Dark polariton solitons in semiconductor microcavities

A. V. Yulin,[1,2] O. A. Egorov,[1,3] F. Lederer,[3] and D. V. Skryabin[1,*]

[1]*Centre for Photonics and Photonic Materials, Department of Physics, University of Bath, Bath BA2 7AY, United Kingdom*
[2]*Department of Engineering Mathematics, University of Bristol, Bristol, BS8 1TR, United Kingdom*
[3]*Institute of Condensed Matter Theory and Solid State Optics, Friedrich Schiller University Jena, Max-Wien-Platz 1,
07743 Jena, Germany*

We report the existence of dark polariton solitons and study their stability and symmetry breaking in semiconductor microcavities operating in the strong-coupling regime. These half-light half-matter solitons are potential candidates for applications in all-optical signal processing. Their excitation time and required pump powers are a few orders of magnitude less than those of their weakly coupled light-only counterparts.

Polaritons are mixed states of photons and material excitations and are well known to exist in many condensed matter, atomic, and optical systems [1–5]. We are dealing below with a semiconductor microcavity, where polaritons exist due to mixing of quantum well excitons and resonant microcavity photons [3–5]. In the strong-coupling regime photons, emitted as a result of electron transitions, excite the medium and are reemitted in a cascaded manner, which gives rise to so-called Rabi oscillations [1,3,4]. This phenomenon results in the two-peak structure of the microcavity absorption spectrum. The measured spectral width of the peaks corresponds to the picosecond polariton lifetime [3]. This is in contrast with the more usual weak-coupling regime [typical for operation of vertical cavity surface emitting lasers (VCSELs) [3]], where the slow (nanosecond) carrier dynamics does not catch up with the fast (picosecond) photon decay. Thereby most of the photons leave the cavity as soon as they are emitted. In this regime the response to a pulse, resonating with a cavity mode, results in a single spectral peak. Thus any potential application of microcavity polaritons in optical information processing leads to a two to three orders of magnitude response time reduction relative to the VCSEL-like operating regimes.

One of the topics of the recent research into the weakly coupled semiconductor microcavities has been the localized structures of light or cavity solitons [6–12], which have demonstrated rich physics and have been proposed for information processing applications [7,8]. In the weak-coupling regime formation of polaritons is irrelevant, since the dispersion of linear excitations is purely photonic. Slow excitation of the light-only cavity solitons is an outstanding problem, which can be rectified in the strong-coupling regime, where potentially much faster, but not yet reported, light-matter solitons are expected.

In the last few years extensive studies of the polaritons in strongly coupled microcavities have been strongly motivated by the smallness of the polariton mass, leading to observation of the polariton Bose-Einstein condensation at few-kelvin temperatures [13,14]. Polaritons have also been recently observed even at room temperatures (see, e.g., [15])

which has further boosted their potential for practical applications. Another very important feature of polaritons in semiconductor microcavities is their strong repulsive interaction (two-body scattering), resulting in a substantial defocusing nonlinearity [2,3]. Among nonlinear effects predicted or observed with microcavity polaritons are optical bistability [16–20] and parametric conversion [18,21–24]. Observation of these effects with polaritons requires pump intensities of ~ 100 W/cm^2 or below (see, e.g., Fig. 1 in [17]), which is less than the typical pump of 10 kW/cm^2 required for semiconductor microcavities operating in the weak-coupling regime [6–11] (see, e.g., Fig. 5 in [9]).

Solitonic effects with polaritons in bulk media have attracted a significant (mostly theoretical) attention since the 1970s until now (see, e.g., Refs. [2,25]). In the latest wave of research on exciton polaritons in strongly coupled microcavities, the solitonic effects have not been much of a focus yet, with an important exception of a recent experimental paper [26]. In this work the authors claim observation of localized structures or cavity solitons in a strongly coupled semiconductor microcavity. Some other papers have reported localization of microcavity polaritons due to linear defects [14,27], as a result of switching between two polarizations [28] or neglecting such important requisites of passive cavities as losses, external pump, and hence bistability [29]. For studies of spatially dependent polariton dynamics, see, e.g., [30]. Our work is aimed at filling an existing gap in the theoretical knowledge about microcavity polariton solitons. This is necessary not only for backing so-far-limited experimental observations [26], but also and mainly for guiding future work in this direction.

The widely accepted dimensionless mean-field model for excitons strongly coupled to the circularly polarized cavity photons is [2,3,21]

$$\partial_t E - i(\partial_x^2 + \partial_y^2)E + (\gamma_c - i\Delta)E = E_p + i\Psi,$$

$$\partial_t \Psi + (\gamma_0 - i\Delta + i|\Psi|^2)\Psi = iE. \qquad (1)$$

Here E and Ψ are the averages of the photon and exciton creation or annihilation operators. Normalization is such that $(\Omega_R/g)|E|^2$ and $(\Omega_R/g)|\Psi|^2$ are the photon and exciton numbers per unit area. Here, Ω_R is the Rabi frequency and g is

*Corresponding author. d.v.skryabin@bath.ac.uk
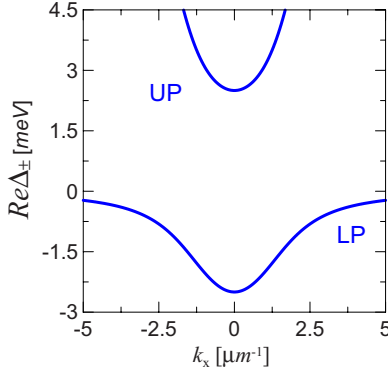
FIG. 1. (Color online) Polariton dispersion calculated from Eq. (2), Re $\Delta_\pm(k_x)$, is rescaled back into physical units. Parameters are $n=3.5$, operating wavelength $\lambda=0.85$ μm, $\gamma_{c,0}=0.1$, and $\hbar\Omega_R =2.5$ meV.

the exciton-exciton interaction constant. $\Delta=(\omega-\omega_r)/\Omega_R$ describes detuning of the pump frequency ω from the identical resonance frequencies of excitons and cavity, ω_r. Time t is measured in units of $1/\Omega_R$. γ_c and γ_0 are the cavity and exciton damping constants normalized to Ω_R. Transverse coordinates x and y are normalized to the value $x_0 =\sqrt{c/2kn\Omega_R}$, where c is the vacuum light velocity, n is the refractive index, and $k=n\omega/c$ is the wave number. The normalized amplitude of the external pump E_p is related to the physical incident intensity I_{inc} as $|E_p|^2=g\gamma_c I_{inc}/\hbar\omega_0\Omega_R^2$ [31]. As a guideline for realistic estimates, one can use parameters for a microcavity with a single InGaAs/GaAs quantum well: $\hbar\Omega_R\simeq 2.5$ meV, $\hbar g\simeq 10^{-4}$ eV μm^2 (see [17,21,31]). Assuming the relaxation times of the photonic and excitonic fields to be 2.5 ps gives $\gamma_{c,0}\simeq 0.1$. In accordance with this set of parameters, the normalized driving amplitude $|E_p|^2=1$ physically corresponds to the external pump intensity ~ 10 kW/cm^2. Optical bistability appears for $|E_p|\sim 0.1$; it gives the input intensity ~ 100 W/cm^2. Experimentally the polariton bistability has been observed for values close or even less than 100 W/cm^2 [16–20].

First, we briefly summarize important aspects of the linear dispersion and bistability properties of the above equations. Assuming that $E,\Psi\sim e^{ik_x x+ik_y y}$ and neglecting pump and nonlinearity, we find the dispersion law of cavity polaritons:

$$\Delta_\pm = \frac{k^2-i(\gamma_c+\gamma_0)}{2}\pm\sqrt{1+\frac{[k^2+i(\gamma_0-\gamma_c)]^2}{4}}, \quad (2)$$

where $k^2=k_x^2+k_y^2$. Re Δ_+ corresponds to the frequency of the upper polariton (U polariton) and Re Δ_- to the lower polariton (L polariton) branch (see Fig. 1). In the strong-coupling regime the gap between U and L polaritons is greater than the linewidth of the branch due to Im $\Delta_\pm\neq 0$.

If $E_p\neq 0$, then solitons can exist only on a finite-amplitude background [$E(\pm\infty)\neq 0$], simply because the zero homogeneous solution is absent. Therefore we proceed with a brief consideration of spatially homogeneous solutions (HSs) and their stability. Then we report the existence of various cavity polariton solitons (CPSs) and study their stability and instability scenarios. HSs having bistable depen-
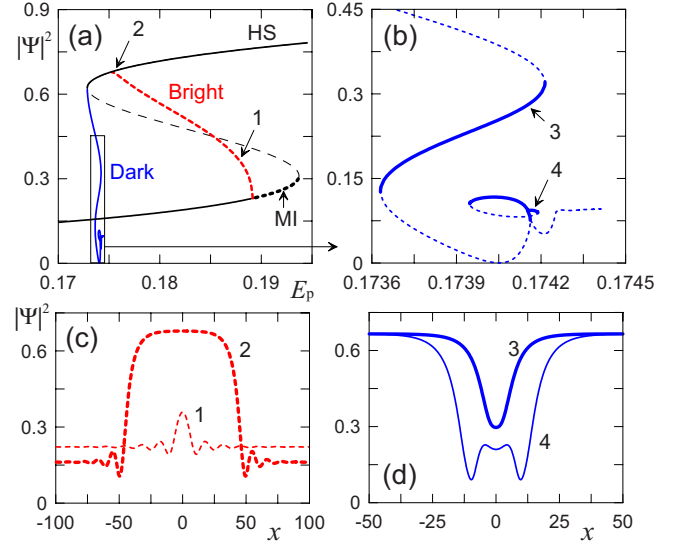

FIG. 2. (Color online) (a) Amplitude of the homogeneous state (HS) (black line), max $|\Psi(x,y)|^2$ for bright solitons (red line) and min $|\Psi(x,y)|^2$ for dark solitons (blue line), shown as functions of E_p: $\Delta=-0.7$, $\gamma_{0,c}=0.1$. (b) is a close-up of the rectangular area from (a) showing bifurcations of the dark solitons. (c), (d) Exciton density distribution $|\Psi(x,y=0)|^2$ across the bright (c) and dark (d) solitons for the points marked by 1, 2, 3, and 4. Solid and dashed lines in (a)–(d) mark stable and unstable solutions, respectively.

dence from E_p is an important prerequisite for the soliton existence. $E(E_p)$ is multivalued provided that $f(\Delta)>0$, where

$$f(\Delta) \equiv \Delta(\Delta^2+\gamma_c^2-1)-\sqrt{3}\gamma_0\left(\Delta^2+\gamma_c^2+\frac{\gamma_c}{\gamma_0}\right). \quad (3)$$

The cumbersome expressions for the roots of $f(\Delta)=0$ simplify for $\gamma_c=\gamma_0=0$ and give two bistability intervals $\Delta>1$ and $-1<\Delta<0$. These two intervals overlap with the Δ intervals allowed by the dispersion relation [see Eq. (2) and Fig. 1]. The bistability in the interval $-1<\Delta<0$ appears because of the nonlinear resonance of the pump with the L polaritons, whereas the bistability in the semi-infinite interval is associated with the nonlinear resonance of the pump with the U polaritons. Weakly coupled cavities with defocusing nonlinearities exhibit bistability only for $\Delta>0$ (see, e.g., [12]). Below we focus our attention on the solitons linked to L polaritons; therefore, our studies are unique to the strong-coupling regime. Stability analysis of the HS L polaritons ($\Delta<0$) has been previously reported for example in Ref. [21]. The lower state of the L-polariton bistability loop can be modulationally unstable within some interval of E_p, while the upper state is generally stable [see Figs. 2(a) and 3(a)]. Here, modulational instability (MI) we understand as the growth of linear perturbations in the form $e^{ik_x x+ik_y y+\kappa t}$ (Re κ is the growth rate). As Δ is changing from the bottom of the L-polariton branch towards the linear exciton resonance, $\Delta=0$, the point of MI is moving towards the left edge of the bistability loop and finally goes beyond the latter [cf. Figs. 2(a) and 3(a)].
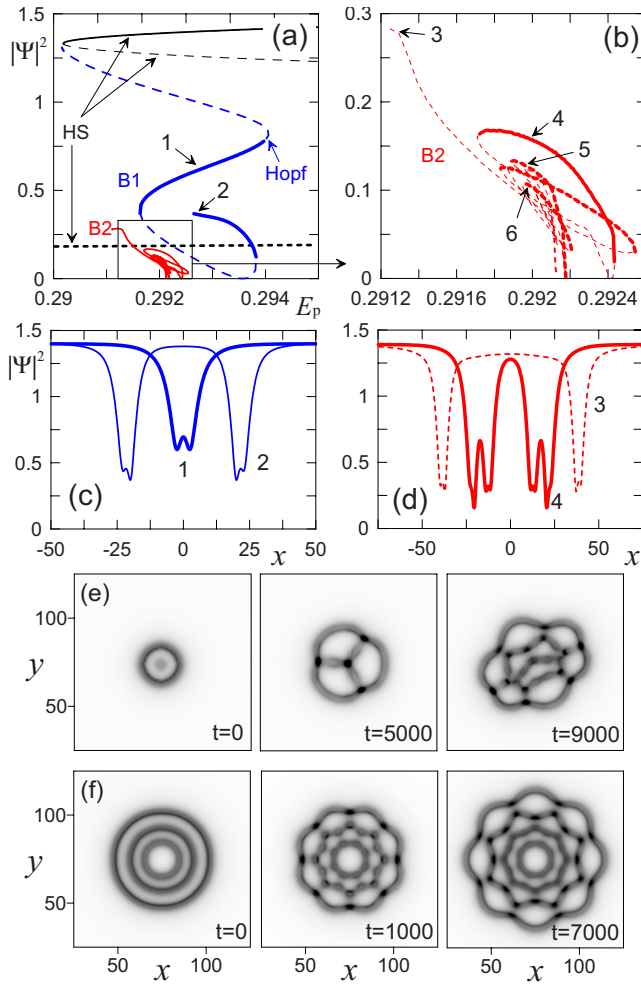
FIG. 3. (Color online) (a) Amplitude of the homogeneous state (HS) (black line) and min $|\Psi(x,y)|^2$ for dark solitons (red and blue lines) shown as functions of E_p: $\Delta=-0.5$, $\gamma_{0,c}=0.1$. $B1$ and $B2$ mark two branches of dark CPSs. (b) is a close-up of the rectangular area from (a) showing bifurcations of the $B2$ dark CPSs. (c), (d) Exciton density distribution $|\Psi(x,y=0)|^2$ across $B1$ (c) and $B2$ (d) CPSs for the points marked by 1, 2, 3, and 4 in panels (a) and (b). Solid and dashed lines in (a)–(d) mark stable and unstable solutions, respectively. (e), (f) show the development of the symmetry-breaking instabilities of the CPSs marked as 5 and 6 in (b).

Restricting ourselves to the structures independent of the polar angle $[\theta=\arg(x+iy)]$ we find that the time-independent CPSs obey

$$-i\left(\frac{d^2E}{dr^2}+\frac{1}{r}\frac{dE}{dr}\right)+(\gamma_c-i\Delta)E=E_p+i\Psi, \quad (4)$$

where $r=\sqrt{x^2+y^2}$, $\Psi=iE/[(\gamma_0-i\Delta)+iz]$, and $z\equiv|\Psi|^2$ is found solving the real cubic equation $[\gamma_0^2+(z-\Delta)^2]z=|E|^2$. z turns out to be a single-valued function of $|E|^2$ throughout the range of parameters corresponding to the bistability of L polaritons. Thus the potential problem of ambiguity in choosing a root for z is avoided.

We start our analysis of cavity polariton solitons from the case when the MI point of low-state L polaritons is within the bistability interval. In many previously studied models,

bifurcation points of the homogeneous solutions have been the sites where localized structures branch off [12]. Applying the Newton iterative method to Eq. (4), we have found a family of small amplitude bright CPSs emerging from the MI point [see the dashed red line in Fig. 2(a)]. Bright CPSs are the ones nesting on the background of the lower state of the bistability loop with their maximum deviating towards the upper state. The dark CPSs discussed below are opposite to the bright ones in the sense that their background is the upper state and they deviate towards the lower state at their core. Going towards smaller values of E_p, the bright CPSs become more intense [see Fig. 2(c)]. A counterintuitive drop in intensity with increase of the energy supply is a frequent signature of the underlying dynamical instability (see below). The E_p value, at which the lower and upper homogeneous states can be connected by a standing 1D front, is called the Maxwell point, and this is the point where the branch of the bright CPSs terminates ($E_p=0.1748$). When the pump approaches the Maxwell point, the soliton broadens and its peak intensity tends towards the intensity of the upper homogeneous state. We also perform a full two-dimensional (2D) linear stability analysis of the found structures. The linear perturbations are assumed in the general form $\epsilon_+(r)e^{iJ\theta+\kappa t}+\epsilon_-^*(r)e^{-iJ\theta+\kappa^*t}$, where $J=0,1,2,\ldots$ [32]. The resulting Jacobian operator is analyzed using finite differences in r. The linear stability analysis shows that the bright CPSs are unstable with respect to the perturbation with the azimuthal index $J=0$ and that the development of the instability splits the CPS into 2D moving fronts. When E_p is close to the Maxwell point, this instability is relatively weak and bright CPSs can be easily stabilized by the spatial inhomogeneities of the pump or cavity detuning.

Because of the defocusing nature of the polaritonic nonlinearity, dark CPSs (see, e.g., [12]) are expected to be naturally selected by our system and the instability of bright CPSs is not surprising. Dark cavity solitons have been previously studied both theoretically and experimentally for semiconductor microcavities in the weak-coupling regime (see, e.g., [9,33,34]). Unlike fiber solitons, the dark cavity solitons have no conceptual disadvantage over the bright ones as information carriers. The branch of dark CPSs has been found to detach from the left folding point of the bistability loop and tend towards the Maxwell point, [see Fig. 2(a) and the zoomed-in area in (b)]. At the onset of their existence the dark solitons are seen only as a very small deviation from the homogeneous background. As E_p tends towards the Maxwell point from the right, they become much deeper. Near the Maxwell point the dark solitons become very broad and can be roughly considered as superpositions of infinitely separated 1D fronts [the $1/r$ term in Eq. (4) can be disregarded for large distances and the equation becomes effectively 1D]. It is important to note that the relaxation of the fronts towards the upper state happens without oscillations; however, the relaxation towards the lower state is oscillatory [see Fig. 2(c)]. Thus pinning of the two fronts and hence stabilization of CPSs is possible only for the dark structures [see the thick line in Fig. 3(c)]. The stable branches of dark CPSs are shown by solid lines in Fig. 2(b). The unstable ones correspond to the instabilities with $J=0$.

Shifting the detuning parameter towards $\Delta=0$ makes the polariton dispersion more exciton like and boosts the repulsive nonlinear effects in our system. As a result, the lower-branch HS becomes unstable within the whole range of the bistability and the branch of bright CPSs disappears [see Fig. 3 ($\Delta=-0.5$)]. However, we have found two distinct branches of dark CPSs marked as $B1$ and $B2$ in Figs. 3(a) and 3(b). The $B1$ branch bifurcates subcritically from the folding point of the upper homogeneous state. Initially unstable ($J=0$) CPSs become stable after the turning point. Close to this turning point the $B1$ CPSs have a deeplike shape, while later they transform into dark rings of growing radius [Fig. 3(c)]. Note that close to the turning points additional destabilization of dark CPS happens due to linear eigenmodes with complex κ and $J=0$ [Hopf instability; see Fig. 3(a)], resulting in the formation of oscillating dark CPSs.

The branch $B2$ consists of ring shaped structures [see Fig. 3(d)]. The linear stability analysis shows that the $B2$ CPSs can be stable [see the interval marked by 4 in Fig. 3(b)]. However, more often, they are unstable with respect to perturbations breaking the radial symmetry—i.e., with $J \neq 0$. An

example of this instability development is shown in Figs. 3(e) and 3(f). The dark ring CPS shown in Fig. 3(e) is unstable against the linear eigenmode with $J=3$. The broader CPSs undergo azimuthal instabilities with larger azimuthal numbers J. For example, $J=8$ for the concentric ring CPS shown in Fig. 3(f).

In summary, following a series of recent experiments on observation of microcavity polaritons, we have studied the formation of spatially localized polariton-soliton structures in the strong-coupling regime. In particular, our results can be used for the qualitative interpretation of the experimental measurements reported in [26], where the polariton Rabi splitting has been observed simultaneously with the formation of various bright and dark localized structures. The microcavity polariton solitons reported here exhibit a picosecond excitation time and can be observed at pump powers a few orders of magnitude lower than those required in the weak-coupling regime of semiconductor microcavities [6–11]. Thus light-matter polariton solitons have potentially significant advantages in all-optical signal processing applications over light-only cavity solitons [7,11,12].

---

[1] *Confined Electrons and Photons: New Physics and Applications*, edited by E. Burstein and C. Weisbuch (Plenum, New York, 1995).

[2] S. A. Moskalenko and D. W. Snoke, *Bose-Einstein Condensation of Excitons and Biexcitons* (Cambridge University Press, Cambridge, England, 2000).

[3] A. V. Kavokin, J. J. Baumberg, G. Malpuech, and F. P. Laussy, *Microcavities* (Oxford University Press, Oxford 2007).

[4] D. G. Lidzey *et al.*, Nature (London) **395**, 53 (1998); J. P. Reithmaier *et al.*, *ibid.* **432**, 197 (2004); A. Wallraff *et al.*, *ibid.* **431**, 162 (2004); G. Khitrova *et al.*, Nat. Phys. **2**, 81 (2006).

[5] C. Weisbuch, M. Nishioka, A. Ishikawa, and Y. Arakawa, Phys. Rev. Lett. **69**, 3314 (1992).

[6] V. B. Taranenko, I. Ganne, R. Kuszelewicz, and C. O. Weiss, Appl. Phys. B: Lasers Opt. **72**, 377 (2001).

[7] S. Barland *et al.*, Nature (London) **419**, 699 (2002).

[8] Y. Tanguy, T. Ackemann, W. J. Firth, and R. Jager, Phys. Rev. Lett. **100**, 013907 (2008).

[9] Y. Larionova, C. O. Weiss, and O. A. Egorov, Opt. Express **13**, 8308 (2005).

[10] M. Brambilla, L. A. Lugiato, F. Prati, L. Spinelli, and W. J. Firth, Phys. Rev. Lett. **79**, 2042 (1997).

[11] F. Pedaci *et al.*, Appl. Phys. Lett. **92**, 011101 (2008).

[12] U. Peschel, D. Michaelis, and C. O. Weiss, IEEE J. Quantum Electron. **39**, 51 (2003); P. Mandel, *Theoretical Problems in Cavity Nonlinear Optics* (Cambridge University Press, Cambridge, England, 1997); N. N. Rosanov, *Spatial Hysteresis and Optical Patterns* (Springer, Berlin, 2002); *Spatial Solitons*, edited by S. Trillo and W. Torruelas (Springer, Berlin, 2001).

[13] H. Deng *et al.*, Science **298**, 199 (2002); J. Kasprzak *et al.*, Nature (London) **443**, 409 (2006).

[14] R. Balili *et al.*, Science **316**, 1007 (2007).

[15] S. Christopoulos *et al.*, Phys. Rev. Lett. **98**, 126405 (2007).

[16] A. Tredicucci, Y. Chen, V. Pellegrini, M. Borger, and F. Bassani, Phys. Rev. A **54**, 3493 (1996).

[17] A. Baas, J. Ph. Karr, H. Eleuch, and E. Giacobino, Phys. Rev. A **69**, 023809 (2004).

[18] N. A. Gippius *et al.*, Europhys. Lett. **67**, 997 (2004).

[19] D. N. Krizhanovskii *et al.*, Phys. Rev. B **77**, 115336 (2008).

[20] D. Bajoni *et al.*, http://xxx.lanl.gov/abs/0809.4758.

[21] I. Carusotto and C. Ciuti, Phys. Rev. Lett. **93**, 166401 (2004).

[22] C. Diederichs *et al.*, Nature (London) **440**, 904 (2006).

[23] R. M. Stevenson *et al.*, Phys. Rev. Lett. **85**, 3680 (2000); P. G. Savvidis *et al.*, *ibid.* **84**, 1547 (2000).

[24] C. Ciuti, P. Schwendimann, and A. Quattropani, Semicond. Sci. Technol. **18**, S279 (2003).

[25] I. B. Talanina, M. A. Collins, and V. M. Agranovich, Solid State Commun. **88**, 541 (1993); D. V. Skryabin, A. V. Yulin, and A. I. Maimistov, Phys. Rev. Lett. **96**, 163904 (2006).

[26] Y. Larionova, W. Stolz, and C. O. Weiss, Opt. Lett. **33**, 321 (2008).

[27] T. C. H. Liew, A. V. Kavokin, and I. A. Shelykh, Phys. Rev. Lett. **101**, 016402 (2008).

[28] I. A. Shelykh, T. C. H. Liew, and A. V. Kavokin, Phys. Rev. Lett. **100**, 116401 (2008).

[29] A. M. Kamchatnov, S. A. Darmanyan, and M. Neviere, J. Lumin. **110**, 373 (2004).

[30] I. A. Shelykh, Y. G. Rubo, G. Malpuech, D. D. Solnyshkov, and A. Kavokin, Phys. Rev. Lett. **97**, 066402 (2006).

[31] M. Wouters and I. Carusotto, Phys. Rev. B **75**, 075332 (2007).

[32] D. V. Skryabin and W. J. Firth, Phys. Rev. E **58**, 3916 (1998).

[33] D. Michaelis, U. Peschel, and F. Lederer, Opt. Lett. **23**, 1814 (1998).

[34] O. A. Egorov and F. Lederer, Phys. Rev. E **75**, 017601 (2007).