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Phase separation in a mixture of a Bose-Einstein condensate and a two-component Fermi gas as a probe of Fermi superfluidity

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We study the phase diagram of the mixture of a Bose-Einstein condensate and a two-component Fermi gas. In particular, we identify the regime where the homogeneous system becomes unstable against phase separation. We show that, under proper conditions, the phase-separation phenomenon can be exploited as a robust probe of "local" Fermi superfluidity.

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Mixtures of superfluids open up possibilities of studying interacting macroscopic quantum systems. The attempt at such studies started decades ago in the system of ^{3}He-^{4}He mixtures. However, the transition into the superfluid phase of ^{3}He atoms in these mixtures occurs at an extremely low temperature and has never been reached in experiment. Realization of superfluids in atomic quantum gases makes such studies possible for the first time.

The atomic analogy of a superfluid ^{3}He-^{4}He system is a mixture of a Bose-Einstein condensate (BEC) and superfluid Fermi gas (i.e., a two-component Fermi gas with attractive interaction). In this paper, we explore the rich phase diagram of this system at zero temperature. By studying the free energy of the homogeneous mixture, we identify the regime where the homogeneous mixture becomes unstable against phase separation. Phase separation is quite a generic phenomenon occurring in trapped atomic mixtures, originating from the interplay between the interactions and the spatial variation induced by the trap, thus allowing different regions of the trap to favor fundamentally different phases [1,2].

Further, we demonstrate an interesting application of the phase-separation phenomenon: a "local" detection of Bardeen-Cooper-Schrieffer (BCS) superfluidity within the Fermi gas [3–6]. While experiments involving the generation of a vortex lattice [6] and rf spectroscopy [4] are able to successfully demonstrate Fermi superfluidity and measure the pairing gap, respectively, they are unable to resolve the pairing property spatially. A "local" measurement technique, such as the one we present in this article, will be instrumental in detecting paired states with nontrivial spatial structure—for example, those responsible for Fulde-Ferell-Larkin-Ovchinnikov (FFLO) superfluidity [7] occurring in polarized fermion mixtures.

The probing concept can be simply understood if we envision a BEC localized to a small region within the Fermi medium. We assume that the Fermi gas is unconfined, which is a valid approximation if the spatial extension of the Fermi cloud is much broader than that of the BEC. In the phase-separation regime, the BEC may exist in a pure form as an isolated bubble surrounded by a Bose-Fermi mixture or a pure Fermi gas. We show that the existence of such a BEC bubble may be made sensitive to the superfluid property of the fermionic medium. Thus, the onset of BCS superfluidity is signaled by the occurrence or disappearance of small BEC bubbles which can be readily detected via a density measurement. Essentially, bosons serve as a matter-wave probe of Fermi superfluid. This idea is illustrated schematically in Fig. 1.

To begin with, we first determine the zero-temperature projected phase diagram of a Bose-Fermi-Fermi mixture comprised of bosons of one species and equal population spin-up and -down fermions of another. Here, by projected we mean a two-dimensional slice of the d-dimensional parameter space by fixing the $d-2$ independent parameters. In the absence of an interfermion interaction, this system can be identically mapped onto a Bose-Fermi mixture and has been previously studied quite extensively [8]. However, the interacting case remains much less explored [9] and is of focal importance to this work. Here, we therefore assume an attractive s-wave interaction between fermions of unlike spins. Furthermore, we assume Bose-Bose and Bose-Fermi interactions to be repulsive. It is convenient to treat all interactions via a pseudopotential modeled by $v_\alpha(\mathbf{r}-\mathbf{x})=\lambda_\alpha\delta(\mathbf{r}-\mathbf{x})$, which is valid for dilute systems, where we indicate different types of interacting atoms by the index $\alpha \in \{BB, B\uparrow, B\downarrow, \uparrow\downarrow\}$. In this work, we make two nonessential simplifications. First, we assume that the Bose-Fermi interaction is spin independent—i.e., $\lambda_{B\uparrow}=\lambda_{B\downarrow}=\lambda_{BF}$. Second, we consider a single spatial dimension. We remark that our theoretical framework is general and the above two restrictions can be straightforwardly removed.

The homogeneous system under study is characterized by a total of five independent parameters: the three interaction strengths $(\lambda_{BB}, \lambda_{BF}, \lambda_{\uparrow\downarrow})$ and two densities (ϱ_B, ϱ_F) where $\varrho_\uparrow=\varrho_\downarrow=\varrho_F/2$. Note that the choice is not unique. For instance, instead of the densities, we may choose chemical

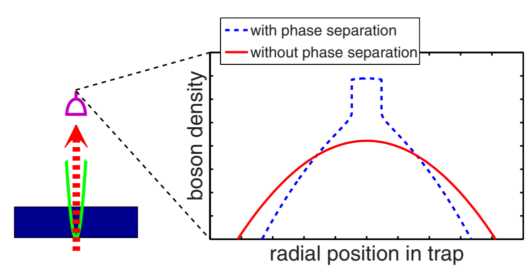

FIG. 1. (Color online) Schematic of the proposed BEC probe. Bosons are confined in the tight trap and are made to interact with the Fermi atoms which are unconfined. Phase separation can be easily detected by measuring the BEC density profile.

potentials (μ_B, μ_F) to be independent parameters. It is instructive to first consider the much simpler case without a Fermi-Fermi interaction, or $\lambda_{\uparrow\downarrow}=0$. As mentioned earlier, this case, for three-dimensional (3D) confinement, has been studied by several authors [8]. A similar analysis can be performed for the 1D case, resulting in the instability criterion given by $\varrho_F < 4\lambda_{BF}^2/(\pi^2\lambda_{BB})$—when this inequality is satisfied, the homogeneous mixture is unstable and tends to phase separate. Thus the BEC density profile in the two regimes (phase separated or not) may be made quite distinct as depicted in the schematic of Fig. 1. However, we emphasize that, since the interaction is density-density, there is no direct connection of the fermionic pairing gap (in the case of interacting fermions) to the *phase-separation* phenomenon. Remarkably, as we shall show, the phase separation *can* be made sensitive to the fermionic pairing, underlying the basis of our proposed BEC probe of Fermi superfluidity.

We begin by writing the free-energy functional of the homogeneous mixture in the form

$$\mathbf{F} = \frac{\lambda_B}{2}\varrho_B^2 - \mu_B\varrho_B + \lambda_{\uparrow\downarrow}\Delta^2\left[\frac{1}{2\pi}\int_0^\infty \frac{1}{\Lambda_k}dk\right]^2,$$

$$+ \frac{4}{2\pi}\int_0^\infty \frac{(\varepsilon_k - \tilde{\mu}_F)}{2}\left[1 - \frac{(\varepsilon_k - \tilde{\mu}_F)}{\Lambda_k}\right]dk, \qquad (1)$$

where we have defined $\varepsilon_k = \hbar^2k^2/2m$, $\tilde{\mu}_F = \mu_F - \lambda_{BF}\varrho_B$, and the fermionic quasiparticle energies are given by $\Lambda_k = \sqrt{(\varepsilon_k - \tilde{\mu}_F)^2 + \Delta^2}$ by neglecting the Hartree contribution since it only leads to a constant energy shift. Also, anticipating the role of Fermi superfluid, we have written the free energy as a function of boson density ϱ_B and BCS pairing gap Δ. The above free-energy functional for the combined Bose-Fermi system is obtained from first principles, the detailed derivation of which will be given in a future article. Clearly, substituting $\varrho_B=0$, Eq. (1) reduces to the usual expression for a two-component superfluid Fermi system at $T=0$.

We will then construct the projected phase diagram in the $\varrho_B-\Delta$ parameter space while fixing the values of three more independent variables to be discussed below.

The thermodynamic ground state is given by the minimum of the free energy and therefore corresponds to the necessary first derivative conditions $\partial\mathbf{F}/\partial\Delta=0$ and $\partial\mathbf{F}/\partial\varrho_B=0$. The first of these conditions essentially reproduces the gap equation in the form $(-\lambda_{\uparrow\downarrow}/2\pi)\int_0^\infty(1/\Lambda_k)dk=1$, while the second fixes the number through the modified bosonic Thomas-Fermi equation $\lambda_B\varrho_B - \mu_B + \lambda_{BF}\varrho_F = 0$. However, the local minimum is guaranteed only if the Hessian matrix $\mathbf{M}$, constructed from the second derivatives of $\mathbf{F}$, is positive definite or the following conditions are satisfied:

$$\mathbf{M}_{\varrho_B\varrho_B} = \partial^2\mathbf{F}/\partial\varrho_B^2 > 0, \quad \mathrm{Det}[\mathbf{M}] > 0. \qquad (2)$$

When condition (2) is violated, the system will necessarily phase separate [10]. Using this simple criterion, one can map the whole phase space of the homogeneous mixture. However, this is an extremely laborious task, given that the total phase space is huge, represented by the set of five independent parameters as we mentioned earlier. We make a judicious choice and use the set $\{\lambda_{BB}, \lambda_{BF}, \Delta, \varrho_B, \mu_F\}$, the motivation behind which will be clear as we proceed further. Other parameters, such as $\lambda_{\uparrow\downarrow}$, ϱ_F, and μ_B, must be calculated self-consistently using the gap equation and the bosonic Thomas-Fermi equation discussed earlier, together with the fermionic number equation.

We remind the reader that our goal here is not to give a complete description of the whole phase space, but focus our study on a small region that is physically meaningful in view of current experimental setups and illustrate a probing technique based on the phase-separation phenomenon. This we do by first picking reasonable values of λ_{BB}, λ_{BF}, and μ_F. We then determine the stable or unstable regions in the $\varrho_B-\Delta$ space via condition (2). For this we need to study the properties of the Hessian matrix $\mathbf{M}$. States that satisfy condition (2) are only guaranteed to be a local minimum of the free energy. To determine whether the state is the ground state of the system, we need to compare the free energies of different homogeneous phases which include the pure BEC and the pure Fermi phase, in addition to their mixture.

Following the above procedure, we have determined the phase space for various values of the fixed parameters. We find that this system exhibits a very rich phase diagram. However, given the lack of space, we restrict ourselves to pointing out some general features that are relevant to this proposal and direct the reader to an upcoming publication for more details.

The projected phase diagram of a homogeneous mixture for a particular set of $\{\lambda_{BF}, \lambda_{BB}, \mu_F\}$ is shown in Fig. 2(a). We note the following: (i) The unstable region corresponds to a partial ellipse cut by the ϱ_B axis [shown by the shaded region in Fig. 2(a)]. (ii) Within this region, the phase space can be further divided into a dynamically unstable region [the yellow (brighter shaded)] at which the free-energy landscape shows a saddle point and a dynamically stable region [the green (darker shaded)] at which the free-energy landscape shows a local but not a global minimum. (iii) The position and the extent of these regions depend on the values of the fixed parameters. Immediately we notice that, in a typical experimental setup with a given value of $\lambda_{\uparrow\downarrow}$, only a small part of the phase space comprised of points on the fixed $\lambda_{\uparrow\downarrow}$ contour—for example, C_1 or C_2 shown in Fig. 2(a)—is physically accessible. The vertical axis of $\Delta=0$ corresponds to a mixture of a BEC with a noninteracting Fermi gas $(\lambda_{\uparrow\downarrow}=0)$. The significance of the particular parameter set chosen earlier is clear since we can now directly obtain the Bose density profile in the probe by mapping the boson chemical potential along this contour, $\mu_B[C_i]$, onto the spatial coordinate in the probe via the local density approximation (LDA) using $\mu_B[C_i]=\mu_B(r)\equiv\mu_B - V(r[C_i])$ where $V(r)$ is the probe trapping potential for the BEC. In Fig. 2(b) we plot the free energy of the mixture as a function of the Bose chemical potential $\mu_B[C_1]$. The same plot also show the free energy for a pure BEC. Now we can easily identify the following special points in Fig. 2(a): the point marked by the circle (triangle) as the point where the free energy of the BEC goes below that of the mixture and thus represents a first-order phase transition in the presence (absence) of pairing, and the point marked by square as the point where μ_B

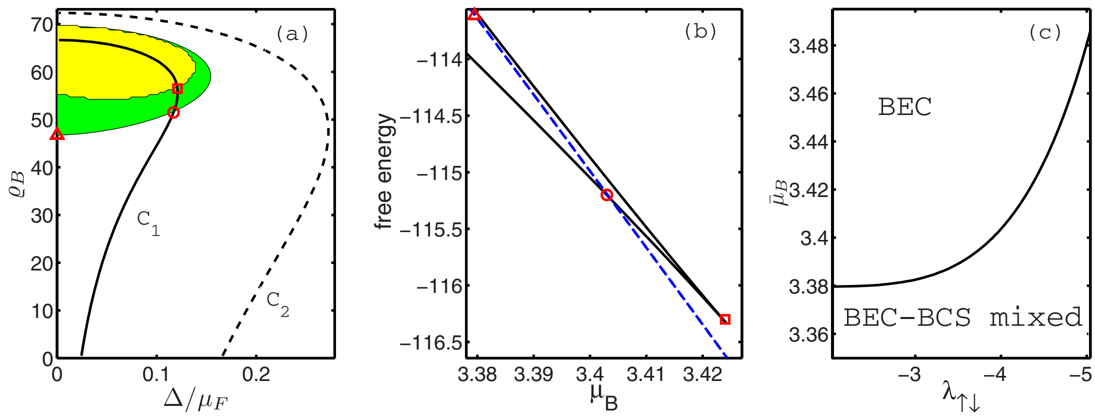

FIG. 2. (Color online) (a) Projected phase space of the Bose-Fermi-Fermi mixture. Here, as well as in other figures, all quantities plotted are made dimensionless by scaling them in units of the probe, represented by a harmonic oscillator with frequency ω_0 and length ℓ_0. Fixed parameters are λ_{BF}=0.44, λ_{BB}=0.05, and μ_F=27.32. The yellow (green) area indicates the region where the Bose-Fermi mixture is dynamically (mechanically) unstable. The contours C_1 and C_2 correspond to $\lambda_{\uparrow\downarrow}=-4$ and -6, respectively. (b) Comparison of the free energy of the mixture (solid curve) along the contour C_1 and the free energy of the pure BEC (dashed curve). The free energy of the pure Fermi gas is much higher and not shown. The points shown by red symbols correspond to those in (a). (c) Critical boson chemical potential $\bar{\mu}_B$ as a function of the inter-Fermi interaction separating the pure BEC and mixed Bose-Fermi phases. Within the LDA, moving vertically upwards along a line parallel to the y axis implies moving from the edge towards the center of the trap.

reaches its maximum value along the contour C_1 and the homogeneous mixture reaches its dynamical instability threshold.

Now we are in the position to discuss how we can take advantage of the phase diagram to detect Fermi superfluidity using the BEC as a probe. To put this idea in context, we note that there is a new avenue in cold-atom research that casts BECs as tools for quantum measurement. A good example is the experiment by Krüger *et al.* [11] where the Thomas-Fermi character of the BEC density profile is exploited to measure the surface potential-energy landscape with exquisite accuracy. Also, very recently, Bhongale and Timmermans have proposed a high-sensitivity force measurement by exploiting a phase-separated two-BEC mixture [12]. The ideas in this article are a next step in this direction.

Since for our system the bosons are confined to the probe, we want a situation such that, on phase separation, the system consists of a pure BEC component near the center of the trap surrounded by a cloud of either mixed bosons and fermions or pure fermions. It is quite intuitive that if $\mu_B(r) > \bar{\mu}_B[\lambda_{\uparrow\downarrow}]$ [the latter being the boson chemical potential along the stable boundary of the green region represented by the circle in Figs. 2(a) and 2(b)], a pure BEC bubble will phase separate out from the mixture. However, in order to use this phenomenon to discern the fermion superfluid property, it is important to connect the occurrence of the BEC bubble with the onset of a nonzero superfluid gap Δ. This is crucial since, as mentioned earlier, phase separation can also occur in a mixture of a BEC and a *normal* Fermi gas due to Bose-Fermi repulsion. For this reason, in Fig. 2(c), we plot the critical boson chemical potential as a function of the interfermion interaction strength. We see that there is a clear separation of regions corresponding to a pure BEC phase above the curve and a mixed phase below. Thus, if we configure the probe such that the center chemical potential is very close to and above the curve in Fig. 2(c) implying a phase-separated pure BEC component near the center of the trap, the appearance of BCS superfluidity on increasing the attractive interfermion interaction is immediately signaled by the disappearance of the phase-separated pure BEC bubble as shown in Fig. 3.

In practice, the sharp density jump will be smoothed out due to the finite kinetic energy contribution; however, the absence of a non-Thomas-Fermi variation in the boson density profile can be easily detected. Also, we claim that this probing method allows for a numeric estimation of the gap Δ by a systematic fitting technique if the exact density profile is obtained via a numerical solution of the coupled Bose-Fermi equations.

Moreover, increasing the attractive interaction beyond a certain value (depending on the value of λ_{BB} and λ_{BF}) results in the absence of a phase-separated regime as depicted by the contour C_2 in Fig. 2(a). In fact, this can be considered as a very strong indication of the pairing phenomenon, the limiting case of which is a homogeneous mixture of molecular (M) and atomic BEC on the repulsive side of the Feshbach resonance, known to be completely stable if $\lambda_{BM} < \sqrt{\lambda_{BB}\lambda_{MM}}$ [1]. However, this involves calculating these additional interaction strength, which will be dealt with in a future article.

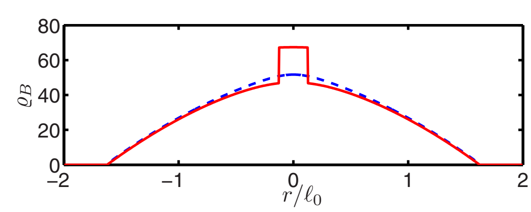

FIG. 3. (Color online) Boson density (in units of $1/\ell_0$) profile within the LDA with (dashed line) and without (solid line) fermion pairing, for a total number of bosons, N_B=101, at $\lambda_{\uparrow\downarrow}=-4$. All other parameters are same as used in the previous plot.

In conclusion, we have studied the phase diagram of a Bose-Fermi-Fermi mixture where both bosons and fermions are in the superfluid phase. The required free-energy functional is derived from first principles, the details of which will be given elsewhere. We exploit the phase diagram and propose a probe for detecting BCS-type superfluidity within a quasi-1D two-component Fermi gas by configuring the system such that the phase separation resulting from thermodynamic instability is sensitive to the interaction between the fermions and hence the BCS pairing. The probe consists of a BEC confined to a relatively tight trap. We have shown that by properly tuning the probe parameters, the density profile of the BEC provides a robust signal of the fermion pairing. The probe idea may be easily extended to 3D systems by identifying the appropriate phase-separation regime. The expressions for the free energy in Eq. (1) and the corresponding Hessian matrix are still valid in 3D with the understanding that the integrals involved are also 3D. One important difference between 3D and 1D is that, in the former, proper renormalization procedures must be taken to remove the ultraviolet divergence in gap equation associated with the contact interaction.

We are aware of a related recent proposal for probing BCS-type superfluidity using a overlapping BEC. However, there, the pairing signal is related to the damping of the BEC acoustic phonons [13]. The strength of our proposal lies in the fact that the signature of pairing is reflected in the BEC density profile, a quantity that can be easily measured in experiment. The proposed probing scheme possesses another important advantage: it probes the local value of Δ. This is crucial in trapped experiments where the gap varies in space due to the trap-induced inhomogeneity and in situations where the gap has an intrinsically nontrivial spatial dependence. The latter arises, for instance, in the case of a FFLO superfluid state in a population-imbalanced Fermi system where the gap varies sinusoidally in space, which may result in density oscillations in the BEC probe. Another natural extension is to study the phase diagram of the Bose-Fermi-Fermi mixture where the Fermi-Fermi interaction is tuned across a Feshbach resonance. How the presence of the bosons affects and probes the BEC-BCS crossover will be an interesting problem to study.

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[1] Eddy Timmermans, Phys. Rev. Lett. **81**, 5718 (1998).

[2] G. B. Partridge *et al.*, Science **311**, 503 (2006).

[3] J. Bardeen, R. E. Coovert, S. R. Morrison, J. R. Schrieffer, and R. Sun, Phys. Rev. **104**, 47 (1956); M. J. Holland, S. J. J. M. F. Kokkelmans, M. L. Chiofalo, and R. Walser, Phys. Rev. Lett. **87**, 120406 (2001).

[4] C. Chin *et al.*, Science **305**, 1128 (2004).

[5] M. Greiner, C. A. Regal, J. T. Stewart, and D. S. Jin, Phys. Rev. Lett. **94**, 110401 (2005).

[6] M. W. Zwierlein *et al.*, Nature (London) **435**, 1047 (2005).

[7] P. Fulde and R. A. Ferrell, Phys. Rev. **135**, A550 (1964); A. I. Larkin and Y. N. Ovchinnikov, Sov. Phys. JETP **20**, 762 (1965).

[8] L. Viverit, C. J. Pethick, and H. Smith, Phys. Rev. A **61**, 053605 (2000); S. K. Yip, *ibid.* **64**, 023609 (2001); H. Pu, W. Zhang, M. Wilkens, and P. Meystre, Phys. Rev. Lett. **88**, 070408 (2002).

[9] S. K. Adhikari and L. Salasnich, Phys. Rev. A **76**, 023612 (2007).

[10] An unstable homogeneous state may also tend to collapse. This possibility, however, is excluded by our choice of repulsive Bose-Bose and Bose-Fermi interactions.

[11] P. Krüger *et al.*, J. Phys.: Conf. Ser. **19**, 56 (2005).

[12] S. G. Bhongale and Eddy Timmermans, Phys. Rev. Lett. **100**, 185301 (2008).

[13] S. Gaudio, B. Mihaila, K. B. Blagoev, K. S. Bedell, and E. Timmermans, Phys. Rev. Lett. **98**, 110407 (2007).