Robustness of multiqubit entanglement against local decoherence

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We investigate the disentanglement dynamics of generalized multiqubit Greenberger-Horne-Zeilinger-type states $|\Psi_{n,N}\rangle=\alpha|1\rangle^{\otimes n}|0\rangle^{\otimes(N-n)}+\beta|0\rangle^{\otimes n}|1\rangle^{\otimes(N-n)}$ (with $1\leqslant n<N$) in a local decoherence environment. By adopting the time after which all the bipartite entanglements vanish or become negligibly small as the criterion of entanglement robustness, we show that states $|\Psi_{n,N}\rangle$ are more robust than states $|\Psi_N\rangle=\alpha|0\rangle^{\otimes N}+\beta|1\rangle^{\otimes N}$ in all respects. Since the two states $|\Psi_N\rangle$ and $|\Psi_{n,N}\rangle$ can be transformed into each other by local operations, we confirm that entanglement robustness can be enhanced by local operations though the amount of entanglement cannot.

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Entanglement can be exploited to perform various intriguing global tasks in quantum computation and quantum communication because it possesses "spooky" distance-independent nonlocality. Scalable quantum computation and network quantum communication require use of large-sized entanglements, i.e., multiqubit entangled states, which are shared among different remote locations. The prerequisite for a quantum task is the persistence of entanglement of initially prepared entangled states during its performance. However, in the process of entanglement distribution and qubit manipulation, each qubit is unavoidably exposed to its own uncontrollable environment. This leads to local decoherences which will sooner or later spoil the necessary entanglement of the shared states. Therefore, the question of robustness of multiqubit entangled states against local decoherences, especially with respect to scaling properties (i.e., the system's size dependence), is of primary significance.

It should be noted that the notion of entanglement robustness is different from that of entanglement amount. Here we adopt the following robustness criterion: an entangled state is said to be more robust if its entanglement amount is sustainable for a longer time before vanishing. Therefore, entanglement robustness can be measured by the time after which an entangled state becomes completely separable or remains entangled but with a negligible entanglement amount. As for the entanglement measure of a multiqubit state, one can deal with bipartitions by dividing the overall system into two arbitrarily chosen subsystems. For any density matrix ρ one can pick up the entanglement of a bipartition $A|B$ with a quantitative measure called negativity $\mathcal{N}^{A|B}$ [1], which is defined as twice the absolute value of the sum of the negative eigenvalues of the partially transposed matrix ρ^{T_A} of ρ, where $\langle i_A j_B|\rho^{T_A}|k_A l_B\rangle=\langle k_A j_B|\rho|i_A l_B\rangle$. When ρ^{T_A} has at most one negative eigenvalue, one has $\mathcal{N}^{A|B}(\rho)=2\max\{0,-\lambda_{\min}\}$, with $\lambda_{\min}$ the lowest eigenvalue of ρ^{T_A}.

Despite a considerable number of current important publications (see, e.g., [2–5]) a comprehensive understanding of the question of robustness of multiqubit entangled states against local decoherences is still missing. Intuitively, on the one hand, different decoherence channels would affect the robustness of a multiqubit entangled state differently. On the other hand, under one and the same kind of decoherence the disentanglement dynamics might be dependent on the kind as well as the size of the multiqubit entangled state. Among other entangled states such as spin squeezed states [6], *W* states [7], graph states [8], etc., the *N*-qubit Greenberger-Horne-Zeilinger (GHZ) states [9] $|\psi_N\rangle=(|0\rangle^{\otimes N}+|1\rangle^{\otimes N})/\sqrt{2}$ have been most intensively studied [2–4] since these states are crucial for many application purposes (see, e.g., [10]) and can already be generated in the laboratory [11]. Recently, *N*-qubit GHZ-type states $|\Psi_N\rangle=\alpha|0\rangle^{\otimes N}+\beta|1\rangle^{\otimes N}$, with $\alpha,\beta\in\mathcal{C}$ such that $|\alpha|^2+|\beta|^2=1$, have also been dealt with in [5]. In this work we further consider the so-called generalized *N*-qubit GHZ-type states of the form

$$|\Psi_{n,N}\rangle=\alpha|1^n0^{N-n}\rangle+\beta|0^n1^{N-n}\rangle, \tag{1}$$

with $1\leqslant n\leqslant N/2$. [Here *N* is assumed even for simplicity. For *N* odd, $N/2$ should be replaced by $(N-1)/2$ throughout.] In Eq. (1) we use the shorthand notation $|j^m\bar{j}^n\rangle\equiv|j\rangle^{\otimes m}\otimes|\bar{j}\rangle^{\otimes n}$ with $j\in\{0,1\}$ and $\bar{j}=1-j$. The results for $N/2\leqslant n\leqslant N-1$ can be obtained from those for $1\leqslant n\leqslant N/2$ with α and β interchanged. Note that $|\Psi_N\rangle$ could formally be denoted as $|\Psi_{0,N}\rangle$ but we do not do so here to emphasize that $|\Psi_N\rangle$ and $|\Psi_{n,N}\rangle$ belong to qualitatively different classes regarding their disentanglement dynamics, as will be seen later. At variance with $|\Psi_N\rangle$ in which all the *N* qubits are equivalent, in the states $|\Psi_{n,N}\rangle$, Eq. (1), there are two groups of permutationally invariant qubits: group 1, denoted by G_1, comprises the first *n* qubits and group 2, denoted by G_2, comprises the remaining $N-n$ qubits. Remarkably, for a given *N*, states $|\Psi_N\rangle$ and states $|\Psi_{n,N}\rangle$ with any *n* are all equivalent in the sense that they can be converted to each other by means of local operations. They all possess the same amount of entanglement measured by the bipartition scheme with negativities as mentioned above, and thus would be equally helpful for certain quantum tasks. Interestingly, however, if each of the *N* qubits experiences an independent local decoherence mechanism, then $|\Psi_N\rangle$ and $|\Psi_{n,N}\rangle$ will undergo completely different disentanglement dynamics. We obtain quite general results: states $|\Psi_{n,N}\rangle$ are more robust than states $|\Psi_N\rangle$ in the whole range of involved parameters. Our results solidly confirm the important fact that entangle-

ment robustness can be enhanced merely by local operations though the entanglement amount itself cannot. Finally, we also investigate the dependence on the system size of robustness of $|\Psi_{n,N}\rangle$, $|\Psi_N\rangle$, and $|W_N\rangle$ [see Eq. (10) later for $|W_N\rangle$] which exhibit very distinct scaling laws. Our results might provide practical guidance in employing multiqubit entangled states to cope with local decoherence.

In this work we treat a qubit as a two-level atom spanned by two orthonormal basis states $|0\rangle$ and $|1\rangle$ corresponding to the ground and excited states, respectively. To be concrete, for the local decoherence let us consider the amplitude-damping (AD) channel at zero temperature, which for an atom j is described by a superoperator $\mathcal{S}_j:\rho_j(t)=\mathcal{S}_j\rho_j(0)$ with $\rho_j(0)$ and $\rho_j(t)$ the initial and the evolved reduced density matrices of atom j. The electromagnetic field initially in the vacuum state acts as the dissipative environment. In the course of time evolution, the atom decays from its excited state $|1\rangle$ to its ground state $|0\rangle$ by emitting a photon, with a probability $p=1-\exp(-\Gamma t)$, where Γ is the decay rate. For convenience, we can parametrize the time dependence in terms of p instead of t noticing that $p=0$ when $t=0$ and $p\to 1$ when $t\to\infty$, i.e., $p\in[0,1]$ for $t\in[0,\infty]$. The action of $\mathcal{S}_j$ on elements of the reduced density matrix of atom j reads $|0\rangle_{jj}\langle 0|\to|0\rangle_{jj}\langle 0|$; $|0\rangle_{jj}\langle 1|\to\sqrt{1-p}|0\rangle_{jj}\langle 1|$; $|1\rangle_{jj}\langle 0|\to\sqrt{1-p}|1\rangle_{jj}\langle 0|$; $|1\rangle_{jj}\langle 1|\to p|0\rangle_{jj}\langle 0|+(1-p)|1\rangle_{jj}\langle 1|$. Then under the action of N independent local AD channels $\mathcal{S}=\mathcal{S}_1\mathcal{S}_2\cdots\mathcal{S}_N$ the initial density matrix $\rho_0^{n,N}=\rho^{n,N}(0)=|\Psi_{n,N}\rangle\langle\Psi_{n,N}|$ is mapped onto $\rho^{n,N}=\rho^{n,N}(p)$, which can be derived as

$$\rho^{n,N}=d|0^n1^{N-n}\rangle\langle 1^n0^{N-n}|+d^*|1^n0^{N-n}\rangle\langle 0^n1^{N-n}|+\sum_{k=1}^{n}\nu_k\mathcal{P}_1[|0^{n-k}1^k\rangle\langle 0^{n-k}1^k|]\otimes|0^{N-n}\rangle\langle 0^{N-n}|+\sum_{k=1}^{N-n}\mu_k|0^n\rangle\times\langle 0^n|\otimes\mathcal{P}_2[|0^{N-n-k}1^k\rangle\langle 0^{N-n-k}1^k|]+\xi|0^N\rangle\langle 0^N|, \quad (2)$$

where

$$d=\alpha^*\beta(1-p)^{N/2},\quad \xi=|\alpha|^2p^n+|\beta|^2p^{N-n}, \quad (3)$$

$$\nu_k=|\alpha|^2p^{n-k}(1-p)^k,\quad \mu_k=|\beta|^2p^{N-n-k}(1-p)^k, \quad (4)$$

with $\mathcal{P}_1$ accounting for all possible permutations from a state of the first n qubits within group G_1, and $\mathcal{P}_2$ accounting for all possible permutations from a state of $N-n$ last qubits within group G_2. Physically, the underlying quantum map $\mathcal{S}_j$ implies that only the excited state $|1\rangle$ interacts with the environment, namely, it can decay to the ground state $|0\rangle$, while the ground state $|0\rangle$ remains completely unchanged. At $t=0$ each of $\rho^N(0)=|\Psi_N\rangle\langle\Psi_N|$ and $\rho^{n,N}(0)=|\Psi_{n,N}\rangle\langle\Psi_{n,N}|$ has only two nonzero diagonal matrix elements. For $t>0$, however, they evolve differently. For the states $|\Psi_N\rangle$, all the N qubits could be populated in their excited states $|1\rangle$ with a probability $|\beta|^2$; thus all of them would have interactions with their environments. As a consequence, all the 2^N diagonal matrix elements of $\rho^N(t)$ become nonzero. On the contrary, for the states $|\Psi_{n,N}\rangle$, the excited states $|1\rangle$ may be populated either by n qubits (with probability $|\alpha|^2$) or by $N-n$ qubits (with probability $|\beta|^2$), but never by all the N qubits. Thus the number of qubits that can interact with the environment is always less than N. As a consequence, not all of the 2^N diagonal matrix elements of $\rho^{n,N}(t)$ become nonzero. In fact, the calculation reveals that $\rho^{n,N}(t)$ contains just $2^n+2^{N-n}-1$ nonzero diagonal matrix elements, as can be verified from Eq. (2). As will be shown by detailed calculations based on the negativities defined above, it is the different structures of $\rho^{n,N}(t)$ and $\rho^N(t)$ that bring about new dynamical features of $|\Psi_{n,N}\rangle$ as compared with $|\Psi_N\rangle$.

Now consider a bipartition $k|N-k$ with $k=1,2,\ldots,N/2$. Since the state $|\Psi_{n,N}\rangle$ has two asymmetric qubit groups G_1 and G_2, for a fixed k we should specify it as $k=k_1+k_2$ where $k_1\in\{0,1,\ldots,n\}$ ($k_2\in\{0,1,\ldots,N-n\}$) is the number of qubits taken from group G_1 (G_2). The partial transposes of $\rho^{n,N}$ of the form (2) have at most one nonpositive eigenvalue. The minimum eigenvalues of the states' partial transposition denoted by $\lambda_{n,k}^{(k_1,k_2)}$ for a fixed bipartition $k|N-k$ with $k=k_1+k_2$ can be derived and classified into three types as

$$\lambda_{n,k}^{(k_1,k_2)}\equiv\lambda_{n,k}^{(k,0)}=\frac{1}{2}(\nu_{n-k}-\sqrt{\nu_{n-k}^2+4|d|^2}) \quad (5)$$

for $k_1\neq 0$, $k_2=0$,

$$\lambda_{n,k}^{(k_1,k_2)}\equiv\lambda_{n,k}^{(0,k)}=\frac{1}{2}(\mu_{N-n-k}-\sqrt{\mu_{N-n-k}^2+4|d|^2}) \quad (6)$$

for $k_1=0$, $k_2\neq 0$, and

$$\lambda_{n,k}^{(k_1,k_2)}\equiv\lambda=-|d| \quad (7)$$

for $k_1\neq 0$, $k_2\neq 0$. It is worth emphasizing that, due to the different structures of $\rho^{n,N}$ and ρ^N, the form of minimum eigenvalues $\lambda_{n,k}^{(k_1,k_2)}$ for $\rho^{n,N}$ in Eqs. (5)–(7) differs very much from that of Λ_k for ρ^N in Eq. (5) of Ref. [5], in which $\Lambda_k=\delta_k-\sqrt{\delta_k^2-\Delta_k}$ with $\delta_k=|\beta|^2[p^{N-k}(1-p)^k+p^k(1-p)^{N-k}]/2$ and $\Delta_k=|\beta|^2(1-p)^N(|\beta|^2p^N-|\alpha|^2)$. This will trigger different degrees of robustness between $|\Psi_{n,N}\rangle$ and $|\Psi_N\rangle$. From Eqs. (5)–(7) one can clearly see that all three types of minimum eigenvalue $\lambda_{n,k}^{(k_1,k_2)}$ are nonpositive, independent of the parameters involved. Hence, according to the concrete values of k_1 and k_2 for the bipartition $k|N-k$, the corresponding negativities $\mathcal{N}_{n,k}^{(k_1,k_2)}$ can be expressed via the minimum eigenvalues as $\mathcal{N}_{n,k}^{(k_1,k_2)}\equiv\{\mathcal{N}_{n,k}^{(k,0)},\mathcal{N}_{n,k}^{(0,k)},\mathcal{N}\}=\{-2\lambda_{n,k}^{(k,0)},-2\lambda_{n,k}^{(0,k)},-2\lambda\}$. From Eqs. (5)–(7), we note that λ and thus $\mathcal{N}$ are independent of both n and the concrete value of k, whereas $\lambda_{n,k}^{(k,0)}$ ($\lambda_{n,k}^{(0,k)}$) and thus $\mathcal{N}_{n,k}^{(k,0)}$ ($\mathcal{N}_{n,k}^{(0,k)}$) are explicitly related to both n and k. For a fixed k, negativities $\mathcal{N}_{n,k}^{(k,0)}$ ($\mathcal{N}_{n,k}^{(0,k)}$) increase with n ($N-n$). The optimal value for n is $N/2$, which yields the maximal negativities $\mathcal{N}_{N/2,k}^{(k,0)}$ ($\mathcal{N}_{N/2,k}^{(0,k)}$) for a fixed k. It can be verified that

$$\mathcal{N}_{n,k}^{(k,0)},\mathcal{N}_{n,k}^{(0,k)}\leqslant\mathcal{N} \quad (8)$$

which is true all the time for all k and n (with the equality holding for $p=0$ or $p=1$). When the largest bipartite entanglement vanishes, the state becomes completely separable since all the other smaller bipartite entanglements have already vanished earlier. The important relationship (8) indicates that $\mathcal{N}$ is surely the largest negativity, i.e., $\max\{\mathcal{N}_{n,k}^{(k_1,k_2)}\}=\mathcal{N}$, so in what follows we shall concentrate only on it (rather than on $\mathcal{N}_{n,k}^{(k,0)}$ or/and $\mathcal{N}_{n,k}^{(0,k)}$) because the

system entanglement associated with $\mathcal{N}$ is sustained longest against the AD channel.

Our first result comes from the observation that, independent of the involved parameters, all the negativities $\mathcal{N}_{n,k}^{(k_1,k_2)} \equiv \{\mathcal{N}_{n,k}^{(k,0)}, \mathcal{N}_{n,k}^{(0,k)}, \mathcal{N}\}$ tend asymptotically to zero only in the limit $p \to 1$, i.e., $t \to \infty$. This fact implies the absence of the so-called entanglement sudden death (ESD) [12], in clear contrast with the states $|\Psi_N\rangle$ for which ESD occurs whenever $|\alpha| < |\beta|$ [5]. Since ESD means the disappearance of entanglement in a finite time, we can say that, for $|\alpha| < |\beta|$, states $|\Psi_{n,N}\rangle$ are more robust than states $|\Psi_N\rangle$ because it takes an infinite time to kill their initial entanglement. We recall that this qualitative distinguishable disentanglement dynamics between $|\Psi_{n,N}\rangle$ and $|\Psi_N\rangle$ originates from the qubit symmetry possessed by the two states, leading to distinct evolutions of their density matrices' diagonal elements, which in turn bring strongly different forms of the minimum eigenvalues of the states' partial transpositions. For $|\alpha| \geqslant |\beta|$, however, neither $|\Psi_N\rangle$ nor $|\Psi_{n,N}\rangle$ suffers from ESD. For the density matrix ρ^N the largest bipartite entanglement $\mathcal{N}_k \equiv 2|\Lambda_k|$ corresponds to the bipartition $k|N-k$ with $k=N/2$, i.e., $\max\{\mathcal{N}_k\}=\mathcal{N}_{N/2}$ [5]. As for the density matrix $\rho^{n,N}$, the largest bipartite entanglement is $\mathcal{N}$ [see (7) and (3)] which corresponds to the bipartition $k|N-k$ with any k greater than 1, provided that qubits of both asymmetric groups G_1 and G_2 must contribute to it, i.e., $k=k_1+k_2$ with both $k_1 \in G_1$ and $k_2 \in G_2$ being nonzero. Of course, this situation can happen only for $|\Psi_{n,N}\rangle$ and never for $|\Psi_N\rangle$.

Our second result comes from the observation that, when $|\alpha| \geqslant |\beta|$, the inequality

$$\frac{\max(\mathcal{N}_k)}{\max(\mathcal{N}_{n,k}^{(k_1,k_2)})} = \frac{\mathcal{N}_{N/2}}{\mathcal{N}} = 1 - \frac{|\beta|}{|\alpha|} p^{N/2} < 1 \tag{9}$$

is always true for $0 < p < 1$. This implies that at any moment in the course of evolution the sustained entanglement amount of $\rho^{n,N}$ is larger than that of ρ^N despite the fact that both states get fully disentangled asymptotically in time. In this sense, we can say that states $|\Psi_{n,N}\rangle$ are more robust than states $|\Psi_N\rangle$ also for $|\alpha| \geqslant |\beta|$.

To see how the disentanglement process scales with size we adopt the criterion for entanglement robustness introduced in Ref. [5] as follows. Let p_c be a critical value of p which is defined by the equation $\mathcal{N}(p_c)=\varepsilon\mathcal{N}(0)$ for $|\Psi_{n,N}\rangle$ [or $\mathcal{N}_{N/2}(p_c)=\varepsilon\mathcal{N}_{N/2}(0)$ for $|\Psi_N\rangle$], with ε some small positive constant. Physically, p_c characterizes the time it takes for the initial entanglement amount to decrease by ε times. It is reasonable to say that a state with a larger p_c is more robust than a state with a smaller p_c. For a sufficiently small value of ε the entanglement amount left after p_c will be negligible, which is generally either not useful for any practical purposes or should be distilled with extremely high cost in quantum and classical resources. Therefore, we can treat $\mathcal{N}(p \geqslant p_c)=0$ [$\mathcal{N}_{N/2}(p \geqslant p_c)=0$] and say that p_c is in a quasi-ESD (QESD) regime, to distinguish it from the true ESD. The dependence of p_c on N for both $|\Psi_{n,N}\rangle$ and $|\Psi_N\rangle$ is plotted in Fig. 1. Although the robustness of both $|\Psi_N\rangle$ and $|\Psi_{n,N}\rangle$ decreases with size, i.e., with the number N of qubits, the figure shows that $|\Psi_N\rangle$ is less robust than $|\Psi_{n,N}\rangle$ for all N.

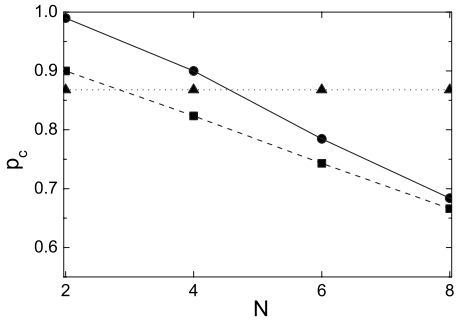

FIG. 1. Critical value $p_c=1-\exp(-\Gamma t_c)$ characterizing the states' robustness as a function of N for $|\Psi_{n,N}\rangle$ (circles), $|\Psi_N\rangle$ (squares), and $|W_N\rangle$ (triangles). The parameters used are $|\alpha|=|\beta|=1/\sqrt{2}$ and $\varepsilon=0.01$.

For a given N, the difference of the critical values $\Delta p_c = p_c^{|\Psi_{n,N}\rangle} - p_c^{|\Psi_N\rangle}$ increases with decreasing $|\alpha/\beta|$. For a given ratio $|\alpha/\beta|$ it decreases with increasing N, remaining positive for all N, with the property $\Delta p_c \to 0$ in the limit $N \to \infty$.

The observed scaling laws of $|\Psi_N\rangle$ and $|\Psi_{n,N}\rangle$ in Fig. 1 are by no reason generic for every multipartite entangled state. For a comparison let us take another kind of state, say, the well-known N-qubit W state of the form [7]

$$|W_N\rangle = \frac{1}{\sqrt{N}}(|10\cdots0\rangle + |01\cdots0\rangle + \cdots + |00\cdots1\rangle). \tag{10}$$

In the course of time evolution under the AD channel, both the off-diagonal and diagonal elements of the density matrix $|W_N\rangle\langle W_N|$ are multiplied by a factor $(1-p)$, but there appears a new diagonal term $|0^N\rangle\langle 0^N|$ weighted by the coefficient p. For a bipartition $k|N-k$, the minimal eigenvalue of the states' partial transposition is

$$L_k(p) = \frac{1}{2}\left(p - \sqrt{p^2 + \frac{4k(N-k)}{N^2}(1-p)^2}\right). \tag{11}$$

Obviously, for all $k \in [1, N/2]$, $L_k(p) \leqslant 0$ with the equality holding only for $p=1$ (i.e., in the limit $t \to \infty$), which implies the nonexistence of ESD, as in the case of $|\Psi_{n,N}\rangle\langle\Psi_{n,N}|$. It is not difficult to verify that

$$|L_1(p)| \leqslant |L_2(p)| \leqslant \cdots \leqslant |L_{N/2}(p)|. \tag{12}$$

Since

$$\max\{L_k(p)\} = |L_{N/2}(p)| = \frac{1}{2}[p - \sqrt{p^2 + (1-p)^2}] \tag{13}$$

manifests dependence neither on k nor on N, the value of p_c for the W state is an invariant, i.e., its stability with respect to complete separability is unaffected by the system size, in transparent contrast with the scaling laws of the states $|\Psi_{n,N}\rangle$ and $|\Psi_N\rangle$. For $\varepsilon=0.01$ we have $p_c \simeq 0.868$, which is also shown in Fig. 1 for visual comparison. The fact that $|W_N\rangle$ are far more robust (with respect to the system size) than $|\Psi_{n,N}\rangle$ and $|\Psi_N\rangle$ does not generally imply that $|W_N\rangle$ are superior or more useful: each of them is suitable to its own domain of application.

In conclusion, we have investigated disentanglement dynamics for the generalized multiqubit GHZ-type states $|\Psi_{n,N}\rangle$ whose individual members are exposed to independent amplitude-damping channels. By deriving and analyzing negativities associated with all possible bipartitions of the system we have found that ESD never occurs for $|\Psi_{n,N}\rangle$, in sharp contrast to $|\Psi_N\rangle$, which suffers ESD if $|\alpha|<|\beta|$. Furthermore, we have also shown that, when $|\alpha|\geqslant|\beta|$, $|\Psi_N\rangle$ still lose their entanglement (in the sense of QESD) sooner than $|\Psi_{n,N}\rangle$ for any n and N. Thus, in all respects $|\Psi_{n,N}\rangle$ are more robust than $|\Psi_N\rangle$. The reason has been explained by the underlying quantum map $\mathcal{S}_j$ responsible for the AD mechanism combined with the different structures of the density matrices of the two states. Remarkably, with simple local bit-flip operations one is always able to transform $|\Psi_N\rangle$ to $|\Psi_{n,N}\rangle$ or vice versa. Our results suggest that it is better to use $|\Psi_{n,N}\rangle$ rather than $|\Psi_N\rangle$, though both possess the same amount of initial entanglement. Experimentally, the ESD phenomenon has been observed in the laboratory for entangled photon pairs [13] and atomic ensembles [14]. To test our results, one specific physical realization may be N identical two-level atoms prepared in states $|\Psi_N\rangle$ or $|\Psi_{n,N}\rangle$ which are resonantly coupled to N independent empty cavities. For a large N it is a demanding job, but for N up to six such states have been generated [11] in both atomic and photonic systems.

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