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Multiparticle entanglement under the influence of decoherence

O. Gühne,[1,2] F. Bodoky,[3] and M. Blaauboer[3]
[1]*Institut für Quantenoptik und Quanteninformation, Österreichische Akademie der Wissenschaften, 6020 Innsbruck, Austria*
[2]*Institut für Theoretische Physik, Universität Innsbruck, Technikerstraße 25, 6020 Innsbruck, Austria*
[3]*Kavli Institute of Nanoscience, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands*

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We present a method to determine the decay of multiparticle quantum correlations as quantified by the geometric measure of entanglement under the influence of decoherence. With this, we compare the robustness of entanglement in Greenberger-Horne-Zeilinger (GHZ), cluster, *W*, and Dicke states of four qubits and show that the Dicke state is the most robust. Finally, we determine the geometric measure analytically for decaying GHZ and cluster states of an arbitrary number of qubits.

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The decoherence of quantum states is a process in quantum dynamics that is relevant for the discussion of fundamental issues like the transition from quantum to classical physics [1]. Also from a practical point of view decoherence phenomena have to be studied as they occur in experiments involving entanglement and their suppression is of vital importance for any implementation of quantum information processing.

Due to this importance, the influence of decoherence on the entanglement of multiparticle systems has been studied from several perspectives [2,3]. These investigations concerned either the lifetime of entanglement or the entanglement properties of the bipartite system which arise if the multiparticle system is split into two parts. The lifetime of entanglement, however, gives no quantitative information about the decay of entanglement [3]. Moreover, as a highly entangled multiparticle state may be separable with respect to each bipartition [4], considering bipartite aspects only may not lead to a full understanding of the decoherence process. It is therefore highly desirable to investigate a full multipartite entanglement measure under the influence of decoherence. Unfortunately, all known entanglement measures for multiparticle entanglement are defined via complicated optimization procedures [5], which makes it practically impossible to compute them for a given mixed quantum state.

In this paper we present a method to investigate the decay of quantum correlations which can be used to overcome these difficulties. We study different four-qubit states and use our method to compare their robustness against decoherence, using a phenomenological model described below. Our approach allows us to compute the entanglement for Greenberger-Horne-Zeilinger (GHZ) and cluster states of an arbitrary number of qubits and thereby to investigate the scaling behavior for these states under decoherence. As we will further see, our results can be directly tested in nowadays experiments with photons or trapped ions. Finally, from the viewpoint of pure quantum information theory, our results represent one of the few cases where the computation of a relevant entanglement measure for mixed states can be performed [6].

We consider the following situation: a pure quantum state $|\psi\rangle$ is prepared at time $t=0$ and in the presence of noise evolves to a mixed state $\varrho(t)$. Our task is to quantitatively investigate the time evolution of the entanglement $E(t)=E[\varrho(t)]$ and its dependence on the initial state and the number of qubits.

As entanglement quantifier, we use the *geometric measure of entanglement* [7]. This is a popular entanglement monotone for multiparticle systems, which is related to the discrimination of multiparticle states with local means [8] and has been investigated from several perspectives [9–11]. For pure states, it is defined as

$$E_G(|\psi\rangle) = 1 - \max_{|\phi\rangle=|a\rangle|b\rangle|c\rangle\cdots} |\langle\phi|\psi\rangle|^2, \quad (1)$$

i.e., as one minus the maximal overlap of $|\psi\rangle$ with fully separable states $|\phi\rangle$. It is extended to mixed states by the convex roof construction

$$E_G(\varrho) = \min_{p_k,|\phi_k\rangle} \sum_k p_k E_G(|\phi_k\rangle), \quad (2)$$

where the minimization is taken over all convex decompositions of ϱ—i.e., over all probabilities p_k and states $|\phi_k\rangle$ which fulfill $\Sigma_k p_k|\phi_k\rangle\langle\phi_k|=\varrho$. Clearly, the optimization in Eq. (1) and especially in Eq. (2) is difficult to perform.

Our method can be summarized as follows: since any set of probabilities p_k and states $|\phi_k\rangle$ in Eq. (2) results in a valid upper bound, we obtain a good upper bound by choosing them appropriately. Then we use the results of Refs. [12,13] to obtain a lower bound on $E_G(t)$. There it has been shown how the geometric measure can be estimated if the mean value of a single or a few observables is given [14]. We show that the lower and upper bounds coincide for the multiparticle states we investigate below, allowing for a precise determination of $E_G(t)$.

The noise we consider is described by a master equation for the matrix elements ϱ_{kl} as they are used in phenomenological models of decoherence for, e.g., electron spin qubits [15]:

$$\partial_t \varrho_{kl} = \begin{cases} \sum_{i\neq k} (W_{ik}\varrho_{ii} - W_{ki}\varrho_{kk}) & \text{for } k=l, \\ -V_{kl}\varrho_{kl} & \text{for } k\neq l. \end{cases} \quad (3)$$

We consider a global dephasing process, where the relaxation of the diagonal elements plays no role ($W_{ij}=0\,\forall\, i,j$) and the off-diagonal terms are affected by a global dephasing rate $V_{kl}\equiv\gamma$. This process leads to exponentially decaying off-diagonal components $\varrho_{kl}(t)=x\varrho_{kl}(0)$, where here and in the following $x\equiv e^{-\gamma t}$. At the end of the paper we will discuss

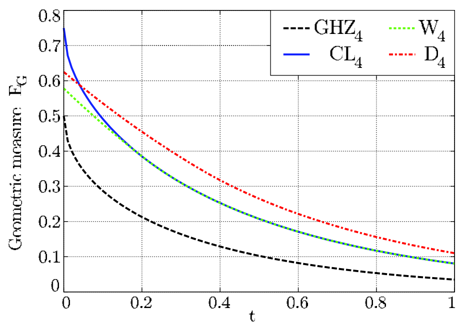

FIG. 1. (Color online) The geometric measure $E_G(t)$ for four-qubit GHZ, cluster, W, and Dicke states for the case $\gamma=1$. For the W and Dicke states, the upper bounds are shown, as the curves of the lower bounds coincide with that. For $t \gtrsim 2$ the values for the W and cluster states coincide [see Eqs. (8) and (10)].

extensions of our method to other (e.g., local) decoherence models.

Before presenting our results for different four-qubit states in detail, note that by construction the geometric measure is a convex quantity—i.e., $E_G[\lambda\varrho_1+(1-\lambda)\varrho_2] \leqslant \lambda E_G(\varrho_1)+(1-\lambda)E_G(\varrho_2)$. From this, it follows directly that $E_G(t)$ is monotonically decreasing. Moreover, $E_G(x)$ is convex in the parameter x [since $\varrho(x)$ depends linearly on x], and consequently $E_G(t)$ is convex in the time t.

Let us start our discussion with the four-qubit GHZ state [16], given by

$$|\mathrm{GHZ}_4\rangle = (|0000\rangle + |1111\rangle)/\sqrt{2}. \tag{4}$$

The geometric measure of the GHZ states equals 1/2 as the maximal overlap with product states is 1/2 [7].

In order to estimate the entanglement from below, note that the time-dependent fidelity is given by $F(t) = \mathrm{Tr}[\varrho(t)|\mathrm{GHZ}_4\rangle\langle\mathrm{GHZ}_4|]=(1+e^{-\gamma t})/2$. Using the results of Ref. [12] we can estimate the geometric measure from $F(t)$. Explicitly, it has been shown that if for an arbitrary ϱ the fidelity F of a state $|\psi_0\rangle$ is given, then E_G is bounded by

$$E_G(\varrho) \geqslant \sup_{r\geqslant 0}\left\{1 + rF(t) - \frac{1}{2}\left[1 + r + \sqrt{(1+r)^2 - 4rE_G(\psi_0)}\right]\right\}. \tag{5}$$

Applying this to $\varrho(t)$ and the fidelity of the GHZ state leads to (using $x=e^{-\gamma t}$)

$$E_G(\varrho) \geqslant \frac{1}{2}(1 - \sqrt{1-x^2}). \tag{6}$$

In order to obtain an upper bound, we consider the two states $|\phi_1\rangle = c|0000\rangle + s|1111\rangle$ and $|\phi_2\rangle = s|0000\rangle + c|1111\rangle$, with $c=\cos(\alpha)$ and $s=\sin(\alpha)$, and write $\varrho(t) =(1/2)\Sigma_{k=1,2}|\phi_k\rangle\langle\phi_k|$. Then, $x/2=(2cs)/2$ has to hold, and using the fact that the geometric measure for the states $|\phi_k\rangle$ is given by $E_G=\min\{s^2,c^2\}$ [17] one arrives at an upper bound for $E_G(t)$ which is given by the right-hand side of Eq. (6). Hence, $E_G(\varrho)=(1-\sqrt{1-x^2})/2$ for the four-qubit GHZ state (4) under the influence of noise. This function is shown in Fig. 1.

Second, let us discuss the four-qubit cluster state [18]

$$|\mathrm{CL}_4\rangle = (|0000\rangle + |0011\rangle + |1100\rangle - |1111\rangle)/2, \tag{7}$$

which has a geometric measure of $E_G(|\mathrm{Cl}_4\rangle)=3/4$ [10]. For this state, the decay of the fidelity is given by $F(t)=(1+3x)/4$, which [combined with Eq. (5)] leads to the lower bound

$$E(\varrho) \geqslant \frac{3}{8}[1 + x - \sqrt{1+(2-3x)x}]. \tag{8}$$

For the upper bound, we consider four trial vectors for the decomposition. The first is given by $|\phi_1\rangle=c|0000\rangle +s(|1100\rangle+|0011\rangle-|1111\rangle)/\sqrt{3}$, and the other three are obtained from this by permuting the four terms. We choose $c \geqslant s$; then, any of the four states has a geometric measure of $E_G(|\phi_i\rangle)=s^2$. With the ansatz $\varrho(t)=(1/4)\Sigma_{k=1}^4|\phi_k\rangle\langle\phi_k|$ we obtain as a condition on s and c that $x/4=[(2cs/\sqrt{3}) +(2s^2/3)]/4$. From this, c can be determined. This leads after a short calculation to the insight that the right-hand side of Eq. (8) also constitutes an upper bound on the entanglement and thus describes exactly the time evolution of the entanglement.

Third, a four-qubit W state is given by [19]

$$|W_4\rangle = (|0001\rangle + |0010\rangle + |0100\rangle + |1000\rangle)/2, \tag{9}$$

for which the geometric measure equals 37/64 [7]. Let us first derive the upper bound. We take as test states the state $|\phi_1\rangle=c|1000\rangle+s(|0100\rangle+|0010\rangle+|0001\rangle)/\sqrt{3}$ and permutations thereof. Using a symmetry argument [9], their geometric measure is determined to be $E_G(|\phi_i\rangle)=[5+3c^2(c^4+c^2-3)]/[(3-4c^2)^2]$ for $c\leqslant 1/\sqrt{2}$ and $E_G(|\phi_i\rangle)=1-c^2$ for $c \geqslant 1/\sqrt{2}$ [17]. Then, one can derive an upper bound as for the cluster state.

It turns out, however, that this upper bound is not convex in x. Since we know that E_G has to be convex, we can take the convex hull (in x) of this upper bound:

$$E_G \leqslant \frac{37(81x-37)}{2816} \text{ for } x \geqslant x_0,$$

$$E_G \leqslant \frac{3}{8}[1 + x - \sqrt{1+(2-3x)x}] \text{ for } x \leqslant x_0, \tag{10}$$

with $x_0=2183/2667$. Physically, taking the convex hull just means that for short times (when $x \geqslant x_0$) the optimum in the convex roof in Eq. (2) is of the form $\varrho(x)=p|W_4\rangle\langle W_4| + (1-p)\varrho(x_0)$, with $\varrho(x_0)=(1/4)\Sigma_k|\phi_k\rangle\langle\phi_k|$. Note that for longer times ($x\leqslant x_0$) the upper bound (10) is the same as for the cluster state.

In order to see that this upper bound is optimal, let us derive a lower bound. Here, we not only take the fidelity of the W state into account, but we use as a second observable the projector onto the space with one excitation, $\mathcal{P}_1 = |0001\rangle\langle 0001| + |0010\rangle\langle 0010|+|0100\rangle\langle 0100|+|1000\rangle\langle 1000|$. Using the fact that the fidelity of $|W_4\rangle$ is given by $F(t) =(1+3x)/4$ and that it is always in the space spanned by $\mathcal{P}_1$ (i.e., $\mathrm{Tr}[\varrho(t)\mathcal{P}_1]=1$), we can use the methods of Ref. [12] to obtain a lower bound from these two expectation values [20]. It turns out that, within numerical accuracy, the lower bound

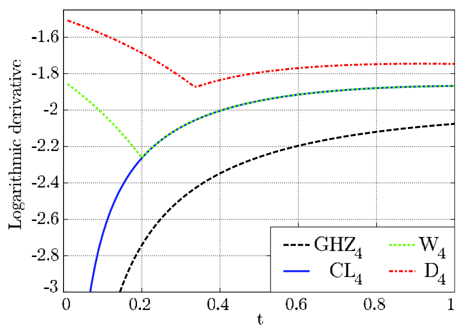

FIG. 2. (Color online) Logarithmic derivative of $E_G(t)$ (for $\gamma=1$) for different four-qubit states. The nonanalytic behavior for the Dicke and W states stems from the convex hull in Eqs. (10) and (12).

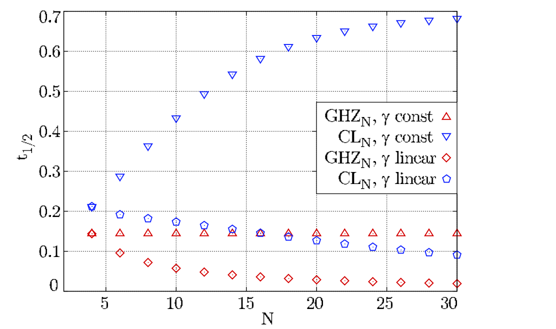

FIG. 3. (Color online) Comparison between the GHZ state and the cluster state as a function of the number of qubits N. The half lifetime $t_{1/2}$ is shown for the case that $\gamma=4$ and for the case that $\gamma=N$ increases linearly with the number of qubits.

coincides with the upper bound, giving strong numerical evidence that Eq. (10) is the exact expression for the decay of quantum correlations in the W state (9).

As a last example of a four-qubit state, let us discuss the symmetric Dicke state [21] given by

$$|D_4\rangle = (|0011\rangle + |0101\rangle + |1001\rangle + |1100\rangle + |0110\rangle + |1010\rangle)/\sqrt{6}, \tag{11}$$

which has a geometric measure $E_G=5/8$ [7]. To obtain the upper bound, we proceed similarly as for the W state: We take six states, the first one being $|\phi_1\rangle = c|0011\rangle + s(|0101\rangle + |1001\rangle + |1100\rangle + |0110\rangle + |1010\rangle)/\sqrt{5}$ and other ones obtained by permuting the terms. Then we make the ansatz $\varrho(t)=(1/6)\Sigma_{k=1}^{6}|\phi_k\rangle\langle\phi_k|$. This leads to an upper bound which is not convex in x, and subsequently taking the convex hull leads to

$$E_G \leqslant \frac{5(3x-1)}{16} \text{ for } x \geqslant \frac{5}{7},$$

$$E_G \leqslant \frac{5}{18}[1+2x-\sqrt{1+(4-5x)x}] \text{ for } x \leqslant \frac{5}{7}. \tag{12}$$

The lower bound is found analogously to the W state as well, with the projector $\mathcal{P}_2$ onto the space with two excitations as second observable. The resulting bound coincides again with the upper bound, giving strong evidence that Eq. (12) describes the time evolution of the entanglement.

For a comparison between the different states, we consider the logarithmic derivative $\eta=\partial_t(\ln[E_G(t)])=\partial_t E_G(t)/E_G(t)$, which describes the relative decay of entanglement [22]. The values of this quantity are plotted in Fig. 2. One can clearly see that the Dicke state is the most robust state, while the GHZ state is the most fragile state [23]. It is an interesting open question as to which properties of the Dicke state are responsible for the high robustness.

For N qubits, we restrict our attention to GHZ and linear cluster states, as they are highly relevant for applications like quantum metrology or measurement-based quantum computation [24]. In the decoherence model, one might keep the dephasing rate $\gamma=\gamma_0$ constant for any number of qubits (where γ_0 is the single-qubit dephasing rate) or scale it as $\gamma=N\gamma_0$ (as would occur for the GHZ state in a local dephasing model). For the GHZ state, nothing changes and all the formulas obtained in the above for the four-qubit case apply. Concerning the cluster state, we consider linear cluster states with $N=2n$ qubits. The linear cluster state is given by

$$|CL_N\rangle = \bigotimes_{k=1}^{n}[|00\rangle + |11\rangle(\sigma_x \otimes \mathbb{1})]/\sqrt{2}, \tag{13}$$

where this formula should be understood as an iteration, with the operator $(\sigma_x\otimes\mathbb{1})$ acting on the Bell state of the next two qubits. Explicitly, we have $|\mathrm{CL}_2\rangle=(|00\rangle+|11\rangle)/\sqrt{2}$ and $|\mathrm{CL}_4\rangle \sim |00\rangle(|00\rangle+|11\rangle)+|11\rangle[\sigma_x\otimes\mathbb{1}(|00\rangle+|11\rangle)] = |0000\rangle + |0011\rangle + |1110\rangle + |1101\rangle$ [25]. Note that the maximal overlap of the cluster state with fully separable states is $1/2^n$ [10].

We calculate the geometric measure for this state under the effect of decoherence in the same way as for the four-qubit case. The lower bound is obtained from the fidelity $F(t)=[1+(2^n-1)x]/2^n$ and yields $E_G \geqslant \frac{1}{2^N}((2-3\times 2^n+2^N)x + 2(2^n-1)\{1-\sqrt{(1-x)[1+(2^n-1)x]}\})$. For the upper bound, we consider 2^n test states similar to the ones before and arrive at an upper bound which coincides again with the lower bound.

To investigate the scaling behavior, we consider the time $t_{1/2}$ when the entanglement has decreased to half of the initial value. These times can be directly computed for the N-particle GHZ and cluster states. Figure 3 shows $t_{1/2}$ as a function of the number of qubits N for the constant and linear models of γ. In both cases the time $t_{1/2}$ of the cluster state is, in the limit $N\to\infty$, larger than that of the GHZ state by a factor of $\ln(4)/\ln(4/3)\approx 4.82$, giving quantitative evidence for the higher robustness against dephasing of the cluster state.

In the calculations presented in this paper we concentrated on a global decoherence model. However, our results can also be applied to other models. First, for the W and GHZ states, our model is equivalent to a local dephasing noise, as occurs in multiphoton experiments. Given the experimental availability of W and GHZ states with photons [26], our results can be directly tested with present-day technology. Second, for local decoherence models where relaxation is the dominant process, a small modification of our scheme allows the calculation of the geometric measure for certain states, such as the W state [27]. The fact that this type of decoherence is dominant in ion traps [28] combined with the possibility to generate such states [29] opens another way for an experimental test.

To conclude, our results provide a versatile method to determine the decay of multiparticle entanglement for quantum states under the influence of decoherence. Our results can be tested in multiqubit experiments and may therefore lead to a better understanding of decoherence as a fundamental obstacle in quantum information processing.

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