PHYSICAL REVIEW A **78**, 063836 (2008)

# Off-resonance slow light

R. N. Shakhmuratov[1,2] and J. Odeurs[2]
[1]*Kazan Physical-Technical Institute, Russian Academy of Sciences, 10/7 Sibirsky Trakt, Kazan 420029, Russia*
[2]*Instituut voor Kern- en Stralingsfysica, Katholieke Universiteit Leuven, Celestijnenlaan 200 D, B-3001 Leuven, Belgium*

(Received 14 October 2008; published 31 December 2008)

We consider the propagation of a light pulse in a medium with a single resonance. If the frequency of the pulse is tuned far from resonance and the pulse duration is much shorter than the lifetime of the excited state of the resonant particles in the medium (atoms in a gas, impurity ions in a solid, etc.), the group velocity of the pulse is appreciably reduced. It is shown that the slowing down of the group velocity of the pulse is accompanied with a pulse chirp, which produces a pulse broadening in time. It is proposed to use two samples in sequence with opposite chirps (up chirp and down chirp or vice versa) compensating the pulse broadening. Then the pulse can be delayed with almost no losses, distortion, and broadening. However, there is a maximum distance, beyond which the pulse experiences corruption. Pumping with an auxiliary laser beam can control the delay time of the light pulse in the medium. Conditions to eliminate the contribution of the dephasing processes in the pulse propagation are considered.

DOI: 10.1103/PhysRevA.78.063836 PACS number(s): 42.50.Gy

## I. INTRODUCTION

The tremendous progress in telecommunication and computing demands the development of new principles and devices, which employ laser pulses to transmit and process information with high bit rate. In this context all-optical signal processing is of great importance. A tunable all-optical delay line with wide bandwidth and a small size is one of the basic elements in optical information processing, e.g., buffering of optical data packets. The size of the delay line can be appreciably reduced with the help of the "slow light" phenomenon. For example, electromagnetically induced transparency (EIT), see a recent review on the topic in Ref. [1], can slow down the group velocity of the pulse to 17 m/sec [2]. It differs from the speed of light in vacuum, $c$, by a factor of $10^7$, which allows one to reduce significantly the size of the delay line based on EIT. However, in most cases the width of the EIT window usually does not exceed 15 MHz (see, for example, Ref. [3] and references therein). Therefore, pulses shorter than 10 ns cannot be delayed with EIT, if the coupling laser, creating the transparency window, is not extremely strong. There are other obstacles limiting the maximum fractional delay of the pulse, i.e., the ratio of the delay to the pulse duration, with EIT [4,5]. To date, according to Ref. [5], delays of only four pulse widths were observed in EIT [6]. This is, of course, without the use of the light-pulse storage technique [7,8], which can delay the pulse as long as 1 ms. A problem to obtain a large fractional delay in EIT is caused by the pulse broadening accompanied by an intensity drop of the pulse [9]. To increase the bandwidth of the "slow light" device, it was proposed to use spectral hole burning [3]. This idea was successfully tested in a hot rubidium vapor, where 5.8 ns pulses were delayed up to 43 ns [10]. Recently, slow light propagation was observed in rubidium [11–13] and cesium [14] vapors without their special preparation by coupling or pumping laser fields. 2 ns pulses were delayed up to 106 ns in four 10 cm long cells with rubidium vapor [12] and 275 ps (740 ps) pulses were delayed up to 6.8 ns (59 ns) in a 10 cm long cell with cesium vapor [14]. The delay is produced by tuning the frequency of the signal pulse between two strongly absorbing and widely spaced resonances (in the middle of the spectral doublet). This delay can be controlled by changing the temperature of the cell or by depletion of the atomic ground states by optical pumping. Control of the delay by optical pumping is made tunable with a fast reconfiguration time of hundreds of ns. This method is proved to be powerful and even allowed an all-optical delay of two-dimensional images [15]. In Ref. [16] we considered the physical processes involved in the slowing down of a laser pulse and analyzed the basic limitations for the pulse delay in a medium with two resonances. We also proposed to use solids with a doublet structure in the absorption spectrum to slow down pulses of picosecond duration or even shorter. Meanwhile, appropriate doublet structures in an absorption spectrum are not frequently available. Therefore, it could be a problem to find such a doublet if an operating device is restricted by a certain wavelength, e.g., 1350 nm or 1550 nm used practically in optical communication systems. In this paper we propose a way to avoid the use of a doublet structure and obtain the same excellent performance as the doublet structure provides. We verify and give further development of an old idea to slow down light pulses whose frequency is detuned from the resonant frequency of a medium. This idea was proposed and experimentally tested in rubidium vapor by Grischkowsky [17]. We found limitations of this method and propose a method to increase the maximum pulse delay in the off-resonant medium. We discuss the effect of dephasing mechanisms on the pulse propagation and we propose a way to minimize their effect.

The paper is organized as follows. In Sec. II we recall the adiabatic following condition and the adiabatic solution for the atomic coherence, and we find the exact and approximate solutions for the pulse propagation. In Sec. III we find the conditions when relaxation processes do not affect the pulse propagation. In Sec. IV the method of compensation of the pulse chirp is considered.

## II. PULSE PROPAGATION IN AN OFF-RESONANT MEDIUM

In this section we consider the off-resonant pulse propagation in a medium if its excitation is adiabatic. The adiabatic excitation means that the medium is left in the ground state after the pulse is gone and the contribution of the irreversible processes, such as incoherent scattering in all directions, is minimized.

### A. Adiabatic following concept

Following Grischkowsky [17] we introduce a pseudospin to describe the evolution of an atom, excited by a near resonant field $E(z,t)=E_s(z,t)\exp(-i\omega_s t+i\mathbf{k}_s\cdot\mathbf{z})$, where $\omega_s$ and $\mathbf{k}_s$ are the frequency and the wave vector of the field, $E_s(z,t)$ is a slowly varying amplitude. We assume that the field is a transform-limited bell-shaped pulse, i.e., its spectral phase is frequency independent. Feynman, Vernon, and Hellwarth [18] proposed the use of a pseudospin $\mathbf{S}$ ($S=1/2$) to describe the evolution of two atomic states interacting with a resonant field. In terms of conventional operators of spin 1/2, $\hat{S}_x$, $\hat{S}_y$, and $\hat{S}_z$, the Hamiltonian, which includes only two states, is

$$\mathcal{H}=\hbar\omega_0\hat{S}_z-2\hbar\Omega(\hat{S}_x\cos\omega_s t+\hat{S}_y\sin\omega_s t), \tag{1}$$

where $\omega_0$ is the resonant frequency, $\Omega=d_{eg}E_s(z,t)/2\hbar$, and $d_{eg}$ is the dipole matrix element for the transition from the ground state $g$ to the excited state $e$. For simplicity of notations we do not show the phase factors $\mathbf{k}_s\cdot\mathbf{z}$ in Eq. (1). By a canonical transformation by means of the unitary operator $U=\exp(i\omega_s t\hat{S}_z)$, the Hamiltonian (1) is transformed to $\widetilde{\mathcal{H}}=U\mathcal{H}U^{-1}+i\dot{U}U^{-1}$. The new Hamiltonian

$$\widetilde{\mathcal{H}}=\hbar\Delta\hat{S}_z-2\hbar\Omega\hat{S}_x \tag{2}$$

is time independent if $\Omega$ is constant, where $\Delta=\omega_0-\omega_s$ is the detuning of the field frequency $\omega_s$ from the resonant frequency $\omega_0$. Such an operation with the atomic-basis states is equivalent in nuclear magnetic resonance to the transformation to the rotating reference frame, where the rf field becomes constant [19].

If the detuning $\Delta$ would be zero, the spin would precess around the rf field amplitude, which is parallel to the $x$ axis in the rotating reference frame. As a result the spin moves up and down between the states $g$ and $e$, which means absorption and reemission of an RF quantum. Here, coordinates $x$, $y$, and $z$ are defined in a pseudospace, which is different from the real space where the pulse propagates (along $\mathbf{z}$ direction). If $|\Delta|\gg\Omega$ the spin precesses around the effective field $H_{eff}=\sqrt{\Delta^2+4\Omega^2}$, which makes a small angle $\theta=\tan^{-1}(2\Omega/\Delta)$ with the quantization axis $z$. For a pulsed field with a bell-shape envelope, whose coupling parameter $\Omega(t)$ smoothly rises to a maximum value and then smoothly decreases to zero, the direction of the effective field $H_{eff}$ undergoes a small excursion from the $z$ axis, making a maximum angle $\theta_{\max}=\tan^{-1}(2\Omega/\Delta)_{\max}$ at the pulse maximum, and then coming back to the $z$ axis.

One can assume that, if the change rate of the pulse, $\Delta_s$, is much smaller than the absolute value of the detuning $\Delta$, the spin will adiabatically follow the changing effective field, satisfying the relation

$$\langle S_x(t)\rangle\approx\frac{\Omega(t)}{\sqrt{\Delta^2+4\Omega^2(t)}}, \tag{3}$$

while the other components are $\langle S_z(t)\rangle\approx-\Delta/[2\sqrt{\Delta^2+4\Omega^2(t)}]$ and $\langle S_y(t)\rangle\approx0$. According to these relations, the atom is excited reversibly and no excitation is left when the pulse is gone. Only the dispersive (in-phase) component of the atomic response $\sim\langle S_x(t)\rangle$ is present, when the pulse is on, and the absorptive (out-of-phase) component $\sim\langle S_y(t)\rangle$ is negligible. In this case one can expect only a refractive index change with no dissipation of energy of the pulse. This is the first order of the adiabatic approximation.

### B. Analytical solution of the master equations

To find the higher-order adiabatic corrections along with the nonadiabatic contribution from the dissipative processes such as the spontaneous decay of the excited state and the dephasing of the atomic coherence, we derive the master equations for the spin components. They are

$$\dot{u}=-\Delta v-\gamma_d u, \tag{4}$$

$$\dot{v}=\Delta u-\Omega-\gamma_d v, \tag{5}$$

where $u=\langle S_x(t)\rangle$, $v=\langle S_y(t)\rangle$, and the change of $\langle S_z(t)\rangle$ is neglected since we apply the linear response approximation ($\langle S_z(t)\rangle\approx-1/2$). $\gamma_d=\gamma+\gamma_m$ is the decay rate of the atomic coherence, where $\gamma$ is produced by the spontaneous decay of the excited state and $\gamma_m$ is the contribution from the random local fields experienced by the atom from the environment. We use here the notations $u$ and $v$ for the atomic coherence, which are conventional in coherent spectroscopy, see, for example, Ref. [20].

These equations originate from the master equation for the slowly varying component $\sigma_{eg}=\rho_{eg}\exp(i\omega_s t-ik_s z)$ of the atomic density matrix $\rho$,

$$\dot{\sigma}_{eg}=-(\gamma_d+i\Delta)\sigma_{eg}+i\Omega(z,t). \tag{6}$$

By means of the Fourier transform

$$F(\nu)=\int_{-\infty}^{+\infty}f(t)e^{i\nu t}dt, \tag{7}$$

Eq. (6) for the bell-shaped pulse $[E_s(z,\pm\infty)=0]$ is reduced to an algebraic equation that can be solved easily. The solution is

$$\sigma_{eg}(\nu)=\frac{i\Omega(z,\nu)}{\gamma_d-i\nu+i\Delta}. \tag{8}$$

If the pulse changes much more slowly than the precession rate $\Delta$ ($\Delta_s\ll\Delta$), the adiabatic expansion [3,9,16] is applicable,

$$\sigma_{eg}(\nu)=\frac{i\Omega(z,\nu)}{\gamma_d+i\Delta}\sum_{k=0}^{\infty}\frac{(i\nu)^k}{(\gamma_d+i\Delta)^k}. \tag{9}$$

The inverse transform of Eq. (9) is

$$\sigma_{eg}(z,t) = \frac{i}{\gamma_d + i\Delta}\sum_{k=0}^{\infty}\frac{(-1)^k\partial^k\Omega(z,t)/\partial t^k}{(\gamma_d + i\Delta)^k}. \tag{10}$$

If the pulse duration is short compared with the dephasing time $T_2 = 1/\gamma_d$ but long compared with the precession period $T_p = 2\pi/\Delta$ (i.e., $\Delta \gg \Delta_s \gg \gamma_d$), only a few terms of expansion (10) have to be considered. Taking only the first three terms gives

$$\sigma_{eg}(z,t) = \frac{\Omega}{\Delta} + i\frac{1}{\Delta^2}\frac{\partial\Omega}{\partial t} - \frac{1}{\Delta^3}\frac{\partial^2\Omega}{\partial t^2} + \cdots . \tag{11}$$

Then the $u$ and $v$ components of the atomic coherence are

$$u \approx \frac{\Omega}{\Delta} - \frac{1}{\Delta^3}\frac{\partial^2\Omega}{\partial t^2}, \tag{12}$$

$$v \approx -\frac{1}{\Delta^2}\frac{\partial\Omega}{\partial t}. \tag{13}$$

Comparing this result with Eq. (3), we see that the next order adiabatic corrections give an additional small term $-\partial^2\Omega/\Delta^3\partial t^2$ to the $u$ component and a nonzero contribution to the $v$ component. For the leading edge of the pulse up to its maximum we have $\partial\Omega/\partial t > 0$, and hence the $v$ component is negative, which results in the pulse absorption. In this stage the energy of the pulse is stored in the excited state ($e$) atoms. For the trailing edge of the pulse, starting from its maximum, we have $\partial\Omega/\partial t < 0$, and hence the $v$ component is positive. This results in the pulse amplification by the atoms that stored the energy of the pulse during its rising stage. Both processes are balanced such that the pulse is not distorted but only delayed.

In the next section we show analytically that the $v$ component, Eq. (13), gives a reduction of the group velocity of the pulse. The $u$ component, Eq. (12), gives a phase shift of the pulse, due to the first term of the right-hand side (RHS) of Eq. (12), and a pulse chirp, due to the second term of the RHS of Eq. (12). Grischkowsky [17] derived Eq. (13) from Eq. (4) since it can be read as $v = -(\dot{u} + \gamma_d u)/\Delta$, and if $u$ is known from Eq. (3), obtained from qualitative speculations, then $v$ is easily found. The chirp-inducing term $-\partial^2\Omega/\Delta^3\partial t^2$ in the $u$ component was not found in Ref. [17]. In the next section we show that this term is crucial in achieving maximum pulse delay in an off-resonant medium.

### C. General solution of the wave equations

If the duration of the pulse is much longer than the oscillation period $T_\omega = 2\pi/\omega_s$ of its carrier frequency, then the propagation of the pulse in a resonant medium is described by the wave equation for the slowly varying amplitude $E_s(z,t)$,

$$\hat{L}E_s(z,t) = i\hbar\frac{\alpha\sigma_{eg}(z,t)}{d_{eg}}, \tag{14}$$

where $\hat{L}$ is the differential operator $\hat{L} = \partial_z + c^{-1}\partial_t$, $\alpha = 4\pi\omega_s N|d_{eg}|^2/\hbar c$, and $N$ is the concentration of resonant particles in the medium. To combine it with the matter equation (6), we use the wave equation for the coupling parameter $\Omega(z,t)$,

$$\hat{L}\Omega(z,t) = i\frac{\alpha}{2}\sigma_{eg}. \tag{15}$$

With the Fourier transform and the solution (8), this equation is reduced to

$$\left(\frac{\partial}{\partial z} - \frac{i}{c}\nu + A(\nu)\right)\Omega(z,\nu) = 0, \tag{16}$$

where

$$A(\nu) = \frac{\alpha/2}{\gamma_d - i\nu + i\Delta}. \tag{17}$$

Its solution is

$$\Omega(z,\nu) = \Omega(0,\nu)\exp[(i\nu z/c) - A(\nu)z], \tag{18}$$

where $\Omega(0,\nu)$ is the spectrum of the coupling parameter of the pulse [or of the pulse amplitude since $\Omega(0,\nu) \sim E_s(0,\nu)$] at the input of the sample $z=0$. The inverse Fourier transform of Eq. (18) gives the pulse envelope at a distance $z$ in the medium, if the Fourier components of the pulse at the input of the sample are known. This envelope is

$$E_s(z,t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} E_s(0,\nu)\exp[-i\nu(t - z/c) - A(\nu)z]d\nu. \tag{19}$$

This is the exact solution of the matter (6) and wave (14) equations, obtained without the adiabatic following approximation. We have to point out that, since Eq. (6) is derived in the linear response approximation, the solution (19) is applicable only for weak pulses.

### D. Gaussian pulse propagation in adiabatic conditions

We consider a pulse with a Gaussian envelope at the input: $E_s(0,t) = E_0\exp[-(\Delta_s t/2)^2]$. Its Fourier transform is $E_s(0,\nu) = (E_0 2\sqrt{\pi}/\Delta_s)\exp[-(\nu/\Delta_s)^2]$. If $|\Delta| \gg \Delta_s \gg \gamma_d$, we can apply the adiabatic following approximation and take only the first terms of expansion Eq. (9) up to $\nu^2$. Then

$$A(\nu) \approx -i\frac{\alpha}{2}\left(\frac{1}{\Delta} + \frac{\nu}{\Delta^2} + \frac{\nu^2}{\Delta^3}\right). \tag{20}$$

As already discussed in the previous section, the second term of this expansion is defined by the contribution of the $v$ component, and the third term is defined by the adiabatic correction $-\partial^2\Omega/\Delta^3\partial t^2$ to the $u$ component. We introduce the following parameters: $\varphi = \alpha z/2\Delta$, $t_d = \alpha z/2\Delta^2$, and $\Delta_{ch} = \sqrt{2|\Delta|^3/\alpha z}$. In terms of these parameters, $A(\nu)z$ in the integral (19) is

$$A(\nu)z = -i\varphi - i\nu t_d \mp i\frac{\nu^2}{\Delta_{ch}^2}, \tag{21}$$

where the minus sign before the last term is for $\Delta > 0$, and the plus sign is for $\Delta < 0$. With this approximation, the integral in Eq. (19) is calculated by means of the saddle-point method, Ref. [21]. The result is

$$E_s(z,t)=\frac{E_0\exp\left[i\varphi-\left(\frac{\Delta_{\mathrm{out}}^2}{4}\pm ib\right)(t-t_d)^2\right]}{\sqrt{1\mp iR}},\tag{22}$$

where

$$\Delta_{out}=\frac{\Delta_s}{\sqrt{1+R^2}}\tag{23}$$

is the spectral half-width of the output pulse, $R=(\Delta_s/\Delta_{ch})^2$, and

$$b=\frac{\Delta_s^4}{4\Delta_{ch}^2(1+R^2)}\tag{24}$$

is the pulse-chirp parameter. The output pulse acquires a linear chirp $b$, which is quantified in units of Hertz per second. This chirp is due to the group velocity (delay) dispersion, i.e., to the fact that different spectral components of the pulse acquire different phases in the medium not according to the linear law $\sim \nu t_d$. It is also possible to specify the amount of group delay dispersion (in units of sec$^2$/length) referred to a medium with unit length. Then the group delay dispersion $D_2=\partial^2 A(\nu)/\partial\nu^2|_{\nu=0}$ is the second derivative of the change in spectral phase of the output pulse with respect to the angular frequency in a medium of 1 cm or 1 mm length. According to Eq. (20) this value is $D_2=\mp\alpha/\Delta^3$.

From the approximation equation (21) it follows that the phase distribution has a parabolic dependence $\pm\nu^2/\Delta_{ch}^2$ with the parabola looking up for $\Delta>0$, and looking down for $\Delta<0$. As a result we have a chirp up (the instantaneous frequency rises with time) if $\Delta>0$ and a chirp down (the instantaneous frequency decreases with time) if $\Delta<0$. The amplitudes of the spectral components of the Gaussian pulse are distributed according to the exponent whose argument is the parabola $-\nu^2/\Delta_s^2$. If $\Delta_{ch}^2\ll\Delta_s^2$, which means that the parabola $\pm\nu^2/\Delta_{ch}^2$ varies much faster than $-\nu^2/\Delta_s^2$, then $R\gg1$ and the duration of the output pulse is appreciably increased (the pulse is stretched) and its amplitude is decreased. According to Eq. (23), for $R\to\infty$ the spectral half-width of the output pulse $\Delta_{out}$ tends to $\Delta_{ch}^2/\Delta_s$, which is much smaller than the spectral half-width of the input pulse, $\Delta_s$. In this case the chirp parameter $b$ tends to $\Delta_{ch}^2/4$.

The intensity of the output pulse, $I_s(z,t)=|E_s(z,t)|^2$, is

$$I_s(z,t)=I_0\frac{\exp\left[-\frac{1}{2}\Delta_{out}^2(t-t_d)^2\right]}{\sqrt{1+R^2}},\tag{25}$$

where $I_0=|E_0|^2$. According to this equation, the intensity of the pulse drops as $(\Delta_{ch}/\Delta_s)^2$, if $\Delta_{ch}^2\ll\Delta_s^2$.

We conclude that the first term of the adiabatic expansion (20) gives the phase shift $\varphi$ of the pulse (due to the main part of the $u$ component), the second term produces the pulse delay $t_d$ (due the $v$ component), and the third term results in the pulse chirp with the rate $2b$ (due to the small correction to the $u$ component).

## III. ELIMINATION OF THE DEPHASING MECHANISMS

In the previous section we considered the pulse propagation if $|\Delta|\gg\Delta_s\gg\gamma_d$. On this condition the contribution of the dephasing processes can be neglected and the evolution of the atomic coherence becomes almost dynamical, which is well described by the solution given by Eqs. (12) and (13). In real absorptive media there are many dephasing processes whose contribution exceeds many times the dephasing induced by the natural broadening of the absorption line, see, for example, Ref. [22]. Therefore, to satisfy the inequality $\Delta_s\gg\gamma_d$, one has to work with very short pulses. In this section we consider conditions when the contribution of extra broadening mechanisms on top of the natural broadening can be eliminated and the limitation imposed upon the pulse duration is moderated.

### A. Inhomogeneous broadening

In atomic vapors absorption lines are inhomogeneously broadened due to the Maxwell distribution of velocities of the atoms. This results in Doppler broadening due to the Gaussian distribution of resonant frequencies $\omega_0$,

$$f_{inh}(\omega_0-\omega_c)=\frac{\exp[-(\omega_0-\omega_c)^2/\Gamma_{inh}^2]}{\sqrt{\pi}\Gamma_{inh}},\tag{26}$$

with typical values of $\Gamma_{inh}=500$ MHz for hot alkaline vapors, which is two orders of magnitude larger than the half-width $\gamma_d$ of the homogeneous line (see, for example, Ref. [12]). The Fourier transform of the response of a unit volume of the vapor to the field excitation, i.e., of the RHS of Eq. (14), is the average

$$N\langle\sigma_{eg}(\nu)\rangle=\frac{iN\Omega(\nu)}{\sqrt{\pi}\Gamma_{inh}}\int_{-\infty}^{+\infty}\frac{e^{-(x/\Gamma_{inh})^2}}{\gamma_d+i(x+\Delta-\nu)}dx,\tag{27}$$

over all spectral packets with resonant frequencies $\omega_0$ distributed according to Eq. (26) around the central frequency $\omega_c$, where $x=\omega_0-\omega_c$ and $\Delta=\omega_c-\omega_s$ is the detuning of the field frequency with respect to $\omega_c$.

If $\Delta=0$, the frequency dependence of the absorptive, $N\,\mathrm{Im}\langle\sigma_{eg}(\nu)\rangle$, and dispersive, $N\,\mathrm{Re}\langle\sigma_{eg}(\nu)\rangle$, components of the average response are typical for an inhomogeneously broadened Gaussian absorption line in the domain $(\omega_c-\Gamma_{inh},\omega_c+\Gamma_{inh})$ near the central frequency of the line. This is because close to the central frequency the homogeneous component $1/[\gamma_d+i(x-\nu)]$ varies much faster than the inhomogeneous component $\exp[-(x/\Gamma_{inh})^2]$ in the convolution integral, Eq. (27), and hence $\exp[-(x/\Gamma_{inh})^2]$ can be removed from the integral as a constant factor with argument $x=\nu$. Therefore, for example, the imaginary part of Eq. (27) reproduces the frequency dependence of the inhomogeneous component, i.e., $N\,\mathrm{Im}\langle\sigma_{eg}(\nu)\rangle\approx\sqrt{\pi}N\Omega(\nu)\exp[-(\nu/\Gamma_{inh})^2]/\Gamma_{inh}$. As a result the maximum of the inhomogeneously broadened absorption line is reduced as $\approx\sqrt{\pi}\gamma_d/\Gamma_{inh}$ with respect to the maximum of the homogeneous line.

If the detuning $\Delta$ is much larger than $\Gamma_{inh}$, the homogeneous component $1/[\gamma_d+i(x+\Delta-\nu)]$ varies much more slowly than the inhomogeneous component $\exp[-(x/\Gamma_{inh})^2]$

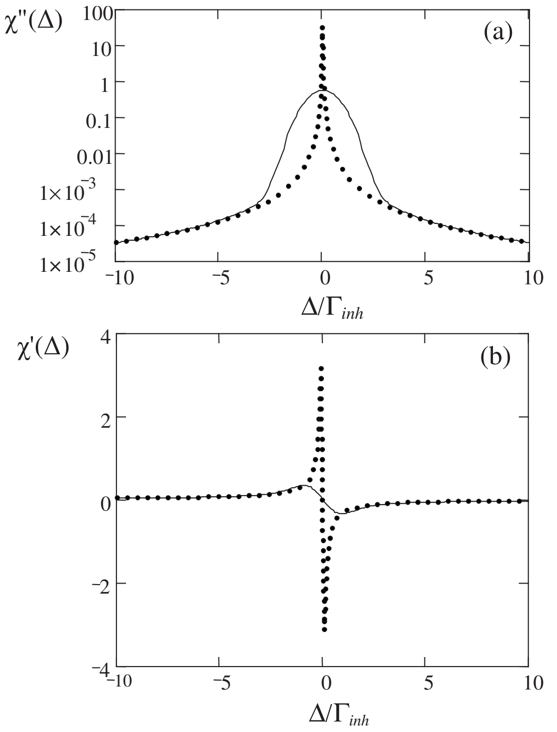

FIG. 1. Detuning $\Delta = \omega_c - \omega_s$ dependence of the absorptive (a) and dispersive (b) components of the atomic response. The dots are for the homogeneous line, i.e., for $\chi''(\Delta) = \gamma_d / \pi(\Delta^2 + \gamma_d^2)$ in (a) and for $\chi'(\Delta) = -\Delta / \pi(\Delta^2 + \gamma_d^2)$ in (b). The solid line is for the inhomogeneous line, i.e., for $\langle \chi''(\Delta) \rangle$ in (a) and for $\langle \chi'(\Delta) \rangle$ in (b). Here for simplicity of notations we drop the common factor $\pi N |d_{eg}|^2$ in $\chi'(\Delta)$ and $\chi''(\Delta)$, used in the conventional definition of the atomic susceptibility. The ratio of the homogeneous to the inhomogeneous half-width is $\gamma_d / \Gamma_{inh} = 10^{-2}$. The vertical scale for (a) is logarithmic. In both plots the vertical scales are in units of $1/\gamma_d$.

in the convolution integral, Eq. (27). As a result the homogeneous component can be removed from the integral in Eq. (27) as a constant factor with $x = 0$, and then the atomic response coincides with the response of a homogeneous ensemble. Figure 1 shows the frequency dependence of the absorptive (a) and dispersive (b) components of the atomic response. It is clearly seen that if $|\Delta| > 3\Gamma_{inh}$ the frequency dependence of these components coincides with the homogeneous one and the inhomogeneous broadening becomes ineffective.

### B. Homogeneous broadening

In solids with impurity ions the absorption line at room temperature is mostly broadened by phonons. If elastic scattering of phonons by an impurity is the dominant contribution to the line broadening, the line is narrow (it is the so-called zero phonon line). For example, in ruby the linewidth of the $R_1$ line is 11 $\mathrm{cm}^{-1}$ at 300 K [23]. Therefore, to obtain slow-light propagation one has to use pulses shorter than a picosecond, detuned at least by 100 $\mathrm{cm}^{-1}$ from resonance. At the temperature of liquid nitrogen (77 K), the linewidth narrows to $\sim 0.1$ $\mathrm{cm}^{-1}$ [24] because the phonon contribution drops with the temperature decrease. With further temperature decrease, the linewidth almost does not change anymore since it is inhomogeneously broadened due to crystal imperfections, while the phonon contribution becomes insignificant [24]. At liquid helium temperature (4 K), the linewidth of the homogeneous components of the absorption line is mostly defined by the magnetic interactions of the electron and the nuclear spins of the impurity with the environment and with nearby impurities, but not by phonons.

These interactions produce a fast fluctuation of the resonant frequency $\omega_0 + \delta(t)$ with $\langle \delta(t) \rangle = 0$, where $\langle \cdots \rangle$ means the time average. If the correlation time, $\tau_c$, of this random process is short with respect to $1/\sqrt{\langle \delta^2(t) \rangle}$, then the decay rate of the optical coherence due to this random process is $\gamma_m = \langle \delta^2(t) \rangle \tau_c$. If $\tau_c$ is comparable or longer than $1/\sqrt{\langle \delta^2(t) \rangle}$, the decay rate is defined by $\sqrt{\langle \delta^2(t) \rangle}$, see, for example, Ref. [25] and references therein.

For a homogeneous spectral packet of the inhomogeneously broadened line whose resonant detuning $\Delta$ is zero, the atom-field interaction Hamiltonian in the "rotating reference frame," Eq. (2), is

$$\tilde{\mathcal{H}} = -2\hbar\Omega\hat{S}_x. \tag{28}$$

Then the atomic pseudospin is quantized along the $x$ direction. Actually, there is also a term $\hbar\delta(t)\hat{S}_z$ in this Hamiltonian due to the magnetic interactions. If the resonant field is strong and the precession frequency $\Omega$ of the pseudospin around the $x$ direction is much faster than the fluctuation rate $1/\tau_c$ of the random field $\delta(t)$, its contribution is averaged out and $\gamma_m$ tends to zero [26–28].

For a homogeneous spectral packet whose detuning is large ($|\Delta|\tau_c \gg 1$, $|\Delta| \gg \sqrt{\langle \delta^2(t) \rangle}$), one could expect that the excitation by a weak laser field ($\Omega \ll |\Delta|$ and $\Omega\tau_c \ll 1$) also experiences a negligible contribution of the magnetic interactions to the coherence dephasing. The argument to support this hypothesis is that the fast precession of the pseudospin with frequency $|\Delta|$ should average to zero the contribution of a small amplitude ($\sqrt{\langle \delta^2(t) \rangle} \ll |\Delta|$) jitter of the resonant frequency whose rate $1/\tau_c$ is also small with respect to $\Delta$. To prove this, in the next section we consider the simplest model of the dephasing process, described by the random-telegraph-signal model [29,30].

### C. Master equations

Assume that a spin 1/2 (nuclear or electron), which is in the close vicinity of an impurity ion, undergoes random flips with a rate $1/2\tau_c$. When this spin is up, it induces a shift of the resonant frequency of the ion to $\omega_0 + \delta$. When it flips down, the frequency of the ion is $\omega_0 - \delta$. The correlation function of the random frequency shift $\delta(t)$, which takes two values $+\delta$ and $-\delta$, is $\langle \delta(t)\delta(0) \rangle = \delta^2 \exp(-t/\tau_c)$ [29,30]. For this process one can introduce the partial density matrix with elements $\sigma_{eg}^{(+)}$ and $\sigma_{eg}^{(-)}$, which correspond to the $+\delta$ and $-\delta$ frequency-shift events, respectively (see, for example, Refs. [25,31] and references therein). According to Eq. (6) and the general formalism of the telegraph-noise model, they are described by two equations,

$$\dot{\sigma}_{eg}^{(\pm)} = -M_{\pm}\sigma_{eg}^{(\pm)} + \frac{\sigma_{eg}^{(\mp)}}{2\tau_c} + \frac{i\Omega}{2}, \tag{29}$$

where $M_{\pm}=\gamma+(1/2\tau_c)+i(\Delta \pm \delta)$, $\gamma$ is the spontaneous decay rate of the coherence $\sigma_{eg}$, and the factor $1/2$ in the last term of the equation appears because the probability of each state, plus or minus, is $1/2$. The "in" and "out" terms with coefficients $1/2\tau_c$ in the equations describe input and output flows of the particles to and from the partial ensemble ($+$ or $-$), respectively. The "in" term is the second term in the RHS of Eq. (29) and the "out" term is contained in $M_{\pm}$. These terms describe the random walk of an impurity ion between the two states with resonant frequencies $\omega_0+\delta$ and $\omega_0-\delta$. These equations model the contribution $\gamma_m$ from the magnetic interactions.

One can introduce the value $\langle\sigma_{eg}(t)\rangle=\sigma_{eg}^{(+)}(t)+\sigma_{eg}^{(-)}(t)$, which is the net coherence of an ion whose resonant frequency is subject to the random telegraph signal $\delta(t)$. This value can be found from the solution of the two equations,

$$\langle\dot{\sigma}_{eg}\rangle = -(\gamma+i\Delta)\langle\sigma_{eg}\rangle - i\delta\langle\sigma_{eg}\rangle_A + i\Omega, \tag{30}$$

$$\langle\dot{\sigma}_{eg}\rangle_A = -\left(\gamma+\frac{1}{\tau_c}+i\Delta\right)\langle\sigma_{eg}\rangle_A - i\delta\langle\sigma_{eg}\rangle, \tag{31}$$

where $\langle\sigma_{eg}(t)\rangle_A=\sigma_{eg}^{(+)}(t)-\sigma_{eg}^{(-)}(t)$.

To know how this random process affects any spectral component $E_s(\nu)$ of the pulse $E_s(t)$, we consider, for example, one component with $\nu=0$. It corresponds to a cw field with constant amplitude ($\Omega=$constant) and resonant detuning $\Delta$. For this component one can neglect the time derivatives in Eqs. (30) and (31) and find the solution for $\langle\sigma_{eg}(\nu)\rangle|_{\nu=0}=\langle\sigma_{eg}\rangle$, which is

$$\langle\sigma_{eg}\rangle = \frac{i\Omega}{\gamma+\gamma_{tlg}+i\Delta}, \tag{32}$$

where

$$\gamma_{tlg} = \frac{\delta^2\tau_c}{1+(\gamma+i\Delta)\tau_c} \tag{33}$$

is a complex number. Comparing this solution with Eq. (8), we conclude that the real part of this number,

$$\mathrm{Re}(\gamma_{tlg}) = \delta^2\tau_c\frac{1+\gamma\tau_c}{(1+\gamma\tau_c)^2+\Delta^2\tau_c^2}, \tag{34}$$

is just the dephasing rate $\gamma_m$, induced by the magnetic interactions, and the imaginary part,

$$\mathrm{Im}(\gamma_{tlg}) = -\delta^2\tau_c\frac{\Delta\tau_c}{(1+\gamma\tau_c)^2+\Delta^2\tau_c^2}, \tag{35}$$

is the contribution of the dephasing process to the resonant frequency $\omega_0$. If $\Delta^2\tau_c^2 \gg 1$, the contribution of the magnetic interactions to the coherence dephasing is cancelled.

### D. Numerical example

Here we consider the propagation of the Gaussian pulse in a solid containing impurity ions and compare our approximation given by Eqs. (22) and (25) with the exact solution, Eq. (19), for weak pulses. At low temperature the contribution of phonons to the line broadening of impurity ions is not dominant. Then, three physical parameters are of importance, i.e., (i) the lifetime of the excited state, (ii) the inhomogeneous broadening, and (iii) the homogeneous dephasing induced by magnetic interactions.

(i) For rare-earth ions and for the iron group (chromium in particular), incorporated into a solid, the lifetime of the excited state ranges from hundreds of $\mu$s (for example, 500 $\mu$s for $\mathrm{Pr}^{3+}$:$\mathrm{LaF}_3$, Ref. [26]) up to several ms (4 ms for ruby, Ref. [27]).

(ii) In a perfect crystal, the inhomogeneous width can be as small as a few GHz.

(iii) At liquid helium temperature the homogeneous dephasing time $T_2$, caused by magnetic interactions, can be as long as 22 $\mu$s for $\mathrm{Pr}^{3+}$:$\mathrm{LaF}_3$ [26] and 16 $\mu$s for ruby [27]. However such a long dephasing time is typical for dilute samples where the concentration of impurity ions is very low. With concentration increase, the dephasing time $T_2$ shortens. Usually, to slow down the spin dynamics of the host spin, influencing an impurity ion, weak permanent magnetic fields of the order of 100 G are applied [22].

To give a hint what would be the propagation of an off-resonant pulse, we consider low-temperature ruby with inhomogeneous half-width $\Gamma_{inh}=3$ GHz. If we take a detuning $\Delta=10$ GHz, then, according to Sec. III A, the inhomogeneous broadening is ineffective. Assume that the concentration of the chromium ions is not low and a weak permanent magnetic field is applied along the symmetry axis of the crystal. Then, we can take for $\gamma_m$, induced by the magnetic interactions, the maximum possible value of $\gamma_m=20$ MHz, which corresponds to 40 MHz full width of the homogeneous line. Then, if we take $\gamma_m=\delta^2\tau_c$ and assume that $\delta \sim 1/\tau_c$, the contribution of the magnetic dephasing according to Eq. (34) reduces for a resonant detuning of $\Delta=10$ GHz down to 80 Hz, which is comparable with the natural broadening for ruby, $\gamma$. In the simulations below we take $\gamma_d=100$ Hz.

At room temperature the phonon broadened $R_1$ line in ruby has a half linewidth $\Gamma_{ph}=165$ GHz. A typical value of the absorption coefficient at 300 K for 0.5 weight percent of $\mathrm{Cr_2O_3}$:$\mathrm{Al_2O_3}$, corresponding to a chromium density $N=1.62\times10^{20}/\ \mathrm{cm}^3$, is $\alpha_0=4\ \mathrm{cm}^{-1}$. This coefficient is related to $\alpha$, defined in Sec. II, as $\alpha_0=\alpha/\Gamma_{ph}$. Then the delay of the pulse in a sample of length $z$ will be $t_d=\alpha_0 z\Gamma_{ph}/2\Delta^2$. If we take $z=10$ cm, then the delay time of the pulse is 5.25 ns, which corresponds to its group velocity $v_g=c/16$.

We consider the propagation of two pulses of duration $t_{p1}=375$ ps and $t_{p2}=187$ ps (full width at half maximum of the pulse intensity, $t_{p_{1,2}}=2\sqrt{2\ln 2}/\Delta_{s_{1,2}}$). We take $\alpha_0 z\Gamma_{ph}=6.6$ THz, which corresponds to the parameters taken above ($\alpha_0=4\mathrm{cm}^{-1}$, $z=10$ cm, and $\Gamma_{ph}=165$ GHz). Then, the delay of the pulses, detuned from resonance by $\Delta=10$ GHz, is $t_d=5.2$ ns. The pulse-chirp parameter is $\Delta_{ch}=550$ MHz, which is 1.8 (3.6) times smaller than the $\Delta_s$ parameter of the pulse with duration 375 ps (187 ps). Therefore, the pulses will be broadened and drop in intensity. According to the analytical solution, given by Eq. (22), the pulse with duration 375 ps (187 ps) will be broadened 3.5 (13.2) times. With no pulse

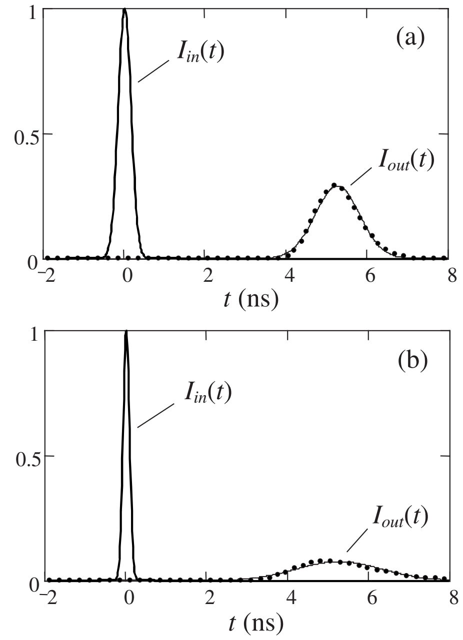

FIG. 2. Time dependence of the intensity of the pulse at the input of the sample, $I_{in}(t)=E(0,t)E^{*}(0,t)$ (thick solid line), and at the output of the sample, $I_{out}(t)=E(z,t)E^{*}(z,t)$. The dots correspond to the exact solution, Eq. (19), and the thin solid line corresponds to the approximation, given by Eq. (22), and hence Eq. (25). All plots are normalized to the maximum intensity of the input pulse, $I_{in}(0)$. Plot (a) is for the input pulse with duration 375 ns, and plot (b) is for the 187 ns pulse.

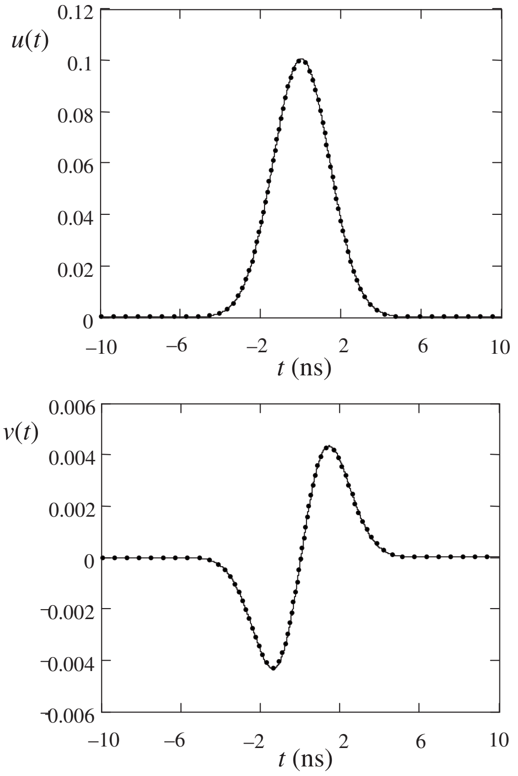

FIG. 3. Comparison of the numerical solution of the matter equations (4) and (5) for the $u$ and $v$ components (solid line) with the analytical approximation, Eqs. (12) and (13) (dots). The vertical scale is in units $\Omega/\Delta_s$.

broadening, its fractional delay (the ratio of the pulse delay to its duration) would be 14 and 28 for the 375 ps and 187 ps pulses, respectively. If the broadening is taken into account, then the fractional delay is only 4 (2) for the 375 ps (187 ps) pulse. Figure 2 shows the time evolution of the intensity of the output pulse. The bold solid line is the intensity of the input pulse. The exact solution [module squared of Eq. (19)] is shown by dots and the approximate solution, Eq. (25), is shown by the thin solid line. In both plots we disregard $z/c$, which is small. The exact and approximate solutions coincide excellently. From these plots it is clearly seen that a decrease of the pulse intensities and an increase of their widths are produced by the chirp of the pulses. The contribution of the irreversible processes due to coherence decay is negligibly small. In the next section we show how to eliminate a pulse chirp and avoid pulse broadening.

Concluding this section we show that the polarization of the medium ($u$ and $v$ components) is well described by the approximation given in Eqs. (12) and (13). Figure 3 shows the comparison of the numerical solution of the matter equations (4) and (5), with our analytical approximation (12) and (13), for the pulse with duration 375 ps and frequency, detuned from resonance by $\Delta=10$ GHz. The decay rate of the atomic coherence is $\gamma_d=100$ Hz. The difference between two solutions is indistinguishable. It should be noted that the third-order adiabatic correction, which gives an additional small term $-\partial^2\Omega/\Delta^3\partial t^2$ in Eq. (12), is approximately $2\times 10^2$ times smaller than the first main term, $\Omega/\Delta$. In spite of its smallness this term produces pulse stretching and chirping in a thick sample.

## IV. COMPENSATION OF A PULSE CHIRP

If we would have a doublet structure in a spectrum and the pulse frequency is tuned in between the two resonances, a pulse chirp would not appear. This is because the red and blue shifted resonances produce a pulse chirp with opposite sign and, hence, they are compensated [16].

If such a doublet is not available in the spectrum, one could consider the possibility to place another sample in the beam path. The estimated group delay dispersion for the sample, taken in the previous section as an example, is $D_2=-1.6\times 10^9$ fs$^2$/mm. For example, the group delay dispersion of silica is 36 fs$^2$/mm at the 800 nm wavelength. Therefore, to compensate the pulse chirp, produced in our sample, one needs $4.6\times 10^3$ km of silica, which is unrealistic. There is another way to solve the problem, if one can find a resonance in a sample with a different impurity. If the pulse frequency is red detuned for our sample [sample (a)] and blue detuned for the resonance in sample (b), then by an appropriate adjustment of the length of the sample (b), one can compensate the pulse chirp.

Following Sec. II it is easy to show that the pulse propagation through two samples in a row is described by the equation

$$E_s(z_s,t)=\int_{-\infty}^{+\infty}\frac{E_s(0,\nu)}{2\pi}e^{-i\nu(t-z_{ab}/c)-A_a(\nu)z_a-A_b(\nu)z_b}d\nu, \quad (36)$$

where $z_a$ and $z_b$ the physical lengths of samples (a) and (b), respectively, $z_{ab}=z_a+z_b$, and

$$A_{a,b}(\nu) = \frac{\alpha_{a,b}/2}{\gamma_{a,b} - i\nu + i\Delta_{a,b}}. \tag{37}$$

The parameters $\alpha_{a,b}$ and $\Delta_{a,b}$ are assumed to be different for both samples. For simplicity, in the subsequent simulations we take $\gamma_a=\gamma_b=100$ Hz, since their contribution is negligible. For sample (a) we take the parameters that are used for our numerical example in Sec. III. For sample (b) we take $\Delta_b$ and $z_b$ such that $\alpha_a z_a/\Delta_a^3=-\alpha_b z_b/\Delta_b^3$. Then, one can expect that the pulse chirp, acquired in sample (a), is compensated in sample (b), and the pulse coming out of the composite sample $(a)+(b)$ resumes its shape and amplitude.

For simplicity we take $\alpha_a=\alpha_b=\alpha$. To satisfy the adiabatic condition, $|\Delta_{a,b}| \gg \Delta_s \gg \gamma_d$, we choose the detuning for sample (b) $\Delta_b=-\Delta_a/2$, which is 5 times larger than $\Delta_s$ for a 375 ns pulse and 2.5 times larger than $\Delta_s$ for a 187 ns pulse. For such a detuning from resonance, the group velocity of the pulse, $V_g$, in sample (b) is 63 times smaller than $c$. For a complete compensation of the pulse chirp the physical length of sample (b) is to be $z_b=z_a/8=1.25$ cm. To calculate analytically the integral in Eq. (36), we apply the same approximation as used in the derivation of the solution, given by Eq. (22). We find that the amplitude of the output pulse is then described by the expression

$$E_s(z_s,t) = E_0 \exp[i\varphi_{ab} - \Delta_s^2(t-t_{ab})^2/4], \tag{38}$$

where the phase of the field is

$$\varphi_{ab} = \alpha\left(\frac{z_a}{2\Delta_a} + \frac{z_b}{2\Delta_b}\right), \tag{39}$$

and the net delay time of the pulse, acquired in the two samples, is

$$t_{ab} = \alpha\left(\frac{z_a}{2\Delta_a^2} + \frac{z_b}{2\Delta_b^2}\right). \tag{40}$$

Figure 4 shows the comparison of the analytical approximation, Eq. (38), with the exact solution, Eq. (36), for the 375 ns pulse. The plots clearly demonstrate that the pulse acquires in sample (b) an additional delay of 2.6 ns. The pulse almost resumes its amplitude and width. However, because of the fourth term of expansion $A_{a,b}(\nu)$, which is $-i\alpha_{a,b}\nu^3/\Delta_{a,b}^4$ in samples (a) and (b) [not shown in Eq. (20)], the pulse is distorted (see, Refs. [9,16] for details). The contributions of these terms are not compensated. This is because they are even functions of the resonant detunings $\Delta_a$ and $\Delta_a$. Therefore, their contribution is additive, i.e., accumulated, and it is impossible to suppress them.

It is convenient to combine them in one term,

$$[A_a(\nu)z_a + A_b(\nu)z_b]^{(3)}_{\nu=0} = \frac{-2i}{\Delta_{dst}^3}, \tag{41}$$

where the left-hand side of Eq. (41) is the third derivative of the change in spectral phase of the pulse for $\nu=0$ and $\Delta_{dst}$ is the distortion parameter, introduced in Refs. [9,16]. If $\Delta_s \lesssim \Delta_{dst}$, the pulse distortion is small. Otherwise the pulse is corrupted.

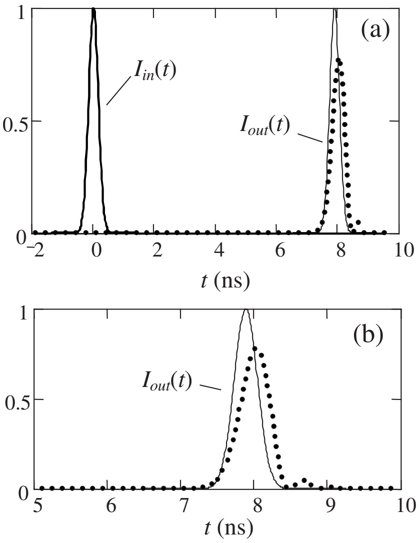

FIG. 4. Time evolution of the intensity, $I_{out}(t)$, of a 375 ns pulse at the output of the composite sample. The thin solid line is the result of the approximation, Eq. (38), and the dots correspond to the exact solution, Eq. (36). Plot (b) is a zoom in of plot (a).

For a 375 ns pulse we have $\Delta_s=1$ GHz and $\Delta_{dst}=696$ MHz, which means that $\Delta_s/\Delta_{dst}=1.44$. Therefore, the pulse experiences some corruption. For a 187 ns pulse we have $\Delta_s=2$ GHz, which is almost three times larger than $\Delta_{dst}=696$ MHz ($\Delta_s/\Delta_{dst}=2.9$). Figure 5 shows the 187 ns pulse propagation through the compound medium with the same parameters as in Fig. 4. The pulse experiences a breakup and its intensity drops.

In Refs. [9,16] it is shown that this breakup is mainly caused by the third-order dispersion and the output pulse is well described by the expression

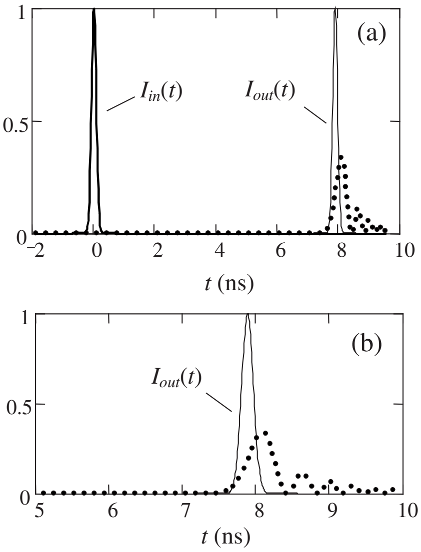

FIG. 5. Time evolution of the intensity of a 187 ns pulse at the output of the composite sample. The parameters of the samples and notations are the same as in Fig. 4.

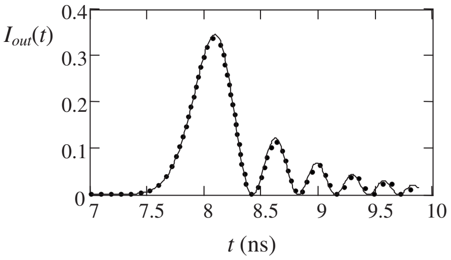

FIG. 6. Comparison of the exact solution (dots), Eq. (36), with the analytical approximation (solid line), Eq. (42), for the intensity $I_{out}(t)$ of a 187 ns pulse at the output of the composite sample. The parameters of the samples and notations are the same as in Fig. 5.

$$E_s(z_s,t) = \Delta_{dst} E_0 e^{i\varphi_{ab}} \int_{-\infty}^{+\infty} E_s(0,t-t_{ab}-\tau)\mathrm{Ai}(-\Delta_{dst}\tau)d\tau, \tag{42}$$

where $\mathrm{Ai}(-\Delta_{dst}\tau)$ is the Airy function. Figure 6 clearly shows that this approximation fits well the exact solution, Eq. (36). Therefore, no higher-order dispersions higher than the third order contribute appreciably.

There is a trade off between the pulse broadening due to the pulse chirp and the pulse corruption due to the distortion induced by the fourth term of the adiabatic expansion, Eq. (20). One can make a not complete compensation of the pulse broadening such that the narrowed width $\Delta_{out}$, Eq. (23), is smaller than the distortion parameter, $\Delta_{dst}$. Then the pulse is essentially not corrupted but only broadened. In both cases its intensity reduces.

## V. CONCLUSION

We considered the propagation of a pulse in an optically dense medium, having a single resonance, if the pulse frequency is far detuned from this resonance. Conditions if relaxation processes are ineffective and the atoms in the medium evolve purely dynamically are found. The condition for the adiabatic evolution of the atomic coherence it specified. It is shown that even if the atomic dynamics is adiabatic, the output pulse is broadened in time. This is due to the group delay dispersion, which produces a chirp of the pulse frequency and a stretching of its length. We propose to use a composite medium with two components having different resonant frequencies. If the pulse frequency is red detuned with respect to one resonance and blue detuned with respect to the other, then it is possible to choose proper lengths of the two components or the right concentrations of the resonant particles in the components to compensate the pulse chirp. Then the pulse is not broadened. However, there is a critical length (optical density) of the material, beyond which the pulse experiences a corruption. One can avoid this corruption, but at the expense of pulse broadening.

A delay line, based on this effect, can be controlled by optical pumping out of the ground state atoms with an auxiliary laser beam that is resonant to a different transition in the same way it was done in Ref. [14]. One can also consider the possibility to work with a fiber doped with trivalent erbium ions ($\mathrm{Er}^{+3}$). The transition between the ground state ${}^4I_{15/2}$ and ${}^4I_{13/2}$ multiplet is in the region of 1540 nm, which is practically used in telecommunication. Cooling the fiber to decrease the interaction of phonons with the impurity ion may help to make the evolution of the optical coherence induced between the ground and excited states of the ion purely dynamical.

## ACKNOWLEDGMENT

This work was supported by the FWO Vlaanderen.

[1] M. Fleischhauer, A. Imamoglu, and J. P. Marangos, Rev. Mod. Phys. **77**, 633 (2005).

[2] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature (London) **397**, 594 (1999).

[3] R. N. Shakhmuratov, A. Rebane, P. Mégret, and J. Odeurs, Phys. Rev. A **71**, 053811 (2005).

[4] A. B. Matsko, D. V. Strekalov, and L. Maleki, Opt. Express **13**, 2210 (2005).

[5] R. W. Boyd, D. J. Gauthier, A. L. Gaeta, and A. E. Willner, Phys. Rev. A **71**, 023801 (2005).

[6] A. Kasapi, M. Jain, G. Y. Yin, and S. E. Harris, Phys. Rev. Lett. **74**, 2447 (1995).

[7] D. F. Phillips, A. Fleischhauer, A. Mair, R. L. Walsworth, and M. D. Lukin, Phys. Rev. Lett. **86**, 783 (2001).

[8] C. Liu, Z. Dutton, C. H. Behroozi, and L. V. Hau, Nature (London) **409**, 490 (2001).

[9] R. N. Shakhmuratov and J. Odeurs, Phys. Rev. A **71**, 013819 (2005).

[10] R. M. Camacho, M. V. Pack, and J. C. Howell, Phys. Rev. A **74**, 033801 (2006).

[11] H. Tanaka, H. Niwa, K. Hayami, S. Furue, K. Nakayama, T. Kohmoto, M. Kunitomo, and Y. Fukuda, Phys. Rev. A **68**, 053801 (2003).

[12] R. M. Camacho, M. V. Pack, and J. C. Howell, Phys. Rev. A **73**, 063812 (2006).

[13] M. R. Vanner, R. J. McLean, P. Hannaford, and A. M. Akulshin, J. Phys. B **41**, 051004 (2008).

[14] R. M. Camacho, M. V. Pack, J. C. Howell, A. Schweinsberg, and R. W. Boyd, Phys. Rev. Lett. **98**, 153601 (2007).

[15] R. M. Camacho, C. J. Broadbent, I. Ali-Khan, and J. C. Howell, Phys. Rev. Lett. **98**, 043902 (2007).

[16] R. N. Shakhmuratov and J. Odeurs, Phys. Rev. A **77**, 033854 (2008).

[17] D. Grischkowsky, Phys. Rev. A **7**, 2096 (1973).

[18] R. P. Feynman, F. L. Vernon, and R. W. Hellwarth, J. Appl. Phys. **28**, 49 (1957).

[19] A. Abragam, *The Principles of Nuclear Magnetism* (Clarendon Press, Oxford, 1961).

[20] L. Allen and J. H. Eberly, *Optical Resonance and Two-level Atoms* (Wiley, New York, 1975).

[21] J. Mathews and R. L. Walker, *Mathematical Methods of Physics* (Benjamin, Inc., New York, 1965), p. 111.

[22] F. R. Graf, A. Renn, G. Zumofen, and U. P. Wild, Phys. Rev. B **58**, 5462 (1998).

[23] J. P. van der Ziel and N. Bloembergen, Phys. Rev. **138**, A1287 (1965).

[24] I. D. Abella, N. A. Kurnit, and S. R. Hartmann, Phys. Rev. **141**, 391 (1966).

[25] A. R. Kessel', R. N. Shakhmuratov, and L. D. Eksin, Sov. Phys. JETP **67**, 2071 (1988).

[26] R. G. DeVoe and R. G. Brewer, Phys. Rev. Lett. **50**, 1269 (1983).

[27] A. Szabo and T. Muramoto, Phys. Rev. A **39**,. 3992 (1989).

[28] A. Szabo and R. Kaarli, Phys. Rev. B **44**,. 12307 (1991).

[29] K. Wódkiewicz, B. W. Shore, and J. H. Eberly, Phys. Rev. A **30**, 2390 (1984).

[30] K. Wódkiewicz and J. H. Eberly, Phys. Rev. A **32**, 992 (1985).

[31] R. N. Shakhmuratov and A. Szabo, Phys. Rev. A **58**, 3099 (1998).