Increase of the fractional delay of the pulse in an electromagnetically-induced-transparency medium

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We propose a nonadiabatic change of the control-field amplitude to restore the temporal shape of a signal pulse, which usually experiences an appreciable temporal broadening in a conventional regime of electromagnetically-induced-transparency. This broadening sets a major limit on the maximum time delay achievable in a slow-light medium. A circumvention of this limit may be useful for the application of a slow-light medium as a practical, controllable delay line.

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I. INTRODUCTION

All-optical signal processing, which includes pulse splitting and shaping, tunable delay lines and buffers used for data synchronization, etc., is of great interest in modern optics. Electromagnetically-induced-transparency (EIT) opens wide perspectives in this domain [1]. However, there are serious hindrances setting limits on the maximum time delay of the pulse achievable on propagation through an EIT medium [2]. One of them is the broadening of the temporal profile of the pulse due to the absorption of the wings of its spectrum detuned from the center of the EIT hole (transparency window) [2–4]. Recently, we showed [5] that an abrupt rise of the coupling field amplitude produces a narrowing of the temporal profile of the pulse and amplifies its amplitude. Under appropriate conditions this procedure does not distort the shape of the pulse. In Ref. [5] we did not consider the effect of the pulse broadening due to the absorption of its spectrum wings. In this paper we develop an analytical theory, which takes into account the pulse broadening, and we show that the pulse almost resumes its shape. This allows one to appreciably extend the maximum pulse delay in an EIT medium.

The paper is organized as follows. In Sec. II we give a brief summary of our previous theory considering the mechanisms of the pulse broadening and its distortion. In Sec. III we give a qualitative explanation of the pulse transformation in an EIT medium if an instantaneous change of the coupling field is applied. Section IV presents the analysis of the solution of the matter equations describing the evolution of the atom in our scenario of the pulse transformation. In Sec. V the analytical solution of the wave equation along with its analysis are given. Sections VI and VII are devoted to the comparison of our model with existing models, to a consideration of other mechanisms of pulse delay, and to give a short list of available experimental data.

II. TEMPORAL BROADENING OF THE PULSE WITH DISTANCE IN AN EIT MEDIUM

In this section we briefly outline the mechanism and consequences of the pulse broadening in an EIT medium. We follow our previous paper [4] where a comprehensive analytical theory of this effect is presented. It is based on the solution of the Maxwell-Bloch equations that is valid for the propagation of the signal field in the linear response approximation, i.e., for a weak signal field. To be specific we address to the Λ-excitation scheme [see inset in Fig. 1(a)] and introduce the following parameters: $\Omega=|d_{me}E_{c0}|/2\hbar$ is the coupling parameter for the coupling field $E_c(z,t)=E_{c0}\exp(-i\omega_c t+ik_c z)$ and $\Phi=|d_{ge}E_{s0}(z,t)|/2\hbar$ is the coupling parameter for the signal field $E_s(z,t)=E_{s0}(z,t)\exp(-i\omega_s t+ik_s z)$. For simplicity we consider the case of exact resonance: $\omega_c=\omega_{em}$, $\omega_s=\omega_{eg}$, where ω_{em} and ω_{eg} are resonant frequencies of the transitions between the metastable (m) and excited (e) states, and the ground (g) and excited (e) states, respectively. Here d_{me} and d_{ge} are matrix elements of the dipole transitions $m-e$ and $g-e$, respectively. We focus on the most practical case when the halfwidth Γ of the absorption line, corresponding to the transition $g-e$, is much larger than the coupling parameter Ω. Then, in the broad absorption line of the signal field the narrow EIT hole with the halfwidth $\Delta_h=\Omega^2/\Gamma$ appears. This narrow hole is accompanied by steep dispersion, i.e., a fast variation of the refractive index. The latter results in a substantial increase of the group index and a decrease of the group velocity of the signal field. Therefore, to reduce the group velocity as much as possible, one needs to create a transparency window as narrow as possible.

For the signal field with a Gaussian envelope $E_{s0}(0,t)=E_0\exp(-\Delta_{\text{in}}^2 t^2/4)$ at the input, we have the following expression at the output of the medium with physical length L and Beer's absorption coefficient α_B at frequency ω_{eg}:

$$E_{s0}(L,t)=E_0\frac{\Delta_{\text{out}}}{\Delta_{\text{in}}}\exp[-\Delta_{\text{out}}^2(t-t_d)^2/4], \tag{1}$$

where Δ_{in} is the spectrum halfwidth of the pulse at the input of the sample $(z=0)$ and

$$\Delta_{\text{out}}=\frac{\Delta_{\text{in}}}{\sqrt{1+(\Delta_{\text{in}}/\Delta_{\text{eff}})^2}} \tag{2}$$

is the spectrum halfwidth of the pulse at the output of the sample $(z=L)$. These expressions are valid if $\Delta_{\text{in}}<\Delta_h$. There $t_d=L/V_g\approx\alpha_B L/2\Delta_h$ is the time delay due to the reduced group velocity of the pulse

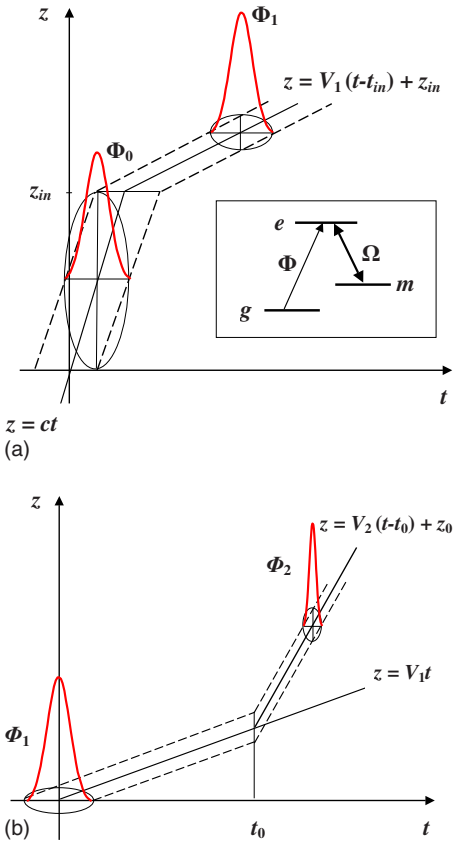

FIG. 1. (Color online) (a) Transformation of the signal pulse crossing at time t_{in} and coordinate z_{in} an interface between two media where the pulse has different group velocities c and V_1, respectively ($c \gg V_1$). The solid lines $z=ct$ and $z=V_1(t-t_{in})+z_{in}$ show the spatiotemporal evolution of the pulse maximum in these media, where the pulse has shapes Φ_0 and Φ_1, respectively. The dashed lines and ellipses indicate the spatiotemporal spread of the pulses Φ_0 and Φ_1 in the media. The temporal profile of the pulse does not change when crossing the interface, while the spatial profile appreciably narrows. Λ-excitation scheme of the three-level atom is shown in the inset. (b) Spatiotemporal evolution of the pulse if at time t_0 all its spatial components change the group velocity from V_1 to V_2 ($V_2 \gg V_1$). In this case the spatial spread of the pulse does not change while the temporal profile narrows.

$$V_g = (c^{-1} + \alpha_B/2\Delta_h)^{-1}. \tag{3}$$

The parameter Δ_{eff} describes the effective width of the pulse spectrum at the output of a thick sample ($\alpha_B L \gg 1$)

$$\Delta_{\text{eff}} = \frac{\Delta_h}{\sqrt{\alpha_B L/2}}. \tag{4}$$

With the increase of the sample thickness L, the delay time t_d increases as $\sim L$ while the temporal width of the pulse $\sim 1/\Delta_{\text{eff}}$ broadens as $\sim\sqrt{L}$. Therefore the product $\Delta_{\text{eff}} t_d$ increases as $\sim\sqrt{L}$. This product shows the actual delay time of the pulse (fractional delay) reflecting the spreading of its trailing edge. The pulse spreading is a natural cause of the worsening of the performance of the EIT medium as a delay line. With the pulse broadening, its amplitude drops as the ratio $\Delta_{\text{out}}/\Delta_{\text{in}} = 1/\sqrt{1+(\Delta_{\text{in}}/\Delta_{\text{eff}})^2}$. In a thick sample we have $\Delta_{\text{out}}/\Delta_{\text{in}} \sim \Delta_h/\Delta_{\text{in}}\sqrt{\alpha_B L/2}$, i.e., it drops as $1/\sqrt{L}$. Thus, if the pulse delays $\sqrt{\alpha_B L/2}$ times with respect to its broadened duration, which can be identified as an actual fractional delay, the amplitude of the pulse drops $\Delta_h/\Delta_{\text{in}}\sqrt{\alpha_B L/2}$ times. To avoid this drop, we have to use pulses with large ratio $\Delta_h/\Delta_{\text{in}} \sim \sqrt{\alpha_B L/2} \gg 1$, which implies the use of spectrally narrow pulses with long duration. This is in contradiction with the requirement of applications to work with pulses that are as short as possible.

The above speculations are applicable if $\Omega \leqslant \Gamma/2$ (see, for example, Ref. [4]). It is remarkable that the same arguments and the same formulas (1)–(4) are applicable if $\Omega > \Gamma/2$. The only difference is in the halfwidth of the transparency window. If $\Omega > \Gamma/2$, instead of an EIT hole we have the Autler-Townes splitting of the absorption line for the signal field with a distance 2Ω between peaks [6]. In this case, to have a reduced absorption, the pulse halfwidth Δ_{in} is to be smaller than Ω. The expansion (11) in Ref. [4] converges if $\Delta_{\text{in}} < \Omega$ and hence the approach developed in Ref. [4] is applicable. If $\Omega > \Gamma/2$ the parameter $\Delta_h = \Omega^2/\Gamma$ is still valid in Eqs. (1)–(4), but it has no meaning of the halfwidth of the transparency window, which is Ω. The common condition of induced transparency for both domains of Ω ($\Omega \lessgtr \Gamma/2$) is $\Omega^2 \gg \gamma\Gamma$, where γ is the decay rate of the low-frequency coherence $g-m$. If $\Omega \gg \Delta_{\text{in}} \gg \Gamma$, the pulse broadening is ineffective and there is a critical value of the propagation distance L_c, beyond which the pulse breakup takes place [4].

There is also an extra loss coming from the decay of the low-frequency coherence $g-m$, which is the backbone of EIT. This coherence reflects the dynamical population trapping in the dark state [7,8] evolving in time. Due to the decay of this coherence, the pulse amplitude drops exponentially [4]

$$\exp\left(-\frac{\alpha_B L}{2}\frac{\gamma\Gamma}{\gamma\Gamma+\Omega^2}\right) \approx \exp\left(-\frac{\alpha_B L}{2}\frac{\gamma}{\Delta_h}\right). \tag{5}$$

Concluding this section, we emphasize that, due to the absorption of the wings of the pulse spectrum and the decay of the low-frequency coherence, the energy of the output pulse or its time integrated intensity $I_s(z,t)=|E_s(z,t)|^2$ is reduced as [4]

$$U_s(L) = \int_{-\infty}^{\infty} I_s(L,t)dt = \frac{\Delta_{\text{out}}}{\Delta_{\text{in}}}\exp\left(-\frac{\alpha_B L\gamma}{\Delta_h}\right)U_s(0), \tag{6}$$

where $U_s(0)$ is the pulse energy at the input.

III. GENERAL ARGUMENTS

In this section we give a qualitative explanation of the pulse transformation in an EIT medium. Assume that at time t_{in} the maximum of the pulse arrives at the input of an EIT medium with coordinate $z_{\text{in}}=ct_{\text{in}}$, see Fig. 1(a). The solid line $z=ct$ in Fig. 1(a) shows the spatiotemporal evolution of the pulse maximum in free space. At the front face of the medium the pulse evolves in time as $\Phi_0(z_{\text{in}},t)=\Phi^0(t-z_{\text{in}}/c)$ and its duration is T_0. When the signal pulse enters an EIT medium, its group velocity is reduced to V_1. Then the maximum of the pulse evolves along the line $z=V_1(t-t_{\text{in}})+z_{\text{in}}$. The

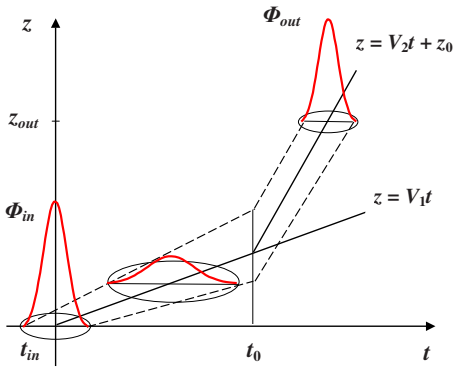

FIG. 2. (Color online) Transformation of the signal pulse in the EIT medium after an abrupt change of the group velocity of the pulse from V_1 to V_2 at t_0. Pulse Φ_{in} enters an EIT medium at t_{in}. It experiences a time broadening and drop of its amplitude. Pulse Φ_{out} narrows in time and its amplitude rises. The value of z_0 is defined as $z_0=V_1t_0$. Other notations are the same as in Fig. 1.

pulse must satisfy the boundary condition at $z=z_{\text{in}}$. Therefore, the pulse, propagating inside the medium with group velocity V_1 and satisfying the boundary condition at z_{in}, must change its spatiotemporal dependence to $\Phi_1(z,t)=\Phi^0[t-t_{\text{in}}-(z-z_{\text{in}})/V_1]$. According to the boundary condition the pulse duration T_1 inside the EIT medium is to be the same as it was in free space, i.e., $T_1=T_0$. Meanwhile the spatial length of the pulse $L_0=T_0c$ changes to $L_1=T_1V_1$. If $V_1 \ll c$, this length reduces appreciably in the EIT medium.

Let us at time t_0 increase instantaneously the group velocity of the pulse from V_1 to V_2 $(V_2>V_1)$ for all its spatial components. From then on the maximum of the pulse evolves along the line $z=V_2(t-t_0)+z_0$, where $z_0=V_1t_0$ is the coordinate of the pulse maximum when the change of the group velocity took place, see Fig. 1(b). Here, for simplicity, we take $z_{\text{in}}=0$ and $t_{\text{in}}=0$. The pulse evolution, which is described by the function $\Phi_2(z,t)$ after t_0, must satisfy the initial condition $\Phi_2(z,t_0)=\Phi^0(t_0-z/V_1)$. This means that the spatial length of the pulse does not change. Therefore, the temporal duration T_2 of the pulse after t_0 must satisfy the condition $T_1V_1=T_2V_2$. According to this relation, the pulse duration shortens as $T_2=T_0V_1/V_2$ and the pulse evolution should be described by the function $\Phi_2(z,t)=\Phi^0[(V_2/V_1)(t-t_0)-(z-z_0)/V_1]$, which is $\Phi^0\{(V_2/V_1)[t-t_0-(z-z_0)/V_2]\}$.

In a previous paper [5] we developed an analytical theory of the signal pulse transformation in an EIT medium when the coupling field amplitude instantaneously increases. It induces an instantaneous change of the group velocity and amplitude of the pulse. We did not consider the spread of the temporal profile of the pulse in the EIT medium due to the absorption of the spectral wings of the pulse. Figure 2 schematically shows the pulse spreading in an EIT medium along with the modification of the pulse when its group velocity is abruptly changed. In the next sections we consider the pulse transformation in an EIT medium with the pulse broadening taken into account.

IV. SOLUTION OF THE MATTER EQUATIONS

In this section we consider the atomic evolution in the linear response approximation, which is valid if $\Phi \ll \Gamma, \Omega$. We analyze the solution of the matter equations if the spectral width of the pulse $2\Delta_{\text{in}}$ is smaller than the width of the transparency window $2\Delta_h$ for $\Omega<\Gamma/2$, or $2\ \Omega$ for $\Omega>\Gamma/2$. The solution will be presented in terms of a perturbation theory [4,9], which is equivalent to the expansion in a power series of $\Delta_{\text{in}}/\Delta_h$ for the Fourier transforms of the atomic variables. If $\Omega>\Gamma/2$ and hence $\Delta_h=\Omega^2/\Gamma>\Omega>\Delta_{\text{in}}$, this expansion converges even more rapidly. Each time we will specify how many terms of the expansion are taken into account. Throughout the paper we neglect the decay γ of the low-frequency coherence $g-m$.

First we consider the atomic response on an abrupt change of the coupling field at time $t=t_0$. We assume that the amplitude of the coupling field changes stepwise such that $\Omega(z,t)=\Omega[1+h\Theta(t-t_0-z/c)]$, where $\Theta(t)$ is the Heaviside step function. This change of the amplitude of the control field propagates in the sample with velocity c. In the linear response approximation $(\Phi \ll \Gamma, \Omega)$, the atomic response can be found from the solution of two approximate equations [4]

$$\dot{\sigma}_{eg} = -\Gamma\sigma_{eg} + i\Phi(z,t) + i\Omega(z,t)\sigma_{mg}, \tag{7}$$

$$\dot{\sigma}_{mg} = i\Omega(z,t)\sigma_{eg}, \tag{8}$$

where $\sigma_{eg}=\rho_{eg}\exp(i\omega_s t-ik_s z)$ and $\sigma_{mg}=\rho_{mg}\exp[i(\omega_s-\omega_c)t-i(k_s-k_c)z]$ are slowly varying amplitudes of the nondiagonal elements of the three-level atom density matrix ρ_{ij}. Here, as already mentioned, for simplicity we neglect the decay γ of the low-frequency coherence σ_{mg}.

For $t<t_z=t_0+z/c$, i.e., before the change of the coupling field arrives at the atoms with coordinate z, the approximate solution of Eqs. (7) and (8) is [4]

$$i\sigma_{eg} \approx -\frac{\Phi_t^{(1)}(z,t)}{\Omega^2} + \frac{\Gamma\Phi_t^{(2)}(z,t)}{\Omega^4} - \frac{(\Gamma^2-\Omega^2)\Phi_t^{(3)}(z,t)}{\Omega^6}, \tag{9}$$

$$\sigma_{mg} \approx -\frac{\Phi(z,t)}{\Omega} + \frac{\Gamma\Phi_t^{(1)}(z,t)}{\Omega^3} - \frac{(\Gamma^2-\Omega^2)\Phi_t^{(2)}(z,t)}{\Omega^5}, \tag{10}$$

where $\Phi_t^{(1)}(z,t)=\partial\Phi(z,t)/\partial t$, $\Phi_t^{(2)}(z,t)=\partial^2\Phi(z,t)/\partial t^2$, and $\Phi_t^{(3)}(z,t)=\partial^3\Phi(z,t)/\partial t^3$. This solution is obtained expanding the Fourier transforms $i\sigma_{eg}(\nu)$ and $\sigma_{mg}(\nu)$ in power series near frequency $\nu=0$. Then, the inverse Fourier transform of the expansion is performed, see Ref. [4] for details. Each term of the expansion is $\Delta_{\text{in}}/\Delta_h$ times smaller than the preceding one if $\Delta_{\text{in}}/\Delta_h<1$. Meanwhile, it is enough to take into account only the first three terms of the expansion. Substituting the solution (9) into the wave equation, one obtains that the first two terms of the expansion (9) describe the delay and time broadening of the pulse, respectively. The third term of this expansion is responsible for the breakup of the pulse. It is effective if $\Omega \gg \Delta_{\text{in}} \gg \Gamma$. Otherwise its contribution is insignificant.

To find the atomic response on an abrupt change of the coupling field, we use the Laplace transform

$$f(p) = \int_{t_z}^{\infty} e^{-pt} F(t) dt. \tag{11}$$

We neglect the difference between t_z and t_0, assuming that the value z/c is small with respect to the duration of the pulse T_0. This approximation is valid if $V_g \ll c$. For convenience we take $t_0=0$. Later, in the final result we can substitute t by $t-t_0$ for arbitrary t_0. If at $t=0$ the coupling field changes from Ω to $\sqrt{a}\Omega$, where $\sqrt{a}=1+h$ and $h\Omega$ is the amount of the jump of the coupling field amplitude, equations (7) and (8) can be reduced with the help of the Laplace transform to algebraic equations for the image functions $\sigma_{eg}(z,p)$ and $\sigma_{mg}(z,p)$

$$p\sigma_{eg} - \sigma_{eg}(z,0) = -\Gamma\sigma_{eg} + i\Phi + i\sqrt{a}\Omega\sigma_{mg}, \tag{12}$$

$$p\sigma_{mg} - \sigma_{mg}(z,0) = i\sqrt{a}\Omega\sigma_{eg}, \tag{13}$$

where the arguments z and p of the image functions are not shown explicitly while the arguments of the original functions $\sigma_{eg}(z,0)$ and $\sigma_{mg}(z,0)$ are shown explicitly to emphasize that they define the initial condition for the solution at $t=0$. Since we consider the case if the coupling field amplitude jumps up, the value of a is larger than 1 by definition. Solving Eqs. (12) and (13) we obtain

$$i\sigma_{eg} = \frac{-p\Phi + pi\sigma_{eg}(z,0) - \sqrt{a}\Omega\sigma_{mg}(z,0)}{p^2 + \Gamma p + a\Omega^2}. \tag{14}$$

The original function $\sigma_{eg}(z,t)$ of the image function $\sigma_{eg}(z,p)$, Eqs. (14), is easily found

$$i\sigma_{eg}(z,t) = F_1(z,t) + F_2(z,t) + F_3(z,t), \tag{15}$$

where

$$F_1(z,t) = -\Theta(t-t_0)\int_{t_0}^{t} \Phi(z,\tau)K_0(t-\tau)d\tau, \tag{16}$$

$$F_2(z,t) = i\sigma_{eg}(z,t_0)K_0(t-t_0), \tag{17}$$

$$F_3(z,t) = -\sqrt{a}\Omega\sigma_{mg}(z,t_0)K_1(t-t_0). \tag{18}$$

Here, it is already taken into account that time t varies from t_0, which is not necessarily 0. The functions $K_0(x)$ and $K_1(x)$ are

$$K_0(x) = \Theta(x)\frac{\Gamma_+ e^{-\Gamma_+ x} - \Gamma_- e^{-\Gamma_- x}}{\Gamma_+ - \Gamma_-}, \tag{19}$$

$$K_1(x) = \Theta(x)\frac{e^{-\Gamma_- x} - e^{-\Gamma_+ x}}{\Gamma_+ - \Gamma_-}, \tag{20}$$

where

$$\Gamma_\pm = \frac{\Gamma}{2} \pm \sqrt{\frac{\Gamma^2}{4} - a\Omega^2}. \tag{21}$$

Below we consider the case if $\Gamma/2 \geqslant \sqrt{a}\Omega$. Then $\Gamma_\pm$ are real and transients, induced by the abrupt change of the coupling field, are exponentially decaying. It is easy to generalize the consideration to the case if $\Gamma/2 \leqslant \sqrt{a}\Omega$. Then, the transients are oscillatorily decaying.

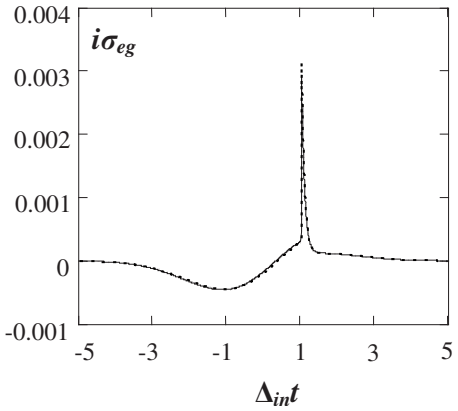

FIG. 3. Time evolution of the coherence $i\sigma_{eg}(z,t)$ induced by the signal field with $\Phi(z,t)=\Phi_0 \exp(-\Delta_{\text{in}}^2 t^2/4)$, where Φ_0 is the coupling amplitude of the signal field. For simplicity, the z dependence is not shown explicitly. Time scale is given in units of $\Delta_{\text{in}}t$. The time of the jump of the coupling field amplitude is $\Delta_{\text{in}}t_0=1$. The thin solid line is the result of the numerical solution of the matter equations and the dash-dot line is the analytical solution.

Taking the integral in Eq. (16) by parts iteratively and retaining only the two main terms, we find

$$F_1(z,t) \approx -\Phi(z,t_0)K_1(t-t_0) - \frac{\Phi_t^{(1)}(z,t) - \Phi_t^{(1)}(z,t_0)K_2(t-t_0)}{a\Omega^2}, \tag{22}$$

where

$$K_2(x) = \Theta(x)\frac{\Gamma_+ e^{-\Gamma_- x} - \Gamma_- e^{-\Gamma_+ x}}{\Gamma_+ - \Gamma_-}. \tag{23}$$

As will be shown in the next section, these terms specify the transients after the abrupt change of the coupling field amplitude and the new group velocity of the pulse. The third and fourth terms of the expansion [not shown in Eq. (22)] describe the pulse broadening and distortion. The reason why they are neglected will be discussed in our numerical example, which follows Eq. (24).

For $t > t_0$, the main contribution to the solution (15) is given by the functions $F_1(z,t)$ and $F_3(z,t)$. The contribution of $F_2(z,t)$ is $\sim\Delta_{\text{in}}/\Delta_h$ times smaller. Approximating $\sigma_{mg}(z,t_0)$ in Eq. (18) by the first term of expansion (10): $\sigma_{mg}(z,t_0) \approx -\Phi(z,t_0)/\Omega$, we obtain

$$i\sigma_{eg}(z,t) \approx h\Phi(z,t_0)K_1(t-t_0) - \frac{\Phi_t^{(1)}(z,t) - \Phi_t^{(1)}(z,t_0)K_2(t-t_0)}{a\Omega^2}. \tag{24}$$

Figure 3 shows a comparison of the numerically found solution of Eqs. (7) and (8) for $i\sigma_{eg}(z,t)$ with the analytical solution given by Eq. (9) for $t < t_0$ and Eq. (24) for $t \geqslant t_0$. In our example we take $\Delta_{h1}=3\Delta_{\text{in}}$, $\Omega=30\Delta_{\text{in}}$, $\Gamma=300\Delta_{\text{in}}$, and $h=1$ (the coupling field amplitude doubles at $t=t_0$). Here, $\Delta_{h1}=\Omega^2/\Gamma$ is the halfwidth of the transparency window for $t < t_0$. Even in this case where the spectral width of the input

pulse $2\Delta_{in}$ is three times smaller than the width of the transparency window $2\Delta_{h1}$ all three terms of expansion (9) for $i\sigma_{eg}(z,t)$ have to be taken into account. For $t \geqslant t_0$ the half-width of the transparency window is $\Delta_{h2}=a\Omega^2/\Gamma=a\Delta_{h1}$, where $a=(h+1)^2$. In our example it broadens by a factor of 4. Therefore, it is enough to take into account only the first time derivative in Eq. (24) since the higher time derivatives of $\Phi(z,t)$ give much smaller contributions. In the next section we will not take into account the second time derivative in the response function of the atoms after the abrupt change of the coupling field amplitude. This approximation is valid because after t_0 this term gives a much smaller contribution. However, to secure the smallness of further broadening of the pulse after t_0, we impose the additional condition that the signal pulse does not propagate a long distance in the EIT medium but leaves the sample shortly after t_0.

It is important to notice that, before the instantaneous change of the coupling field amplitude, the atoms are predominantly in the time-dependent dark state [7,8]

$$|d_1(t)\rangle = \cos\,\theta_1|g\rangle - \sin\,\theta_1|m\rangle, \tag{25}$$

where $\theta_1=\tan^{-1}[\Phi(z,t)/\Omega]$. If $\Phi(z,t) \ll \Omega$, then $\cos\,\theta_1 \approx 1$ and $\sin\,\theta_1 \approx \Phi(z,t)/\Omega$. From this expression it immediately follows that $\sigma_{mg} \approx -\Phi(z,t)/\Omega$. An atom in this state cannot be excited by the signal and coupling fields because of destructive interference of the excitation paths $g \rightarrow e$ and $m \rightarrow e$ [7,8]. There is a counterpart of the dark state, the so called bright state

$$|b_1(t)\rangle = \sin\,\theta_1|g\rangle + \cos\,\theta_1|m\rangle. \tag{26}$$

The interference of the excitation paths is constructive for this state, hence the atom leaves state $|b_1(t)\rangle$ [7,8]. (see also Refs. [10,11]). These states are defined as follows. In the interaction representation the Hamiltonian of a three-level atom interacting with the signal and coupling fields is

$$H = -\Phi\hat{P}_{eg} - \Omega\hat{P}_{em} + \text{H.c.}, \tag{27}$$

where we set $\hbar=1$ for simplicity of notations. The operators $\hat{P}_{mn}=|m\rangle\langle n|$ are defined in the basis of the eigenfunctions $|g\rangle$, $|m\rangle$, $|e\rangle$ of the unperturbed atom in the interaction representation (i.e., without time exponents). In the basis of the dark $|d_1\rangle$, bright $|b_1\rangle$, and excited $|e\rangle$ states (they are orthogonal and normalized) this Hamiltonian is transformed to

$$H^{(1)}_{dbg} = -B_1(\hat{P}_{b_1e} + \hat{P}_{eb_1}), \tag{28}$$

where $B_1=\sqrt{\Omega^2+\Phi^2}$, see, for example, Ref. [11]. From this expression it is clearly seen that the dark state is uncoupled from the excited state and the bright state is coupled to excited state with the coupling parameter B_1, which is $B_1 \approx \Omega$ if $\Omega \gg \Phi$.

If the coupling field parameter Ω suddenly changes to $\sqrt{a}\Omega$, the definition of the dark and bright states changes as

$$|d_2(t)\rangle = \cos\,\theta_2|g\rangle - \sin\,\theta_2|m\rangle, \tag{29}$$

$$|b_2(t)\rangle = \sin\,\theta_2|g\rangle + \cos\,\theta_2|m\rangle, \tag{30}$$

where $\theta_2=\tan^{-1}[\Phi(z,t)/\sqrt{a}\Omega]$. The corresponding Hamiltonian in this new basis is $H^{(2)}_{dbg}=-B_2(\hat{P}_{b_2e}+\hat{P}_{eb_2})$, where $B_2=\sqrt{a\Omega^2+\Phi^2}$. The atom, adiabatically following the slowly changing dark state $|d_1(t)\rangle$ before the jump ($t<t_0$), just after the jump ($t>t_0$) is described by the state $|d(t)\rangle=|d_1(t)\rangle$, which is not a pure dark state anymore. With the help of Eqs. (25), (29), and (30) one can show that

$$|d_1(t)\rangle = \cos(\theta_1-\theta_2)|d_2\rangle - \sin(\theta_1-\theta_2)|b_2\rangle, \tag{31}$$

where $\sin(\theta_1-\theta_2) \approx \Phi(z,t)/\Omega - \Phi(z,t)/\sqrt{a}\Omega = h\Phi(z,t)/\sqrt{a}\Omega$. Thus, the atomic state $|d(t)\rangle=|d_1(t)\rangle$ instantly acquires a bright state component, $|b_2\rangle$, after the sudden change of the coupling field amplitude. Therefore, the atom can now be excited and the probability amplitude of the excitation into state e becomes proportional to the probability amplitude of the bright state $\sim h\Phi(z,t)$. This excitation is mostly produced by the coupling field, inducing the transition $m \rightarrow e$ to bring the population of state m, which was $|\Phi(z,t)/\Omega|^2$, down to the value $|\Phi(z,t)/\sqrt{a}\Omega|^2$. Decay of the excited state e and depopulation of the bright state $|b_2(t)\rangle$ by the excitation B_2 bring the atom in a new equilibrium state $|d_2(t)\rangle$. In the next section, we consider the signal field modification caused by transients following an abrupt change of the coupling field amplitude.

V. SOLUTION OF THE WAVE EQUATION

The propagation of the signal field in an EIT medium is described by the wave equation

$$\left(\frac{\partial}{\partial z} + \frac{1}{c}\frac{\partial}{\partial t}\right)\Phi(z,t) = \frac{\alpha}{2} i\sigma_{eg}(z,t), \tag{32}$$

where $\alpha=4\pi\omega_s N_a|d_{eg}|^2/\hbar c$ is a coupling parameter, related to Beer's constant as $\alpha=\alpha_B\Gamma$, and N_a is the concentration of the resonant atoms in the sample. Substituting the solution (9) of the matter equations into the right-hand side of Eq. (32), one can find for a Gaussian pulse at the input the solution (1) at the output $z=L$ [4]. The first time derivative of $\Phi(z,t)$ in Eq. (9) modifies the group velocity of the pulse to the value V_1. The second time derivative of $\Phi(z,t)$ in Eq. (9) produces a time broadening of the pulse. The third time derivative of $\Phi(z,t)$ in Eq. (9) results in a distortion of the pulse shape. If $\Delta_{in}<\Delta_{h1}$, the contribution of the third time derivative can be neglected [4]. Solution (1) is valid for $t<t_0$. It can be expressed for arbitrary z as

$$\Phi_1(z,t) = \Phi_0\frac{\Delta_{out}}{\Delta_{in}}\exp[-\Delta^2_{out}(t-z/V_1)^2/4], \tag{33}$$

where Φ_0 is the maximum amplitude of the coupling parameter for the signal field and Δ_{out} is defined in Eq. (2), where $\Delta_{eff}=\Delta_{h1}/\sqrt{\alpha_B z/2}$.

Taking into account the solution (24) of the matter equations, we express the wave equation (32) for $t>t_0$ as

$$\left(\frac{\partial}{\partial z}+\frac{1}{V_2}\frac{\partial}{\partial t}\right)\Phi(z,t)=\frac{\alpha h}{2}\Phi_1(z,t_0)K_1(t-t_0), \quad (34)$$

where $V_2=(c^{-1}+\alpha_B\Gamma/2a\Omega^2)^{-1}$ is a new group velocity, which is nearly a times higher than V_1, i.e., $V_2\approx aV_1$. Here, in the atomic response function, Eq. (24), we neglect the third term, proportional to $K_2(t-t_0)$, because of its smallness $\sim\Delta_{\text{in}}(\Gamma_+-\Gamma_-)/a\Omega^2$ with respect to the first term, proportional to $\Phi(z,t_0)$.

The fundamental solution $\Phi_2(z,t)$ of Eq. (34) can be found as the sum of the fundamental solution $\Phi_{2h}(z,t)$ to the homogeneous equation

$$\left(\frac{\partial}{\partial z}+\frac{1}{V_2}\frac{\partial}{\partial t}\right)\Phi(z,t)=0, \quad (35)$$

and a particular solution $\Phi_{2i}(z,t)$ to the inhomogeneous equation (34). This sum must satisfy the initial condition $\Phi_2(z,t_0)=\Phi_1(z,t_0)$ (see Fig. 2). We construct a particular solution as follows:

$$\Phi_{2i}(z,t)=\frac{\alpha h}{2}\int_0^z K_1\left(t-t_0-\frac{z-z'}{V_2}\right)\Phi_1(z',t_0)dz'. \quad (36)$$

It satisfies Eq. (34). With the substitution $\tau=t-t_0-(z-z')/V_2$, Eq. (36) can be reduced to

$$\Phi_{2i}(z,t)=\frac{\alpha hV_2}{2}\int_0^{t-t_0} K_1(\tau)\Phi_1[z-V_2(t-t_0-\tau),t_0]d\tau, \quad (37)$$

where the lower limit of the integration is obtained by taking into account that the function $K_1(x)$ contains the Heaviside step function $\Theta(x)$. From this expression it is obvious that $\Phi_{2i}(z,t)\to 0$ when $t\to t_0+0$. Taking the integral in Eq. (37) by parts and retaining the first main term, we obtain

$$\Phi_{2i}(z,t)=\frac{\alpha hV_2}{2a\Omega^2}\{\Phi_1[z-V_2(t-t_0),t_0]-K_2(t-t_0)\Phi_1(z,t_0)\}. \quad (38)$$

The other terms are at least $\Delta_{\text{in}}/\Delta_h$ times smaller. For simplicity we disregard the variation of $\Delta_{\text{out}}(z)$ along the cut line $t=t_0$ in Fig. 2, assuming that the spectral half width of the pulse at $t=t_0$ is equal to $\Delta_1=\Delta_{out}(z_0)$, where $z_0=V_1t_0$. Then, taking into account that $V_2\approx 2a\Omega^2/\alpha$, the function $\Phi_{2i}(z,t)$ for the Gaussian input pulse can be expressed as

$$\Phi_{2i}(z,t)=\frac{h\Phi_0\Delta_1}{\Delta_{\text{in}}}[e^{-(\Delta_1^2V_2^2/4V_1^2)\{t-t_0-[(z-z_0)/V_2]\}^2} - K_2(t-t_0)e^{-(\Delta_1^2/4)[t_0-(z/V_1)]^2}]. \quad (39)$$

This particular solution to the inhomogeneous equation (34) is zero for $t=t_0$. Thus, we have to find the fundamental solution to the homogeneous equation (35) satisfying the initial condition $\Phi_{2h}(z,t_0)=\Phi_1(z,t_0)$. For a Gaussian input pulse, this solution is (see the Appendix)

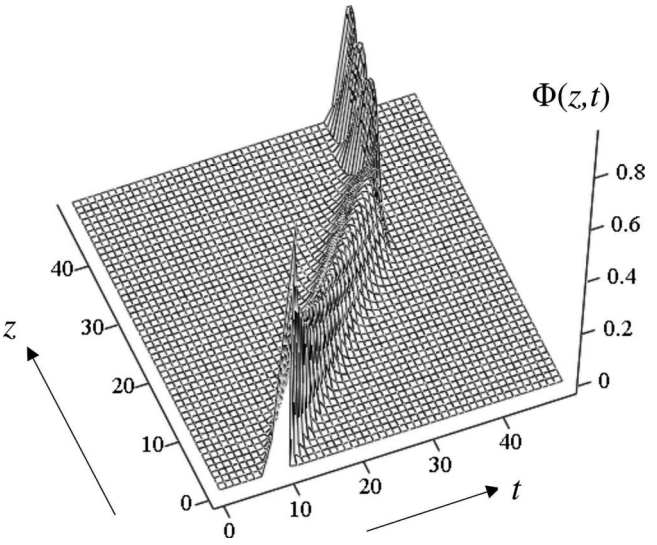

FIG. 4. 3D plot of the pulse $\Phi(z,t)$ evolution in the EIT medium with $\alpha_B=40$ cm^{-1}. The parameters of the pulse and the medium are $\Delta_{h1}=3\Delta_{\text{in}}$, $\Omega=30\Delta_{\text{in}}$, $\Gamma=300\Delta_{\text{in}}$, and $h=1.2$. Time t varies from $\Delta_{\text{in}}t=-20$ to $\Delta_{\text{in}}t=100$. Coordinate z varies from 0 to 15 cm. z and t axes are scaled in arbitrary units, which are relevant to line number in a grid formed by 50×50 lines. The amplitude of the coupling field suddenly rises at time $\Delta_{\text{in}}t_0=66.6$ and $h=1.2$. This rise exactly compensates the pulse broadening.

$$\Phi_{2h}(z,t)=\frac{\Phi_0\Delta_1}{\Delta_{\text{in}}}e^{-(\Delta_1^2V_2^2/4V_1^2)[t-t_0-(z-z_0)/V_2]^2}. \quad (40)$$

Figure 4 shows a three-dimensional (3D) plot of the solution of the wave equation for the Gaussian input pulse, which is described by $\Phi_1(z,t)$ for $t<t_0$ and by $\Phi_2(z,t)=\Phi_{2i}(z,t)+\Phi_{2h}(z,t)$ for $t\geqslant t_0$. The latter,

$$\Phi_2(z,t)=\frac{\Phi_0\Delta_1}{\Delta_{\text{in}}}[\sqrt{a}e^{-(\Delta_1^2V_2^2/4V_1^2)[t-t_0-(z-z_0)/V_2]^2} - hK_2(t-t_0)e^{-(\Delta_1^2/4)(t_0-z/V_1)^2}], \quad (41)$$

tends to

$$\Phi_2(z,t)=\sqrt{a}\frac{\Delta_1}{\Delta_{\text{in}}}\Phi_0\exp\left[-\frac{\Delta_1^2V_2^2}{4V_1^2}\left(t-t_0-\frac{z-z_0}{V_2}\right)^2\right], \quad (42)$$

for $t>t_0+\Gamma_-^{-1}$. It is obvious that the signal pulse narrows since its spectral width broadens as $\Delta_1V_2/V_1\approx a\Delta_1$. For our numerical example we choose a such that $\Delta_1V_2/V_1=\Delta_{\text{in}}$. Therefore, the transformed pulse resumes its original width, which was at the input. The group velocity of the pulse V_2 rises a times, which means that the ratio V_2/V_1 is equal to the increase factor of the coupling field intensity. In this case the amplitude of the signal pulse rises only $\sqrt{a}$ times, see Eq. (42), while, according to the imposed condition, the second factor in this equation is $\Delta_1/\Delta_{\text{in}}=V_1/V_2=1/a$. Therefore, the amplitude of the transformed output pulse is still $\sqrt{a}$ times smaller than the amplitude of the input pulse, i.e., $\sqrt{a}\Delta_1/\Delta_{\text{in}}=1/\sqrt{a}$. According to the definition of the time integrated intensity of the pulse [see Eq. (6)], this value for the

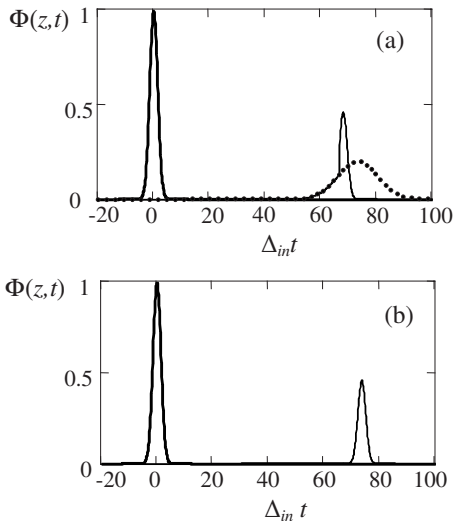

FIG. 5. Time dependence of $\Phi(z,t)$ in the EIT medium after a change of the coupling field amplitude (thin solid line). Dotted line in (a) shows the signal pulse without the change of the coupling field amplitude. Time dependence of the signal field travelling the same distance in free space is shown by the thick solid line. $z=11$ cm in (a) and $z=15$ cm in (b). Other parameters are the same as in Fig. 4.

modified pulse $\Phi_2(z,t)$, i.e., the pulse energy, does not change after the jump of the coupling field amplitude and equals $U_s(z)=\Delta_1/\Delta_{\text{in}}U_s(0)$ in spite of the increase of the pulse amplitude and the restoration of its duration.

Figure 5 demonstrates the time dependence of $\Phi(z,t)$ for different z in the EIT medium (thin solid line). This dependence is compared with the one one obtains if the pulse propagates the same distance in free space (thick solid line). Figure 5(a) is plotted for z, where the jump of the coupling field has not yet reached the pulse front and the change of the pulse shape is not complete. Figure 5(b) shows the pulse when its transformation is complete. The pulse evolution, if the coupling field does not change, is shown by the dotted line in Fig. 5(a) for comparison.

VI. COMPARISON WITH EXISTING MODELS

The instantaneous processing of slow light, discussed in this paper, is consistent with the dark-state polariton concept introduced in Refs. [12,13]. As was shown in Ref. [5], any change of the coupling field amplitude can be considered as adiabatic for the dark-state polariton if the group velocity of the signal pulse is many orders of magnitude smaller than the speed of light in vacuum. The atoms and the slow signal pulse are described in this concept by a coherent superposition of the atomic low-frequency coherence σ_{gm} and the signal pulse amplitude E_s. This superposition state satisfies the wave equation with modified group velocity V_g. The value of the polariton velocity is defined by the coupling field intensity. A change of the group velocity of the polariton, introduced by a slow or fast variation of the coupling field intensity, changes the dark-state polariton parameters but the compound system, atoms+field, remains in this polariton state.

The results of our paper can also be explained by means of the adiabaton picture of EIT [14,15]. According to the adiabaton model, the signal pulse, entering the EIT medium, is transformed to the coupling field and leaves the sample with group velocity c [14,15]. Then a dip and a bump in the temporal profiles of the coupling and signal fields are formed, both propagating with reduced group velocity V_g. This correlated structure is called adiabaton [14]. It is produced by the "spin wave," which is the coherence $\sigma_{mg}(z-V_g t)$, propagating with group velocity V_g [15]. In contrast to Ref. [14], a weak signal pulse is considered in Ref. [15]. The spin-wave propagation is described by the equation

$$\left(\frac{\partial}{\partial z}+\frac{1}{V_1}\frac{\partial}{\partial t}\right)\theta_1(z,t)=0, \tag{43}$$

where $\theta_1(z,t)=\tan^{-1}[\Phi(z,t)/\Omega(z,t)]$ defines the mixing angle in the dark state (25). The propagation of this spin wave means that the probability amplitude of the metastable state m, which is proportional to $\sin\theta_1\approx\theta_1$, moves from atom to atom. Before arrival of this wave to point z, state m is empty and the coupling field does not interact with the atom. The appearance of the population of the metastable state causes the absorption of the coupling field, which induces the transition $m\rightarrow e$. Simultaneously, the signal field appears induced by the transition $e\rightarrow g$. If the amplitude of the coupling field is suddenly increased, the spin-wave amplitude does not change, but its group velocity increases. The amplitude of the induced signal field increases also because of the speedup of the cascade $m\rightarrow e\rightarrow g$. The spreading of the temporal profile of the signal field in the EIT medium with a fast relaxation of the excited state e can be explained as follows. With the solution of the Schrödinger equation for a three-level atom given in Eq. (13) of Ref. [15], one can derive the wave equation for $\theta(z,t)$, which is

$$\left(\frac{\partial}{\partial z}+\frac{1}{V_1}\frac{\partial}{\partial t}\right)\theta_1(z,t)=\frac{\alpha\Gamma}{2\Omega^4}\frac{\partial^2\theta_1(z,t)}{\partial t^2}. \tag{44}$$

It differs from Eq. (43) by the right hand side, containing the second time derivative of $\theta_1(z,t)$. One can apply the method of solution of such an equation developed in Ref. [4] and obtain

$$\theta_1(z,t)=\theta_{10}\frac{\Delta_{\text{out}}(z)}{\Delta_{\text{in}}}\exp[-\Delta_{\text{out}}^2(z)(t-z/V_1)^2/4], \tag{45}$$

where $\theta_{10}=\tan^{-1}(\Phi_0/\Omega)$. This solution clearly shows that the spin wave spreads with propagation distance and decreases in amplitude.

In our proposal we do not compensate the energy loss of the signal pulse after the restoration of its shape. Therefore, it is reasonable to use an amplifier at the output of the EIT medium. Meanwhile, the restoration of the pulse duration helps to increase appreciably the maximum pulse delay, which can be achieved with EIT.

VII. EXPERIMENTAL DATA

Most of the experiments to produce slow light with EIT were performed with gases. Among them are experiments in

a rubidium vapor cell excited by a signal field with wavelength 780 nm and a coupling field with wavelength 775.8 nm, Ref. [16]; in a buffer-gas cell of hot Rb atoms excited by a signal pulse with wavelength 795 nm and a coupling field with the same frequency but orthogonal polarization (see, for example, Ref. [17]) or with a frequency offset 6.8 GHz, Ref. [18]; in Pb vapor excited by a signal pulse with wavelength 283 nm and by a coupling field with wavelength 1064 nm, Ref. [19] or 406 nm, Ref. [20]. A steep dispersion and group velocity below $c/3000$ was found in a cesium vapor cell at 852 nm if it was excited by a strong coupling field with a frequency offset 9.192 GHz (the cesium ground-state hyperfine splitting) with respect to the signal field (see, for example, Ref. [21]). A record low group velocity of 17 m/s was found in an ultracold gas of Na atoms excited by a signal field, 589 nm, and a coupling field with a frequency offset 1.8 GHz, Ref. [22]. The signal pulse is delayed by 7.05 μs in a 229-μm-long cloud of sodium atoms. There are few examples where optically dense crystals were employed, i.e., Pr-doped Y_2SiO_5 (Pr:YSO) whose 605.9 nm transition is imposed to the resonant probe field and to the coupling field with a frequency offset 10.2 MHz, Ref. [23] or 8.65 MHz, Ref. [24].

Practical optical communication systems typically make use of light with wavelengths of around 1350 or 1550 nm. Recently, for this communication band a slowing down of the light propagation was observed due to a single gain induced by stimulated Brillouin scattering (SBS) in a standard single-mode fiber and a dispersion-shifted optical fiber (see, for example, Refs. [25,26]), and due to narrow band partially degenerated optical parametric amplification (OPA) and stimulated Raman scattering (SRS) in highly nonlinear dispersion-shifted fiber (see, for example, Refs. [27,28]). In these experiments the gain-variable fractional delay of the pulse does not exceed 1.3 for a 500-m-long fiber with SBS [26] and 2.3 for a 2-km-long fiber with SRS [28]. Small fractional delays in a long fiber are due to the small group index change, of the order of 10^{-3}, which corresponds to a decrease of the speed of light by ~0.1%. Resonant media at EIT conditions demonstrate a much higher group velocity refractive index (group index), $n_g=c/V_g$. For example, in solids $n_g=6\times10^6$ was measured in Ref. [23] (compare with $\Delta n_g=10^{-3}$ for SBS and SRS, where $\Delta n_g=n_g-n_p$ and n_p is the phase index of the fiber). It should be mentioned that pulse distortion and the broadening mechanism for EIT radically differ from SBS and SRS, where the broadening and distortion of the pulse increase appreciably with gain rise.

The lack of EIT experiments for pulses with wavelength around 1550 nm seems not to be due to technical limitations. For example, the laser materials $YAlO_3$:Er, erbium in both silicate and phosphate glasses have optical transitions between the ground state $^4I_{15/2}$ and the $^4I_{13/2}$ multiplet in the region of 6500 cm^{-1} (1540 nm), see, for example, Refs. [29,30]. These transitions are good candidates for EIT since they have hyperfine splitting and hence a simultaneous excitation by the probe and coupling fields is capable to create the low-frequency coherence among ground-state sublevels, which is the backbone of EIT.

VIII. CONCLUSION

We considered the temporal broadening of the pulse during its propagation in an EIT medium. The pulse duration broadens with distance L as $\sim\sqrt{L}$ and its maximum intensity drops as $\sim1/L$. Since the maximum of the pulse delays as $\sim L$, the fractional delay of the pulse (the ratio of the pulse delay to its duration) increases with distance as $\sim L/\sqrt{L}=\sqrt{L}$. The pulse broadening and its amplitude decrease set a limit for the maximum pulse delay achievable in EIT medium. We propose to make an abrupt increase of the intensity of the coupling field just before the signal field leaves the EIT medium. The value of the increase is chosen such that the signal pulse resumes its initial temporal width. However, the intensity of the signal pulse, being increased after the abrupt increase of the intensity of the coupling field, does not resume its input value. From the value $\sim1/L$, reduced by the pulse broadening, the maximum intensity of the pulse rises to the value $\sim1/\sqrt{L}$ after the increase of the coupling field. We propose to increase the coupling field when the front edge of the signal field is close to the exit of the sample. Then, the maximum delay is achieved since almost all the length of the EIT medium is used to delay the pulse. After the increase of the coupling field, the group velocity of the signal pulse is appreciably increased and the transparency window significantly widens. Therefore, the further pulse broadening during the process of leaving the sample is almost insignificant.

Our consideration is applicable to the case if the spectral width of the signal field $2\Delta_{in}$ is smaller than the half width of the transparency window, which is $2\Omega^2/\Gamma$ for $\Omega\leqslant\Gamma/2$ or $2\ \Omega$ for $\Omega>\Gamma/2$. If the Rabi frequency of the coupling field Ω, the spectral width of the pulse Δ_{in}, and the decay rate of the coherence Γ induced by the signal field between the ground state g and excited state e, satisfy the condition $\Omega\gg\Delta_{in}\gg\Gamma$, the pulse broadening is ineffective. In this case, there is a maximum propagation distance, beyond which the signal pulse experiences a breakup. This distortion of the pulse is irreversible and any manipulations with the coupling field do not restore the shape of the signal pulse.

In our consideration we disregarded the decay γ of the low-frequency coherence $g-m$, induced by the signal and coupling fields in a two-quantum process. It can be shown that this decay gives an additional drop of the signal field intensity, which is governed by $\exp(-d\gamma\Gamma/\Omega^2)$, where $d=\alpha_B L$ is the optical thickness of the EIT medium.

Recently, a paper by Irina Novikova *et al.* [31] has been published, where the authors proposed and experimentally proved a method of circumvention of the pulse broadening in an EIT medium. The core idea is a gradual rise of the coupling field amplitude in an EIT medium with integrated gain. According to theoretical proposals [32–35] a linear increase in time of the coupling field amplitude narrows the pulse shape. The integrated gain compensates the drop of the pulse amplitude with propagation distance. However, the pulse narrowing in such a regime distorts its shape since the leading edge of the pulse is shortened much less than the trailing edge. Also, the integrated gain, considered in Ref. [31], is applicable only for a degenerate excitation scheme when the signal and coupling fields have the same frequencies but they

are different in polarization. Therefore, we believe that our scheme has some advantages and/or is complementary to the method proposed in Ref. [31]. Extra pulse distortion is not present in our proposal and it can be applied not only for a degenerate excitation scheme. It should be mentioned that pulse broadening during the retrieval process in stored light was discussed in Ref. [33]. For write-storage-reading experiments with EIT [17,36] a compensation of this broadening acquired in the reading regime was considered in Refs. [33,37].

ACKNOWLEDGMENTS

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APPENDIX

Assume we have a pulse $\Phi(z,t)=\Phi_0(\Delta_1/\Delta_{\text{in}})\exp[-\Delta_1^2(t-z/V_1)^2/4]$ whose propagation in a medium is described by the equation

$$\left(\frac{\partial}{\partial z}+\frac{1}{V_1}\frac{\partial}{\partial t}\right)\Phi(z,t)=0, \tag{A1}$$

where V_1 is the group velocity of the pulse. Let at time $t_0=0$ the group velocity suddenly changes to V_2. To find the pulse behavior after t_0 we apply the Laplace transform (11). Then for $t>t_0$, the wave equation (A1) is reduced to

$$\frac{\partial}{\partial z}\Phi(z,p)+\frac{1}{V_2}[p\Phi(z,p)-\Phi(z,0)]=0, \tag{A2}$$

where $\Phi(z,p)$ is the image function and $\Phi(z,0)$ is the original function for $t=0$. The solution of this equation is

$$\Phi(z,p)=\frac{1}{V_2}\int_{-\infty}^{z} e^{-p(z-z')/V_2}\Phi(z',0)dz'. \tag{A3}$$

Substituting the function $\Phi(z',t_0)=\Phi_0(\Delta_1/\Delta_{\text{in}})\exp[-\Delta_1^2(t_0-z'/V_1)^2/4]$, where $t_0=0$, into Eq. (A3) and integrating, we obtain

$$\Phi(z,p)=\Phi_0\frac{\sqrt{\pi}V_1}{\Delta_{\text{in}}V_2}e^{A^2(p-Bz)^2-Cz^2}\operatorname{erfc}[A(p-Bz)], \tag{A4}$$

where $A=V_1/(\Delta_1V_2)$, $B=\Delta_1^2V_2/(2V_1^2)$, $C=\Delta_1^2/(2V_1)^2$, and $\operatorname{erfc}(x)$ is the error function [38]. One can find in Ref. [38] that the image function $\exp(A^2p^2)\operatorname{erfc}(Ap)$ corresponds to the original function $(1/A\sqrt{\pi})\exp(-t^2/4A^2)$. In addition, there is a relation showing that the image function $f(p-Bz)$ corresponds to the original function $\exp(Bzt)F(t)$. Taking this into account, we find the original function for $\Phi(z,p)$, which is

$$\Phi(z,t)=\Phi_0\frac{\Delta_1}{\Delta_{\text{in}}}\exp\left[-\frac{\Delta_1^2}{4}\left(\frac{V_2}{V_1}t-\frac{z}{V_1}\right)^2\right]. \tag{A5}$$

For arbitrary t_0, not necessarily equal to 0, we have

$$\Phi(z,t)=\Phi_0\frac{\Delta_1}{\Delta_{\text{in}}}\exp\left[-\left(\frac{\Delta_1V_2}{2V_1}\right)^2\left(t-t_0-\frac{z-z_0}{V_2}\right)^2\right], \tag{A6}$$

where $z_0=V_1t_0$.

For the pulse $\Phi(z,t)$ with an arbitrary envelope but with a limited spatial length it is possible to derive a general solution. Taking by parts the integral in Eq. (A3) for the image function, one can find that

$$\Phi(z,p)=\frac{1}{p}\Phi(z',0)\,e^{-p(z-z')/V_2}\Big|_0^z - \frac{1}{p}\int_0^z e^{-p(z-z')/V_2}\Phi_{z'}^{(1)}(z',0)dz', \tag{A7}$$

where $\Phi_{z'}^{(1)}(z',0)=d\Phi(z',0)/dz'$. Without loss of generality we can extend the lower limit in the integration, $z'=0$, to minus infinity, $z'=-\infty$. If the pulse has a limited spatial length, then $\Phi(-\infty,0)=0$. Repeating the integration by parts in Eq. (A7) iteratively and imposing the condition that $\lim_{z'\to-\infty}d^n\Phi(z',0)/d(z')^n=0$, we obtain

$$\Phi(z,p)=\sum_{n=0}^{\infty}(-1)^n\frac{V_2^n}{p^{n+1}}\Phi_z^{(n)}(z,0), \tag{A8}$$

where $\Phi_z^{(n)}(z,0)=d^n\Phi(z,0)/dz^n$. The original function of the image function $\Phi(z,p)$ in Eq. (A8) is

$$\Phi(z,t)=\sum_{n=0}^{\infty}(-1)^n\frac{V_2^nt^n}{n!}\Phi_z^{(n)}(z,0), \tag{A9}$$

which is the Taylor expansion of $\Phi(z-V_2t,0)$. If for $t<0$ the signal is described by $\Phi(z,t)=\Phi^0(t-z/V_1)$, then $\Phi(z,0)=\Phi^0(-z/V_1)$. According to solution (A9), for $t>0$ the signal is described by the function $\Phi^0[(V_2t-z)/V_1]=\Phi^0[(V_2/V_1)(t-z/V_2)]$. For $t_0\neq0$ we have, respectively, $\Phi(z,t)=\Phi^0(t-z/V_1)$ if $t<t_0$ and $\Phi(z,t)=\Phi^0\{(V_2/V_1)[t-t_0-(z-z_0)/V_2]\}$ if $t>t_0$, where $z_0=V_1t_0$.

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