Adiabatic shape preservation and solitary waves in a five-level atomic medium

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Adiabatically shape-preserving-wave and solitary-wave solutions for five-level atomic systems are presented. They give the idealized situations for the shape preserving propagation of optical waves through a five-level atomic medium, and provide some useful comparisons with the electromagnetically induced transparency experiments performed by Harris and his collaborators on atoms with hyperfine structure.

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The problem of rendering a multilevel optical medium optically transparent has been a subject of considerable interest for many years. Much theoretical and experimental study has been done on the problem of sending two optical waves through an atomic medium that interact coherently with three levels of the atoms in the "Λ-type" energy-level configuration. Experimental work on this study has been pioneered by Harris and his collaborators and many references on their earlier work can be found in Ref. [1]. They succeeded in getting what they called electromagnetically induced transparency (EIT) in many of their experiments. On the theoretical front, exact analytical solutions for the so-called matched optical solitary waves usually require some very specific wave forms in order for the two optical pulses to preserve their shapes [2]. However, Grobe *et al.* [3] found an exact solution for the two coupled nonlinear Maxwell-Schrödinger equations under the adiabatic condition, that allows waves of arbitrary shapes that can propagate, after some initial reshaping, with their shapes relatively unchanged. The resulting pair of shape-preserving waves are called adiabatons.

Harris and his collaborators [4] have extended their experimental work on EIT to atoms with hyperfine structure that essentially have five participating energy levels. Exact analytic solitary-wave solutions for five-level systems exist but are somewhat scattered [2,5], whereas adiabatons for five-level systems have never been presented, even though the general conditions for the closely related concept of population trapping in multilevel system [6] have been given a long time ago.

In this paper, we shall present a simple case of adiabatons (when the adiabatic condition applies which will be explained later) as well as an exact solitary-wave solution for a specific five-level system. As is the case with many theoretical results, they require conditions (concerning the coupling strengths between the levels and the relative strengths of the electric fields used, etc.) that appear to be rather stringent compared to the experimental situations encountered. This is understandable because theoretically the wave forms are required to be completely shape invariant while experimentally it often suffices for the optical medium to only appear transparent.

We shall confine our attention to a "double lambda (Λ)" type five-level system as shown in Fig. 1 that arises from a Λ type three-level system in which the two upper levels are doubly split. This is the type of atomic system with the hyperfine structure experimentally studied by Xia *et al.* [4]. The ground level is numbered as level 1 as usual, but the two pairs of split upper levels are numbered levels 2, 4 and 3, 5, and are not numbered in ascending order according to their energies as Xia *et al.* did. This is in line with the numbering system in our previous papers on the adiabatons and solitary waves for which the nonzero couplings are between odd- and even-numbered levels only and we can ignore the odd-odd or even-even connections that are excluded because of the selection rule. One or more odd-even connections may still be excluded in addition to the exclusion of the odd-odd and even-even connections, as we shall see. Denoting time and coordinate by t and z, the wave function of level m by k_m, and $\alpha_{mn}(z,t) \equiv d_{mn}E_{mn}(z,t)/\hbar = \alpha^*_{nm}(z,t)$, where d_{mn} is the dipole matrix element and $E_{mn}(z,t)$ is the slowly varying amplitude of the electric field connecting levels m and n, the Schrödinger equation can be written as

$$i\frac{\partial}{\partial t}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \end{bmatrix} = -\begin{bmatrix} 0 & \alpha_{12} & 0 & \alpha_{14} & 0 \\ \alpha_{21} & \Delta_2 & \alpha_{23} & 0 & \alpha_{25} \\ 0 & \alpha_{32} & 0 & \alpha_{34} & 0 \\ \alpha_{41} & 0 & \alpha_{43} & \Delta_4 & \alpha_{45} \\ 0 & \alpha_{52} & 0 & \alpha_{54} & 0 \end{bmatrix}\begin{bmatrix} k_1 \\ k_2 \\ k_3 \\ k_4 \\ k_5 \end{bmatrix}, \quad (1)$$

where Δ's are the two-photon detunings that are assumed small and will be ignored in this study. The Maxwell equations can be written as

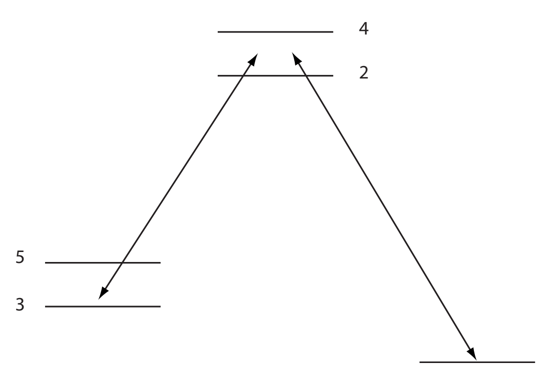

FIG. 1. Five-level system.

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$$(\partial/\partial z + c^{-1}\partial/\partial t)\alpha_{mn} = -i\mu_{mn}k_m k_n^*$$

$$\text{for } (m,n) = (1,2) \text{ and } (1,4) \tag{2}$$

and

$$(\partial/\partial z + c^{-1}\partial/\partial t)\alpha_{mn} = i\mu_{mn}k_m k_n^*$$

$$\text{for } (m,n) = (2,3),\ (4,5),\ (2,5),\ \text{and } (4,3), \tag{3}$$

where $\mu_{mn}=2\pi D d_{mn}^2\omega_{mn}/\hbar c$, D is the density of the atoms, c is the speed of light, and ω_{mn} is the laser frequency connecting levels m and n. Note that we have used α_{mn} instead of the usual Rabi frequency defined by $2d_{mn}E_{mn}/\hbar$ to eliminate factors of 2 in the equations.

We now consider the adiabatic approximation for the solutions of Eqs. (1)–(3). If the Hamiltonian matrix on the right-hand side of Eq. (1) is denoted by H, and $\eta(t)$ is the smallest frequency difference between two distinct eigenfrequencies of H at time t, the adiabatic condition generally states that the rates of change of the detunings and amplitudes of the incident laser fields must satisfy $\dot{\Delta}_j(t)$, $\dot{\alpha}_{jk}(t) \ll \eta(t)^2$ at all times so that a quasi steady state is maintained throughout the process. As it was pointed out in Refs. [7,8], this adiabatic condition leads to the condition $\Omega_{\text{eff}}T \gg 1$, where Ω_{eff} is some "effective" Rabi frequency that in many cases can be approximated by $(\Sigma|\alpha_{jk}|^2)^{1/2}$, and T is the pulse duration; and this has a nontrivial consequence, because it follows that $T\to\infty$ is not required to reach the asymptotic limit of adiabatic following. Rather, for given T, the limit can be approached by increasing Ω_{eff}, and this is exactly the point of interest for experiments with intense laser. Another important result that came from consideration of adiabatic following was the suggestion of using the counterintuitive order of incidence for the laser fields (examples of which will be seen below) and way of adjusting the detunings [7–9]. The use of this procedure that relies on the initial creation of a coherence or a population trapping state with subsequent adiabatic evolution has been one of the most successful methods for population transfer among quantum states of atoms and molecules [7], and the same principle has been made use of, often implicitly, in many other atomic experiments.

We use the pulse-localized coordinates $\zeta=z$ and $\tau=t-z/c$. In Eq. (1), t is replaced by τ and in Eqs. (2) and (3), the operator $\partial/\partial z+c^{-1}\partial/\partial t$ is replaced by $\partial/\partial\zeta$, $\alpha_{mn}(\zeta,\tau)$ and $k_m(\zeta,\tau)$ are now functions of ζ and τ. If the 5×5 matrix on the right-hand side of Eq. (1) is denoted by $H(\zeta,\tau)$, following Hioe and Carroll [6], the adiabatic solution is given by $H(\zeta,\tau)\vec{k}(\zeta,\tau)=\vec{0}$, giving, for the above five-level system, the following normalized $\vec{k}(\zeta,\tau)$:

$$k_1 = \frac{1}{\Xi}(\alpha_{23}\alpha_{45} - \alpha_{43}\alpha_{25}),$$

$$k_3 = -\frac{1}{\Xi}(\alpha_{21}\alpha_{45} - \alpha_{41}\alpha_{25}),$$

$$k_5 = -\frac{1}{\Xi}(\alpha_{23}\alpha_{41} - \alpha_{43}\alpha_{21}), \tag{4}$$

where

$$\Xi^2(\zeta,\tau) = |\alpha_{21}\alpha_{45} - \alpha_{41}\alpha_{25}|^2 + |\alpha_{23}\alpha_{41} - \alpha_{43}\alpha_{21}|^2 + |\alpha_{23}\alpha_{45} - \alpha_{43}\alpha_{25}|^2. \tag{5}$$

The two other components of $\vec{k}$, k_2 and k_4 are assumed small but not zero as they are required for the propagation equations (2) and (3), and they are given by

$$\begin{aligned} k_2 &= -i\left(\frac{\alpha_{54}\partial k_3/\partial\tau - \alpha_{34}\partial k_5/\partial\tau}{\alpha_{32}\alpha_{54} - \alpha_{34}\alpha_{52}}\right) \\ &= -i\left(\frac{\alpha_{54}\partial k_1/\partial\tau - \alpha_{14}\partial k_5/\partial\tau}{\alpha_{12}\alpha_{54} - \alpha_{14}\alpha_{52}}\right) \\ &= -i\left(\frac{\alpha_{34}\partial k_1/\partial\tau - \alpha_{14}\partial k_3/\partial\tau}{\alpha_{12}\alpha_{34} - \alpha_{14}\alpha_{32}}\right), \end{aligned}$$

$$\begin{aligned} k_4 &= -i\left(\frac{\alpha_{32}\partial k_5/\partial\tau - \alpha_{52}\partial k_3/\partial\tau}{\alpha_{32}\alpha_{54} - \alpha_{34}\alpha_{52}}\right) \\ &= -i\left(\frac{\alpha_{12}\partial k_5/\partial\tau - \alpha_{52}\partial k_1/\partial\tau}{\alpha_{12}\alpha_{54} - \alpha_{14}\alpha_{52}}\right) \\ &= -i\left(\frac{\alpha_{12}\partial k_3/\partial\tau - \alpha_{32}\partial k_1/\partial\tau}{\alpha_{12}\alpha_{34} - \alpha_{14}\alpha_{32}}\right). \end{aligned} \tag{6}$$

The equalities in the expressions for k_2 and k_3 in Eq. (6) can be shown to be consistent with the relation $\partial(|k_1|^2+|k_2|^2+|k_3|^2)/\partial\tau=0$. Substituting these k's from Eqs. (4) and (6) into the propagation equations (2) and (3) gives six coupled nonlinear differential equations for the α's, and the solutions correspond to the adiabatic approximation for the waves for the coupled Maxwell-Schrödinger equations.

In order to see some basic features of these solutions, we make the following simplifications that correspond partially to the situation in the experiments of Xia *et al.* [4].

(1) We assume that $\alpha_{mn}(\zeta,\tau)=c_{mn}f(\zeta,\tau)$ for $(m,n)=(1,2)$ and (1,4), and $\alpha_{mn}(\zeta,\tau)=c_{mn}g(\zeta,\tau)$ for $(m,n)=(2,3)$, (4,5), (2,5), and (4,3), where $f(\zeta,\tau)$ and $g(\zeta,\tau)$ are arbitrary coordinate- and time-varying functions, and the c's are constants that depend on the field amplitudes and couplings between the levels. That is, only two laser fields are used and the fields that connect levels $1\leftrightarrow2$ and $1\leftrightarrow4$ have the same dependence on (ζ,τ), and the fields that connect levels 2,4 and levels 3,5 have the same dependence on (ζ,τ).

(2) We assume that the transition matrix elements for (2,5) and (4,3) are zero. In the experiment of Xia *et al.* [4], the coupling for (2,5) is zero because of the selection rule but the coupling for (4,3) is not zero even though it is considerably smaller than those for (2,3) and (4,5) connected by the same laser field.

The expressions for the k's then simplify considerably and they are given by

$$k_1 = \frac{1}{\Omega(\zeta,\tau)}c_{23}c_{45}g(\zeta,\tau),$$

$$k_3 = -\frac{1}{\Omega(\zeta,\tau)} c_{21}c_{45}f^*(\zeta,\tau),$$

$$k_5 = -\frac{1}{\Omega(\zeta,\tau)} c_{23}c_{41}f^*(\zeta,\tau), \tag{7}$$

where

$$\Omega^2(\zeta,\tau) = C_f^2|f(\zeta,\tau)|^2 + C_g^2|g(\zeta,\tau)|^2, \tag{8}$$

with

$$C_f^2 = |c_{21}c_{45}|^2 + |c_{23}c_{41}|^2,$$

$$C_g^2 = |c_{23}c_{45}|^2,$$

and

$$k_2 = -i\left(\frac{\partial k_3/\partial\tau}{c_{32}g^*}\right) = -i\left(\frac{c_{54}g^*\partial k_1/\partial\tau - c_{14}f\partial k_5/\partial\tau}{c_{12}c_{54}fg^*}\right),$$

$$k_4 = -i\left(\frac{\partial k_5/\partial\tau}{c_{54}g^*}\right) = -i\left(\frac{c_{12}f\partial k_3/\partial\tau - c_{32}g^*\partial k_1/\partial\tau}{-c_{14}c_{32}fg^*}\right). \tag{9}$$

The four propagation equations for $\alpha_{12}(\zeta,\tau) = c_{12}f(\zeta,\tau), \alpha_{14}(\zeta,\tau) = c_{14}f(\zeta,\tau), \alpha_{23}(\zeta,\tau) = c_{23}g(\zeta,\tau), \alpha_{45}(\zeta,\tau) = c_{45}g(\zeta,\tau)$, become two equations for $f(\zeta,\tau)$ and $g(\zeta,\tau)$

$$\frac{\partial f}{\partial\zeta} = -\frac{\mu_{12}|c_{45}|^2}{\Omega}\frac{\partial}{\partial\tau}\left(\frac{f}{\Omega}\right), \tag{10}$$

$$\frac{\partial g}{\partial\zeta} = -\frac{\mu_{23}|c_{45}|^2}{\Omega}\frac{\partial}{\partial\tau}\left(\frac{g}{\Omega}\right) - \frac{\mu_{23}|c_{14}|^2}{\Omega}\frac{f}{g^*}\frac{\partial}{\partial\tau}\left(\frac{f^*}{\Omega}\right), \tag{11}$$

provided that the following three relations are satisfied:

$$\mu_{12}|c_{45}|^2 = \mu_{14}|c_{23}|^2,$$

$$\mu_{23}|c_{45}|^2 = \mu_{45}|c_{23}|^2,$$

$$\mu_{23}|c_{14}|^2 = \mu_{45}|c_{12}|^2. \tag{12}$$

A special case of interest is given by the following set of μ's and c's that satisfies the three above conditions:

$$\mu_{23} = \mu_{45} = 2\mu_{12} = 2\mu_{14} \tag{13}$$

and

$$|c_{23}|^2 = |c_{45}|^2 = 2|c_{12}|^2 = 2|c_{14}|^2. \tag{14}$$

These values of μ's and c's result in $C_f^2 = C_g^2 \equiv C^2$ for Eq. (8). Multiplying Eq. (10) by f^* and Eq. (11) by g^*, and similarly for equations of their conjugates, and then adding, we find that $\partial(|f|^2+|g|^2)/\partial\zeta = 0$. It follows that $V^2(\zeta,\tau) \equiv |f(\zeta,\tau)|^2 + |g(\zeta,\tau)|^2$ and $\Omega^2(\zeta,\tau) = C^2(|f(\zeta,\tau)|^2 + |g(\zeta,\tau)|^2) = C^2V^2(\zeta,\tau)$ do not depend on ζ and thus depend only on the input fields, and they can be written as $V^2(\zeta,\tau) = V^2(0,\tau)$ and $\Omega^2(\zeta,\tau) = \Omega^2(0,\tau)$. Even though the coupled nonlinear equations (10) and (11) are somewhat different from the corresponding equations for the three-level case [3], and the conditions given by Eqs. (13) are also different from the corresponding condition for the three-level case (for which it is required that $\mu_{12} = \mu_{23}$ that was not explicitly stated in Ref. [3]), that $|f(\zeta,\tau)|^2 + |g(\zeta,\tau)|^2$ is independent of ζ made the adiabaton solution possible and similar for the two cases. Here we have a pair of adiabatons characterized by $f(\zeta,\tau)$ and $g(\zeta,\tau)$ that are given by

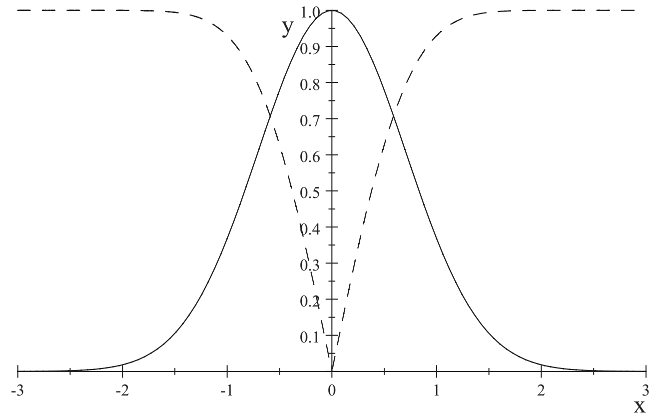

FIG. 2. $f(x)$ (solid) and $g(x)$ (dash) for $\Omega = C = 1$.

$$f(\zeta,\tau) = \Omega(0,\tau)F(Z(\tau) - \varepsilon\zeta), \tag{15}$$

$$g(\zeta,\tau) = \Omega(0,\tau)\{C^{-2} - F^2(Z(\tau) - \varepsilon\zeta)\}^{1/2}, \tag{16}$$

where

$$\varepsilon \equiv \mu_{12}|c_{45}|^2 = 2\mu_{12}|c_{12}|^2, \text{ etc.} \tag{17}$$

$$Z(\tau) = \int_{-\infty}^{\tau} d\tau'\,\Omega^2(0,\tau'), \tag{18}$$

and where F is an arbitrary function of $Z(\tau) - \varepsilon\zeta$. Equations (15) and (16) can be verified to satisfy Eqs. (10) and (11) by direct substitutions provided that Eqs. (13) and (14) are satisfied. Note that the two pulse shapes $f(\zeta,\tau)$ and $g(\zeta,\tau)$ are related by $|g(\zeta,\tau)|^2 = V^2(0,\tau) - |f(\zeta,\tau)|^2 = C^{-2}\Omega^2(0,\tau) - |f(\zeta,\tau)|^2$.

For some specific examples, consider real f and g. After a sufficient time for the pulse reshaping to complete, the initial pulse shape $f(0,\tau)$ evolves into its shape-preserving adiabaton $f(\zeta,\tau) = \Omega F[\Omega^2\tau - \varepsilon\zeta + \beta]$ and its partner from its initial form $g(0,\tau)$ into $g(\zeta,\tau) = [V^2(0,\tau) - f^2(\zeta,\tau)]^{1/2} = \Omega\{C^{-2} - F^2[\Omega^2\tau - \varepsilon\zeta + \beta]\}^{1/2}$, where Ω, C and β are constants. Let $x \equiv \Omega^2\tau - \varepsilon\zeta$, and let $F(x)$ be a Gaussian shape. We have presented the plots of $y(x) = f(x) = \Omega\exp(-x^2)$ (solid line) and $y(x) = g(x) = \Omega[C^{-2} - \exp(-2x^2)]^{1/2}$ (dash line) for two arbitrary sets of $(\Omega, C) = (1,1)$ and $(1,1/2)$ (in some arbitrary units) in Figs. 2 and 3, respectively. In each case, it has the two pulses sent in the so-called counterintuitive order [7–9] such that the fields that connect the initially unpopulated levels (2,3) and (4,5) preceed the fields that connect the initially populated ground level 1 to levels (2,4). The speed v of the propagation of these adiabatons is given by

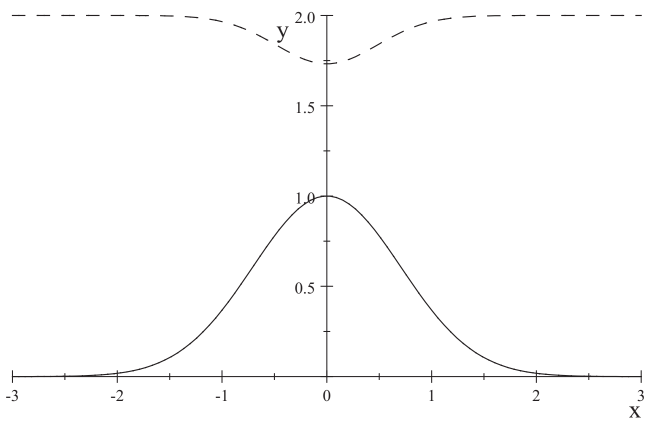

FIG. 3. $f(x)$ (solid) and $g(x)$ for $\Omega=1$, $C=1/2$.

$$1/v = 1/c + \varepsilon/\Omega^2 \tag{19}$$

and it can be much smaller than the vacuum speed c of light. The evolution to a pair of pulse shapes that eventually become ones shown in Fig. 3, say, is similar to that for the three-level case shown in Ref. [3]. For the five-level system considered here, remember that four parameters α_{12}, α_{14}, α_{23}, and α_{45} are in play involving two pulse shapes $f(\zeta,\tau)$ and $g(\zeta,\tau)$, and that conditions (13) and (14) are necessary for the eventual shape-preserving propagation.

It is to be noted that I have chosen the simple pulse shapes $f(x)$ and $g(x)$ [remembering that $f^2(x)$ and $g^2(x)$ represent their intensities] that are shown in Figs. 2 and 3 as they are very similar to the so-called bright and dark solitons, and the bright and gray solitons, respectively, that have been experimentally created and observed long ago in optical fibers [10], even though the nonlinear differential equations governing the propagation of optical solitons in fibers are the nonlinear Schrödinger equations [11] rather than the nonlinear Maxwell-Schödinger equations for the adiabatons here. Although many other examples of pulse shapes can be given, the principal feature of the pair of pulse shapes for the adiabatons is clear. It consists of a $g^2(x)$ pulse that connects the initially unpopulated levels (2,4) to (3,5) and that is much stronger in the beginning than the $f^2(x)$ that connects the initially populated ground state 1 to the initially unpopulated upper levels (2,4), where $g^2(x)$ and $f^2(x)$ are related by the simple relation given by Eqs. (15) and (16). For the pair of pulses to propagate through the atomic medium and emerge from it with their shapes unchanged from the time they entered, and with all the atoms returned to their ground states, the $g^2(x)$ pulse needs to return to its initial value while the $f^2(x)$ needs to decrease and vanish. The initial turn-on and final turn-off of the $g^2(x)$ pulse are not shown in the figures.

In the EIT experiments of Xia *et al.* [4], the pulse shapes may look very different from the examples we presented since they did not strictly require the pulse shapes to emerge completely unchanged; only that the atomic medium would appear to be transparent. However, the qualitative requirement that $g^2(x) \gg f^2(x)$ for their pulses at the beginning and at the end was probably satisfied, and it was possible that the adiabatic condition $\Omega_{\text{eff}}T \gg 1$ as well as the counterintuitive order of incidence for the laser fields were partly or fully utilized.

We now present an exact solitary-wave solution of the coupled equations (1)–(3) for the five-level system without assuming the adiabatic condition. This result is one of the many possible cases given in Ref. [2] but it is not one that was explicitly presented. We shall present it here in the form that can be directly compared with the result for the adiabatons just presented.

Assume $\Delta_2=\Delta_4=0$, $\alpha_{34}=\alpha_{25}=0$, and $\mu_{34}=\mu_{25}=0$ in Eqs. (1) and (2). If the pulses are shape invariant and propagate through the medium with velocity v, then they depend on t and z through $\xi \equiv (t-z/v)/\kappa$, where $\kappa>0$ is the pulse length (a scaling parameter). The operator $\partial/\partial z + c^{-1}\partial/\partial t$ is replaced by $\partial/\partial\xi$. The solitary-wave pair characterized by

$$f(\xi) = \sec h(\xi),$$

$$g(\xi) = \tanh(\xi), \tag{20}$$

for the waves are given by

$$\alpha_{12}(\xi) = \alpha_{14}(\xi) = Bf(\xi),$$

$$\alpha_{23}(\xi) = \alpha_{45}(\xi) = Ag(\xi), \tag{21}$$

while the atomic variables are given by

$$k_1 = -g(\xi),$$

$$k_2 = k_4 = \frac{i}{2B\kappa} f(\xi),$$

$$k_3 = k_5 = \frac{A}{2B} f(\xi), \tag{22}$$

where the constants A and B that represent the amplitudes of the waves must satisfy the relation

$$(2B^2 - A^2)\kappa^2 = 1. \tag{23}$$

In addition, the propagation constants μ's must satisfy the following condition:

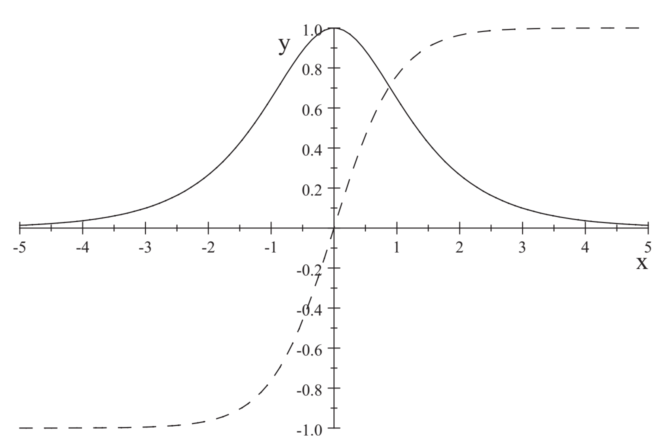

FIG. 4. sech(x) (solid) and tanh(x) (dash).

$$\mu_{23} = \mu_{45} = 2\mu_{12} = 2\mu_{14}. \tag{24}$$

The speed v of the waves is given by

$$\frac{1}{v} = \frac{1}{c} + \frac{\mu_{12}}{2B^2}. \tag{25}$$

The atoms of the medium, initially in their ground states $|k_1(-\infty)|^2=1$, $k_2(-\infty)=k_3(-\infty)=k_4(-\infty)=k_5(-\infty)=0$, return to their ground states $|k_1(+\infty)|^2=1$, $k_2(+\infty)=k_3(+\infty)=k_4(+\infty)=k_5(+\infty)=0$, after the pulses propagate through the medium.

This exact analytic solitary-wave solution [Eqs. (20)–(25)] can be verified by direct substitutions to Eqs. (1) and (2). It can be seen that these solitary waves, that can propagate through the five-level atomic system with their shapes invariant, bear a great deal of similarity with the the adiabatons given in Eqs. (13)–(19). The plots of of the pulse shapes given by Eq. (20) $y(x)=\sec h(x)$ and $y(x)=\tanh(x)$ are shown in Fig. 4, and they are seen to be similar to the pulse shapes of the adiabatons of Fig. 2 [Remember that we compare the intensities given by $y^2(x)$ and that the negative values of $y(x)$ become positive for $y^2(x)$]. However, the pulse shapes for these exact solitary waves are very specific [Eq. (20)] and satisfy the relation $f^2+g^2=$const independent of ξ; whereas for the adiabatons, one of them $f(\zeta,\tau)$ can be of an arbitrary shape and the shape of its partner is $g(\zeta,\tau)=\{V^2(0,\tau)-f^2(\zeta,\tau)\}^{1/2}$ for an arbitrary $V(0,\tau)$.

In summary, we have presented the adiabaton solution [Eqs. (13)–(19)] and the exact solitary-wave solution [Eqs. (20)–(25)] for a five-level system whose energy levels are a double Λ configuration under some specific conditions (including zero coupling between levels 2 and 5, and 3 and 4). These solitary waves are quite analogous to the bright-dark or more generally the bright-gray solitary wave pairs found in the optical fibers. The difference with and the extra conditions required in addition to those for the corresponding adiabatons and solitary waves for the three-level systems have been noted.

I would like to thank Clark Carroll and Joe Eberly for comments and discussions.

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[11] For more solitary waves from two coupled nonlinear Schödinger equations, see, e.g.. F. T. Hioe, J. Phys. A **36**, 7307 (2003), and references cited.