Quantum-information splitting using multipartite cluster states

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We provide various schemes for the splitting up of quantum information into parts using four- and five-partite cluster states. Explicit protocols for the quantum information splitting (QIS) of single- and two-qubit states are illustrated. It is found that the four-partite cluster state can be used for QIS of an entangled state and the five-partite cluster state can be used for QIS of an arbitrary two-qubit state. The schemes considered here are also secure against certain eavesdropping attacks.

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I. INTRODUCTION

Quantum entanglement, a feature not available in classical physics, has led to many advances in communication theory and cryptography. It has found many practical applications in teleportation, dense coding, and secret sharing. Quantum teleportation is a technique for transfer of information between parties, using a distributed entangled state and a classical communication channel [1]. It also serves as an elementary operation for a number of quantum communicational protocols and has been experimentally demonstrated for arbitrary single- and two-qubit systems using pairs of Bell states [2,3]. With better understanding of multipartite entanglement, several authors have devised protocols for teleportation using multiparticle entangled channels [4–9].

Quantum secret sharing (QSS) is the generalization of classical secret sharing schemes to the quantum scenario [10]. In QSS the owner who possesses and wishes to transmit the secret information splits it among various parties such that the original information can only be reconstructed by a specific subset of the parties. QSS plays a key role in transmitting and protecting both classical as well as quantum information and hence it can be further divided into two branches, namely, secret sharing of classical information and quantum information.

Hillery *et al.* [11] described the first scheme for secret sharing of quantum-information by extending quantum key sharing to quantum-information splitting by the use of teleportation. In their scheme, the three parties, namely, Alice, Bob, and Charlie, share an entangled three-qubit Greenberger-Horne-Zeilinger (GHZ) state:

$$|\mathrm{GHZ}\rangle_{ABC} = \frac{1}{\sqrt{2}}(|000\rangle \pm |111\rangle)_{ABC}, \quad (1)$$

each possessing one qubit. Alice has an unknown single qubit of information, $|\psi_a\rangle = \alpha|0\rangle + \beta|1\rangle$, which she wants Bob and Charlie to share. Initially, Alice combines $|\psi_a\rangle$ with her qubit in the entangled state and performs a Bell measurement on her pair of qubits and conveys the outcome of her measurement to Charlie via two classical bits, after which the Bob-Charlie system collapses into an entangled state given by

$$U_x \otimes I(\alpha|00\rangle + \beta|11\rangle)_{BC}, \quad (2)$$

where $U_x \in (I, \sigma_x, i\sigma_y, \sigma_z)$. Hence, the unknown single-qubit information is locked between Bob and Charlie in such a way that neither of them can obtain the unknown qubit completely, by locally operating on their own qubits. Now, Bob can perform a single-qubit measurement in the basis $(1/\sqrt{2})(|0\rangle \pm |1\rangle)$ on his qubit and convey the outcome of his measurement to Charlie via one classical bit. Knowing the outcomes of both their measurements, Charlie can obtain $|\psi_a\rangle$ by performing a suitable unitary operation. This technique of splitting and sharing quantum information among two or more parties such that none of them can retrieve the information fully by operating on their own qubits is usually referred to as quantum-information splitting (QIS). The QIS of $|\psi_a\rangle$ involving the tripartite $|\mathrm{GHZ}\rangle_3$ as an entangled channel along with many other probabilistic schemes has been extensively studied and discussed in [12]. The in-principle feasibility of experimental realization of QIS of $|\psi_a\rangle$ has been demonstrated through a pseudo-GHZ state [13]. Later, an experimental scheme using single-photon sources for splitting up of $|\psi_a\rangle$ was demonstrated [14]. Recently, attention has turned toward the usage of different types of multipartite entangled channels for QIS. Four-party secret sharing has been experimentally shown, using a special four-photon polarization entangled state [15]. QIS of $|\psi_a\rangle$ has also been carried out using an asymmetric W state [16] given by

$$|W_a\rangle = \left(\frac{1}{2}|001\rangle + \frac{1}{2}|010\rangle + \frac{1}{\sqrt{2}}|001\rangle\right)_{ABC} \quad (3)$$

and experimentally realized in ion-trap systems. Later, schemes have been devised which used cavity QED to split $|\psi_a\rangle$ via $|W_a\rangle$ [17,18]. Many other QIS schemes have been proposed, in which the particle carrying the information that

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is to be teleported needs to be initially entangled with other particles [19,20]. Recently, a generalization of QIS has been made from qubits to qudits [21]. With the increase in the number of qubits, the scenario becomes complicated, owing to the lack of proper understanding of multiparticle entanglement.

The QIS of an arbitrary two-qubit state given by

$$|\psi_b\rangle = \alpha|00\rangle + \mu|10\rangle + \gamma|01\rangle + \beta|11\rangle \tag{4}$$

was initially carried out using four Bell pairs as an entangled resource [22]. Later, we proposed a robust scheme [23], involving a lesser number of particles for the splitting up of $|\psi_a\rangle$ and $|\psi_b\rangle$. In this paper, we shall present alternate schemes for splitting up of arbitrary single- and two-qubit states using four- and five-partite cluster states and discuss its advantages over other schemes. Up to six-partite cluster states have been experimentally realized in laboratory conditions. These states were mentioned in Ref. [24] for linear-optic one-way quantum computation. Like the previous ones, our schemes are also secure against certain eavesdropping attacks.

In general, an N-qubit cluster state is given by [24]

$$|C_N\rangle = \frac{1}{2^{N/2}} \otimes_{a=1}^{N} (|0\rangle_a \sigma_z^{a+1} + |1\rangle_a), \tag{5}$$

with $\sigma_z^{N+1}=1$. These states show a strong violation of local reality and are shown to be robust against decoherence [25,26]. Much work has been done in trying to characterize the entanglement exhibited by these states, owing to their promising usefulness in quantum-information theory [27,28]. These states have been identified as task-oriented maximally entangled states [29]. In the case of two- and three-partite scenarios, the cluster state is the same as the Bell and the GHZ states, respectively, under local operations and classical communication and for higher-qubit systems, the state is not locally equivalent to the GHZ states and exhibits remarkably different entanglement properties [24]. Recently, up to six-particle cluster states have been experimentally created [30]. This motivates us to investigate the usefulness of these states for QIS of single- and two-qubit states.

The scheme proposed here does not require the initial qubit to be entangled with other qubits, as in Ref. [19]. Hence it is as useful as the QIS schemes involving the $|\text{GHZ}_3\rangle$ and $|W_a\rangle$ states for splitting up of $|\psi_a\rangle$. Moreover, any N-partite GHZ state and the asymmetric W states [16] cannot be used for the QIS of an arbitrary two-qubit state, while we show here that higher-dimensional cluster states ($N>4$) can be used for this purpose. Previous schemes for the QIS of an arbitrary two-qubit state required four Bell pairs, but the same is achieved here using a five-partite entangled state. Unlike the Brown state, cluster states have been realized in laboratory conditions. Hence, our scheme using a five-qubit cluster state is more advantageous than the known schemes [22,23] for the QIS of an arbitrary two-qubit state. Like the previous ones, our schemes are also secure against certain types of eavesdropping attacks.

TABLE I. The outcome of the measurement performed by Alice and the state obtained by Bob and Charlie.

Outcome of the measurement	State obtained
$\frac{1}{\sqrt{2}}(\|00\rangle+\|11\rangle)_{a1}$	$(\alpha(\|000\rangle+\|110\rangle)+\beta(\|001\rangle-\|111\rangle))_{234}$
$\frac{1}{\sqrt{2}}(\|00\rangle-\|11\rangle)_{a1}$	$(\alpha(\|000\rangle+\|110\rangle)-\beta(\|001\rangle-\|111\rangle))_{234}$
$\frac{1}{\sqrt{2}}(\|01\rangle+\|10\rangle)_{a1}$	$(\alpha(\|001\rangle-\|111\rangle)+\beta(\|000\rangle+\|110\rangle))_{234}$
$\frac{1}{\sqrt{2}}(\|01\rangle-\|10\rangle)_{a1}$	$(\alpha(\|001\rangle-\|111\rangle)-\beta(\|000\rangle+\|110\rangle))_{234}$

This paper is organized as follows. In the following section, we devise protocols for splitting up of an unknown single- and entangled two-qubit states using $|C_4\rangle$ as an entangled channel. Subsequently, in Sec. III, we devise schemes for splitting of an unknown single- and two-qubit states using $|C_5\rangle$ as an entangled channel. The conclusion follows in Sec. IV, where we summarize our scheme and point out the direction for future work.

II. $|C_4\rangle$ FOR QIS

The four-partite cluster state is given by

$$|C_4\rangle = \frac{1}{2}(|0000\rangle + |0110\rangle + |1001\rangle - |1111\rangle)_{1234}. \tag{6}$$

$|C_4\rangle$ has been proven to be of immense use in one-way quantum computation [24,31] and also for quantum error correction [32]. One-way quantum computation has been experimentally demonstrated [26]. There has been extensive investigation regarding the usefulness of these states for teleportation and dense coding [29,33]. It has been shown that this state could be used for perfect teleportation of an arbitrary two-qubit state and that its superdense coding capacity reaches the Holevo bound, i.e., one can send four classical bits by sending only two quantum bits using two ebits of entanglement [29]. We shall now demonstrate the usefulness of this state for QIS of single- and two-qubit systems.

A. Single-qubit state

We let Alice possess qubit 1, Bob possess qubits 2 and 3, and Charlie possess 4. Alice combines $|\psi_a\rangle$ with her qubit in the entangled state and performs a Bell measurement. The outcome of the measurement performed by Alice and the entangled state obtained by Bob and Charlie are shown in Table I.

Instead of a Bell measurement, Alice can also perform two single-particle measurements in the basis $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$. Alice can communicate the results of her measurement to Charlie using two cbits of information. Bob then performs a

TABLE II. The outcome of the measurement performed by Alice and the state obtained by Bob and Charlie.

Outcome of the measurement	State obtained
$\frac{1}{\sqrt{2}}(\|000\rangle+\|111\rangle)_{aa'1}$	$(\alpha(\|000\rangle+\|110\rangle)+\beta(\|001\rangle-\|111\rangle))_{423}$
$\frac{1}{\sqrt{2}}(\|000\rangle-\|111\rangle)_{aa'1}$	$(\alpha(\|000\rangle+\|110\rangle)-\beta(\|001\rangle-\|111\rangle))_{423}$
$\frac{1}{\sqrt{2}}(\|001\rangle+\|110\rangle)_{aa'1}$	$(\alpha(\|001\rangle-\|111\rangle)+\beta(\|000\rangle+\|110\rangle))_{423}$
$\frac{1}{\sqrt{2}}(\|001\rangle-\|110\rangle)_{aa'1}$	$(\alpha(\|001\rangle-\|111\rangle)-\beta(\|000\rangle+\|110\rangle))_{423}$

measurement in the basis $(|00\rangle_{23},|11\rangle_{23})$ and communicates the outcome of his results to Charlie via one cbit of information. Knowing the outcomes of both their measurements, Charlie can obtain $|\psi_a\rangle$ by applying a suitable operator on his qubit. For instance, had the Bob-Charlie system evolved into the first state shown in Table II and if the outcome of Bob's measurement is $|00\rangle_{23}$, then Charlie's state collapses to $(\alpha|0\rangle+\beta|1\rangle)_4$. Instead, if the outcome of Bob's measurement is $|11\rangle_{23}$, then Charlie's state collapses to $(\alpha|0\rangle-\beta|1\rangle)_4$. Charlie can obtain $|\psi_a\rangle$ by applying a suitable unitary operator on his qubit.

Now, let us discuss the security of this protocol against an eavesdropper (say Eve). We assume that Eve has managed to entangle an ancilla with a qubit possessed by Bob in the four-qubit cluster state, so that she can measure the ancilla to gain information about the unknown qubit state. Suppose all the three participants are unaware of this attack by Eve; then after Alice performs a Bell measurement, the combined state of Bob, Charlie, and Eve collapses into a four-partite entangled state. However, after Bob performs the two-particle measurement, the Charlie-Eve system collapses into a product state, leaving Eve with no information about the unknown qubit. To see this scenario more explicitly, let us assume that Eve has managed to entangle an ancilla $|0\rangle$ to the second qubit of the entangled channel. If Alice performs a measurement in the first basis, then the combined state of Bob, Charlie, and Eve is

$$\alpha(|0000\rangle+|1101\rangle)+\beta(|0010\rangle-|1111\rangle)_{BCE}. \qquad (7)$$

Now if Bob performs a measurement in the basis $|00\rangle$, then the Charlie-Eve system collapses to $(\alpha|0\rangle+\beta|1\rangle)_C|0\rangle_E$. Hence, Eve's state is unaltered, leaving no chance for her to gain any information about the unknown qubit state. This is due to the fact that entanglement is monogamous [34].

B. Two-qubit state

The $|C_4\rangle$ state can be used for the QIS of an entangled state of type $(\alpha|00\rangle+\beta|11\rangle)_{aa'}$. The protocol goes as follows. We let Alice possess qubit 1, Bob possess qubit 4, and Charlie possess 2 and 3. Alice combines the entangled state $(\alpha|00\rangle+\beta|11\rangle)_{aa'}$ with her particles and performs a three-partite GHZ measurement. The outcome of the measurement performed by Alice and the entangled state obtained by Bob and Charlie are shown in Table II.

Instead of making a three-particle measurement, Alice can also perform a two-particle measurement followed by a single-particle measurement. Alice can send the outcome of her measurement to Charlie via two cbits of information. Now, Bob and Charlie can meet up and convert their state to $(\alpha|000\rangle+\beta|111\rangle)_{423}$ by joint unitary operations on their particles. Bob can perform a single-partite Hadamard measurement in the basis $\frac{1}{\sqrt{2}}(|0\rangle\pm|1\rangle)_4$ and convey the outcome of his measurements to Charlie via 1 cbit of information. If Bob measures in the basis $\frac{1}{\sqrt{2}}(|0\rangle\pm|1\rangle)$, then Charlie's state evolves into $(\alpha|00\rangle\pm\beta|11\rangle)_{23}$. Knowing the outcomes of both their measurements, Charlie can get the state $(\alpha|00\rangle+\beta|11\rangle)_{23}$ by performing an appropriate unitary operation on his qubits.

Let us now consider the security of this scheme against an eavesdropper Eve. Assume that Eve has been able to entangle a qubit to Bob's system. After Alice performs the measurement, when Bob and Charlie meet up and perform unitary operations on their combined state, the ancilla is left

TABLE III. The outcome of the measurement performed by Alice and the state obtained by Bob and Charlie.

Outcome of the measurement	State obtained
$\frac{1}{\sqrt{2}}(\|000\rangle+\|111\rangle)_{a12}$	$(\alpha(\|000\rangle+\|111\rangle)+\beta(\|101\rangle+\|010\rangle))_{345}$
$\frac{1}{\sqrt{2}}(\|000\rangle-\|111\rangle)_{a12}$	$(\alpha(\|000\rangle+\|111\rangle)-\beta(\|101\rangle-\|010\rangle))_{345}$
$\frac{1}{\sqrt{2}}(\|011\rangle+\|100\rangle)_{a12}$	$(\alpha(\|101\rangle+\|010\rangle)+\beta(\|000\rangle+\|111\rangle))_{345}$
$\frac{1}{\sqrt{2}}(\|011\rangle-\|100\rangle)_{a12}$	$(\alpha(\|101\rangle+\|010\rangle)-\beta(\|000\rangle+\|111\rangle))_{345}$

TABLE IV. The outcome of the measurement performed by Bob and the state obtained by Charlie.

Outcome of the measurement	State obtained
$\frac{1}{\sqrt{2}}(\|00\rangle+\|10\rangle)_{34}$	$(\alpha\|0\rangle+\beta\|1\rangle)_5$
$\frac{1}{\sqrt{2}}(\|00\rangle-\|10\rangle)_{34}$	$(\alpha\|0\rangle-\beta\|1\rangle)_5$
$\frac{1}{\sqrt{2}}(\|01\rangle+\|11\rangle)_{34}$	$(\beta\|0\rangle+\alpha\|1\rangle)_5$
$\frac{1}{\sqrt{2}}(\|01\rangle-\|11\rangle)_{34}$	$(\beta\|0\rangle-\alpha\|1\rangle)_5$

TABLE V. The outcome of the measurement performed by Alice and the state obtained by Bob and Charlie (the coefficient is removed for convenience).

Outcome of the measurement	State obtained
$\|0000\rangle+\|1001\rangle+\|0111\rangle+\|1110\rangle$	$\alpha\|000\rangle+\mu\|011\rangle+\gamma\|110\rangle+\beta\|101\rangle$
$\|0000\rangle+\|1001\rangle-\|0111\rangle-\|1110\rangle$	$\alpha\|000\rangle+\mu\|011\rangle-\gamma\|110\rangle-\beta\|101\rangle$
$\|0000\rangle-\|1001\rangle+\|0111\rangle-\|1110\rangle$	$\alpha\|000\rangle-\mu\|011\rangle+\gamma\|110\rangle-\beta\|101\rangle$
$\|0000\rangle-\|1001\rangle-\|0111\rangle+\|1110\rangle$	$\alpha\|000\rangle-\mu\|011\rangle-\gamma\|110\rangle+\beta\|101\rangle$
$\|0001\rangle+\|1000\rangle+\|0110\rangle+\|1111\rangle$	$\alpha\|011\rangle+\mu\|000\rangle+\gamma\|101\rangle+\beta\|110\rangle$
$\|0001\rangle+\|1000\rangle-\|0110\rangle-\|1111\rangle$	$\alpha\|011\rangle+\mu\|000\rangle-\gamma\|101\rangle-\beta\|110\rangle$
$\|0001\rangle-\|1000\rangle+\|0110\rangle-\|1111\rangle$	$\alpha\|011\rangle-\mu\|000\rangle+\gamma\|101\rangle-\beta\|110\rangle$
$\|0001\rangle-\|1000\rangle-\|0110\rangle+\|1111\rangle$	$\alpha\|011\rangle-\mu\|000\rangle-\gamma\|101\rangle+\beta\|110\rangle$
$\|0011\rangle+\|1010\rangle+\|0100\rangle+\|1101\rangle$	$\alpha\|110\rangle+\mu\|101\rangle+\gamma\|000\rangle+\beta\|011\rangle$
$\|0011\rangle+\|1010\rangle-\|0100\rangle-\|1101\rangle$	$\alpha\|110\rangle+\mu\|101\rangle-\gamma\|000\rangle-\beta\|011\rangle$
$\|0011\rangle-\|1010\rangle+\|0100\rangle-\|1101\rangle$	$\alpha\|110\rangle-\mu\|101\rangle+\gamma\|000\rangle-\beta\|011\rangle$
$\|0011\rangle-\|1010\rangle-\|0100\rangle+\|1101\rangle$	$\alpha\|110\rangle-\mu\|101\rangle-\gamma\|000\rangle+\beta\|011\rangle$
$\|0010\rangle+\|1011\rangle+\|0101\rangle+\|1100\rangle$	$\alpha\|101\rangle+\mu\|110\rangle+\gamma\|011\rangle+\beta\|000\rangle$
$\|0010\rangle+\|1011\rangle-\|0101\rangle-\|1100\rangle$	$\alpha\|101\rangle+\mu\|110\rangle-\gamma\|011\rangle-\beta\|000\rangle$
$\|0010\rangle-\|1011\rangle+\|0101\rangle-\|1100\rangle$	$\alpha\|101\rangle-\mu\|110\rangle+\gamma\|011\rangle-\beta\|000\rangle$
$\|0010\rangle-\|1011\rangle-\|0101\rangle+\|1100\rangle$	$\alpha\|101\rangle-\mu\|110\rangle-\gamma\|011\rangle+\beta\|000\rangle$

unchanged. Now, if Bob performs a single-partite measurement on his qubit, the Charlie-Eve system collapses into a product state, leaving the unknown qubit information with Charlie. Note that $|C_4\rangle$ cannot be used for the QIS of an arbitrary two-qubit state.

III. $|C_5\rangle$ FOR QIS

The five-qubit cluster state is given by

$$|C_5\rangle = \frac{1}{2}(|00000\rangle + |00111\rangle + |11101\rangle + |11010\rangle). \quad (8)$$

Like $|C_4\rangle$, the $|C_5\rangle$ state turns out be an important resource for quantum communication. $|C_5\rangle$ can also be used for teleportation of unknown single- and two-qubit states. The superdense coding capacity of this state reaches the Holevo bound, allowing five cbits to be sent using only three qubits. We shall now proceed to study the usefulness of this state for QIS of single- and two-qubit systems.

A. Single-qubit state

We let Alice possess qubits 1 and 2, Bob possess qubits 3 and 4, and Charlie possess 5. Alice combines the unknown qubit $\alpha|0\rangle+\beta|1\rangle$ with her qubits and performs a three-particle GHZ measurement and communicates its outcome to Charlie via two cbits. The outcome of the measurement performed by Alice and the entangled state obtained by Bob and Charlie are shown in Table III.

Now Bob can perform a two-particle measurement and convey its outcome to Charlie via two cbits of information; thus Charlie's particle evolves into a single-qubit state. For instance, had the Bob-Charlie system evolved into the first state in Table III, then the outcome of the measurement performed by Bob and the state obtained by Charlie are shown in Table IV.

Knowing the outcomes of both their measurements, Charlie can obtain the unknown qubit by performing a suitable unitary operation on his qubit.

We now investigate the security of this protocol. As in the previous cases, let us assume that Eve has managed to entangle an ancilla to the channel. After Alice performs a three-partite measurement, the combined system of Bob, Charlie, and Eve evolves into an entangled state. Now, if Bob performs a measurement in the Bell basis, then the Charlie-Eve system collapses into a product state. For instance, if Alice and Bob measure along the first basis, then the Charlie-Eve system collapses into $(\alpha|0\rangle+\beta|1\rangle)_C(|0\rangle+|1\rangle)_E$. Hence, Charlie gets the unknown qubit information and the protocol is secure against these types of eavesdropping attacks.

B. Arbitrary two-qubit state

In this section, we demonstrate a scheme for QIS of an arbitrary two-qubit state using the five-qubit cluster state. Alice has an arbitrary two-qubit state $|\psi_b\rangle$ which she wants Bob and Charlie to share. We now demonstrate the utility of $|C_5\rangle$ for the QIS of an arbitrary two-qubit state. We let Alice possess particles 1 and 5, Bob possess particle 2, and Charlie possess particles 3 and 4 in $|C_5\rangle$. Alice first combines the state $|\psi_b\rangle$ with $|C_5\rangle$ and performs a four-particle von Neumann measurement and conveys the outcome of her measurement to Charlie by four cbits of information. The outcome of the measurement made by Alice and the entangled state obtained by Bob and Charlie are shown in Table V.

It should be noted that each four-partite measurement basis in Table V can be further broken down into Bell and single-partite measurements, making the scheme experimentally feasible. For instance, the first measurement basis can be written as

$$(|\psi_+\rangle(|0\rangle + |1\rangle) + |\psi_-\rangle(|0\rangle - |1\rangle))|0\rangle + (|\phi_-\rangle(|0\rangle - |1\rangle) + |\phi_+\rangle(|0\rangle + |1\rangle))|1\rangle, \tag{9}$$

where $|\psi_+\rangle$, $|\psi_-\rangle$, $|\phi_+\rangle$, and $|\phi_-\rangle$ refer to the Bell states. Bob performs a measurement in the basis $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ and communicates the outcome of his measurement to Charlie who then performs local unitary transformations to get the state $|\psi_b\rangle$. For instance, if the Bob-Charlie system is the first state, then, after Bob's measurement, Charlie's state collapses to $(\alpha|00\rangle \pm \mu|11\rangle \pm \gamma|10\rangle + \beta|01\rangle)$, which can be converted to $|\psi\rangle$ by an appropriate unitary operation. This completes the protocol for QIS of an arbitrary two-qubit state using $|C_5\rangle$. The security of this protocol against eavesdropping attacks will require further investigation. Nevertheless, the protocol is significant, as the threshold number of qubits that an entangled channel should possess for QIS of an arbitrary two-qubit state in the case where Bob and Charlie need not meet up is five.

IV. CONCLUSION

Cluster states are some of the most widely discussed multipartite states in quantum-information theory. Applications such as one-way quantum computing, teleportation, and superdense coding using the cluster states have already been discussed in detail. In this paper, we show the efficacy of cluster states for quantum-information splitting. We discussed different scenarios in which four- and five-partite cluster states can be used for the QIS of single- and two-qubit states. The schemes considered are secure against certain types of eavesdropping attacks. Any experimental ventures for splitting up arbitrary two-qubit information can make use of our scheme. A detailed cryptoanalysis of our work and an investigation of the robustness of all the protocols considered in this paper against particle loss is currently under investigation. We are also looking forward to generalizing our schemes for QIS of an arbitrary two-qubit state using the N-dimensional cluster state as an entangled channel. Further, it remains to be proved that one can devise $(N-4)$ protocols for this purpose.

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