PHYSICAL REVIEW A **78**, 062101 (2008)

Casimir effect in a superconducting cavity and the thermal controversy

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(Received 18 July 2008; revised manuscript received 9 September 2008; published 2 December 2008)

One of the most important and still unresolved problems in the field of dispersion forces, is that of determining the influence of temperature on the Casimir force between two metallic plates. While alternative theoretical approaches lead to contradictory predictions for the magnitude of the effect, no experiment has yet detected the thermal correction to the Casimir force. In this paper we show that a superconducting cavity provides a new opportunity to investigate the problem of the thermal dependence of the Casimir force in real materials, by looking at the change of the Casimir force determined by a small change of temperature. The actual feasibility of the proposed scheme is briefly discussed.

DOI: 10.1103/PhysRevA.78.062101 PACS number(s): 12.20.Ds, 03.70.+k, 42.50.Lc

I. INTRODUCTION

Dispersion forces have been the subject of intense theoretical and experimental investigations for a long time. This is not surprising, because these weak intermolecular forces have a truly pervasive impact, from biology to chemistry, from physics to engineering [1]. It may therefore come as a surprise to know that there still exist, in this well established field, unresolved problems of a fundamental character. This is indeed the case with respect to the problem of determining the van der Waals–Casimir interaction between two metallic bodies at finite temperature [2–4]. As of now, people simply do not know how to compute it, and the numerous recent literature on this subject provides contradictory recipes, which give widely different predictions for its magnitude [2–4]. Apart from its intrinsic interest as a problem in the theory of dispersion forces, addressing this problem is important because many experiments on non-Newtonian forces at the submicron scale use metallic surfaces at room temperature [5]. We remark that, while the Casimir pressure has now been measured accurately in a number of experiments [5,6], there is at present only one experiment that detected the influence of temperature on the Casimir-Polder interaction of a Bose-Einstein condensate with a dielectric substrate [7]. There are, however, no experiments so far that detected the temperature correction to the Casimir force between two metallic bodies, and indeed there are at present several ongoing and planned experiments to measure it [8]. Since the theoretical solution of this difficult problem is still far, it is of the greatest importance to do experiments to probe the effect. A recent accurate experiment using a micromechanical oscillator [5] seems to favor one of the theoretical approaches that have been proposed, but this claim is disputed by other researchers [3].

In this paper, we propose an experiment using superconducting Casimir devices, that could help to find an answer to the question. Superconducting cavities are valuable tools to explore important aspects of Casimir physics. Some time ago we proposed to use rigid superconducting cavities to obtain the first direct measurement of the change of Casimir energy that accompanies the superconducting transition [9]. Here, we argue that it might be possible to shed some light on the controversial problem of the thermal Casimir effect in metallic systems, by measuring how the magnitude of the Casimir force changes as an effect of a change of temperature of the superconducting apparatus. Indeed we show that certain extrapolations to the superconducting state of existing alternative approaches to the thermal Casimir effect for normal metals, that have appeared in the recent literature, lead to strikingly different predictions for the magnitude of the effect. A distinctive feature of the proposed scheme is that, being a difference force measurement, it might achieve a better sensitivity than absolute measurements of the Casimir force. The idea of exploiting difference force measurements to probe the features of the thermal Casimir effect is not new indeed. Already in Ref. [10] the possibility of observing the difference in the thermal Casimir force in a normal (i.e., nonsuperconducting) metallic system, at two different temperatures, was considered, opening up an opportunity for the investigation of the thermal Casimir effect in the submicron separation range.

The plan of the paper is a follows. In Sec. II we briefly review the thermal Casimir effect, and explain in some detail the theoretical problems that arise in the case of metallic cavities. In Sec. III we describe our superconducting Casimir devices and explain the general ideas on which our proposed experiment is based. In Sec. IV we discuss the important issue of the possible prescriptions for the reflection coefficient of quasistatic magnetic fields by a superconducting plate, and in Sec. V we apply these prescriptions to the computation of the predicted change in the Casimir force, in a plane-parallel system, as the temperature of the superconductor is varied. In Sec. VI we discuss the experimentally important plate-sphere geometry, and finally in Sec. VII, we present our conclusions and briefly discuss the prospects of the proposed scheme of being experimentally feasible.

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II. THE THERMAL CASIMIR EFFECT IN METALLIC CAVITIES

All current approaches to the thermal Casimir effect are based on the theory of the Casimir effect developed long ago by Lifshitz [11], on the basis of Rytov's general theory of e.m. fluctuations [12]. According to this theory, the Casimir pressure P between two plane-parallel (possibly stratified) slabs at temperature T, separated by an empty gap of width a, is

$$P(a,T)=\frac{k_BT}{2\pi^2}\sum_{l\geqslant 0}{}' \int d^2\mathbf{k}_\perp q_l \sum_{\alpha=\mathrm{TE,TM}}\left(\frac{e^{2aq_l}}{r_\alpha^{(1)}r_\alpha^{(2)}}-1\right)^{-1}, \quad (1)$$

where the plus sign corresponds to an attraction between the plates. In this equation, the prime over the l sum means that the $l=0$ term has to taken with a weight one half, T is the temperature, $\mathbf{k}_\perp$ denotes the projection of the wave vector onto the plane of the plates and $q_l=\sqrt{k_\perp^2+\xi_l^2/c^2}$, where $\xi_l=2\pi k_BTl/\hbar$ are the Matsubara frequencies. The quantities $r_\alpha^{(j)}\equiv r_\alpha^{(j)}(i\xi_l,\mathbf{k}_\perp)$, with $j=1,2$, denote the reflection coefficients for the two slabs, for α polarization (for simplicity, we do not consider the possibility of a nondiagonal reflection matrix), evaluated at imaginary frequencies $i\xi_l$. It is of the outmost importance at this point to make a remark on the range of validity of Eq. (1): Lifshitz original derivation assumed that spatial dispersion is absent, so that the slabs can be modeled by permittivities $\epsilon^{(j)}(\omega)$ and $\mu^{(j)}(\omega)$ depending only on the frequency ω, and the reflection coefficients $r_\alpha^{(j)}(i\xi_l,\mathbf{k}_\perp)$ have the familiar Fresnel expression. Nowadays it is well known [13] that Eq. (1) is actually also valid for spatially dispersive media (separated by an empty gap), such as metals at low temperature and obviously superconductors, provided only that the appropriate reflection coefficients are used.

We now come to the main topic of the present paper, i.e., the application of Lifshitz formula to nonmagnetic metals (we set $\mu=1$ from now on), which is the experimentally important case. We suppose for simplicity that the plates are made of the same metal. For our purposes, it is convenient first to write Eq. (1) as a sum of three terms $P_0^{(\mathrm{TE})}$, $P_0^{(\mathrm{TM})}$, and P_1, which include, respectively, the TE zero mode, the TM zero mode and all nonvanishing Matsubara modes

$$P(a,T)=P_0^{(\mathrm{TE})}(a,T)+P_0^{(\mathrm{TM})}(a,T)+P_1(a,T). \quad (2)$$

We consider first $P_1(a,T)$. This term poses no particular problems. At room temperature (300 K), the frequency ξ_1 of the first nonvanishing Matsubara mode that enters in Eq. (1) is around $\xi_1=0.159$ eV$/\hbar$, and belongs to the IR region of the spectrum. On the other hand, the presence of the exponential factor $\exp(2aq_l)$ cuts off the Matsubara modes with frequencies greater than a few times the characteristic cavity (angular) frequency $\omega_c=c/(2a)$. For the relevant experimental separations in the range from 100 nm to one or two microns, the frequency ω_c falls somewhere in the range from the near UV to the IR. In the frequency range extending from ξ_1 to a few times ω_c, spatial dispersion is negligible and one can use the Fresnel formula for the reflection coefficients, in terms of the permittivity $\epsilon(i\xi_n)$. The values of $\epsilon(i\xi_n)$ can be obtained, using dispersion theory, from optical data of the plates [see, however, the footnote before Eq. (10) below] or, alternatively, can be computed using analytical models, such as, for example, the plasma model of IR optics

$$\epsilon_P(\omega)=1-\Omega_P^2/\omega^2, \quad (3)$$

where Ω_P is the plasma frequency. If needed, one can possibly augment the above simple model by oscillator contributions accounting for interband transitions that may become sizable at higher frequencies [5]. If desired, one can also take account of dissipation, via the inclusion of a suitable set of relaxation frequencies in the various terms. For the experimentally relevant separations a, the inclusion of all these corrections changes a bit the magnitude of $P_1(a,T)$, and poses no problems at all.

We consider now the two terms in Eq. (2) that have $l=0$, namely, $P_0^{(\mathrm{TE})}(a,T)$ and $P_0^{(\mathrm{TM})}(a,T)$ (we shall refer to these terms as zero modes). Obviously, a vanishing complex imaginary frequency is the same as a real vanishing frequency, and therefore the zero modes involve the reflection coefficients of the slabs for static e.m. fields. Let us examine first $P_0^{(\mathrm{TM})}(a,T)$. A zero-frequency TM mode represents a static electric field. Since a metal expels an electrostatic field, we clearly have $r_{\mathrm{TM}}(0,\mathbf{k}_\perp)=1$. Inserting this value of r_{TM} into the $l=0$ term of Eq. (1) we see that in metals $P_0^{(\mathrm{TM})}(T)$ attains its maximum possible value

$$P_0^{(\mathrm{TM})}(a,T)=\frac{k_BT}{8\pi a^3}\zeta(3), \quad (4)$$

where $\zeta(3)\simeq 1.20$ is the Riemann zeta function.

Consider now the TE zero mode $P_0^{(\mathrm{TE})}(a,T)$. Surprisingly, this single term has become the object of controversy in the last ten years, and no consensus has been reached yet about its correct value. The whole crisis of the thermal Casimir effect for metallic bodies in fact stems from this very term, and it is therefore useful to review briefly what the problem is. A zero-frequency TE mode represents a static magnetic field. Since nonmagnetic metals are transparent to magnetic fields, it seems natural to suppose that we should take

$$r_{\mathrm{TE}}\,(0,\mathbf{k}_\perp)|_{\mathrm{Dr}}=0. \quad (5)$$

When this value is inserted in Eq. (1) one finds easily that the TE zero mode gives a vanishing contribution to the Casimir pressure

$$P_0^{(\mathrm{TE})}|_{\mathrm{Dr}}=0. \quad (6)$$

This result is dubbed in the Casimir community as the Drude-model prediction [14], because $r_{\mathrm{TE}}(0,\mathbf{k}_\perp)=0$ is the value that is obtained by inserting into the Fresnel formula for the TE reflection coefficient the familiar Drude extrapolation of the permittivity of a metal to low frequencies:

$$\epsilon_D(\omega)=1-\Omega_P^2/[\omega(\omega+i\gamma)]. \quad (7)$$

In this formula, γ is the relaxation frequency accounting for Ohmic conductivity. Despite its physical plausibility, this Drude approach has been much criticized in the recent literature. The first problem that was encountered is theoretical, as it was shown that, in the case of perfect lattices, the Drude model value of the TE zero mode is apparently inconsistent with Nernst theory [15]. The problem is subtle though, and the actual existence of this inconsistency is still disputed. For a thorough discussion of different points of view on this issue we refer the reader to the recent review [3] (see also Ref. [16]). In addition, and perhaps more importantly, it has been claimed that the Drude model is also inconsistent with recent experiments [5]. The solution that has been proposed [5] to cure both difficulties is very puzzling: in evaluating the low-frequency contributions of Eq. (1), and in particular the TE zero mode, instead of the physically plausible Drude model, one should use the plasma model of IR optics, Eq. (3), extrapolated without modifications all the way to zero frequency. For the TE zero mode, this prescription leads to the following expression for the reflection coefficient:

$$r_{\mathrm{TE}}\,(0,\mathbf{k}_\perp)|_{\mathrm{pl}} = \frac{\sqrt{\Omega_P^2/c^2 + k_\perp^2} - k_\perp}{\sqrt{\Omega_P^2/c^2 + k_\perp^2} + k_\perp}, \tag{8}$$

and we shall refer to this as the plasma model prescription. For the relevant values of $k_\perp \simeq 1/(2a)$, and for typical values of $\Omega_P \simeq 9$ eV$/\hbar$ (corresponding to gold), it is easy to verify that $r_{\mathrm{TE}}\,(0,\mathbf{k}_\perp)|_{\mathrm{pl}}$ is close to 1. For example, for $a=200$ nm, we obtain $r_{\mathrm{TE}}\,[0,1/(2a)]|_{\mathrm{pl}}=0.90$. Since $r_{\mathrm{TE}}\,(0,\mathbf{k}_\perp)|_{\mathrm{pl}} \neq 0$, $P_0^{(\mathrm{TE})}(a,T)$ does not vanish anymore, and we let $P_0^{(\mathrm{TE})}\,(a,T;\Omega_P)|_{\mathrm{pl}}$ its magnitude, as determined by the plasma prescription. For $\omega_c/\Omega_P \ll 1$, $P_0^{(\mathrm{TE})}\,(a,T;\Omega_P)|_{\mathrm{pl}}$ has the following expansion:

$$P_0^{(\mathrm{TE})}\,(a,T;\Omega_P)|_{\mathrm{pl}} \simeq \frac{k_B T}{8\pi a^3}\zeta(3)\left(1 - 6\frac{\delta}{a} + 24\frac{\delta^2}{a^2}\right), \tag{9}$$

where $\delta = c/\Omega_P$. We see that, for sufficiently large separations a, the magnitude of $P_0^{(\mathrm{TE})}\,(a,T;\Omega_P)|_{\mathrm{pl}}$ is comparable to $P_0^{(\mathrm{TM})}(a,T)$. Recalling that the Drude prescription predicts instead $P_0^{(\mathrm{TE})}|_{\mathrm{Dr}}=0$, we see that the predicted Casimir pressures for the two prescriptions differ by a quantity $\widetilde{\Delta P}(a,T) \equiv P(a,T)|_{\mathrm{Dr}} - P(a,T)|_{\mathrm{pl}}$ [22]:

$$\widetilde{\Delta P}(a,T) = -P_0^{(\mathrm{TE})}\,(a,T;\Omega_P)|_{\mathrm{pl}}. \tag{10}$$

The minus sign an the right-hand side means that the Drude model predicts a smaller Casimir pressure than the plasma model.

The disagreement between the two prescriptions is most evident at large separations. Indeed, for $k_B T a/(\hbar c) \gg 1$, $P_1(a,T)$ becomes negligible and the entire Casimir pressure is given by the zero modes contribution only [2,3]. Since in this limit $P_0^{(\mathrm{TE})}\,(a,T;\Omega_P)|_{\mathrm{pl}} \simeq P_0^{(\mathrm{TM})}(T)$, we see at once that the pressure predicted by the Drude prescription is one-half that predicted by the plasma model prescription Obviously then the straightforward way to clarify the problem would be to measure the Casimir pressure at large separations, say around 5 μm, for room temperature. Unfortunately, at such large separations the Casimir pressure is very small, and as a result all attempts made so far to measure it have been unsuccessful.

At the time of this writing, the most significant experiments are the ones reported in Ref. [5], in which the Casimir pressure was measured for separations in the range from 160 to 750 nm. For such small separations, the quantity $\widetilde{\Delta P}(a,T)$ is only a small fraction of the absolute Casimir pressure (for gold plates at room temperature, about 2–3% at separations of around 200–300 nm, where the experiment in the second of Ref. [5] was most sensitive). Observing such a small correction via an absolute Casimir measurement is very difficult because one needs to measure with high precision the plates separation and control a number of systematic errors, similar to residual electrostatic attractions [17], roughness of the plates, etc. After an accurate analysis of these possible sources of systematic errors, the authors of Ref. [5] conclude that the Drude approach is inconsistent with the experiment at high confidence level. However, this statement has been disputed by other researchers (see Ref. [2]). In this regard, it has been remarked recently [18] that obtaining an accurate theoretical prediction, at the percent level, for the magnitude of Casimir force is barely possible, unless one has accurate optical data, extending over a wide range of frequencies around ω_c, taken on the actual surfaces used in the experiment.

III. A SUPERCONDUCTING CASIMIR EXPERIMENT

In light of the above considerations, we wondered if one can conceive a type of experiment specifically devised to probe the magnitude of the contribution from the TE zero mode. As we see below, a superconducting Casimir apparatus is, in principle, a good tool to do that.

Consider then two metallic plates made of a superconducting metal, and imagine cooling them at a temperature $T_1 < T_c$, where T_c is the critical temperature. We suppose that the equilibrium temperature of the system is rapidly changed from the temperature T_1 to a higher temperature $T_2 < T_c$. It is our aim to estimate the difference ΔP between the Casimir pressures at the two temperatures

$$\Delta P(a|T_2,s;T_1,s) \equiv P(a|T_2,s) - P(a|T_1,s). \tag{11}$$

Let us explain the meaning of the new notation used in the above equation. The notation is used to remind the reader of the twofold dependence of the Casimir pressure on the temperature T. Indeed, by looking at Eq. (1) we see that, on one side, P has an explicit temperature dependence, determined by the overall T factor, in front of the sum symbol, and by the T dependence of the Matsubara frequencies $\xi_l = 2\pi k_B T l/\hbar$. More importantly for our purposes, there is also an implicit dependence of P on T, through the reflection coefficients, which may themselves depend on the temperature. The notation in Eq. (11) stresses this fact by making explicit reference to the dependence of P on the state of the plates, which can either be normal (n) or superconducting (s). We remark that probing the explicit T dependence of the Casimir pressure, by letting pretty large changes of temperature (around 50 K), was the goal of the room-temperature experiment proposed in Ref. [10]. In that experiment, the temperature dependence of the reflection coefficients was completely negligible. In our case the situation is much the opposite: the main expected cause of variation of $P(a,T)$ is now the temperature dependence (in a certain frequency range) of the reflection coefficients of the plates in the superconducting state.

After these remarks, we can go now to the main point: why should the proposed experiment be capable of telling us anything about the thermal Casimir problem in metallic systems? This is so because the superconducting transition affects the optical properties of a metal only at frequencies ω smaller than a few times $k_B T_c/\hbar$ [20]. As a result, the magnitude of ΔP turns out to be almost completely determined by the TE zero mode, and therefore it is very sensitive to the prescription adopted for it. We shall indeed show below that the two prescriptions considered in the previous section, lead to sharply different magnitudes for ΔP.

Before we embark in the computation of ΔP, a key problem to address is to decide what prescription are we going to use for the reflection coefficient of the TE zero mode in the superconducting state. This is the subject of the next section.

IV. THE TE ZERO MODE IN A SUPERCONDUCTOR

According to the Lifshitz formula, which we recall is also valid for superconductors, in order to estimate the contribution of the TE zero mode to the Casimir pressure for a superconducting cavity, we need to determine what expression for the reflection coefficient $r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp)$ should be used in Eq. (1), when a plate is superconducting. Addressing this problem is the purpose of this section.

We have seen in Sec. II that there is a theoretical uncertainty in the value of the reflection coefficient $r_{\rm TE}^{(n)}(0,\mathbf{k}_\perp)$ of a metallic plate, in the normal state, depending on whether we include or not, in the permittivity of the metal, the effect of Ohmic dissipation. If dissipation is included, we end up with the Drude prescription, (5), if dissipation is ignored we instead obtain the plasma prediction, (8). What about a superconductor? For sure, static currents flow in a superconductor without any measurable dissipation, and so there should really be no room for ambiguity now: as far as we know, in the dc limit, superconductors are strictly dissipationless. This is reflected in the plasmalike form of the permittivity function $\epsilon_s(\omega)$ that can be used to describe the response of superconductors to quasistatic electromagnetic fields, in the local London limit

$$\epsilon_s(\omega) = -[c/(\lambda_L \omega)]^2. \tag{12}$$

In this equation, λ_L represents the penetration depth, that can be expressed in terms of the plasma frequency Ω_P as $\lambda_L = (n/n_s)^{1/2} c/\Omega_P$, where n_s/n represents the fractional density of superconducting electrons. The temperature dependence of n_s/n is rather well described by the phenomenological law $n_s/n = 1-(T/T_c)^4$ [19]. If we insert $\epsilon_s(\omega)$ into the Fresnel formula for the TE reflection coefficient at zero frequency we obtain

$$r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp|T) = \frac{\sqrt{1/\lambda_L^2 + k_\perp^2} - k_\perp}{\sqrt{1/\lambda_L^2 + k_\perp^2} + k_\perp}, \quad \text{for } T \leqslant T_c, \tag{13}$$

where we stressed that $r_{\rm TE}^{(s)}$ is temperature dependent, because λ_L is. The above form of $r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp)$ is physically plausible, and it correctly describes the Meissner effect. It should be noted that $r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp)$ has the same form as the plasma-model prescription $r_{\rm TE}^{(n)}(0,\mathbf{k}_\perp)|_{\rm pl}$ for the normal state, apart from the replacement of Ω_P^2/c^2 by $1/\lambda_L^2$. In fact, taking account of the temperature dependence of λ_L, $r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp|T)$ provides a smooth interpolation between $r_{\rm TE}^{(n)}(0,\mathbf{k}_\perp)|_{\rm pl}$ and $r_{\rm TE}^{(n)}(0,\mathbf{k}_\perp)|_{\rm Dr}$, as the temperature is increased from zero towards T_c:

$$r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp|T) \to r_{\rm TE}^{(n)}(0,\mathbf{k}_\perp)|_{\rm pl} \quad \text{for } T/T_c \to 0, \tag{14}$$

$$r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp|T) \to r_{\rm TE}^{(n)}(0,\mathbf{k}_\perp)|_{\rm Dr} = 0 \quad \text{for } T/T_c \to 1. \tag{15}$$

Now we are faced with the following question: depending on what prescription we choose in the normal state, what do we do in the superconducting state? We cannot offer a certain answer to this question, but the following choices appear to us as reasonable. Consider first the Drude prescription. Since the superconducting phase transition is of second order, the reflection coefficients must be smooth across T_c. Now, the Drude prescription asserts that $r_{\rm TE}^{(n)}(0,\mathbf{k}_\perp)$ is zero, and therefore we should fix the prescription in the superconducting phase in such a way that $r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp|T)$ approaches zero for $T \to Tc$. Of course, we could achieve this by taking $r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp|T)$ identically zero for $T < T_c$. However, such an ansatz looks to us physically wrong, because it is in conflict with the Meissner effect. The prescription that we choose then is to use Eq. (13), with λ_L temperature dependent, as our Drude recipe in the superconducting phase

$$r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp|T)|_{\rm Dr} = r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp|T) \quad \text{for } T \leqslant T_c. \tag{16}$$

This choice ensures smoothness of the reflection coefficient at the phase transition, while describing correctly the Meissner effect. Importantly, within this prescription, the reflection coefficient is temperature dependent in the superconducting phase.

Consider now the plasma prescription. Again, the reflection coefficient must be smooth through the transition. If we further make the plausible assumptions that the reflection coefficient cannot decrease as we reduce the temperature, and that the plasma frequency sets up an upper limit on the quantity c/λ_L, we are led to conclude that, within the plasma prescription, the reflection coefficient $r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp|T)$ has to be temperature independent and equal to its normal-state value

$$r_{\rm TE}^{(s)}(0,\mathbf{k}_\perp|T)|_{\rm pl} = r_{\rm TE}^{(n)}(0,\mathbf{k}_\perp)|_{\rm pl} \quad \text{for } T \leqslant T_c. \tag{17}$$

It should be noted that, with this prescription, the reflection coefficient does not change at all across the transition, and it is temperature independent in the superconducting phase. This feature marks a crucial difference between the plasma prescription and our Drude prescription in Eq. (16). Having defined our prescriptions for the TE reflection coefficient in the superconducting state, we are ready now to proceed with the computation of ΔP according to the two approaches.

V. Computing ΔP

In this section, we undertake the computation of ΔP. We shall consider two possible geometries: The plane-parallel configuration and the sphere-plate configuration, which is the one usually adopted in current experiments. For either geometry, it is useful to consider two different setups for the system: the first one consists of two superconducting plates, while the second setup has just one superconducting plate, the other plate being made of a normal metal. To be definite, we shall consider Nb as our superconducting material, and Au as the normal metal. The choice of Nb is due to the fact that, among the classic metallic superconductors, it is the one with the highest critical temperature $T_c = 9.2$ K, a feature that will be seen to ensure the largest difference between the Drude and the plasma predictions for ΔP. It is assumed that the Nb plates have a thickness much larger than the penetration depth λ_L of magnetic fields. In this section we shall only consider the plane-parallel case. The sphere-plate case will be treated in Sec. VI below.

It is convenient to split Eq. (11) in a way analogous to Eq. (2):

$$\Delta P = \Delta P_0^{(\mathrm{TE})} + \Delta P_0^{(\mathrm{TM})} + \Delta P_1, \tag{18}$$

where the meaning of the symbols is obvious. Leaving aside for the moment $\Delta P_0^{(\mathrm{TE})}$, we consider first $\Delta P_0^{(\mathrm{TM})}$ and ΔP_1. Since $r_{\mathrm{TM}}(0,\mathbf{k}_\perp)$ is one for metals, no matter whether normal of superconducting, we surely have

$$\Delta P_0^{(\mathrm{TM})}(a|T_2,s;T_1,s) = \Delta P_0^{(\mathrm{TM})}(a|T_2,n;T_1,n). \tag{19}$$

Consider now ΔP_1. Recall that ΔP_1 includes only the contributions from non vanishing Matsubara modes. The key experimental fact to consider now is that the optical properties of superconductors are appreciably different from those of the normal state (right above T_c) only for frequencies less than a few times $k_B T_c/\hbar$ [20]. Since $\xi_1(T_2) \simeq \xi_1(T_1) \simeq \xi_1(T_c) = 2\pi k_B T_c/\hbar$, we see that already the first nonvanishing Matsubara mode is over six times larger than $k_B T_c/\hbar$. Therefore, in the range of frequencies that is relevant for ΔP_1, the superconductor can be described, at all temperatures below T_c, by the same set of reflection coefficients of the normal state, at temperatures slightly above T_c (this consideration applies even more to Au, of course). This implies at once that

$$\Delta P_1(a|T_2,s;T_1,s) = \Delta P_1(a|T_2,n;T_1,n). \tag{20}$$

At this point, it is convenient to add and subtract from Eq. (18) the quantity $\Delta P_0^{(\mathrm{TE})}(a|T_2,n;T_1,n)$. In this way, we get

$$\begin{aligned}\Delta P(a|T_2,s;T_1,s) &= [\Delta P_0^{(\mathrm{TE})}(a|T_2,s;T_1,s) - \Delta P_0^{(\mathrm{TE})}(a|T_2,n;T_1,n)] \\ &\quad + [\Delta P_0^{(\mathrm{TE})}(a|T_2,n;T_1,n) + \Delta P_0^{(\mathrm{TM})}(a|T_2,n;T_1,n) \\ &\quad + \Delta P_1(a|T_2,n;T_1,n)].\end{aligned} \tag{21}$$

Now we note that the sum of three terms on the second line of the above equation is nothing but the quantity $\Delta P(a|T_2,n;T_1,n)$ that represents the change of Casimir pressure when the temperature is changed from T_1 to T_2, in a normal metal system, with temperature-independent reflection coefficients. This quantity has been computed already in Ref. [10], both for the Drude and the plasma prescriptions, and we shall later use the formulas derived in that paper to estimate it. Substituting $\Delta P(a|T_2,n;T_1,n)$, we may then rewrite Eq. (21) as

$$\begin{aligned}\Delta P(a|T_2,s;T_1,s) &= [\Delta P_0^{(\mathrm{TE})}(a|T_2,s;T_1,s) - \Delta P_0^{(\mathrm{TE})}(a|T_2,n;T_1,n)] \\ &\quad + \Delta P(a|T_2,n;T_1,n).\end{aligned} \tag{22}$$

The above equation represents the first key result of this paper, because it shows, as announced earlier, that superconductivity has an effect on the Casimir pressure only via the TE zero mode. At this point we consider separately the plasma and the Drude prescriptions.

A. The plasma prescription

According to our plasma prescription for the superconducting state (17), the reflection coefficient for the TE zero mode in the superconducting state has precisely the same expression as in the normal state. This being so, we obviously have

$$\Delta P_0^{(\mathrm{TE})}\,(a|T_2,s;T_1,s)|_{\mathrm{pl}} - \Delta P_0^{(\mathrm{TE})}\,(a|T_2,n;T_1,n)|_{\mathrm{pl}} = 0, \tag{23}$$

both for the Nb-Nb and the Nb-Au setups. When this formula is used in Eq. (22), we see that, in either setup, superconductivity has no effect on ΔP:

$$\Delta P\,(a|T_2,s;T_1,s)|_{\mathrm{pl}} = \Delta P\,(a|T_2,n;T_1,n)|_{\mathrm{pl}}. \tag{24}$$

The quantity $\Delta P\,(a|T_2,n;T_1,n)|_{\mathrm{pl}}$ has been computed in Ref. [10], for the case of two plates made of the same metal. This case is good enough for us, also in the case of the Nb-Au setting, because of the small difference between the plasma frequencies of Nb (8.7 eV/$\hbar$) and Au (9 eV/$\hbar$). In our computations, from now on, we shall than take for both metals the common value $\Omega_P = 9$ eV/$\hbar$. Then, our $\Delta P\,(a|T_2,n;T_1,n)|_{\mathrm{pl}}$ coincides with minus the quantity ΔF_{pp} of Ref. [10] (the minus sign is due to the fact that we consider positive forces to correspond to an attraction, while Ref. [10] uses the opposite convention). Therefore, we can rewrite Eq. (24) as

$$\Delta P\,(a|T_2,s;T_1,s)|_{\mathrm{pl}} = -\Delta F_{pp}. \tag{25}$$

We can obtain an estimate of the effect by using the following perturbative expression for ΔF_{pp} of Ref. [10], that holds for low temperatures $k_B T a/(\hbar c) \ll 1$, and for not too small separations $\omega_c/\Omega_p \ll 1$:

$$\Delta F_{pp}(a,T_1,T_2) = -\Delta^{(1)}F_{pp}(T_1,T_2)\Delta^{(2)}F_{pp}(a,T_1,T_2), \tag{26}$$

where

$$\Delta^{(1)}F_{pp}(T_1,T_2) = \frac{\pi^2 k_B^4 (T_2^4 - T_1^4)}{45\hbar^3 c^3} \tag{27}$$

and

$$\Delta^{(2)}F_{pp}(a,T_1,T_2) = 1 + \frac{90\zeta(3)}{\pi^3}\frac{\delta}{a}\frac{T_{\mathrm{eff}}}{T_1+T_2}\left(1 + \frac{T_1 T_2}{T_1^2 + T_2^2}\right), \tag{28}$$

where $k_B T_{\mathrm{eff}} = \hbar c/(2a)$ and $\delta = c/\Omega_p$. The above formulas show that ΔP (Nb-Nb)$|_{\mathrm{pl}}$ and ΔP (Nb-Au)$|_{\mathrm{pl}}$ are both extremely small. For example, upon taking $T_1=5$ K, $T_2 \simeq T_c$, and $a=100$ nm, we obtain

$$\Delta P|_{\rm pl} \approx 1.4 \times 10^{-9}\ \text{Pa}. \qquad (29)$$

The conclusion that we draw is that, with the plasma prescription, the change with temperature of the Casimir pressure is unmeasurably small, both in the Nb-Nb and in the Nb-Au setups.

B. The Drude prescription

We now consider the Drude prescription. First, we analyze the Nb-Au setup. Since Au is always a normal metal, its Drude reflection coefficient for the TE zero mode is zero, at both temperatures. Therefore, since Lifshitz formula implies only products of reflection coefficients for the two plates, it follows that the TE zero mode gives no contribution, in the Nb-Au setup

$$\Delta P_0^{(\rm TE)}\ (a|T_2,s;T_1,s)|_{\rm Dr}^{\rm Nb\text{-}Au} = \Delta P_0^{(\rm TE)}\ (a|T_2,n;T_1,n)|_{\rm Dr}^{\rm Nb\text{-}Au} = 0. \qquad (30)$$

Upon substituting these formulas into Eq. (22) we obtain

$$\Delta P\ (a|T_2,s;T_1,s)|_{\rm Dr}^{\rm Nb\text{-}Au} = \Delta P\ (a|T_2,n;T_1,n)|_{\rm Dr}^{\rm Nb\text{-}Au}. \qquad (31)$$

Again, we find that superconductivity has no effect on the change of Casimir pressure. The Drude value of the quantity $\Delta P\ (a|T_2,n;T_1,n)|_{\rm Dr}^{\rm Nb\text{-}Au}$ can also be found in Ref. [10], from which we obtain the estimate

$$\Delta P\ (a|T_2,s;T_1,s)|_{\rm Dr}^{\rm Nb\text{-}Au} = -\Delta F_{pp} - \frac{\zeta(3)k_B(T_2-T_1)}{8\pi a^3}\left(1-6\frac{\delta}{a}+24\frac{\delta^2}{a^2}\right). \qquad (32)$$

The expression on the right-hand side of the above equation, differs from the analogous expression obtained within the plasma prescription (25) by the presence of the extra term on the second line. At our cryogenic temperatures, the absolute value of the latter term is larger than $|\Delta F_{pp}|$ by several orders of magnitude, for changes of temperature of a few degrees. We therefore see that heating the system, the Casimir pressure decreases by an amount proportional to the temperature change. We remark once again that this change of Casimir pressure is not an effect of superconductivity, and it only arises from the explicit T dependence of the Lifshitz formula. Indeed, an analogous phenomenon was found to occur in the room temperature setting of Ref. [10].

We finally consider the Nb-Nb setup, the most interesting case indeed. Using the reflection coefficient in Eq. (16), we easily find

$$\Delta P_0^{(\rm TE)}\ (a|T_2,s;T_1,s)|_{\rm Dr}^{\rm Nb\text{-}Nb} = P_0^{(\rm TE)}(a,T_2;c/\lambda_L) - P_0^{(\rm TE)}(a,T_1;c/\lambda_L), \qquad (33)$$

$$\Delta P_0^{(\rm TE)}\ (a|T_2,n;T_1,n)|_{\rm Dr}^{\rm Nb\text{-}Nb} = 0, \qquad (34)$$

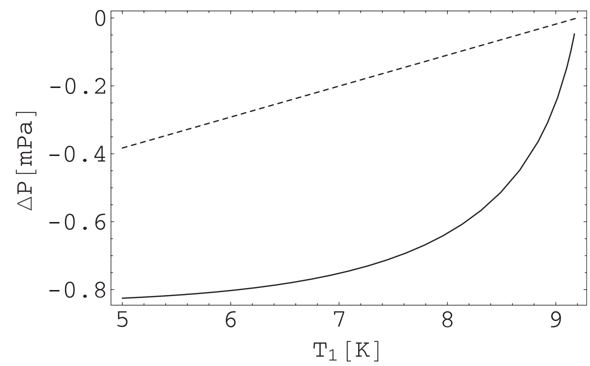

FIG. 1. Plots of ΔP (Nb-Nb)$|_{\rm Dr}$ (solid line) and ΔP (Nb-Au)$|_{\rm Dr}$ (dashed line) (in mPa) for two parallel plates at fixed separation $a=150$ nm, versus temperature T_1 (in K), for fixed $T_2 \simeq T_c$.

where in the first of the two above equations, $P_0^{(\rm TE)}(a,T;c/\lambda_L)$ denotes the magnitude of the TE zero mode, that results after we plug into the $l=0$ term of Lifshitz formula the expression the Drude ansatz for the TE reflection coefficient for the superconductor (16). As for the quantity $\Delta P\ (a|T_2,n;T_1,n)|_{\rm Dr}$, its magnitude is given by the same expression on the right-hand side of Eq. (32). Collecting everything together, Eq. (22) gives

$$\Delta P\ (a|T_2,s;T_1,s)|_{\rm Dr}^{\rm Nb\text{-}Nb} = [P_0^{(\rm TE)}(a,T_2;c/\lambda_L) - P_0^{(\rm TE)}(a,T_1;c/\lambda_L)] - \Delta F_{pp} - \frac{\zeta(3)k_B(T_2-T_1)}{8\pi a^3}\left(1-6\frac{\delta}{a}+24\frac{\delta^2}{a^2}\right). \qquad (35)$$

In Fig. 1 we show plots of $\Delta P\ (a|T_2,s;T_1,s)|_{\rm Dr}^{\rm Nb\text{-}Nb}$ (solid line) and $\Delta P\ (a|T_2,s;T_1,s)|_{\rm Dr}^{\rm Nb\text{-}Au}$ (dashed line), both expressed in mPa, for $a=150$ nm, as a function of the temperature T_1 (in K), for $T_2 \simeq T_c$. In Fig. 2 we show the plots of the same quantities, this time as functions of plates separation a (in microns), for $T_1=5$ K and $T_2 \simeq T_c$. Note that, contrary to what we found with the plasma prescription, our Drude prescription predicts negative changes of pressure, i.e., the Casimir pressure decreases as one goes from the superconducting state towards the normal state. Moreover, it should also

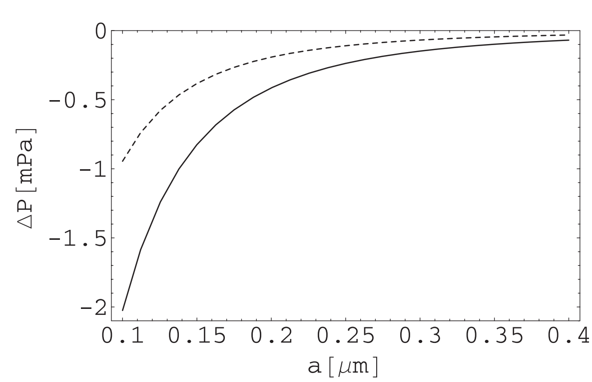

FIG. 2. Plots of ΔP (Nb-Nb)$|_{\rm Dr}$ (solid line) and ΔP (Nb-Au)$|_{\rm Dr}$ (dashed line) (in mPa) for two parallel plates versus separation a (in microns), for $T_1=5$ K and $T_2 \simeq T_c$.

be noted that the change of pressure is larger in the Nb-Nb setup, than in the Nb-Au setup. However, the really striking finding is that, for both setups, the Drude approach predicts changes of Casimir pressure, whose magnitudes are around five or six orders of magnitude larger than the corresponding changes predicted by the plasma approach

VI. THE SPHERE-PLATE CASE

Now we consider the configuration of a sphere placed above a plate, which is the one used in most of the current experiments. We let R the radius of the sphere and a the minimum sphere-plate separation. As is well known, for small separations, i.e., for $a/R \ll 1$, the Casimir force $F_{\rm sp}$ between a sphere and a plate can be obtained by means of the so-called proximity force approximation (PFA) [2]. The relative error introduced by this approximation is of order a/R [2], and therefore it is only a fraction of a percent in typical experimental situations. The PFA expression for $F_{\rm ps}$ is

$$F_{\rm ps} = -\frac{k_B T R}{2\pi}\sum_{l\geqslant 0}{}' \int d^2\mathbf{k}_\perp \sum_{\alpha=\rm TE,TM} \ln(1 - r_\alpha^{(1)} r_\alpha^{(2)} e^{-2aq_l}), \tag{36}$$

where a plus sign for the force again corresponds to attraction. Analogously to what we did in the plane-parallel case, we consider two distinct setups: in the first one, both the plate and the sphere are made of Nb, while in the second setup the sphere is made of Nb and the plate of Au (or vice versa). For either setup, we consider then the change in the sphere-plate force

$$\Delta F_{\rm ps}(a|T_2,s;T_1,s) \equiv F_{\rm ps}(a|T_2,s) - F_{\rm ps}(a|T_1,s), \tag{37}$$

that occurs when the equilibrium temperature of the system is increased from T_1 to T_2 (the notation adopted here is analogous to that used in the plane parallel case). By repeating the reasonings that led us to write Eq. (22), we can obtain the following expression for $\Delta F_{\rm ps}(a|T_2,s;T_1,s)$:

$$\begin{aligned}\Delta F_{\rm ps}(a|T_2,s;T_1,s) &= [\Delta F^{(\rm TE)}_{(\rm ps)0}(a|T_2,s;T_1,s) - \Delta F^{(\rm TE)}_{(\rm ps)0}(a|T_2,n;T_1,n)] \\ &\quad + \Delta F_{\rm ps}(a|T_2,n;T_1,n),\end{aligned} \tag{38}$$

where again the meaning of the symbols is obvious. By similar steps that led us to write Eq. (24), one can show that, both for the Nb-Nb and the Nb-Au setups, the plasma prediction for the change of force in the sphere-plate geometry is

$$\Delta F_{\rm ps}\,(a|T_2,s;T_1,s)|_{\rm pl} = \Delta F_{\rm ps}\,(a|T_2,n;T_1,n)|_{\rm pl}. \tag{39}$$

Once the small difference between the plasma frequencies of Nb and Au is neglected, we can use the explicit expressions provided in Ref. [10] to evaluate $\Delta F_{\rm ps}\,(a|T_2,n;T_1,n)|_{\rm pl}$ (recall that our sign convention for the Casimir force is opposite to that of Ref. [10]):

$$\Delta F_{\rm ps}\,(a|T_2,n;T_1,n)|_{\rm pl} = R\Delta^{(1)}F_{ps}(T_1,T_2)\Delta^{(2)}F_{ps}(a,T_1,T_2), \tag{40}$$

where

$$\Delta^{(1)}F_{ps}(T_1,T_2) = \frac{\zeta(3)k_B^3}{\hbar^2 c^2}(T_2 - T_1)(T_1^2 + T_2^2) \tag{41}$$

and

$$\begin{aligned}\Delta^{(2)}F_{ps}(a,T_1,T_2) &= \left(1 + \frac{T_1T_2}{T_1^2+T_2^2}\right)\left(1 + 2\frac{\delta}{a}\right) \\ &\quad - \frac{\pi^3}{45\zeta(3)}\frac{T_1+T_2}{T_{\rm eff}}\left(1 + 4\frac{\delta}{a}\right).\end{aligned} \tag{42}$$

Analogously to what we found in the plane-parallel case, the effect predicted by the plasma prescription is unmeasurably small. For example, for a sphere of radius $R=200\ \mu$m, at a distance $a=150$ nm from a plane, for $T_1=5$ K and $T_2 \simeq T_c$ the above formulas give a change of Casimir force of about 5.3×10^{-19} N.

We consider now the Drude prescription. Repeating the steps that led to Eq. (31), we obtain for the Nb-Au setup the equation

$$\Delta F_{\rm ps}\,(a|T_2,s;T_1,s)|_{\rm Dr}^{\rm Nb\text{-}Au} = \Delta F_{\rm ps}\,(a|T_2,n;T_1,n)|_{\rm Dr}^{\rm Nb\text{-}Au}. \tag{43}$$

An explicit formula for $\Delta F_{\rm ps}\,(a|T_2,n;T_1,n)|_{\rm Dr}^{\rm Nb\text{-}Au}$ is given in Eq. (10) of Ref. [10]:

$$\begin{aligned}\Delta F_{\rm ps}\,&(a|T_2,n;T_1,n)|_{\rm Dr}^{\rm Nb\text{-}Au} \\ &= R\Delta^{(1)}F_{ps}\Delta^{(2)}F_{ps} - \frac{Rk_B\zeta(3)}{8a^2}(T_2 - T_1)\left(1 - 4\frac{\delta}{a} + 12\frac{\delta^2}{a^2}\right).\end{aligned} \tag{44}$$

The first term on the right-hand side of the above equation is completely negligible with respect to the second term. We therefore see that the Drude model predicts a decrease of Casimir force proportional to $T_2 - T_1$. We repeat that this effect has nothing to do with superconductivity, and it is again analogous to what was found in Ref. [10] [see comments following Eq. (32)].

It remains to discuss the Nb-Nb setup. By proceeding in the same way as we did to derive Eq. (35), one can prove the following equation:

$$\begin{aligned}\Delta F_{\rm ps}\,&(a|T_2,s;T_1,s)|_{\rm Dr}^{\rm Nb\text{-}Nb} \\ &= [F^{(\rm TE)}_{(\rm ps)0}(a,T_2;c/\lambda_L) - F^{(\rm TE)}_{(\rm ps)0}(a,T_1;c/\lambda_L)] \\ &\quad + R\Delta^{(1)}F_{ps}\Delta^{(2)}F_{ps} \\ &\quad - \frac{Rk_B\zeta(3)}{8a^2}(T_2 - T_1)\left(1 - 4\frac{\delta}{a} + 12\frac{\delta^2}{a^2}\right),\end{aligned} \tag{45}$$

where $F^{(\rm TE)}_{(\rm ps)0}(a,T;c/\lambda_L)$ denotes the TE zero mode contribution to the sphere-plate force [the $l=0$ term for TE polarization in Eq. (36)], as it results after we use our Drude prescription for the reflection coefficient of the superconductor (16). In Fig. 3, we plot $\Delta F_{\rm ps}\,(\text{Nb-Nb})|_{\rm Dr}/R$ (solid line), and $\Delta F_{\rm ps}\,(\text{Nb-Au})|_{\rm Dr}/R$ (dashed line), in units of 10^{-10} N/m, as a function of T_1 (in K), for $T_2 \simeq T_c$ and for a plate-sphere separation $a=150$ nm. In Fig. 4 the same quantities are plotted versus plate-sphere separation a (in microns), for fixed tem-

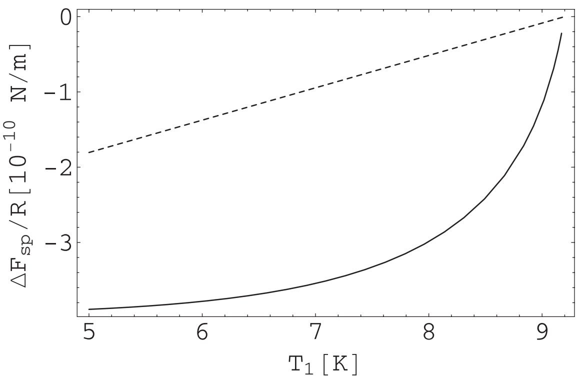

FIG. 3. Plots of $\Delta F_{sp}(\text{Nb-Nb})|_{Dr}/R$ (solid line) and $\Delta F_{sp}(\text{Nb-Au})|_{Dr}/R$ (dashed line), in units of 10^{-10} N/m, versus T_1 (in K), for $T_2 \simeq T_c$ and for fixed sphere-plate separation $a = 150$ nm.

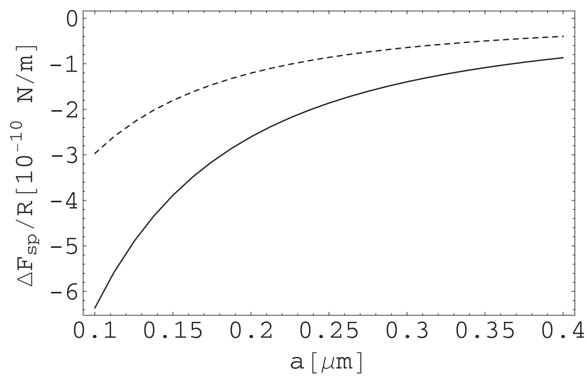

FIG. 4. Plot of $\Delta F_{sp}(\text{Nb-Nb})|_{Dr}/R$ (solid line) and $\Delta F_{sp}(\text{Nb-Au})|_{Dr}/R$ (dashed line), in units of 10^{-10} N/m, versus sphere-plate separation a (in microns), for $T_1 = 5$ K and $T_2 \simeq T_c$.

peratures $T_1 = 5$ K and $T_2 \simeq T_c$. Note that the changes of force are always negative, indicating that the Casimir force decreases as the temperature of the superconducting system is increased from T_1 to T_c. As we see from Fig. 3, for a sphere-plate separation of 150 nm and for a sphere of radius $R = 200$ μm, the Drude model predicts for the Nb-Nb setup a change in the force around -0.8×10^{-13} N, which is over five orders of magnitude larger than the plasma model prediction.

VII. CONCLUSIONS AND DISCUSSION

One of the most important unresolved theoretical problems in the theory of dispersion forces is that of determining the thermal correction to the Casimir force between two conductors. The source of the difficulties stems from the ambiguity in the correct value for the TE zero-mode contribution to the force, corresponding to quasistatic magnetic fields. In this paper, we have proposed a Casimir experiment with superconducting cavities, specifically devised to probe the TE zero mode. The proposal involves measuring the change of Casimir force that occurs in a cavity with one or two superconducting plates, as the temperature of the device is increased towards the normal state. The interest of performing difference force measurements to probe the features of the thermal correction to the Casimir force has already been stressed in Ref. [10], which considered measuring the difference in the Casimir force at two different temperatures in cavities made of ordinary metals at room temperature. Our proposal is very much in the same spirit as Ref. [10], with the significant difference that the change in the Casimir force determined by a small change of temperature is much enhanced (in the Drude case) by superconductivity of the plates. The need of smaller temperature changes, than those required in Ref. [10], might make it easier to achieve them in an experiment.

It is perhaps the case to comment further on the possible advantages and drawbacks that are implicit in the proposed scheme. A potential advantage is that, being a difference force measurement, it might be possible to achieve a better sensitivity than in an absolute force measurement. This, of course, holds true only provided that systematic errors do not change too much when the temperature of the apparatus is varied. Importantly enough, this is the case with regards to the theoretical uncertainty arising from insufficient knowledge of the optical data of the plates, that was estimated in Ref. [18] to be easily as large as 5% in absolute force measurements. This uncertainty is completely irrelevant in our scheme, because as we showed in Sec. III, the magnitude of the effect is determined solely by the TE zero mode. More delicate is the case of the systematic errors arising from residual electrostatic attractions between the plates [17], which constitute a concern in all Casimir experiments. If their magnitude is temperature dependent in the superconducting state, it might be necessary to perform electrostatic calibrations of the apparatus at the considered temperatures.

One of the main experimental issues to address is to find a good mean of varying the temperature of the plates. Perhaps, a convenient way to do this is by illuminating the plates with short laser pulses, as suggested in Ref. [10]. Very likely, this method of heating the plates will reduce to a minimum unwanted thermal expansions or contractions of parts of the apparatus that would inevitably result, were we to warm up the whole system instead. Such expansions would clearly easily alter the separations of the plates by a few nanometers, thus rendering much more difficult a comparison of the Casimir force between the two temperatures.

Finally, we would like to comment on the prospects of the experiment described in this paper of being actually capable of discriminating between the contradictory theoretical approaches to the description of the thermal Casimir force between real materials. According to our computations, the two approaches predict strikingly different magnitudes for the changes of Casimir force in the superconducting state, the plasma approach predictions being always five or six orders of magnitude smaller than those of the Drude approach. While the changes of force resulting from the plasma approach are unmeasurably small, the effect predicted by the Drude approach is, in fact, large enough for an experimental detection to be hopeful. For example, in the sphere-plate geometry, which is the most widely used geometry in current experiments, if both the sphere and the plate are made of Nb, for a sphere radius of 200 μm, and a sphere-plate separation of 150 nm the Drude approach predicts a drop in the Casimir force of 0.8×10^{-13} N, as the system is heated by 2°–3°.

Force oscillations slightly larger than this one have been demonstrated to be measurable by means of an atomic force microscope, with an error about 20% at 95% confidence level, in a recent measurement of the so-called lateral Casimir force [21]. If the same accuracy can be achieved at cryogenic temperatures, there are good chances that the experiment proposed here might actually be able to discriminate the alternative approaches to the description of the thermal Casimir force between real metals.

ACKNOWLEDGMENTS

The author thanks R. Decca, G.L. Klimchitskaya, V. Mostepanenko for their critical reading of the manuscript, and T. Ricciardi for her invaluable support. The authors acknowledges also the kind hospitality of the Kavli Institute for Theoretical Physics in Santa Barbara, where this work was completed. This research was supported in part by the National Science Foundation under Grant No. NSF PHY05-51164.

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[22] The actual difference $\Delta\tilde{P}(a,T)$ between the predicted thermal Casimir pressures is slightly different from the value quoted in Eq. (10). The reason for this is that a low frequency extrapolation of the optical data is required also for the evaluation, by means of dispersion theory, of $\epsilon(i\xi_l)$ for $l>0$, and therefore the two prescriptions predict slightly different magnitudes also for $P_1(a,T)$ (see Ref. [5] for details).