teleportation through noisy channels [10]. However, the results obtained in the weak-coupling limit cannot be extrapolated to the low-temperature regime. Therefore its applicability for solid-state devices operating at extremely low temperatures is not straightforward [11]. Systems operating beyond the Markovian regime are difficult to handle and require special methods. Yet there are models which are exactly solvable. One of them is a model of pure decoherence (called dephasing). It has been exploited to study general aspects of open quantum systems [12], maintenance of coherence in quantum computers [13], entanglement dynamics [14–16], and geometric phases [17]. We formulate an extension of this model by admitting asymmetric coupling to the outer environment. We demonstrate that for such an asymmetric interaction, high-fidelity quantum operations can be improved by means of a control field that determines the qubit energy splitting. The interaction symmetry breaking gives the possibility to slow down the fidelity decay and to work out its control and optimization. Within the analyzed model, it is impossible for the standard symmetric coupling.

The paper is organized as follows. In Sec. II we define the model of the asymmetric dephasing channel and present corresponding exact reduced dynamics of a qubit. In Sec. III we calculate the fidelity of the asymmetric channel. Here, we assume that the environment is initially in vacuum or coherent states. In Sec. IV we extend our discussion to the problem of the entanglement preservation under asymmetric dephasing channel qualified by the entanglement fidelity. Section V contains summary and conclusions.

II. ASYMMETRIC DEPHASING CHANNEL

The system we study is a qubit Q, formed by an arbitrary two-level system coupled to its outer environment. We consider the case when the process of energy dissipation is negligible and only pure decoherence takes place. It leads to an irreversible process of information loss [18]. We model such a system by the Hamiltonian ($\hbar=1$)

$$H = H_Q \otimes \mathbb{I}_B + \mathbb{I}_Q \otimes H_B + H_I, \tag{2}$$

where $\mathbb{I}_Q$ and $\mathbb{I}_B$ are identity operators (matrices) in corresponding Hilbert spaces of the qubit Q and the environment B, respectively. Let the qubit canonical basis be $\{|1\rangle, |-1\rangle\}$. The qubit Hamiltonian H_Q reads

$$H_Q = \varepsilon_+ |1\rangle\langle 1| + \varepsilon_- |-1\rangle\langle -1|, \tag{3}$$

where $\varepsilon_\pm$ are the qubit energy levels. If $\varepsilon_+=-\varepsilon_-=\varepsilon$, then $H_Q=\varepsilon S^z$ is the spin Hamiltonian, where $S^z=|1\rangle\langle 1|-|-1\rangle\langle -1|$ and ε is proportional to amplitude of the magnetic field. The environment is modeled as a collection of bosons and is described by the Hamiltonian H_B of the form

$$H_B = \int_0^\infty d\omega h(\omega) a^\dagger(\omega) a(\omega), \tag{4}$$

where the real-valued spectrum function $h(\omega)$ depends on the kind of environment. The operators $a^\dagger(\omega)$ and $a(\omega)$ are the creation and annihilation boson operators, respectively. The qubit-environment interaction in general is assumed to be asymmetric:

$$H_I = |1\rangle\langle 1| \otimes H_+ + |-1\rangle\langle -1| \otimes H_-,$$

$$H_\pm = \pm \int_0^\infty d\omega [g_\pm^*(\omega) a(\omega) + g_\pm(\omega) a^\dagger(\omega)]. \tag{5}$$

The van Hove operators $H_\pm$ are expressed in terms of the coupling functions $g_\pm(\omega)$, and $g_\pm^*(\omega)$ are the complex conjugate functions to $g_\pm(\omega)$. The Hamiltonian (2) can be recast as

$$H = |1\rangle\langle 1| \otimes H_1 + |-1\rangle\langle -1| \otimes H_{-1}, \tag{6}$$

$$H_{1/-1} = H_B + H_\pm + \varepsilon_\pm. \tag{7}$$

Hamiltonians of the similar structure like (6) have been studied in the context of a quantum kicked rotator [19], chaotic dynamics of a periodically driven superconducting single-electron transistor [20], the Josephson flux qubit [21], and quantum dots [22]. The model may also serve as a component of a simple quantum register [18]. Moreover, it contains, as particular cases, the widely used van Hove model [23] [for $g_+(\omega)=g_-(\omega)$] and the Friedrichs model [24] [for either $g_+(\omega)=0$ or $g_-(\omega)=0$]. The generalized spin-boson model (6) has been applied to analyze the electron-transfer reactions [25] and the interconversion of electronic and vibrational energy [26].

The model (2)–(5) is exactly solvable in the sense that the exact density matrix of the qubit can be obtained provided the initial state of the total system is separable. In the case of symmetric coupling, it has been solved in Refs. [12,13,27,28]. For this paper to be self-contained, below we briefly present a derivation of the reduced dynamics for the case of asymmetric coupling. To this aim, let us notice that in the canonical basis, the Hamiltonian (6) is a diagonal 2×2 matrix reading

$$H = \mathrm{diag}[H_1, H_{-1}]. \tag{8}$$

This form is convenient because we can directly apply results of Ref. [14] and solve the Schrödinger equation with the Hamiltonian (2). To do it, let us specify an initial state of the system assuming a product state: namely,

$$|\Psi(0)\rangle = (b_1|1\rangle + b_{-1}|-1\rangle) \otimes |R\rangle, \tag{9}$$

where b_1 and b_{-1} determine the qubit initial state and $|R\rangle$ is the initial state of the environment.

Time evolution of the state (9) is governed by [14]

$$|\Psi(t)\rangle = b_1 e^{-i\Lambda_1(t)}|1\rangle \otimes D(g_t^+ - g^+) e^{-iH_B t}|R\rangle + b_{-1} e^{-i\Lambda_{-1}(t)}|-1\rangle \otimes D(g^- - g_t^-) e^{-iH_B t}|R\rangle, \tag{10}$$

where the phases $\Lambda_1(t)$ and $\Lambda_2(t)$ have the form

$$\Lambda_1(t) = \varepsilon_+ t - \int_0^\infty d\omega |g^+(\omega)|^2 \{h(\omega)t - \sin[h(\omega)t]\},$$

$$\Lambda_{-1}(t) = \varepsilon_- t - \int_0^\infty d\omega |g^-(\omega)|^2 \{h(\omega)t - \sin[h(\omega)t]\} \tag{11}$$

and the abbreviations

$$g^+(\omega) = \frac{g_+(\omega)}{h(\omega)}, \quad g^-(\omega) = \frac{g_-(\omega)}{h(\omega)} \tag{12}$$

have been introduced. For any function f, the notation f_t means

$$f_t(\omega) = e^{-ih(\omega)t} f(\omega). \tag{13}$$

The displacement operator $D(f)$ reads [29]

$$D(f) = \exp\left\{\int_0^\infty d\omega [f(\omega) a^\dagger(\omega) - f^*(\omega) a(\omega)]\right\} \tag{14}$$

for an arbitrary square-integrable function f.

We do not need to know full information on the total system: qubit+environment. Rather the dynamics of the qubit and the influence of the environment on its behavior are crucial. The qubit dynamics can be obtained for the initial states given by Eq. (9) or, more generally, for a larger class of states defined by the initial statistical operator $\varrho(0)$ of the total system:

$$\varrho(0) = \sum_{i,j=1,-1} p_{ij} |i\rangle\langle j| \otimes |R\rangle\langle R|, \tag{15}$$

where p_{ij} are non-negative parameters. The reduced statistical operator $\rho(t)$ for the qubit only can be expressed in the form

$$\rho(t) = \mathrm{Tr}_B[\varrho(t)] = \sum_{i,j=1,-1} p_{ij} |i\rangle\langle j| \otimes \mathrm{Tr}_B(e^{-iH_i t}|R\rangle\langle R| e^{iH_j t}) = \sum_{i,j=1,-1} p_{ij} c_{ji}(t) |i\rangle\langle j|, \tag{16}$$

where Tr_B denotes partial tracing over the environment, $H_i (i=1,-1)$ is given by Eq. (7), and

$$c_{ji}(t) = \langle e^{-iH_j t}R|e^{-iH_i t}R\rangle = \langle \psi_j(t)|\psi_i(t)\rangle \tag{17}$$

is a scalar product of the functions $|\psi_j(t)\rangle$ and $|\psi_i(t)\rangle$ in the environment Hilbert space. It follows from Eq. (16) that the qubit reduced dynamics can exactly be constructed, provided one is able to evaluate the above scalar product. It is possible at least for two classes of initial states of the environment: for the vacuum state $|R\rangle = |\Omega\rangle$ and the coherent states $|R\rangle = |\alpha\rangle = D(\alpha)|\Omega\rangle$. A coherent state is determined by a complex function $\alpha = \alpha(\omega)$ (analogically as in a one mode case by a complex number). Since $|\Omega\rangle = D(0)|\Omega\rangle$, we consider both the cases simultaneously assuming the initial state to be the coherent one:

$$|R\rangle = |\alpha\rangle = D(\alpha)|\Omega\rangle. \tag{18}$$

For this choice of the initial states of the environment, Eq. (10) takes the form

$$|\Psi(t)\rangle = b_1|1\rangle \otimes |\psi_1(t)\rangle + b_{-1}|-1\rangle \otimes |\psi_{-1}(t)\rangle, \tag{19}$$

where

$$|\psi_1(t)\rangle = e^{-i\Lambda_2(t)}D(g_t^+ - g^+ + \alpha_t)|\Omega\rangle$$

$$|\psi_{-1}(t)\rangle = e^{-i\Lambda_{-2}(t)}D(g^- - g_t^- + \alpha_t)|\Omega\rangle, \tag{20}$$

and the relation

$$D(g)D(f) = e^{i\,\mathrm{Im}\langle g|f\rangle}D(g+f) \tag{21}$$

has been utilized; $\mathrm{Im}\langle g|f\rangle$ is the imaginary part of the scalar product of two functions g and f defined as

$$\langle g|f\rangle = \int_0^\infty d\omega\, g(\omega)f^*(\omega). \tag{22}$$

The phases $\Lambda_2(t)$ and $\Lambda_{-2}(t)$ read

$$\Lambda_2(t) = \Lambda_1(t) - \mathrm{Im}\langle g_t^+ - g^+|\alpha_t\rangle,$$

$$\Lambda_{-2}(t) = \Lambda_{-1}(t) - \mathrm{Im}\langle g^- - g_t^-|\alpha_t\rangle. \tag{23}$$

It is convenient to present an initial state of the qubit $|\theta,\phi\rangle$ as a vector on the Bloch sphere:

$$|\theta,\phi\rangle = \cos(\theta/2)|1\rangle + e^{i\phi}\sin(\theta/2)|-1\rangle, \tag{24}$$

where θ and ϕ are the polar and azimuthal angles, respectively. This parametrization corresponds to $b_1 = \cos(\theta/2)$ and $b_{-1} = e^{i\phi}\sin(\theta/2)$ in Eq. (9). The initial statistical operator $\rho(0)$ for the reduced qubit dynamics takes the form

$$\rho(0) = \begin{pmatrix} \cos^2(\theta/2) & \frac{1}{2}\sin\theta e^{-i\phi} \\ \frac{1}{2}\sin\theta e^{i\phi} & \sin^2(\theta/2) \end{pmatrix}. \tag{25}$$

From Eq. (16) one gets the statistical operator $\rho(t)$ for time $t > 0$. It has the form

$$\rho(t) = \begin{pmatrix} \cos^2(\theta/2) & \frac{1}{2}A(t)\sin\theta e^{-i\phi} \\ \frac{1}{2}A^*(t)\sin\theta e^{i\phi} & \sin^2(\theta/2) \end{pmatrix}. \tag{26}$$

This formula defines the asymmetric dephasing channel

$$T: \rho(0) \to \rho(t), \tag{27}$$

where the influence of the infinite bosonic environment is represented by the function

$$A(t) = \mathrm{w}(t)e^{-i\Phi(t)}, \quad A(0) = 1. \tag{28}$$

The damping part reads

$$\mathrm{w}(t) = \langle\Omega|D(g_t^+ - g^+ + g_t^- - g^-)|\Omega\rangle = e^{-r(t)}, \tag{29}$$

where the decoherence function

$$r(t) = \int_0^\infty d\omega |g^+(\omega) + g^-(\omega)|^2\{1 - \cos[h(\omega)t]\}. \tag{30}$$

The phase part

$$\Phi(t) = \Lambda_1(t) - \Lambda_{-1}(t) - \mathrm{Im}[\langle g_t^+ - g^+|\alpha_t\rangle + \langle g_t^- - g^-|\alpha_t\rangle + \langle g_t^- - g^- - \alpha_t|g_t^+ - g^+ + \alpha_t\rangle]. \tag{31}$$

One can observe that a coherent initial state does not influence the damping part. However, it modifies a phase part of the reduced statistical operator.

In a general case, the functions g^+, g^-, and α are complex functions of a real variable. For the sake of simplicity, we assume from now on that they are real functions. In this case, the total phase reads

$$\Phi(t) = (\varepsilon_+ - \varepsilon_-)t + \Phi_I(t) + \Phi_\alpha(t), \tag{32}$$

where

$$\Phi_I(t) = \int_0^\infty d\omega[g^-(\omega)^2 - g^+(\omega)^2]\{h(\omega)t - \sin[h(\omega)t]\} \tag{33}$$

is the phase induced by the asymmetric interaction. For the symmetric interaction $g^+(\omega) = g^-(\omega)$, this term vanishes. The last part

$$\Phi_\alpha(t) = 2\int_0^\infty d\omega[g^+(\omega) + g^-(\omega)]\alpha(\omega)\sin[h(\omega)t] \tag{34}$$

is the phase related to the initial coherent state. For the initial vacuum state $\alpha(\omega) = 0$, this contribution vanishes.

III. FIDELITY

From Eq. (26), we can calculate the fidelity (1) which, for the considered class of pure initial states (24), reduces to the form

$$F(t) \equiv F(\rho(t),\rho(0)) = \mathrm{Tr}[\rho(t)\rho(0)] \tag{35}$$

and measures the deviation of the evolving state $\rho(t)$ from the initial one $\rho(0)$. Its explicit form reads

$$F(t) = 1 - \frac{1}{2}[1 - e^{-r(t)}\cos\Phi(t)]\sin^2\theta. \tag{36}$$

Now, we discuss general properties of the fidelity $F(t)$. The first observation is the independence of $F(t)$ on the azimuthal angle ϕ. Under some nonrestricted and commonly accepted assumptions on the spectral properties of the environment, one can infer from Eq. (30) that the function $r(t)$ is a nonnegative function of time, bounded by $r(0)=0$ and $r(\infty)>0$. So in some cases this function can tend to infinity, $r(t)\to\infty$ as $t\to\infty$. In this case, $F(\infty)=1-(1/2)\sin^2\theta \geqslant 1/2$ and it does not depend on the phase Φ. The maximal fidelity $F(\infty)=1$ is for two classes of initial states: $|\theta,\phi\rangle=|1\rangle$ ($\theta=0$ and arbitrary ϕ) and $|\theta,\phi\rangle=|-1\rangle$ ($\theta=\pi$ and arbitrary ϕ). The averaged fidelity (on Bloch sphere)

$$\langle F(t)\rangle = \frac{1}{4\pi}\int_0^{2\pi} d\phi \int_0^{\pi} d\theta \sin\theta\, F(t) \tag{37}$$

yields the best possible value $\langle F(\infty)\rangle = 2/3$ obtained by the classical communication [10]. From the formal point of view, the case $r(\infty)=\infty$ is subtle: A ground state of the Hamiltonian (2) has an infinite number of bosons and therefore does not lie in the Hilbert space $C^2\otimes\mathcal{F}$ with $\mathcal{F}$ the symmetric Fock space of bosons [30]. We recall that it takes place for the Ohmic environment [30]. However, we will not discuss this case and the reason is not just the above formal drawback. Much more interesting is the case $r(\infty)=r<\infty$. The time decay of the fidelity can be reduced depending on the phase $\Phi(t)$ in the long-time regime. From Eq. (36) we deduce that the phase $\Phi(t)$ should be as close as possible to a multiple of 2π.

A. Vacuum initial environment state

Let an initial environment state be the vacuum state

$$|R\rangle = |\Omega\rangle. \tag{38}$$

Then the phase takes the form

$$\Phi(t) = (\varepsilon_+ - \varepsilon_-)t + \int_0^\infty d\omega[g^-(\omega)^2 - g^+(\omega)^2] \times\{h(\omega)t - \sin[h(\omega)t]\}. \tag{39}$$

To maximize the fidelity, the phase $\Phi(t)$ should be as close as possible to a multiple of 2π. Let us consider two cases: (i) the system has the degenerate energy levels and (ii) the energy levels are nondegenerate.

(i) Let $\varepsilon_+=\varepsilon_-$. The best value of the phase is $\Phi(t)=0$. It can be attained for the symmetric coupling, $g^+(\omega)=g^-(\omega)$. In the long-time regime, the averaged fidelity is

$$\langle F(\infty)\rangle = 1 - \frac{1}{3}[1-e^{-r}], \tag{40}$$

$$r = \int_0^\infty d\omega |g^+(\omega)+g^-(\omega)|^2. \tag{41}$$

There are no external control parameters and the fidelity cannot be varied.

(ii) Let $\varepsilon_+=-\varepsilon_-=\varepsilon$ as for the spin qubit. In the long-time regime,

$$\Phi(t) \approx \Omega t, \tag{42}$$

$$\Omega = 2\varepsilon + \int_0^\infty d\omega[g^-(\omega)^2 - g^+(\omega)^2]h(\omega). \tag{43}$$

Obviously, the choice $\Omega=0$ optimizes the quantum channel in a sense that the fidelity is maximal and its oscillations are extinguished. For qubits of the spin type, the parameter ε is proportional to the magnetic field B—i.e., $\varepsilon(B)\propto B$. Therefore the phase can be changed by the magnetic field. The best tuning is for B determined by the relation

$$\varepsilon(B) = \frac{1}{2}\int_0^\infty d\omega[g^+(\omega)^2 - g^-(\omega)^2]h(\omega). \tag{44}$$

Under this condition $\Omega=0$, the phase $\Phi=0$ and the averaged fidelity is given by relations (40) and (41). As the magnetic field can be controlled with a very high precision, oscillations can be suppressed and the fidelity can be tuned to the maximal value determined, via the quantity r, by the coupling with the environment. Of course, as is seen from Eq. (41), a weaker coupling results in a better fidelity. However, for the fixed coupling strength, the reduction parameter r depends on spectral properties of couplings and environment as shown in the example below.

In order to illustrate the above general considerations, we present explicit results for a qubit with nondegenerate energy levels $\varepsilon_+=-\varepsilon_-=\varepsilon$, $\varepsilon\neq 0$. We assume that the environment has a linear energy spectrum—i.e., $h(\omega)=\omega$. The coupling to the environment is usually described in terms of the spectral density. In order to introduce the spectral density in our approach, let us first consider the symmetric case $g^+(\omega) = g^-(\omega)$. One can redefine the coupling function defining a new function $J(\omega)$ according to the relation

$$[g^{\pm}(\omega)]^2 = \frac{J(\omega)}{\omega^2}. \tag{45}$$

Then the decoherence function (30) takes the form

$$r(t) = 4\int_0^\infty d\omega \frac{J(\omega)}{\omega^2}[1-\cos(\omega t)]. \tag{46}$$

Comparing this equation with the standard expression for the decoherence function (see, e.g., Eq. (4.51) in Ref. [28]), one can identify $J(\omega)$ as the spectral density. One can also justify the above statement in the following way. In the case of a qubit that is coupled to a discrete set of oscillators, one usually defines the spectral density as [31]

$$J(\omega) = \sum_k |g_k|^2 \delta(\omega-\omega_k). \tag{47}$$

Analogically, for a continuous set of oscillators, one can define

$$J(\omega) = \int_0^\infty d\omega' |g_{\pm}(\omega')|^2 \delta(\omega-\omega'), \tag{48}$$

which is exactly the same as the spectral density defined by Eq. (45). We choose the spectral density which has frequently been used in the literature [12,31]: namely,

$$J(\omega) = J_{\lambda,\mu}(\omega) = \lambda \omega^{1+\mu} \exp(-\omega/\omega_c), \tag{49}$$

with $\mu > -1$. The physical meaning of parameters λ, ω_c, and μ can be explained in the following way. λ determines the coupling strength, whereas the cutoff frequency ω_c is the largest energy scale of the environment. The spectral exponent μ characterizes the low-frequency properties of the environment and defines its various types. According to the classification proposed in Ref. [31], the environment is called sub-Ohmic, Ohmic, and super-Ohmic for $\mu \in (-1,0)$, $\mu=0$, and $\mu \in (0,\infty)$, respectively. For $g^+(\omega) \neq g^-(\omega)$ we introduce independent sets of parameters λ_1, μ_1 and λ_{-1}, μ_{-1} for the states $|1\rangle$ and $|-1\rangle$, respectively. We investigate the effects which follow from the tuning of the energy splitting. Therefore, some other quantity should be taken as the energy unit. Since the cutoff frequency is the largest energy scale, we take $\hbar\omega_c/100$ as the energy unit. Then the time unit is $100/\omega_c$.

In the long-time regime, the sub-Ohmic and Ohmic cases lead to the reduction parameter $r=\infty$. For the super-Ohmic environment, by substituting Eq. (49) into the integrals (30) and (33), we obtain an explicit expression for the fidelity damping:

$$r(t) = \mathcal{L}(\lambda_1,\mu_1;t) + \mathcal{L}(\lambda_{-1},\mu_{-1};t) + 2\mathcal{L}\left(\sqrt{\lambda_1\lambda_{-1}}, \frac{\mu_1+\mu_{-1}}{2};t\right),$$

$$\mathcal{L}(\lambda,\mu;t) = \int_0^\infty d\omega \frac{J_{\lambda,\mu}(\omega)}{\omega^2}[1-\cos(\omega t)]$$

$$= \lambda\Gamma(\mu)\omega_c^{\mu}\left[1 + \frac{\cos[\mu \arctan(\omega_c t)]}{(1+\omega_c^2 t^2)^{\mu/2}}\right], \tag{50}$$

where $\Gamma(z)$ is the Euler gamma function. The phase part of the fidelity can also be calculated. One gets

$$\Phi(t) = 2\varepsilon t + \mathcal{M}(\lambda_1,\mu_1;t) + \mathcal{N}(\lambda_1,\mu_1;t) - \mathcal{M}(\lambda_{-1},\mu_{-1};t) - \mathcal{N}(\lambda_{-1},\mu_{-1};t),$$

$$\mathcal{M}(\lambda,\mu;t) = t\int_0^\infty d\omega \frac{J_{\lambda,\mu}(\omega)}{\omega} = \lambda\omega_c^{1+\mu}\Gamma(1+\mu)t,$$

$$\mathcal{N}(\lambda,\mu;t) = \int_0^\infty d\omega \frac{J_{\lambda,\mu}(\omega)}{\omega^2}\sin(\omega t)$$

$$= \frac{\lambda\Gamma(\mu)\omega_c^{\mu}\sin[\mu\arctan(\omega_c t)]}{(1+\omega_c^2 t^2)^{\mu/2}}. \tag{51}$$

The above two equations (50) and (51) determine the exact reduced dynamics (26) and the noisy channel T. In the long-time regime the above equations can be simplified significantly according to $\mathcal{L}(\lambda,\mu;\infty)=\lambda\Gamma(\mu)\omega_c^{\mu}$, $\mathcal{M}(\lambda,\mu;\infty)=\lambda\Gamma(1+\mu)\omega_c^{1+\mu}$, and $\mathcal{N}(\lambda,\mu;\infty)=0$. Then, the optimal case occurs when the fidelity is weakly damped and nonoscillating; i.e., $\Phi(\infty)=0$ and $r(\infty)$ is small with the additional constraints $\mu_{\pm1}>0$ and $\lambda_{\pm1}\geqslant 0$. To show how the fidelity depends on the low-frequency properties of the environment, let us consider the simplest case $\mu_{\pm1}=\mu$, when the asymmetry results from different values of λ_1 and λ_{-1}. Then the minimal value of $r(\infty)$ is when $\Psi(\mu)+\ln(\omega_c/\varepsilon)=0$, where Ψ is the Euler psi function. For, e.g., $\omega^c=100$, one gets a minimum at $\mu\approx 0.23$, whereas for $\omega_c/\varepsilon\to\infty$ the optimal environment approaches the Ohmic limit $\mu\to 0$. For a given degree of the asymmetry, a suitable tuning of ε extinguishes the oscillations and the fidelity becomes monotonic in the long-time regime, as can be inferred from Fig. 1. On the other hand, for a given $\varepsilon\neq 0$ one can perform a tuning of fidelity by a proper choice of an asymmetry of dephasing as indicated in Fig. 2. Despite this tuning, the fidelity may still be nonmonotonic in the short-time regime, provided the asymmetry is sufficiently strong (see Fig. 3).

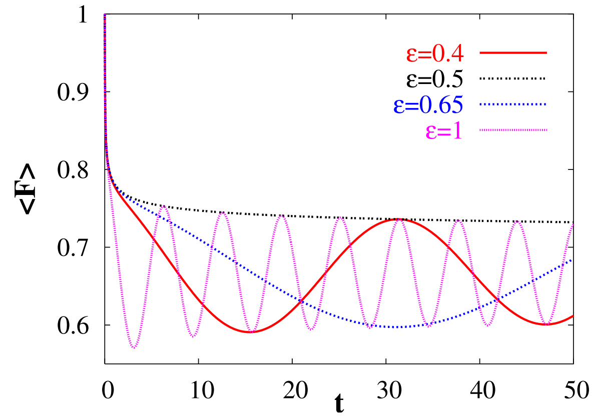

FIG. 1. (Color online) Time evolution of the averaged fidelity of asymmetric dephasing channel T: the influence of qubit level separation ε for fixed $\lambda_1=0.05$, $\mu_{\pm1}=0.1$, and $\lambda_{-1}\approx 0.04$ (within this choice the monotonic fidelity with $\Omega=0$ occurs for $\lambda_1=0.05$ and $\varepsilon=0.5$). The time unit is $100/\omega_c$.

B. Coherent initial preparation of environment

Now, we consider the qubit-environment system the bosonic part of which is initially in a coherent state

$$|R\rangle = D(\alpha)|\Omega\rangle. \tag{52}$$

Contrary to the case of finite bosonic system, coherent states of bosonic field are rather a theoretical construction. There is

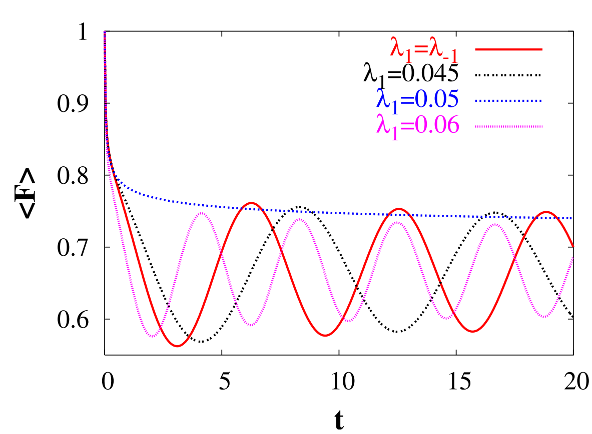

FIG. 2. (Color online) Time evolution of the averaged fidelity of asymmetric dephasing channel T: the effect of asymmetry $\lambda_1 \neq \lambda_{-1}$ resulting in controllable fidelity oscillations. Values of other parameters are the same as in Fig. 1.

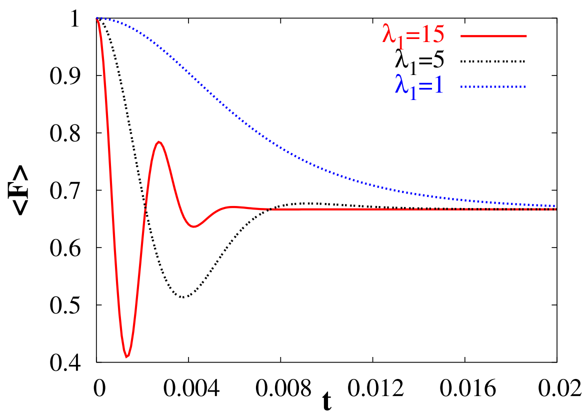

FIG. 3. (Color online) Time evolution of the averaged fidelity of asymmetric dephasing channel T: Initial oscillations of the averaged fidelity for sufficiently large asymmetry. Values of other parameters are the same as in Fig. 1.

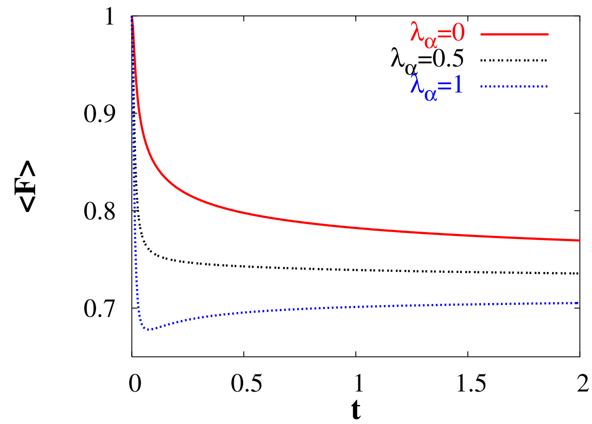

FIG. 4. (Color online) Time evolution of the averaged fidelity of asymmetry dephasing channel T: the effect of the initial coherent state, Eq. (52), for μ_α=0.1. Values of other parameters are the same as in Fig. 1.

no obvious choice of the function $\alpha(\omega)$ in Eq. (18) which could be used for general considerations. The only requirement concerns the square integrability of $\alpha(\omega)$ [29]. However, in order to illustrate the role of the initial state of environment, we consider the class of initial preparations parametrized by

$$\alpha^2(\omega) = J_{\lambda_\alpha,\mu_\alpha}(\omega)/\omega^2, \tag{53}$$

where the function $J_{\lambda_\alpha,\mu_\alpha}(\omega)$ is defined by Eq. (49). Then, the phase $\Phi(t)$ acquires the contribution

$$\Phi_\alpha(t) = 2\mathcal{N}\left(\sqrt{\lambda_1\lambda_\alpha}, \frac{\mu_1+\mu_\alpha}{2};t\right) + 2\mathcal{N}\left(\sqrt{\lambda_{-1}\lambda_\alpha}, \frac{\mu_{-1}+\mu_\alpha}{2};t\right). \tag{54}$$

The long-time dynamics of fidelity is not affected by the initial state since

$$\lim_{t\to\infty} \Phi_\alpha(t) = 0. \tag{55}$$

The short-time dynamics of fidelity is presented in Fig. 4. The deep minimum occurring in this regime indicates that the quality of the asymmetric dephasing channel quantified by the fidelity becomes better when the transmitted qubit is affected by the environment a bit longer.

The impact of the initial preparation beyond the assumed class (53) remains an open problem in the regime of short times. However, for long times, the property (55) holds true for any square-integrable function $\alpha(\omega)$—i.e., for an arbitrary coherent initial state. This statement follows from the properties of the Fourier transforms.

IV. ENTANGLEMENT FIDELITY

There is a certain class of problems in quantum information processing when Bob and Alice share a bipartite entangled state [3]. The parties are often separated and are prepared, at least for one of the parties, in a noisy environment. In this section we investigate to what extent the asymmetric dephasing channel preserves entanglement of a two-qubit state.

We consider two noninteracting qubits: one does not evolve whereas the second is coupled to asymmetric dephasing environment. We assume that initially the bosonic environment is in its vacuum state—i.e., $|R\rangle=|\Omega\rangle$ One of possible measures of a "stiffness" of entanglement against noisy channel T is the *entanglement fidelity* [3,32]. This choice enables a direct comparison with results presented in the proceeding sections for the one-qubit fidelity. We assume that one of the qubits prepared initially in a maximally entangled state is sent through the asymmetric dephasing channel. Again, as the initial state is pure, the entanglement fidelity has a simple computable form [3]

$$F_e = \langle\phi^{(i)}|[(\mathbb{I}_Q \otimes T)|\phi^{(i)}\rangle\langle\phi^{(i)}|]|\phi^{(i)}\rangle, \tag{56}$$

where T is a asymmetric dephasing channel. We limit our attention to the maximally entangled initial states

$$|\phi^{(1)}(0)\rangle = \frac{1}{\sqrt{2}}[|-1,1\rangle + |1,-1\rangle],$$

$$|\phi^{(2)}(0)\rangle = \frac{1}{\sqrt{2}}[|-1,-1\rangle + |1,1\rangle]. \tag{57}$$

for which entanglement fidelity F_e can be calculated in a compact form:

$$F_e = \frac{1}{2}[1 + e^{-r(t)}\cos\Phi(t)], \tag{58}$$

which does not depend on the choice of particular state $|\phi^{(i)}(0)\rangle$, $i=1,2$ in Eq. (57). The properties of the entanglement fidelity (58) are, for the assumed initial preparations, qualitatively the same as those of the one-qubit fidelity of the asymmetric dephasing channel (36). As a result, the entanglement fidelity can be controlled in a similar manner.

V. CONCLUSIONS

We have shown that an asymmetric dephasing quantum channel can transmit quantum information in a nontrivial way. Namely, it allows for an effective controlling of the information loss, quantified by the fidelity calculated for the initial state and the state at a given instant of time. The time evolution of the fidelity strongly depends on the symmetry breaking of the coupling with the outer environment. By adjusting the degree of asymmetry, the time evolution of the fidelity becomes monotonic and approaches asymptotically a steady-state value. This is due to both the time-independent phase of the oscillations and the properties of the super-Ohmic environment studied previously in the context of persistent entanglement [14,16]. The asymmetry-induced asymptotic vanishing of oscillations of the fidelity is an example of *nonperturbative* phenomena in a sense that it does not occur in asymmetric channels *weakly* coupled to the dephasing environment.

Qualitatively similar results for the fidelity have been shown to occur also for some specific class of coherent initial states of the bosonic environment. However, coherent preparation of the dephasing environment affects only the short-time properties of the fidelity. The presented analysis of the entanglement fidelity, limited to a narrow class of initial states of the bipartite system, leads to similar conclusions.

Optimization of quantum channels with respect to the amplitude of fidelity does not need to be justified. However, the asymptotically monotonic time dependence of the fidelity can also be of crucial importance for the information processing. In a generic case of oscillating fidelity, the information loss strongly depends on time, when the final state is measured. Therefore, the time of measurement should precisely be adjusted to match the desired phase of oscillations. We have demonstrated that for the asymmetric coupling this time dependence can completely be tuned out.

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