Rabi cycling of two pulses in a mode-locked ring laser cavity with electro-optical control

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The intensities of the circulating pulses in a mode-locked ring laser are shown to be analogous to the population transfer (Rabi cycle) in a two-level system. We present an experimental demonstration of this analogy.

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I. INTRODUCTION

Various analogies can be found between quantum-mechanical and classical systems. For instance, the quantum-mechanical interaction of a field with a two- or three-level system has been shown to have a simple electrical circuit analogy [1]. The purpose of this study is to investigate the analogy between a two-level system and a mode-locked laser in which two intracavity pulses circulate. The large variety of pulsed interrogation techniques of two-level systems [2], discovered first in nuclear magnetic resonance [3], and applied later to optical transitions, may inspire new detection methods in ring and linear lasers.

A general description of the analogy between a two-level system and a bidirectional ring laser is presented in Sec. II. The system to study this analogy is a mode-locked laser, with two pulses circulating in the cavity, and control over various types of coupling between the two pulses, as discussed in Sec. III. The standard two-level system has generally a well-defined ground state that serves as an initial condition for most interactions. In Sec. IV, it is shown that the density-matrix equations of a two-level system apply to the ring laser, as well as the Bloch vector diagram of Hellwarth-Vernon-Feynman [4]. The means to establish a preferential state of rotation in the case of the ring laser are proposed in Sec. V.

II. RING LASER AND TWO-LEVEL SYSTEM

The atomic or molecular system considered in this analogy has two quantum states with opposite parity, which can be coupled by a dipole transition. The two levels are distinguished by their energy. In the ring laser under consideration, the states $|1\rangle$ and $|2\rangle$ are the two senses of rotation of a pulse in a ring laser. The probability of each direction (clockwise and counterclockwise) is monitored through the pulse energy in the corresponding direction. The two levels of the quantum-mechanical system are coupled by an electromagnetic wave of frequency ω nearly equal to the transition frequency ω_0. Either situation is treated within the slowly varying approximation. This means, in the case of the quantum-mechanical two-level system, that the transition rates and detuning $\Delta\omega=\omega_0-\omega$ are negligible compared to ω or ω_0. In the case of the ring laser, the transition rate between $|1\rangle$ and $|2\rangle$, as well as the difference in cavity resonances for the two senses of rotation, are negligible compared to the carrier frequencies.

A sketch of a laser with two pulses circulating in the cavity is presented in Fig. 1(a), and its two-level analogue in Fig. 1(b). A ring laser is taken as an example, but the analogy applies as well to a linear cavity in which two pulses are made to circulate. In the Schrödinger description of a dipole transition $|1\rangle\to|2\rangle$ interacting near resonance with an electric field $E=\frac{1}{2}\tilde{\mathcal{E}}\exp(i\omega t)+\text{c.c.}$, the wave function ψ is written as a linear combination of the basis: $\psi(t)=a_1(t)|1\rangle+a_2(t)|2\rangle$. Slowly varying coefficients, defined by $a_i=c_i\exp(\pm i\omega_0 t)$, are substituted to the a_i in the Schrödinger equation, resulting in the set of time-dependent coefficient differential equations (see, for instance, Ref. [5]),

$$\frac{d}{dt}\begin{pmatrix}c_1\\c_2\end{pmatrix}=\begin{pmatrix} i\frac{\Delta\omega}{2} & i\frac{1}{2}\kappa\tilde{\mathcal{E}} \\ -i\frac{1}{2}\kappa\tilde{\mathcal{E}}^* & -i\frac{\Delta\omega}{2}\end{pmatrix}\begin{pmatrix}c_1\\c_2\end{pmatrix}, \tag{1}$$

where $\kappa|\tilde{\mathcal{E}}|=(p/\hbar)|\tilde{\mathcal{E}}|$ (p being the dipole moment of the transition) is the Rabi frequency.

In the analogy of the ring laser, the coefficients $c_i(t)$ correspond to the complex field amplitudes $\tilde{\mathcal{E}}_i$ (the tilde indicating a complex quantity) of each pulse circulating in the ring cavity (round-trip time τ_{RT}), as sketched in Fig. 1. The state of the system is also defined by $\psi(t)=\tilde{\mathcal{E}}_1(t)|1\rangle+\tilde{\mathcal{E}}_2(t)|2\rangle$. The evolution equation of these fields is

$$\frac{d}{dt}\begin{pmatrix}\tilde{\mathcal{E}}_1\\ \tilde{\mathcal{E}}_2\end{pmatrix}=\frac{1}{\tau_{\mathrm{RT}}}\begin{pmatrix}\tilde{r}_{11} & \tilde{r}_{12}\\ \tilde{r}_{21} & \tilde{r}_{22}\end{pmatrix}\begin{pmatrix}\tilde{\mathcal{E}}_1\\ \tilde{\mathcal{E}}_2\end{pmatrix}=\frac{1}{\tau_{\mathrm{RT}}}\|R\|\cdot\|E\|. \tag{2}$$

In order to have an equivalence between Eqs. (1) and (2), the matrix $\|R\|$ should be *anti-Hermitian*, which imposes that $\tilde{r}_{21}=-\tilde{r}_{12}^*$ and that $\tilde{r}_{kk}$ be purely imaginary. It can also easily be verified that this is the only form of interaction matrix for which energy is conserved, $d/dt(|\tilde{\mathcal{E}}_1|^2+|\tilde{\mathcal{E}}_2|^2)=0$. The general case in which $\|R\|$ is neither Hermitian nor anti-Hermitian is discussed in Sec. III.

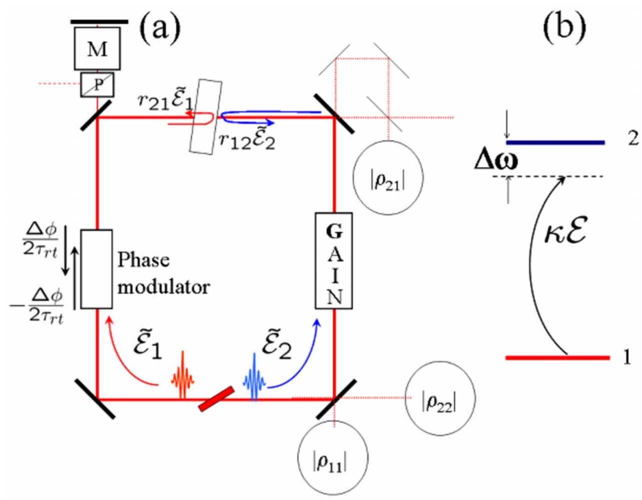

FIG. 1. (Color online) Sketch of the analogy between a ring laser and a two-level system. (a) The bidirectional mode-locked ring laser, where two circulating pulses meet in a saturable absorber jet. An interface, positioned at or near the opposite crossing point of the two pulses, controls the amplitude of the coupling parameter $\tilde{r}_{ij}$. The laser with two pulses circulating in its cavity is the analogue of the two-level system sketched in (b); the circulating intensities in the laser, measured for each direction by quadratic detectors, are the diagonal elements (populations) of the density matrix of the equivalent two-level system. The absence of phase modulation corresponds to the two levels being on resonance, driven at the Rabi frequency $\kappa\mathcal{E}$ by a resonant field (the Rabi frequency $\kappa\mathcal{E}$ corresponds to the frequency $r_{12}/\tau_{\mathrm{RT}}$ in the ring laser analogy). The backscattering at the interface thus provides coherent coupling (Rabi cycling) between the two states, while other noncoherent decays tend to equalize the population in the two directions, and washes out the phase information. The detuning $\Delta\omega$ in (b) corresponds to the phase difference per round trip $\Delta\phi/\tau_{\mathrm{RT}}$ in (a), imposed by an electro-optic phase modulator driven exactly at the cavity round-trip time. A beat note detector measuring the interference between the two fields records the off-diagonal matrix element. A combination of a Pockels cell M and polarizer P controls a feedback of the clockwise pulse into the counterclockwise one.

The real parts of the diagonal elements of the matrix $\|R\|$ represent gain and loss in the cavity. In steady state, the gain and loss are in equilibrium, and the real parts of $\tilde{r}_{kk}$ are zero. A gain (or absorber) with a recovery (relaxation) time longer than $\tau_{\mathrm{RT}}/2$ will cause transients in population. In the cavity sketched in Fig. 1(a), an electro-optic phase modulator imposed an opposite phase shift ($\Delta\phi/2$ and $-\Delta\phi/2$) in either direction, thereby modifying the resonance of the cavity for the pulse $\tilde{\mathcal{E}}_1$ by $\Delta\omega/2=\Delta\phi/(2\tau_{\mathrm{RT}})$, and for pulse $\tilde{\mathcal{E}}_2$ by $-\Delta\omega/2=-\Delta\phi/(2\tau_{\mathrm{RT}})$. These detuning terms contribute to the diagonal terms of the matrix $\|R\|$: $\tilde{r}_{11}=-\tilde{r}_{22}=i\Delta\phi/2$.

III. CONSERVATIVE VERSUS DISSIPATIVE COUPLING

A. Symmetric coupling

The mutual coupling terms $\tilde{r}_{ij}$ have taken numerous forms in the ring laser literature [6,7], which mainly addresses continuous-wave (cw) lasers. This case is considered here for completeness, but, as mentioned above, it is not conservative and does not lead to an equivalence with the two-level equations. The traditional approach has been to represent the scattering coupling distributed in the whole cavity by an equivalent complex "scattering coefficient" $\tilde{r}_{12}=\tilde{r}_{21}=\tilde{\xi}=\xi\exp(i\theta)$. This coefficient couples symmetrically, at each round trip, a fraction ξ of one beam into the other, with an average phase factor θ [6,8,9]. Writing for the fields $\tilde{\mathcal{E}}_1=\mathcal{E}\exp(i\varphi_1)$ and $\tilde{\mathcal{E}}_2=\mathcal{E}\exp(i\varphi_2)$, substituting in Eqs. (2), and separating the real and imaginary parts leads to the standard equations for the laser gyro. In particular, the real part leads to an expression for the total intensity change, which is proportional to

$$\frac{1}{2}\frac{d}{dt}(\tilde{\mathcal{E}}_1\tilde{\mathcal{E}}_1^*+\tilde{\mathcal{E}}_2\tilde{\mathcal{E}}_2^*)=\frac{1}{2}\left(\tilde{\mathcal{E}}_1^*\frac{d\tilde{\mathcal{E}}_1}{dt}+\tilde{\mathcal{E}}_2\frac{d\tilde{\mathcal{E}}_2^*}{dt}+\text{c.c.}\right)$$
$$=\frac{2\xi\cos\theta}{\tau_{\mathrm{RT}}}\,\mathrm{Re}(\tilde{\mathcal{E}}_1\tilde{\mathcal{E}}_2^*). \tag{3}$$

The derivative of the phase difference $\psi=\varphi_2-\varphi_1$ is calculated by extracting the imaginary part of the expression for the derivative of the electric fields,

$$\frac{\partial\psi}{\partial t/\tau_{\mathrm{RT}}}=\Delta\omega-\left\{\xi\left[\frac{\mathcal{E}_1}{\mathcal{E}_2}\sin(\psi-\theta)+\frac{\mathcal{E}_2}{\mathcal{E}_1}\sin(\psi+\theta)\right]\right\}. \tag{4}$$

It is only for $\theta=\pi/2$ that this coupling is conservative, as can be seen from the equation for conservation of energy (3). This particular condition, combined with $\tilde{\mathcal{E}}_1\tilde{\mathcal{E}}_2$, is the one for which the coupling $\xi\exp(i\theta)$ does not introduce any dead band [10], as seen in the expression (4) for the derivative of the differential phase [11].

Unlike the case of a cw laser, the amplitude and phase of the coupling coefficients $\tilde{r}_{12}$ and $\tilde{r}_{21}$ can be easily controlled in the case of the bidirectional mode-locked ring laser. The type of coupling, with $\tilde{\xi}$ purely imaginary, can be created by a thin dielectric layer normal to the beam, in the limit of vanishing thickness. Indeed, such a layer can be described by a polarization $\tilde{P}=N\alpha_p\delta(z)\tilde{E}$, where α_p is the polarizability of the layer of dipoles of density N. Inserting this polarization into Maxwell's wave equation, and integrating across the dielectric layer while taking into account the continuity of the tangential component of the field, leads to the following expressions for the complex reflection coefficient $\tilde{r}_d$ and transmission coefficient $\tilde{t}_d$,

$$\tilde{r}_d=\frac{-i\beta}{1+i\beta}, \tag{5}$$

$$\tilde{t}_d=\frac{1}{1+i\beta}, \tag{6}$$

where $\beta=(2\pi)^2N\alpha_p/\lambda$, λ being the wavelength of the light. It can easily be verified that coupling of the two fields $\tilde{\mathcal{E}}_1$ and $\tilde{\mathcal{E}}_2$ by such a layer conserves the total energy $|\tilde{\mathcal{E}}_1|^2+|\tilde{\mathcal{E}}_2|^2$ consistent with $|r_d|^2+|t_d|^2=1$ and $r_dt_d^*+r_d^*t_d=0$, as long as β is real.

B. Nonsymmetric coupling

The case of mode-locked lasers is particularly interesting, because the localization of the radiation in the cavity enables one to select a truly conservative coupling. Instead of a coupling structure of thickness small compared to the wavelength as considered above, we will consider an interface (that could include a coating) between two media.

The coupling, localized at the crossing point of the two circulating pulses, can be produced by the backscattering at a dielectric interface between two media 1 and 2, for which $\tilde{r}_{12}=\tilde{r}$ and $\tilde{r}_{21}=-\tilde{r}^*$, which corresponds indeed to an anti-Hermitian matrix. It can easily be verified that the total intensity change introduced by this coupling is zero, as expected for a conservative coupling. In fact, the phase relation between the two reflections at either side of the interface is a consequence of energy conservation.

Following the approach of Spreeuw *et al.* [12], we can write the matrix $\|R\|$ in Eq. (2) as a sum of a conservative (here, anti-Hermitian) matrix $\|A\|$ and a dissipative matrix $\|H\|$,

$$\|R\| = \|A\| + \|H\| = \begin{pmatrix} i\dfrac{\Delta\phi}{2} & \tilde{r}_{12} \\ -\tilde{r}_{12}^* & -i\dfrac{\Delta\phi}{2} \end{pmatrix} + \begin{pmatrix} \alpha_1 & -\tilde{g} \\ -\tilde{g}^* & \alpha_2 \end{pmatrix}. \quad (7)$$

In Fig. 1(a), the phase shifts $\pm\Delta\phi/2$ imposed on either pulse 1 and 2 by the electro-optic phase modulator, divided by the round-trip time τ_{RT}, correspond to the detuning of the analogue two-level system. The differential frequency $\Delta\omega$ can also be caused by rotation, Fresnel drag, Faraday effect, etc. In the "dissipative matrix" $\|H\|$, α_1 and α_2 are the net gain (loss) coefficients for the two beams. All phenomena that involve an energy exchange between pulse $|1\rangle$ and $|2\rangle$ contribute to the real part of $\tilde{g}$. These are, for instance, a mutual saturation term in the gain, and coupling between forward and backward pulses in a saturable absorber, because of the population grating induced by the counterpropagating fields [13].

IV. DENSITY-MATRIX EQUATIONS

We can rewrite Eq. (2) in terms of the intensities in either sense of rotation $\rho_{22}=\tilde{\mathcal{E}}_2\tilde{\mathcal{E}}_2^*$ and $\rho_{11}=\tilde{\mathcal{E}}_1\tilde{\mathcal{E}}_1^*$, and the quantities $\rho_{12}=\tilde{\mathcal{E}}_1\tilde{\mathcal{E}}_2^*$ and $\rho_{21}=\tilde{\mathcal{E}}_2\tilde{\mathcal{E}}_1^*$,

$$\frac{d(\rho_{22}-\rho_{11})}{dt/\tau_{RT}} = -4\,\mathrm{Re}(\tilde{r}_{12}\rho_{21}) + 2\alpha_2\rho_{22} - 2\alpha_1\rho_{11}, \quad (8)$$

$$\frac{d\rho_{21}}{dt/\tau_{RT}} = -i\Delta\omega\tau_{RT}\rho_{21} + \tilde{r}_{12}^*(\rho_{22}-\rho_{11})\gamma_t\rho_{21} + (\alpha_1+\alpha_2)\rho_{21} - \tilde{g}^*(\rho_{22}+\rho_{11}), \quad (9)$$

$$\frac{d(\rho_{22}+\rho_{11})}{dt/\tau_{RT}} = 2\alpha_2\rho_{22} + 2\alpha_1\rho_{11} - 4\,\mathrm{Re}(\tilde{g}\rho_{21}). \quad (10)$$

For the pure conservative case, $\|H\|=0$, this system of equations reduces to Eqs. (8) and (9) [with $d/dt(\rho_{11}+\rho_{22})=0$], in which one recognizes Bloch's equation for a two-level system driven off-resonance by a step-function Rabi frequency of amplitude $\tilde{r}/\tau_{RT}$ [4]. The difference in intensities $(\rho_{22}-\rho_{11})$ is the direct analogue of the population difference between the two levels. The off-diagonal matrix element ρ_{21} is the interference signal obtained by beating the two outputs of the laser on a detector. As in the case of the two-level system, one can introduce phenomenological relaxation rates γ for the energy relaxation (diagonal matrix element) and γ_t for the coherence relaxation (off-diagonal matrix elements). The physical meaning of the transverse relaxation time $1/\gamma_t$ is the mutual coherence time of the two pulse trains. For a constant "detuning" $\Delta\omega$, γ_t is related to the beat note bandwidth. Combination of the nonconservative coupling g and the loss α results in energy and coherent relaxation rates γ and γ_t. A general conservative case may involve the dissipative matrix, with the condition that $\rho_{22}+\rho_{11}$ is a constant.

The detuning term in Eq. (9) may include a contribution from the Kerr effect $\Delta\omega_{Kerr}=\Delta\omega_{SPM}+\Delta\omega_{XPM}$. The Kerr-induced self-phase modulation $\Delta\omega_{SPM}$ takes place in all components where the pulses do not cross, such as the gain medium, and the phase modulator in Fig. 1,

$$\Delta\omega_{SPM} = \frac{1}{\tau_{RT}}\left(\frac{2\pi n_{2s}\ell_s}{\lambda}\right)(\rho_{22}-\rho_{11}), \quad (11)$$

where n_{2s} and ℓ_s are the nonlinear index and length of the nonlinear medium involved in self-phase modulation. The Kerr-induced cross-phase modulation $\Delta\omega_{XPM}$ takes place in all components where the pulses do cross, such as the saturable absorber in the example of Fig. 1,

$$\Delta\omega_{XPM} = \frac{1}{\tau_{RT}}\left(\frac{4\pi n_{2x}\ell_x}{\lambda}\right)\sqrt{\rho_{22}\rho_{11}}, \quad (12)$$

where n_{2x} and ℓ_x are the nonlinear index and length of the nonlinear medium involved in cross-phase modulation. The Kerr effect thus introduces two nonlinear terms of the form $C_s(\rho_{22}-\rho_{11})\rho_{21}$ and $C_x\sqrt{\rho_{22}\rho_{11}}\rho_{21}$ in Eq. (9). In the particular experimental situation presented here, these terms are smaller by a factor 10^5 than other contributions and will be neglected. There are, however, situations in which these terms are important and can lead to a determination of the Kerr coefficient [14].

There is a contribution to the beat note bandwidth due to fluctuations in the mirror position (i.e., each pulse sees random differences in the cavity length of the order of the mirror motion over a time of $\tau_{RT}/2$). This broadening mechanism can be considered to be the equivalent of inhomogeneous broadening. The gain (loss) terms α_i can be seen as contributions to the population from other levels. In an inversion driven laser, $\alpha_1\approx\alpha_2=\alpha$ is an intensity-dependent relaxation that exists only in transients, since $\alpha=0$ (gain=loss) when the laser is at equilibrium. This situation occurs also in a two-level system when both levels (1) and (2) are strongly coupled to a third level (laser medium pumped to zero inversion).

V. INITIAL CONDITION

In order to observe Rabi cycling, the system should be initially in a "ground state," i.e., the intensity of one of the pulses dominates. There are several methods possible to achieve this goal:

(i) Insertion of a nonsymmetrical coupling in the cavity.

(ii) Use of a directional gain that can be controlled externally.

(iii) Feedback from one direction into the other outside of the cavity. One direction is extinguished by feeding the pulse back into the other direction, in time and space.

A. Nondirectional coupling

Nondirectional coupling can be achieved with a combination of thin (compared to the wavelength) dielectric or gain (absorbing) layers [15]. Equation (5) gives an expression for the reflection coefficient of a dielectric thin layer. In the case of a gain layer, as in an optically pumped quantum well, $\beta = ib$, where b is a real number, and the expression for the complex reflection field coefficient of the gain layer is

$$\xi_g = \frac{b}{1-b}. \tag{13}$$

A combination of a gain (L) and dielectric layer (R), an eighth of a wavelength apart [total propagation $\exp(-ikz)=\exp(-i\pi/2)=-i$], will have a different reflection coefficient when irradiated from the left ($\tilde{\mathcal{R}}_L$) or from the right ($\tilde{\mathcal{R}}_R$),

$$\tilde{\mathcal{R}}_L \approx i\left(\frac{r_d}{i} - \xi_g\right) \approx 0,$$

$$\tilde{\mathcal{R}}_R \approx \xi_g + \frac{r_d}{i}. \tag{14}$$

This is definitely not a conservative coupling, since the gain layer is providing energy to the light field. This type of nonreciprocal coupling provides an ideal initial condition if it can be applied as a step function. A structure with a single gain layer cannot have sufficient gain for laser operation. Instead, a multiple-quantum-well (MQW) structure with 19 gain layers and 4 dielectric layers was designed and tested in a ring laser [15,16]. Unidirectional and quasiunidirectional (depending on the pump power) operation was demonstrated. Because of the large number of layers, this structure has a too narrow bandwidth (1 nm) to be used with ultrashort pulses.

B. Directional gain and Faraday rotation

In a synchronously pumped optical parametric oscillator (OPO), the gain is traveling with the pump pulse. By inserting the OPO crystal of a ring laser in a linear pump cavity, two countercirculating pulses are created (one at each passage of the pump) [17]. The relative intensity of the pump pulses determines the relative intensity of the circulating signal pulses. This is an interesting system in relation to this analogy, because there is no coupling between the circulating pulses introduced by the gain element.

Another approach to unidirectionality is directional losses, such as can be introduced by a Faraday rotator. The Faraday rotation can be seen as the analogue of fluorescence decay, which transfers energy from the upper level to the lower level, while conserving the total population. In the laser, an approximate conservation of the population results from the saturation properties of the gain medium.

C. External feedback

This is the approach chosen in this work. In order to define the initial condition, the output pulse from one direction is extracted, and fed back ($<1\%$) with a mirror, after appropriate optical delay, into the opposite direction. By using a fast switch (turn-off time of less than the cavity round-trip time of 10 ns) at the Pockels cell, the coupling can be turned off to let the countercirculating fields evolve in the cavity.

VI. EXPERIMENTAL RESULTS

A. The laser

The analogy can be tested with a variety of laser systems, which will differ by the parameters of the gain and loss media. In the case of a synchronously pumped OPO, the gain is of the order of a few percent, and the mutual saturation parameter is zero. In the case of the ring Ti:sapphire-laser chosen for this demonstration, the gain balances the losses for a coefficient α of approximately 0.08.

The experimental system is a ring Ti:sapphire-laser (pump power=5 W, 8% loss/round-trip in a 3.1 m cavity with four prisms [18]) mode-locked with a dye jet as saturable absorber, resulting in 30 ps pulses at 800 nm. The following analogies can be tested:

(i) Rabi cycling of the population difference.

(ii) For the resonant case ($\Delta\omega=0$), the Rabi frequency is proportional to the "driving force" that is the backscattering coefficient $|\tilde{r}|$.

(iii) The Rabi cycling for the population difference ($\rho_{22}-\rho_{11}$) and the off-diagonal element ρ_{21} are 90° out of phase.

(iv) There is a longitudinal and transverse relaxation time.

(v) For the off-resonant case, the Rabi cycling is at frequency $\sqrt{\Delta\omega^2+(|\tilde{r}|/\tau_{\mathrm{RT}})^2}$.

B. Rabi cycling on resonance

In the measurements that follow, the system is "at resonance;" i.e., $\Delta\omega=0$. An example of "Rabi cycling" is shown in Fig. 2. The counterclockwise intensity (ρ_{22}) is plotted as a function of time [Fig. 2(a)]. The clockwise intensity ρ_{11} (not shown) is complementary. The system is prepared so that the ρ_{11} is initially populated ($\rho_{11}=0.8$, $\rho_{22}=0.2$). As the feedback that creates the initial state is switched off at $t=1$ ms, there is a fast (approximately 10 μs) transient. This risetime reflects combined dynamics of the gain and cavity, as the laser adapts to the different (now symmetrical) cavity losses. This risetime corresponds roughly to the fluorescence lifetime of

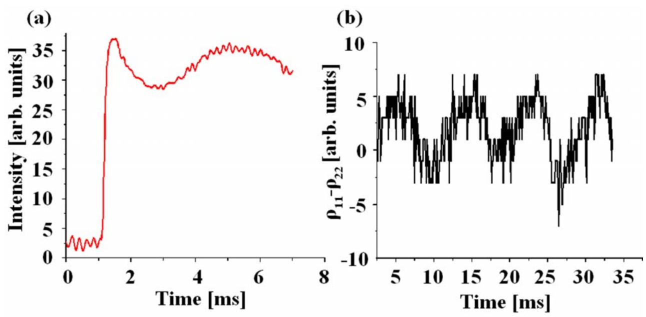

FIG. 2. (Color online) The evolution of the intensities are shown after switching the Pockels cell. (a) The counterclockwise direction is shown—the intensity at clockwise direction is 180° out of phase with this graph, with population dropping from the maximum initial value. The fast initial transient reflects the gain and cavity dynamics associated with the sudden change in cavity loss at the switching time. Thereafter, a slow oscillation due to population transfer or Rabi oscillation between two directions is observed. (b) Population difference showing the Rabi cycling.

the upper state of Ti:sapphire. The "Rabi cycling of the "population difference" $\rho_{22}-\rho_{11}$ is plotted in Fig. 2(b). One can also record the beat note frequency (off-diagonal element $|\rho_{12}|$) as sketched in Fig. 1(a). As can easily be seen from the Bloch vector model of Feynman, Hellwarth, and Vernon [4], the oscillations of the diagonal elements and the off-diagonal element are 90° out of phase. This property can indeed be seen in Fig. 3(a). The Rabi frequency $|\tilde{r}|/\tau_{\mathrm{RT}}$ can be varied by changing the position of the scattering surface, as shown in Fig. 3(b). The maximum value measured [18] for this interface corresponds to a backscattering coefficient of $|\tilde{r}| \approx 1\times 10^{-6}$. Note that the Rabi frequency provides a direct measurement of very minute backscattering coefficients, without the need to trace a complete gyroscopic response as in Refs. [18,19].

In the case of a two-level system, the phenomenological "longitudinal" and "transverse" relaxation times have been identified as energy relaxation time (fluorescence decay) and phase relaxation time (due, for instance, to atomic collisions). Figure 4 shows a measurement of the decay of the Rabi oscillation for the diagonal and off-diagonal elements. The decay is measured by fitting the Fourier transform of the

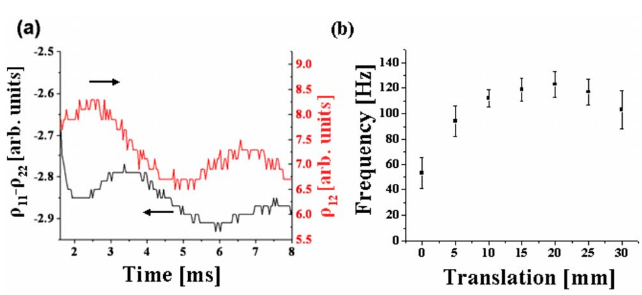

FIG. 3. (Color online) (a) Comparison of the oscillation of the population difference $\rho_{22}-\rho_{11}$ and the off-diagonal element (beat note) ρ_{12}. (b) Rabi frequency as a function of position of the glass at the meeting point of the two directions. Translation of the glass-air interface along the beam result in different values of coupling $\tilde{r}$.

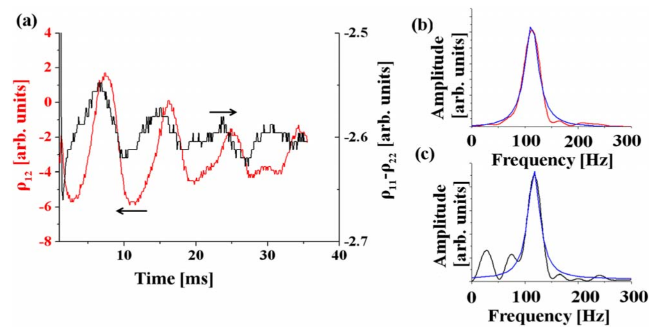

FIG. 4. (Color online) (a) Measurement of the decay of the Rabi oscillations in W and ρ_{21}. (b) The Fourier transforms of the relative measurements are shown on the right.

measurement to a Lorentzian, and measuring its full width at half-maximum. The values are 27 and 30 Hz. As noted previously, there are at least two origins to the decay of the off-diagonal element: vibration of mirrors and coupling through absorption (gain). The latter affects equally the diagonal and off-diagonal elements. The former can be seen as a type of "inhomogeneous broadening," since it has its origin in random cavity length fluctuation, expressed as randomness in the value of $\Delta\omega$. The approximately 30 Hz bandwidth of both decays is consistent with 0.3 μm amplitude vibrations at 100 Hz of cavity components, causing differential cavity fluctuations of 0.3 pm/round trip.

If the radiation of amplitude $\mathcal{E}$ (Rabi frequency $\kappa\mathcal{E}$) is off-resonance with a two-level system by an amount $\Delta\omega$, the Rabi frequency becomes $\sqrt{\kappa^2\mathcal{E}^2+\Delta\omega^2}$. In the case of the ring laser, we can control the off-resonance amount Δ with a Pockels cell (Fig. 1); the initial condition is set favorable to the counterclockwise direction as shown in Fig. 2(a). The Rabi cycling is measured indeed to correspond to $\sqrt{|\tilde{r}|^2/\tau^2+\Delta\omega^2}$. In the resonance case $\Delta\omega=0$, measurement of ρ_{12} leads to $r/\tau_{\mathrm{RT}}=138\pm 15$ Hz. With $\Delta\omega$ of 171 ± 12 Hz, the off-resonant measurement is $r/\tau_{\mathrm{RT}}=237\pm 21$ Hz, which behaves as a two-level system off-resonance. The analogy presented here sheds light on previously unexplained observations on a ring Kerr-lens mode-locked ring laser [20]. Rather than being bidirectional, the operation was observed to switch direction at rates (tens of Hz) that did not seem to correspond to any cavity parameter. The slow switching rate might be due to similar "Rabi cycling," caused by the scattering coefficient of the gain crystal, which is where the pulses meet in the case of pure Kerr-lens mode-locking.

VII. CONCLUSIONS

We have demonstrated analytically and experimentally the analogy between a two-level system and a bidirectional mode-locked ring laser. In the latter, the two "quantum states" $|1\rangle$ and $|2\rangle$ are the sense of circulation of the beams in the ring. The use of short pulses makes it possible to apply purely conservative coupling between the two countercirculating pulses. Resonant interaction between a step-function resonant electromagnetic field and the two-level system leads to Rabi oscillation between the population of the upper and lower states. Similarly, we observe Rabi oscillation between the populations of the "upper" ($\tilde{\mathcal{E}}_2\tilde{\mathcal{E}}_2^*$) and "lower" ($\tilde{\mathcal{E}}_1\tilde{\mathcal{E}}_1^*$) states of the ring laser. The same density-matrix equations that describe the evolution of the two-level system apply to the ring laser, where the diagonal elements represent the intensity of the countercirculating pulses, and the off-diagonal element the interference of the two beams $\tilde{\mathcal{E}}_1\tilde{\mathcal{E}}_2^*$ that is recorded on a detector. The Rabi oscillations of the off-diagonal element are 90° out of phase with those of the population difference. The interaction can be made off-resonance by the use of a phase modulator. The Rabi oscillation frequency increases as expected, as the interaction is detuned from resonance. The Rabi oscillations decay with a time constant that appears to be associated with the mechanical vibrations of the laser support.

The impact of this analogy is in the development of new sensors. Most spectroscopic techniques involve some measurement of $|\rho_{12}|$, as a function of the driving field (measurement of the Rabi frequency $\kappa\mathcal{E}$ leading to the determination of the dipole moment) or detuning $\Delta\omega$. The dependence of the spectrum of $|\rho_{12}|$ on scattering in the ring laser is a very sensitive measurement of the backscattering coefficient $|\tilde{r}|^2 = [(\text{Rabi frequency}) \times \tau_{RT}]^2$ (as small as 0.25×10^{-12} in the lower data point of Fig. 3). The quantity $\Delta\omega$ in the ring laser is an intracavity conversion of a minute phase difference between the two circulating pulses, and results in a modulation observed on ρ_{12}. Any resolution enhancing technique that has been devised in spectroscopy, such as Ramsey fringes [2,3], could be transposed to a laser phase sensor with two intracavity pulses. Pulsed coupling $\tilde{r}$ could be applied (for instance by using a rotating disk) to perform measurements within the beat note bandwidth.

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