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Induced transparency in an ensemble of Λ atoms

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We consider the effect of induced transparency in an ensemble of Λ atoms controlled by high-power radiation. We study the case of an optically dense medium where nonlinear interaction between probe radiation and a Stokes satellite of the drive wave is important. It is shown that the probe radiation and the Stokes satellite form normal biharmonic modes. Each normal mode is characterized by a fixed ratio between the amplitudes of different-frequency partial components and that both partial waves have a common absorption coefficient and a common group velocity. Transparency of an optically dense medium is ensured in the regime of amplification of one of the normal modes. An exact solution (without considering inhomogeneous broadening) is constructed for an arbitrary nonuniform distribution of Λ atoms along the probe radiation path and uniform drive-wave intensity. A geometric-optical approach is developed for the case of a spatially varied intensity of the drive field. The obtained solution contains both new results and a generalization of the results of the previous papers.

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I. INTRODUCTION

There have recently been intense studies of the resonant interaction between multilevel quantum systems and multichromatic radiation. These studies led to a considerable broadening of the concept of the properties of the radiation processes. Discovery of the effect of electromagnetically induced transparency (EIT) is one of important achievements in this field (see review [1]). At present this effect is promising for the realization of high-resolution spectroscopy and magnetometry [2–5] and the creation of optical and quantum memory systems [6,7]. Strong group delay [8] and the so-called "storage of light" [9–11] have been realized in the EIT regime.

The simplest model for a theoretical study of such processes (see Fig. 1) is the so-called Λ scheme in a three-level system [1,12–15]. In this scheme, the frequency ω_p of the probe wave (*P* wave) and the frequency ω_d of the drive wave (*D* wave) satisfy the two-photon resonance condition $\omega_p - \omega_d = \omega_{21}$, where ω_{21} is the frequency of the low-frequency (LF) transition $|2\rangle \rightarrow |1\rangle$. The occurrence of LF coherence leads finally to the suppression of the polarization of a three-level atom at the frequency ω_p, making the medium transparent for the *P* wave.

A classical analog of the EIT effect, as is shown in Ref. [16], is the effect of dynamic damping in a system of coupled oscillators, which is known in oscillation theory [17]. The studies of analogs of the EIT effect in a classical plasma [18,16,19–22] showed that propagation of the *P* wave in a medium that is opaque for it is possible due to the nonlinear coupling of the *P* and *D* waves with any fairly long-lived mode of collective oscillations of the medium. By analogy with Ref. [22], we will call this regime parametrically induced transparency (PIT). Unlike the standard variant of EIT, in the PIT regime the condition of synchronism of wave vectors, which is required for the nonlinear wave processes [23], is not fulfilled automatically, but is considerably dependent on dispersion characteristics of the considered waves.[1] PIT is different from the standard Raman scattering in that the medium is opaque for the P wave in the absence of the nonlinear coupling with other waves.

In some papers (see Refs. [24–26], and references cited therein), their authors investigated experimentally and theoretically specific features of the EIT effect in an optically dense Λ medium, which are stipulated by the nonlinear coupling of the *P* wave with a weakly damped Stokes satellite of the drive field (*S* wave). We will show in what follows that in the Λ system the *S* wave can be undamped or can even be amplified. Thus, induced transparency takes place in this system, both due to the excitation of internal degrees of freedom of a separate three-level atom and the collective electromagnetic excitation of an ensemble of atoms, which leads to a combination of the EIT and PIT effects.

The *S* wave with frequency ω_s in the Λ system is excited under the action of beats between the *D* wave and the LF coherence ($\omega_s = \omega_d - \omega_{21}$). Then the beats between *D* and *S* waves excite again the LF coherence, influencing the *P*-wave propagation and thus forming the four-wave mixing process. The corresponding condition of frequency synchronism has the form

$$\omega_p + \omega_s = 2\omega_d. \tag{1}$$

As in Refs. [24–26], we consider propagation of a pair of waves (*P* and *S*), coupled by an intense *D* wave, in a Λ medium. Confining ourselves to a relatively simple three-level model of the medium in the range of standard EIT parameters, which is adopted in these papers, we studied in detail the corresponding electrodynamic problem. As a result, we obtained a fairly common solution containing both new results and the analytical generalization and interpretation of the experimental and computational results reported

[1]For an ensemble of independent Λ atoms, the LF coherence can be considered as the wave excitation with zero group velocity, which ensures automatic fulfillment of the condition of the wave-vector synchronism in the EIT process.

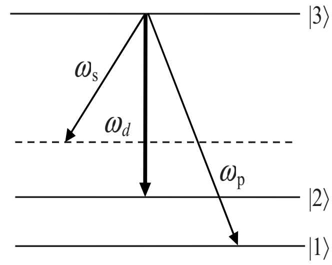

FIG. 1. Λ scheme with Stokes wave excitation.

in the previous papers. From this solution it follows, in particular, that the eigenmodes (normal electromagnetic modes) of the Λ medium in the presence of a high-power D wave are biharmonic waves (combinations of P and S waves), whose properties are, in many respects, similar to the ordinary and extraordinary normal waves in anisotropic media (earlier, such normal modes were found during analysis of propagation of the radiation under conditions of undulator-induced transparency (UIT) in a plasma [21]). The main peculiarities of the experimental data, obtained in Refs. [24–26], are explained by the properties of these normal modes. Some quantitative discrepancies between theoretical and experimental results are caused by a number of simplifications of the theory. The main one is likely that inhomogeneous broadening of spectrum lines was not taken into account, while it is the important factor of mentioned experiments. As for contribution to the theory of EIT, the most important results obtained in the present paper are the following.

The center of the frequency "transparency window" of the PIT is shifted with respect to that for the EIT due to the corresponding condition of wave-vector synchronism (this explains, in particular, why the frequency line of the transmitted P radiation is asymmetric in the case of induced transparency in a dense medium).

Finally, as the optical depth of the Λ layer increases, its induced transparency is always determined by exactly the PIT regime.

One of the normal biharmonic modes (we call it extraordinary by analogy with the theory of waves in anisotropic media) can be amplified in an optically dense layer of Λ atoms due to transformation of D photons into S and P photons, namely, $\hbar\omega_d+\hbar\omega_d \Rightarrow \hbar\omega_s+\hbar\omega_p$.

Large group delay that is characteristic of the EIT effect can also be retained in the PIT regime.

We note that the nonlinear amplification and/or transparency for bichromatic radiation in a dissipative medium of multilevel atoms was also considered earlier for the double Λ scheme [27], in which the LF coherence can be excited by beats between two drive waves. Similar regimes were also discussed for lasing without inversion (LWI) in quantum [12–15] and classical [28,29] systems and for acoustically induced transparency (AIT) in an optically dense ensemble of resonant two-level atoms [30]. Unlike the case being discussed, in all the papers mentioned above the biharmonic (or multiharmonic [30]) combination of waves propagates in a dissipative medium without absorption [30] or is amplified [12–15,28,29] due to interaction with the medium-parameter oscillations excited by some external source.

The paper is organized as follows. In Sec. II we obtain constitutive equations for a three-level ensemble affected by the drive field and its two satellites coupled by the frequency synchronism condition (1). In Secs. III and IV we obtain equations for coupled electromagnetic waves and formulate the procedure of constructing a solution in the form of biharmonic normal modes. In Sec. V we consider various regimes of transmission of the radiation through the Λ layer in the approximation of uniform intensity of the drive field and establish the main dimensionless parameter determining the realization of a particular regime; we analyze conditions for possible experimental observation of the most interesting regime of instability. In Sec. VI we compare our calculations with experimental data, presented in Refs. [24–26].

II. CONSTITUTIVE EQUATIONS IN A THREE-LEVEL MEDIUM

Consider the behavior of an ensemble of three-level atoms in a classical external field (see Fig. 1):

$$\vec{E}=\vec{x}_0 E, \quad E=\frac{1}{2}\left(\sum_{j=p,d,s} E_j \exp(-i\omega_j t)+\text{c.c.}\right), \tag{2}$$

where $\vec{x}_0$ is a unit polarization vector, E_d is the amplitude of the D wave, and $E_{p,s}$ are the amplitudes of the P and S waves, respectively. The eigenfrequencies of the transitions are ω_{31}, ω_{32}, ω_{21}. The wave frequencies are coupled by condition (1).

Assume that the frequency of the D wave is equal to the frequency of one of the HF transitions, i.e., $\omega_d=\omega_{32}$. Then the two-photon resonance detuning $\Delta\omega=\omega_p-\omega_d-\omega_{21}$ is identical to the difference between the frequency of the P wave and the frequency of another HF transition $\omega_p-\omega_{31}$ and the frequency ω_s is determined by the relation $\omega_s=2\omega_{32}-\omega_p$. We also assume that the HF transitions are well resolved with respect to each other and the characteristic relaxation time of quantum coherence in the LF transition is significantly greater than for the HF transitions. Assuming also that the frequencies $\omega_{p,s}$ are close to the corresponding resonant values, we obtain the following group of conditions:

$$\begin{aligned}|\omega_p-\omega_{31}|&=|(\omega_p-\omega_d)-\omega_{21}|=|\omega_s-(2\omega_{32}-\omega_{31})|\\&\ll\omega_{21}\ll\omega_{31,32}, \quad \gamma_{21}\ll\gamma_{31,32}\ll\omega_{21},\end{aligned} \tag{3}$$

where $1/\gamma_{31,32}$ are the characteristic relaxation times for the HF transitions and $1/\gamma_{21}$ is a characteristic relaxation time for the LF transition.

Within the framework of the simplest approximation for the relaxation operator, the equations for the off-diagonal density matrix elements ρ_{ik} have the following form (see Ref. [31]):

$$\dot{\rho}_{31}+(i\omega_{31}+\gamma_{31})\rho_{31}=\frac{i}{\hbar}(d_{31}(N_1-N_3)+d_{32}\rho_{21})E,$$

$$\dot{\rho}_{32}+(i\omega_{32}+\gamma_{32})\rho_{32}=\frac{i}{\hbar}(d_{32}(N_2-N_3)+d_{31}\rho_{21}^{*})E,$$

$$\dot{\rho}_{21} + (i\omega_{21} + \gamma_{21})\rho_{21} = \frac{i}{\hbar}(d_{32}^{*}\rho_{31} - d_{31}\rho_{32}^{*})E, \tag{4}$$

where $N_{1,2,3}=\rho_{11,22,33}$ are the populations of the corresponding levels normalized to the atomic concentration, d_{ik} are the elements of the operator of the dipole moment X projection (for a standard case of the EIT regime $d_{21}=0$).

We will seek the solution of Eqs. (4) in the following way:

$$\rho_{21} = \sigma_{21}\exp[-i(\omega_p - \omega_d)t],$$

$$\rho_{31} = \sum_{j=p,d,s}\sigma_{31}(\omega_j)\exp(-i\omega_j t),$$

$$\rho_{32} = \sum_{j=p,d,s}\sigma_{32}(\omega_j)\exp(-i\omega_j t). \tag{5}$$

Consider the case of relatively weak field E, so that the population distribution is only determined by the relaxation processes $|d_{31,32}E|/2\hbar << \gamma_{31,32}$. Often, the EIT effect is discussed in terms of a simple model corresponding to the limiting case $N_2=N_3=0, N_1\neq 0$ (see Refs. [1,24–26]). A finite (although small) quantity $N_{2,3}$ can be of fundamental importance for one of the Raman schemes of lasing without inversion (LWI)[12,14]. In the present paper, we do not consider the known LWI regimes, so assume $N_2=N_3=0$. Substituting Eqs. (1), (2), and (5) into Eq. (4) yields

$$\sigma_{21} = \frac{d_{32}^{*}(\sigma_{31}(\omega_p)E_d^{*} + \sigma_{31}(\omega_d)E_s^{*}) - d_{31}(\sigma_{32}^{*}(\omega_d)E_p + \sigma_{32}^{*}(\omega_s)E_d)}{2\hbar\Delta_{21}(\omega_l)},$$

$$\sigma_{31}(\omega_p) = \frac{d_{31}N_1E_p + d_{32}\sigma_{21}E_d}{2\hbar\Delta_{31}(\omega_p)}, \quad \sigma_{31}(\omega_s) = \frac{d_{31}N_1E_s}{2\hbar\Delta_{31}(\omega_s)}, \quad \sigma_{31}(\omega_d) = \frac{d_{31}N_1E_d + d_{32}\sigma_{21}E_s}{2\hbar\Delta_{31}(\omega_d)},$$

$$\sigma_{32}(\omega_p) = 0, \quad \sigma_{32}(\omega_s) = \frac{d_{31}\sigma_{21}^{*}E_d}{2\hbar\Delta_{32}(\omega_s)}, \quad \sigma_{32}(\omega_d) = \frac{d_{31}\sigma_{21}^{*}E_p}{2\hbar\Delta_{32}(\omega_d)}, \tag{6}$$

where $\omega_l=\omega_p-\omega_d=\omega_d-\omega_s$, $\Delta_{ik}(\omega_j)=\omega_{ik}-\omega_j-i\gamma_{ik}$, $i,k=1,2,3$, $j=p,d,s,l$.

Taking into account that $|\Delta_{31}(\omega_p)|$, $|\Delta_{32}(\omega_d)|\sim\gamma_{31,32}\ll\omega_{21}\sim|\Delta_{31}(\omega_d)|$, $|\Delta_{32}(\omega_s)|$, $\frac{1}{2}|\Delta_{31}(\omega_s)|$ by virtue of conditions (3), we obtain from Eq. (6) within the framework of the linear approximation over the $E_{p,s}$ fields that

$$\sigma_{21} = \frac{d_{31}d_{32}^{*}N_1}{2\hbar Z}\left(E_pE_d^{*} + E_dE_s^{*}\frac{\Delta_{31}(\omega_p)}{\Delta_{31}(\omega_d)}\right),$$

$$\sigma_{31}(\omega_p) = \frac{d_{31}N_1}{2\hbar Z}\left(E_p\Delta_{21}(\omega_l) + \frac{E_s^{*}\Omega_2^2e^{2i\Psi}}{\Delta_{31}(\omega_d)}\right),$$

$$\sigma_{31}(\omega_s) = \frac{d_{31}N_1E_s}{2\hbar\Delta_{31}(\omega_s)}, \quad \sigma_{32}(\omega_p) = 0,$$

$$\sigma_{32}(\omega_s) = \frac{d_{32}N_1\Omega_1^2}{2\hbar Z^{*}}\left(E_s\frac{\Delta_{31}^{*}(\omega_p)}{\Delta_{32}(\omega_s)\Delta_{31}^{*}(\omega_d)} + E_p^{*}\frac{e^{2i\Psi}}{\Delta_{32}(\omega_s)}\right), \tag{7}$$

where $\Omega_{1,2}=|d_{31,32}E_d|/2\hbar$ are the Rabi frequencies for the drive field $\Psi=\mathrm{Arg}[E_d]$ is the phase of the drive field,

$$Z = (\omega_{21} - \omega_l - i\gamma_{21})(\omega_{31} - \omega_p - i\gamma_{31}) - \Omega_2^2. \tag{8}$$

Substituting the obtained solution for density matrix (7) into relation for the polarization of the medium $P_x=2\mathrm{Re}(d_{31}^{*}\rho_{31}+d_{32}^{*}\rho_{32})$ and then into the expression for the electric displacement vector $\vec{D}=\vec{E}+4\pi\vec{P}$, we obtain the nonlinear (with respect to the field E_d) constitutive equations for the Λ medium

$$D_x(\omega_p) = n_p^2E_p + g_{ps}e^{2i\Psi}E_s^{*},$$

$$D_x(\omega_s) = n_s^2E_s + g_{sp}e^{2i\Psi}E_p^{*}, \tag{9}$$

where $n_{p,s}^2$ are the squares of the refractive indexes of the P and S waves and $g_{ps,sp}$ are the coupling coefficients determining parametric interaction of the waves. Specifying the expressions for $n_{p,s}^2$ and $g_{ps,sp}$, we consider the following parameter range which is characteristic of the EIT regime and corresponds to a significant suppression of the P-wave absorption [1]

$$\frac{\gamma_{31}\gamma_{21}}{\Omega_2^2} \ll 1, \tag{10a}$$

$$(\Delta\omega)^2, |\Delta\omega|\gamma_{31} \ll \Omega_2^2, \tag{10b}$$

where $\Delta\omega=\omega_l-\omega_{21}=\omega_p-\omega_{31}$; Eq. (10b) determines the frequency bandwidth of EIT. For this parameter range, one can put $\Omega_2^2/Z\approx-1$ and $\Omega_1^2/Z\approx-|d_{31}|^2/|d_{32}|^2$ in Eqs. (7), that leads to certain simplifications. In particular, the explicit dependence on the D-wave power in the expressions for n_s^2 and $g_{ps,sp}$ is "reduced" (see also Refs. [24–26]). Allowing, together with Eqs. (10), for conditions (3), we have the following expressions for $n_{p,s}^2$ and $g_{ps,sp}$:

$$n_p^2 = \varepsilon_0 + \frac{\eta(\omega_p - \omega_{31})}{\Omega^2} + i\eta\left[\frac{\gamma_{21}}{\Omega^2} + \gamma_{31}\left(\frac{\omega_p - \omega_{31}}{\Omega^2}\right)^2\right], \tag{11a}$$

$$n_s^2 = \varepsilon_0 + \frac{\eta}{2\omega_{21}} + \frac{\eta[\omega_s - (2\omega_{32} - \omega_{31})]}{4\omega_{21}^2} - i\eta\frac{3\gamma_{31}}{4\omega_{21}^2}, \tag{11b}$$

$$g_{ps} = g_{sp} = g = -\frac{\eta}{\omega_{21}}, \tag{11c}$$

where and elsewhere below $\Omega^2 \equiv \Omega_2^2$, $\eta = 4\pi|d_{31}|^2 N_1/\hbar$ is a characteristic frequency which determines the role of the collective effects (and is expressed through a standard cooperative frequency of an ensemble of two-level atoms $\omega_c^2 = 8\pi|d_{31}|^2\Omega_1 N_1/\hbar$ [32]); ε_0 is the background dielectric permittivity of the medium, which can be determined by other levels (not included into the resonant Λ scheme), the buffer gas, the material of the matrix doped with Λ atoms (in condensed media), etc.

For further analysis we will also need an expression for the refractive index n_d^2 of the D wave, which, in terms of the adopted model, has the following form:

$$n_d^2|_{\omega_d=\omega_{32}} \approx \varepsilon_0 + \frac{\eta}{\omega_{21}} + i\eta\frac{\gamma_{31}}{\omega_{21}^2}. \tag{11d}$$

In the absence of the investigated parametric interaction of probed and Stokes waves the following regimes take place. The expression (11a) determines the standard EIT regime. The measured width $\langle\Delta\omega\rangle_{\mathrm{EIT}}$ of the EIT "transparency window" (see also Refs. [24,33,34]) is determined by the optical depth of the Λ-atom layer for the P wave

$$\tau_p|_{\omega_p=\omega_{31}+\langle\Delta\omega\rangle_{\mathrm{EIT}}} - \tau_p|_{\omega_p=\omega_{31}} \approx 1,$$

where $\tau_p = 2\int_0^L \kappa_p dz$, L is the layer thickness, and κ_p is the absorption coefficient of the P wave, which is determined by Eq. (11a):

$$\kappa_p \approx \frac{\omega_p}{2c\sqrt{\mathrm{Re}\, n_p^2}}\,\mathrm{Im}\, n_p^2. \tag{12}$$

As a result, using Eqs. (12) and (11a) we find

$$\langle\Delta\omega\rangle_{\mathrm{EIT}} \approx \frac{\Omega^2}{\gamma_{31}\sqrt{\tau_{\mathrm{res}}}}, \tag{13}$$

where $\tau_{\mathrm{res}} = \frac{\omega_{31}}{c\gamma_{31}}\int_0^L \frac{\eta(z)}{\sqrt{\mathrm{Re}\varepsilon_0(z)}}dz$ is the optical depth of the resonant layer in the absence of the drive field.

The propagation of the S wave, when the interaction with drive wave only is taken into account, is characterized by the absorption coefficient, determined from Eq. (11b),

$$\kappa_s \approx \frac{\omega_s}{2c\sqrt{\mathrm{Re}\, n_s^2}}\,\mathrm{Im}\, n_s^2 = \frac{\omega_s}{2c\sqrt{\mathrm{Re}\, n_s^2}}\left(\mathrm{Im}\,\varepsilon_0 - \eta\frac{3\gamma_{31}}{4\omega_{21}^2}\right). \tag{14}$$

It can be negative for a not too large background absorption coefficient $\propto \mathrm{Im}\,\varepsilon_0$. This fact is related to another dissipative instability, namely, the decay of a D photon into an S photon and a quantum of excitation of the medium at the frequency of the LF transition $\hbar\omega_d \Rightarrow \hbar\omega_s + \hbar\omega_{21}$. From a comparison of Eqs. (14) and (11d) it follows that the spatial instability growth rate of the S wave cannot be greater than the absorption coefficient of the D wave, i.e., $\kappa_d \approx \omega_d\,\mathrm{Im}\, n_d^2/2c\sqrt{\mathrm{Re} n_d^2}$. Thus, the possible amplification of the S wave is fairly small due to the instability since in the optimal case the medium must be transparent for the D wave. However, it is of principle that interaction with the drive field improves at least the transparency of the layer for the S wave.

III. NORMAL BIHARMONIC MODES IN A THREE-LEVEL MEDIUM

Consider stationary waves (i.e., waves with real frequencies) propagating along the Z axis. From the wave equation for the transverse field

$$c^2\nabla^2 E_x - \frac{\partial^2}{\partial t^2}D_x = 0$$

we obtain, together with Eqs. (9) and (11a)–(11c), the equations for the $E_{p,s}$ fields

$$\left(-\frac{\partial^2}{\partial z^2} - \frac{\omega_p^2}{c^2}n_p^2(\omega_p)\right)E_p = \frac{\omega_p^2}{c^2}gE_s^* e^{2i\Psi},$$

$$\left(-\frac{\partial^2}{\partial z^2} - \frac{\omega_s^2}{c^2}n_s^2(\omega_s)\right)E_s = \frac{\omega_s^2}{c^2}gE_p^* e^{2i\Psi}, \tag{15}$$

where $\omega_s = 2\omega_{32} - \omega_p$.

The solution of system (15) can be found in the form of a superposition of normal biharmonic modes (see also Ref. [21]). Here, by analogy with normal waves in anisotropic media, whose polarization is not varied during propagation in a homogeneous medium, we assume that normal biharmonic modes are pairs of waves for which the amplitude ratio of the partial harmonics is not varied during propagation in a homogeneous medium.

Assign the wave field in the following form:

$$E_d = A_d \exp\left(i\Theta + i\int_0^z q_d dz\right), \quad E_p = A_p \exp\left(i\int_0^z k dz,\right),$$

$$E_s = A_s \exp\left(i\int_0^z (2q_d - k^*)dz,\right), \tag{16}$$

where the quantities q_d and A_d are real [in Eq. (15) we have $\Psi = \Theta + \int_0^z q_d dz$] and $A_{p,s}$ and k are complex. We assume that the possible inhomogeneity of the waves amplitudes is "smooth" enough for using the geometric-optical approximation.

Substituting Eq. (16) into Eqs. (15) and performing complex conjugation of the second equation in system (15), we obtain a dispersion equation for determining the quantity k:

$$\left(k^2-\frac{\omega_p^2}{c^2}n_p^2(\omega_p)\right)\left((2q_d-k)^2-\frac{\omega_s^2}{c^2}[n_s^2(\omega_s)]^*\right)=\frac{\omega_p^2\omega_s^2}{c^4}g^2,$$

$$\omega_s=2\omega_{32}-\omega_p. \tag{17}$$

By analogy with the theory of waves in anisotropic media, we will call the solution corresponding to different roots of Eq. (17) the ordinary (O) and extraordinary (X) modes. We choose the indices such that in the transition to vacuum the O and X modes tend to monochromatic waves with frequencies ω_p and ω_s, respectively, propagating in the direction of the wave vector of the D wave. The modes in which the direction of the energy flux is opposite, will be marked by an additional subscript "-." Thus, the solutions of Eq. (17) are $k=k_J$, where $J=O,X,O_-,X_-$. For each biharmonic mode, one can determine the amplitude ratio of the harmonic components

$$K_J=\frac{A_s^*}{A_p}=\frac{c^2k_J^2-\omega_p^2n_p^2(\omega_p)}{\omega_p^2g}e^{-2i\Theta}$$

$$=\frac{\omega_s^2g}{c^2(2q_d-k_J)^2-\omega_s^2[n_s^2(\omega_s)]^*}e^{-2i\Theta}. \tag{18a}$$

For the E_x field, we obtain a solution in the form of a superposition of biharmonic normal waves

$$E_x=\sum_{J=O,X,O_-,X_-}e^{-\int_0^z\kappa_Jdz}\left[A_J\exp\left(i\int_0^z q_{Jp}dz-i\omega_pt\right)\right.$$

$$\left.+K_J^*A_J^*\exp\left(i\int_0^z q_{Js}dz-i\omega_st\right)\right], \tag{18b}$$

where $\kappa_J=\mathrm{Im}\,k_J$, $q_{Jp}=\mathrm{Re}k_J$, and $q_{Js}=2q_d-\mathrm{Re}k_J$. It is seen that despite the different frequencies and wave vectors of the partial waves, each biharmonic mode is characterized by a common absorption coefficient. Taking into account that $\omega_p+\omega_s=\text{const}$, it can easily be verified that the group velocity for the biharmonic mode is also common:

$$V_J^{\mathrm{gr}}=\left(\frac{\partial q_{Jp}}{\partial\omega_p}\right)^{-1}=\left(\frac{\partial q_{Js}}{\partial\omega_s}\right)^{-1}.$$

Solution (18b) can be used for constructing an exact solution of the corresponding boundary-value problem. For a homogeneous medium and a homogeneous drive field A_J are arbitrary constants. In the case of smooth inhomogeneity, the solution can be constructed within the framework of the WKB (Wentzel-Kramers-Brillouin) approximation (geometric-optical approach). Making use of the procedure described in Refs. [35,36] for similar equations of the wave theory in anisotropic media, from Eqs. (15), (18a), and (18b) we find an expression for the geometric-optical wave amplitudes

$$A_J=\frac{C_J}{\sqrt{k_J-(2q_d-k_J)K_J^2e^{4i\Theta}}}, \tag{18c}$$

where C_J are arbitrary constants.

IV. NORMAL MODES IN THE CASE OF A RELATIVELY SMALL DENSITY OF THREE-LEVEL ATOMS: THE PIT REGIME

Common relationships (17) and (18a)–(18c) can be significantly simplified for the practically important case of a relatively small effective density of resonant atoms where $\eta^2<<\mathrm{Re}\varepsilon_0\omega_{21}^2$. Under this condition, the wave numbers of all three (including the drive) partial waves ($\mathrm{Re}k_j=\pm\mathrm{Re}n_j\omega_j/c$, $j=p,s,d$) differ only weakly from each other. Actually, from Eqs. (11a), (11b), and (11d) we obtain

$$\pm\mathrm{Re}\,k_p\approx\frac{\omega_{31}}{c}n_0+\frac{\omega_p-\omega_{31}}{V_{\mathrm{EIT}}^{\mathrm{gr}}},\quad n_0\approx\sqrt{\mathrm{Re}\,\varepsilon_0},$$

$$V_{\mathrm{EIT}}^{\mathrm{gr}}\approx\frac{c}{n_0+\dfrac{\omega_p}{2n_0}\dfrac{\partial\,\mathrm{Re}(n_p^2)}{\partial\omega_p}}\approx\frac{c}{n_0+\dfrac{\eta\omega_{31}}{2n_0\Omega^2}},$$

$$\pm\mathrm{Re}\,k_s\approx\frac{(2\omega_{32}-\omega_{31})}{c}n_{s0}+\frac{\omega_s-(2\omega_{32}-\omega_{31})}{V_s^{\mathrm{gr}}},$$

$$n_{s0}\approx n_0+\frac{\eta}{4n_0\omega_{21}},$$

$$V_s^{\mathrm{gr}}\approx\frac{c}{n_{s0}+\dfrac{\omega_s}{2n_{s0}}\dfrac{\partial\,\mathrm{Re}(n_s^2)}{\partial\omega_s}}\approx\frac{c}{n_{s0}+\dfrac{(2\omega_{32}-\omega_{31})\eta}{8n_{s0}\omega_{21}^2}},$$

$$\mathrm{Re}\,k_d=q_d\approx\frac{\omega_{32}}{c}n_{d0},\quad n_{d0}\approx n_0+\frac{\eta}{2n_0\omega_{21}}, \tag{19a}$$

where n_0, n_{s0}, and n_{d0} are the real refractive indices of the waves $V_{\mathrm{EIT}}^{\mathrm{gr}}$ and V_s^{gr} are the group velocities of the P and S waves, and the $\pm$ signs correspond to the different directions of wave propagation.

In the considered case of a relatively small effective density of the Λ medium, the right-hand side of algebraic equation (17) is small. Hence, the solutions of this equation can differ radically from the solutions of the dispersion equations for partial waves only if both cofactors on the left-hand side tend to zero simultaneously. Thus, a significant interaction of P and S waves takes place for such frequencies $\omega_{p,s}=\omega_{p,s}^0$ for which the dispersion equations of partial waves are compatible with the conditions of the wave vectors and frequencies synchronism[2]

[2]In the considered process of wave interaction the frequency synchronism condition (1) must be assigned by the analogous in form synchronism condition for the wave numbers (see Ref. [23]), which is the first of conditions (19b).

$$\mathrm{Re}\, k_p(\omega_p)|_{\omega_p=\omega_p^0} + \mathrm{Re}\, k_s(\omega_s)|_{\omega_s=\omega_s^0} = 2q_d, \quad \omega_p^0 + \omega_s^0 = 2\omega_{32}. \tag{19b}$$

It is exactly the case that corresponds to the PIT regime [22].

In the considered situation, the fulfillment of condition (19b) is possible for the O and X modes and is not possible for the O_- and X_- modes.[3] We determine the resonant frequencies $\omega_{p,s}^0$, which satisfy synchronism condition (19b), by substituting Eq. (19a) into Eq. (19b). The following relationships are valid:

$$\omega_s^0 = 2\omega_{32} - \omega_p^0,$$

$$(\omega_p^0 - \omega_{31})\left(\frac{1}{V_{\mathrm{EIT}}^{\mathrm{gr}}} - \frac{1}{V_s^{\mathrm{gr}}}\right) = 2\frac{\omega_{32}}{c}(n_{d0} - n_0) - \frac{(2\omega_{32} - \omega_{31})}{c}(n_{s0} - n_0) \approx \frac{3\eta\omega}{4cn_0\omega_{21}}, \tag{19c}$$

where

$$\left(\frac{1}{V_{\mathrm{EIT}}^{\mathrm{gr}}} - \frac{1}{V_s^{\mathrm{gr}}}\right) = \frac{\eta}{2cn_0}\left(\frac{\omega_{31}}{\Omega^2} - \frac{1}{2\omega_{21}} - \frac{n_0}{n_{s0}}\frac{(2\omega_{32} - \omega_{31})}{4\omega_{21}^2}\right) \approx \frac{\eta\omega}{2n_0 c\Omega^2} \tag{19d}$$

and $\omega \approx \omega_{31,32,p,d,s}$ is a characteristic high frequency.

As a result, we arrive at

$$\omega_p^0 \approx \omega_{31} + \Delta\omega_{\mathrm{PIT}}, \quad \omega_s^0 \approx (2\omega_{32} - \omega_{31}) - \Delta\omega_{\mathrm{PIT}},$$

$$\Delta\omega_{\mathrm{PIT}} = \frac{3\Omega^2}{2\omega_{21}},$$

$$q_p^0 = \mathrm{Re}\, k_p(\omega_p^0) \approx \frac{\omega_{31}}{c}\left(n_0 + \frac{3\eta}{4n_0\omega_{21}}\right),$$

$$q_s^0 = k_s(\omega_s^0) = 2q_d - q_p^0 \approx \frac{(2\omega_{32} - \omega_{31})}{c}\left(n_0 - \frac{\eta}{4n_0\omega_{21}}\right). \tag{19e}$$

Thus, the resonant frequencies corresponding to the wave-number synchronism do not depend on the effective density N_1 of resonant atoms nor the parameter ε_0 of the background medium, although the corresponding values of the "resonant" wave numbers depend on these parameters and, in particular, can vary in space. We note that the optimal frequency for the wave synchronism (the center of the PIT line) is shifted with respect to the EIT line center, which explains the asymmetry of the frequency transparency window for the P wave under conditions of the S-wave generation, which was observed in Ref. [24].

[3] In addition, it is worth mentioning that the backward propagating modes are strongly absorbed in the inhomogeneously broadened medium.

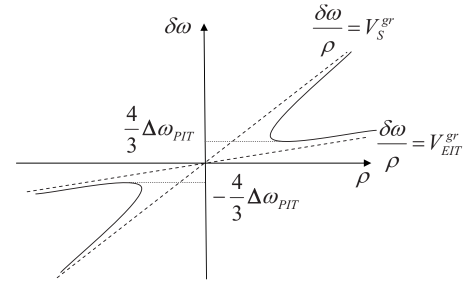

FIG. 2. The dispersion curve for normal modes of the Λ system in the region of convective instability.

The dispersion equation for the O and X modes in the case of a relatively small effective density of Λ atoms can be obtained by expansion of common relationship (17) with respect to the small quantity $\rho = k - q_p^0$ with allowance for square-law terms. Using Eqs. (11a), (11b), (11d), (12), (14), (19a), and (19e) and assuming that the parameters $\mathrm{Im}\,\varepsilon_0/\mathrm{Re}\,\varepsilon_0$, η/ω_{21}, ω_{21}/ω, and γ_{31}/ω_{21} are small, we obtain

$$\left(\rho - \frac{\delta\omega}{V_{\mathrm{EIT}}^{\mathrm{gr}}} - i\kappa_{\mathrm{EIT}} - i\kappa^0\right)\left(\rho - \frac{\delta\omega}{V_s^{\mathrm{gr}}} + i\mu_s - i\kappa^0\right) + \chi_{\mathrm{PIT}}^2 = 0, \tag{19f}$$

where $\delta\omega = \omega_p - \omega_p^0 = \omega_s^0 - \omega_s$ is the frequency detuning of the PIT resonances

$$\chi_{\mathrm{PIT}} = \frac{\omega_p\omega_s}{2c^2\sqrt{q_p^0 q_s^0}}|g| \approx \frac{\omega\eta}{2cn_0\omega_{21}}.$$

The dissipative effects are determined by the following terms of the dispersion equation [see also Eqs. (12) and (14)]: $\mu_s \approx 3\omega\eta\gamma_{31}/8cn_0\omega_{21}^2$, which is the spatial growth rate of the S-wave instability, and

$$\kappa_{\mathrm{EIT}} \approx \frac{\eta\omega}{2cn_0\Omega^2}\left(\gamma_{21} + \gamma_{31}\frac{(\omega_p - \omega_{31})^2}{\Omega^2}\right), \tag{19g}$$

which is the absorption coefficient of the P wave on Λ atoms in the EIT regime $\kappa^0 \approx \omega\,\mathrm{Im}\,\varepsilon_0/2cn_0-$ background absorption coefficient.

In the weak dissipation limit (i.e., for $\chi_{\mathrm{PIT}} \gg \kappa_{\mathrm{EIT}}, \kappa^0, \mu_s$) the dispersion curve $\rho(\delta\omega)$ determined from Eq. (19f) has a form characteristic of convectively unstable systems (see Fig. 2).

The instability region corresponds to the frequency interval

$$-\frac{4}{3}\Delta\omega_{\mathrm{PIT}} \leqslant \delta\omega \leqslant \frac{4}{3}\Delta\omega_{\mathrm{PIT}}, \tag{19h}$$

and the maximum spatial growth rate is given by

$$-\mathrm{Im}\,\rho|_{\delta\omega=0} \approx \chi_{\mathrm{PIT}}.$$

For a more detailed analysis, it is useful to obtain the system of differential equations corresponding to dispersion equation (19f). To do this, it is convenient to introduce the new variables $I_{p,s}$:

$$\frac{c\sqrt{q^0_{p,s}(z)}}{\omega_{p,s}}E_{p,s}(z)=I_{p,s}(z)\exp\left(i\int_0^z q^0_{p,s}dz\right). \tag{20a}$$

The phases of the new complex variables are determined by the difference of the wave numbers of the partial waves from the resonant values $q^0_{p,s}$, and the absolute values determine the photon fluxes

$$N_{p,s}=\frac{c}{8\pi\hbar\omega_{p,s}}\,\mathrm{Re}[\vec{E}_{p,s}\times\vec{H}^*_{p,s}]\approx\frac{|I_{p,s}|^2}{8\pi\hbar},$$

where $\vec{H}_{p,s}\approx(cq^0_{p,s}/\omega_{p,s})\vec{E}_{p,s}$ are the monochromatic components of the magnetic field.

We now substitute Eq. (20a) into system (15), performing complex conjugation of the second equation of the system and using Eqs. (19a) and (19e). With accuracy up to square-law and bilinear corrections over small parameters $c/n_0\omega l$, $\mathrm{Im}\,\varepsilon_0/\mathrm{Re}\,\varepsilon_0$, η/ω_{21}, ω_{21}/ω, and γ_{31}/ω_{21}, where l is the scale of the wave-amplitude inhomogeneity, we have

$$\left(\frac{\partial}{\partial z}-i\frac{\delta\omega}{V^{\mathrm{gr}}_{\mathrm{EIT}}}+\kappa_{\mathrm{EIT}}+\kappa^0\right)I_p=-i\chi_{\mathrm{PIT}}e^{2i\Theta}I^*_s,$$

$$\left(\frac{\partial}{\partial z}-i\frac{\delta\omega}{V^{\mathrm{gr}}_s}-\mu_s+\kappa^0\right)I^*_s=+i\chi_{\mathrm{PIT}}e^{-2i\Theta}I_p. \tag{20b}$$

From Eqs. (20b) we have the photon-flux variation law

$$\left(\frac{\partial}{\partial z}+2\kappa_{\mathrm{EIT}}+2\kappa^0\right)N_p-\left(\frac{\partial}{\partial z}-2\mu_s+2\kappa^0\right)N_s=0,$$

which, in the absence of dissipative processes (for $\kappa_{\mathrm{EIT}}=\mu_s=\kappa^0=0$), converts to the Manley-Rowe relationship for the four-quantum process $\hbar\omega_d+\hbar\omega_d\Leftrightarrow\hbar\omega_p+\hbar\omega_s$ corresponding to synchronism condition (1), namely $N_p-N_s=\mathrm{const}$.

System (20b) reduces to the second-order equation

$$U''_{\xi\xi}+\left[\frac{1}{4}(\sigma^2-1)-\frac{i}{2}\sigma'_\xi\right]U=0, \tag{20c}$$

where

$$U=I_p\exp\left\{-\frac{1}{2}\int_0^Z\left[i\left(\frac{\delta\omega}{V^{\mathrm{gr}}_{\mathrm{EIT}}}+\frac{\delta\omega}{V^{\mathrm{gr}}_s}\right)-\kappa_{\mathrm{EIT}}+\mu_s-2\kappa^0\right]dz\right\},\quad \xi=\frac{\omega}{c\omega_{21}}\int_0^Z\frac{\eta(z)}{n_0(z)}dz, \tag{20d}$$

$$\sigma=\frac{\dfrac{\delta\omega}{V^{\mathrm{gr}}_{\mathrm{EIT}}}-\dfrac{\delta\omega}{V^{\mathrm{gr}}_s}+i(\kappa_{\mathrm{EIT}}+\mu_s)}{\dfrac{\eta\omega}{cn_0\omega_{21}}}\approx\frac{3\delta\omega}{4\Delta\omega_{\mathrm{PIT}}}+i\left\{\frac{1}{2G}\left[1+\frac{\gamma_{31}\gamma_{21}}{\Omega^2}\left(\frac{\delta\omega}{\gamma_{21}}+\frac{3}{2}G\right)^2\right]+\frac{3\gamma_{31}}{8\omega_{21}}\right\}, \tag{20e}$$

$$G=\frac{\Omega^2}{\omega_{21}\gamma_{21}}=\frac{2\Delta\omega_{\mathrm{PIT}}}{3\gamma_{21}} \tag{20f}$$

[for obtaining the expression for σ we used Eqs. (19d) and (19g)]. It is seen in Eq. (20e) that the quantity σ does not depend on the effective density of Λ atoms nor the dielectric properties of the background medium.

Equation (20c) is an equation with constant coefficients in the case $\Omega^2=\mathrm{const}$ even for the spatially varied parameters $\eta(z)$ and $\varepsilon_0(z)$; thus, in the approximation of a constant D-wave intensity we find an exact solution of system (20b):

$$I_p=\sum_{J=O,X}F_J e^{\int_0^z(i\delta q_J-\kappa_J)dz},$$

$$I_s=\sum_{J=O,X}Q^*_J F^*_J e^{\int_0^z(-i\delta q_J-\kappa_J)dz},$$

$$Q_{O,X}=(\sigma\mp\sqrt{\sigma^2-1})e^{-2i\Theta},$$

$$\kappa_{O,X}=\frac{\kappa_{\mathrm{EIT}}-\mu_s}{2}+\kappa^0\pm\frac{\eta\omega}{2cn_0\omega_{21}}\,\mathrm{Im}\,\sqrt{\sigma^2-1},$$

$$\delta q_{O,X}=\frac{\delta\omega}{2}\left(\frac{1}{V^{\mathrm{gr}}_{\mathrm{EIT}}}+\frac{1}{V^{\mathrm{gr}}_s}\right)\pm\frac{\eta\omega}{2cn_0\omega_{21}}\,\mathrm{Re}\,\sqrt{\sigma^2-1}, \tag{20g}$$

where $F_{O,X}$ are arbitrary constants and $Q_OQ_X=e^{-4i\Theta}$. The group velocities of the normal modes are determined by the expression $V^{\mathrm{gr}}_J=[\partial(\delta q_J)/\partial(\delta\omega)]^{-1}$. It can easily be verified that the expressions for the complex wave vectors $\rho=\delta q_{O,X}+i\kappa_{O,X}$ are solutions of dispersion equation (19f).

In the limit $|\sigma|^2\gg1$, Eqs. (20g) become considerably simpler:

$$\kappa_O\approx\kappa^0+\kappa_{\mathrm{EIT}}+\frac{\kappa_{\mathrm{EIT}}+\mu_s}{4|\sigma|^2},\quad \delta q_O\approx\frac{\delta\omega}{V^{\mathrm{gr}}_{\mathrm{EIT}}},$$

$$Q_O\approx\frac{1}{2\sigma}e^{-2i\Theta},\quad V^{\mathrm{gr}}_O\approx V^{\mathrm{gr}}_{\mathrm{EIT}},$$

$$\kappa_X\approx\kappa^0-\mu_s-\frac{\kappa_{\mathrm{EIT}}+\mu_s}{4|\sigma|^2},\quad \delta q_X\approx\frac{\delta\omega}{V^{\mathrm{gr}}_s},$$

$$Q_X\approx2\sigma e^{-2i\Theta},\quad V^{\mathrm{gr}}_X\approx V^{\mathrm{gr}}_s.$$

It can easily be seen that the ordinary (O) mode always corresponds to the process of predominant transformation of P and S photons into D photons

$$\hbar\omega_p+\hbar\omega_s\Rightarrow\hbar\omega_d+\hbar\omega_d:\quad \frac{\partial}{\partial z}(N_p+N_s)<-2\kappa^0(N_p+N_s),$$

and the extraordinary (X) mode always corresponds to the inverse process

$$\hbar\omega_d+\hbar\omega_d\Rightarrow\hbar\omega_p+\hbar\omega_s:\quad \frac{\partial}{\partial z}(N_p+N_s)>-2\kappa^0(N_p+N_s).$$

In the absence of the background absorption ($\kappa_0\propto\mathrm{Im}\,\varepsilon_0\to0$), the X mode is always amplified.

Thus, in the case of an optically depth layer of Λ atoms its transparency is determined exclusively by the excitation efficiency of the X mode.

In the case of nonuniform intensity of the D wave, the amplitudes $F_{O,X}$ become variable. One can construct an asymptotic WKB solution of Eq. (20c), which determines the amplitude variation law[4]

$$F_{O,X} = \frac{C_{O,X}}{\sqrt{\pm 2(\sqrt{\sigma^2-1})(\sigma \mp \sqrt{\sigma^2-1})}} = \frac{C_{O,X}}{\sqrt{1-Q_{O,X}^2(z)e^{4i\Theta}}}, \tag{21a}$$

where $C_{O,X}$=const.

A common criterion of applicability of the WKB asymptotics during solution of Eq. (20c) is the fulfillment of the inequality

$$|\sigma'_\xi| \ll |\sigma^2-1|. \tag{21b}$$

It is easily seen that inequality (21b) is equivalent of the condition $|\Delta k^{-1} dQ_{O,X}/dz| \ll |Q_{O,X}|$, where $\Delta k = (\delta q_X + i\kappa_X) - (\delta q_O + i\kappa_O)$. Thus, in addition to the standard condition of geometrical-optics applicability $|k^{-2}k'_z| \ll 1$, an additional constraint is imposed for systems of coupled wave equations. The coefficients $Q_{O,X}$ characterizing the structure of the normal modes must vary only weakly on the length of the beats. In this sense, the coefficients $Q_{O,X}$ are completely similar to the polarization coefficients of monochromatic waves in an anisotropic medium, whose relatively rapid variation can be responsible for the violation of the geometric-optical solution in anisotropic media (see Refs. [35,36]).

It follows from Eqs. (21a) and (21b) that even a relatively small variation in the D-wave intensity inside the Λ layer can be of basic importance in the vicinity of the singularity $\sigma^2 \to 1$. However, this singularity can be insignificant by virtue of the dissipative effects. Indeed, in the case $G \ll 1$ we obtain $|\sigma^2-1|_{\min} \sim G^{-2} \gg 1$. At the same time, for $G \gg 1$ we have $|\sigma^2-1|_{\min} \sim \min[1/G, \gamma_{31}/\omega_{21}] \ll 1$. In this case, the WKB asymptotic can be inadequate when passing through the boundary of the unstable region (19h) where $\sigma^2 \to 1$. Here, we will not consider the construction of a solution in this domain and only note that for the linear approximation (corresponding, e.g., to weak absorption of the drive wave), such that $\Omega^2 \approx \Omega_0^2[1-\alpha(\xi-\xi_0)]$, Eq. (20c) can be reduced to the well-known parabolic-cylinder equation [37].

V. TRANSMISSION OF THE RADIATION THROUGH THE Λ LAYER

In this section, we limit ourselves to exact solution (20g), which is valid for arbitrary inhomogeneous profiles[5] $\eta(z)$ and $\varepsilon_0(z)$ and a constant drive intensity since the solution allowing for the inhomogeneity of the WKB drive field in the region of its applicability does not lead to radically new effects. Consider the incidence of a signal wave with frequency ω_p and amplitude corresponding to the quantity $I_p|_{z=o} = I_p^0$ from vacuum on a Λ layer of thickness L. Neglecting the reflection of waves from the layer, we obtain the following expressions for arbitrary constants of solution (20g):

$$F_O = \frac{Q_X I_p^0}{Q_X - Q_O}, \quad F_X = -\frac{Q_O I_p^0}{Q_X - Q_O}. \tag{21c}$$

Formally, solution (20g) with found constants contains the above-discussed singularity for $\sigma^2 \to 1$ when $Q_O \to Q_X$. However, since solution (20g) is exact, the corresponding correct passage to the limit leads to the expressions not containing singularities

$$I_p \to I_p^0\left(1 - \frac{i\omega}{2c\omega_{21}}\int_0^z \frac{\eta}{n_0}dz\right)\exp\left(\int_0^z (i\delta q_O - \kappa_O)dz\right),$$

$$I_s \to I_p^{0*}\frac{i\omega}{2c\omega_{21}}\int_0^z \frac{\eta}{n_0}dz \exp\left(-\int_0^z (i\delta q_O + \kappa_O)dz\right).$$

Using Eqs. (20g) with arbitrary constants found above, we obtain expressions for the ratio of incident and transmitted photons.

Neglecting the small factors $2\int_0^L \kappa^0 dz$ and $2\int_0^L \mu_s dz$ in a medium that is transparent for the drive field, we find

$$\frac{N_p(z=L)}{N_p(z=0)} \approx \frac{1}{4|\sigma^2-1|}\Big(|\sigma + \sqrt{\sigma^2-1}|^2 e^{-(1/2\tau_{\mathrm{EIT}}+\Gamma_{\mathrm{PIT}})} - 2\cos(\Delta\Phi + 2\vartheta)e^{-(1/2)\tau_{\mathrm{EIT}}} + |\sigma - \sqrt{\sigma^2-1}|^2 e^{(\Gamma_{\mathrm{PIT}}-1/2\tau_{\mathrm{EIT}})}\Big),$$

$$\frac{N_s(z=L)}{N_p(z=0)} \approx \frac{1}{4|\sigma^2-1|}\Big(e^{-(1/2\tau_{\mathrm{EIT}}+\Gamma_{\mathrm{PIT}})} - 2\cos(\Delta\Phi)e^{-(1/2)\tau_{\mathrm{EIT}}} + e^{(\Gamma_{\mathrm{PIT}}-1/2\tau_{\mathrm{EIT}})}\Big), \tag{22a}$$

where

$$\tau_{\mathrm{EIT}} = 2\int_0^L \kappa_{\mathrm{EIT}} dz,$$

$$\sigma \approx \frac{3\delta\omega}{4\Delta\omega_{\mathrm{PIT}}} + i\frac{1}{2G}\left[1 + \frac{\gamma_{31}\gamma_{21}}{\Omega^2}\left(\frac{\delta\omega}{\gamma_{21}} + \frac{3}{2}G\right)^2\right],$$

$$\Gamma_{\mathrm{PIT}} = \int_0^L (\kappa_O - \kappa_X)dz = \phi \,\mathrm{Im}\sqrt{\sigma^2-1},$$

$$\Delta\Phi = \int_0^L (\delta q_O - \delta q_X)dz = \phi \,\mathrm{Re}\sqrt{\sigma^2-1},$$

[4]Such a result can be obtained also by constructing the WKB solution of original system (20b) or by the corresponding passage to the limit in Eq. (18c).

[5]Certainly, the profiles must be gradual on the scale of the wavelength $\frac{2\pi c}{\omega n_0}$.

$$\phi = \frac{\omega}{c\omega_{21}}\int_0^L \frac{\eta(z)}{n_0(z)}dz, \quad \vartheta = \mathrm{Arg}[\sigma + \sqrt{\sigma^2 - 1}] \tag{22b}$$

(the features of the behavior of the solution for $\sigma^2 \to 1$ were analyzed above). The right-hand sides of Eqs. (22a) represent the photon fluxes in the O and X modes (the first and last terms in the parentheses) and an interference term which can appear significant for comparable amplitudes of the O and X modes at the layer output.

For $|\sigma|^2 \gg 1$, the asymptotics

$$\Gamma_{\mathrm{PIT}} \approx \frac{1}{2}\tau_{\mathrm{EIT}}\left(1 + \frac{1}{2|\sigma|^2}\right) \tag{22c}$$

is valid, within the framework of which the simpler expressions can be obtained from Eqs. (22a) and (22b). The found expressions give a fairly complete solution of the problem of transmission of the P and S waves through the Λ layer. First, it is seen that in any case, the PIT effect ensures the appearance of power-law "tails" of the frequency transparency line outside the region of exponential decay of an EIT signal. Other features of the signal transmission are determined by the dimensionless parameter G, introduced in Eq. (20f), and the minimum optical depth for the EIT regime, which is given by $\tau_{\mathrm{EIT}}^{\min} = \tau_{\mathrm{EIT}}|_{\omega_p=\omega_{31}} = \phi G^{-1}$.

(i) $G \ll 1$. In this case, Eqs. (22c) apply for all frequencies. For the EIT parameter range ($|\delta\omega| \leq \Omega^2/\gamma_{31}$) we have

$$\frac{1}{4|\sigma|^2} \approx G^2\left[1 + \left(\frac{\delta\omega}{\gamma_{21}}\right)^2\right]^{-1}. \tag{22d}$$

Displacement of the PIT line center ($\omega_p = \omega_p^0$) with respect to the center of the EIT line ($\omega_p = \omega_{31}$) is much smaller than the width of the PIT line $\langle\delta\omega\rangle_{\mathrm{PIT}} \sim \gamma_{21}$. For $\tau_{\mathrm{EIT}}^{\min} \ll 4\ln(1/G)$, the generation of the S wave affects only weakly the transmission of the P wave. For $\tau_{\mathrm{EIT}}^{\min} \gg 4\ln(1/G)$, transmission of the radiation through the layer is due exclusively to the PIT effect; the group velocity of the X mode in this regime is close to the group velocity V_s^{gr} of the S wave. In the transmitted radiation, the S wave is about a factor G^{-2} more intense than the P wave at the line center and decays significantly weaker as the frequency detuning increases. In Fig. 3 we present the output intensities of P and S waves numerically calculated on the basis of Eq. (22a) in the corresponding regimes. In the region $\tau_{\mathrm{EIT}}^{\min} \sim 4\ln(1/G)$, the contributions of the O and X modes in the expression for the transmitted intensity of the P wave are of the same order of magnitude. Their interference results in oscillations of the frequency dependence of the transmitted-radiation intensity, which can be observed if the oscillations' period $\approx 8\pi\Delta\omega_{\mathrm{PIT}}/3\phi$ is less than the EIT line width, so that the condition $2\pi\sqrt{\gamma_{31}/\omega_{21}\phi} < 1$ is fulfilled. This regime is presented in Fig. 4.

(ii) $G \gg 1$. In this case, a parametric instability takes place for the extraordinary mode in the frequency band $\langle\delta\omega\rangle_{\mathrm{PIT}} \approx \frac{8}{3}\Delta\omega_{\mathrm{PIT}} = 4G\gamma_{21}$ (19h). The maximum amplification coefficient (at the line center) is given by $\frac{1}{4}\exp(\Gamma_{\mathrm{PIT}} - \frac{\tau_{\mathrm{EIT}}^{\min}}{2}) \approx \frac{1}{4}\exp(\phi - \frac{\phi}{2G})$. The ratio between characteristic widths of the frequency passbands $\langle\Delta\omega\rangle_{\mathrm{EIT}}$ (13) and $\langle\delta\omega\rangle_{\mathrm{PIT}}$ is

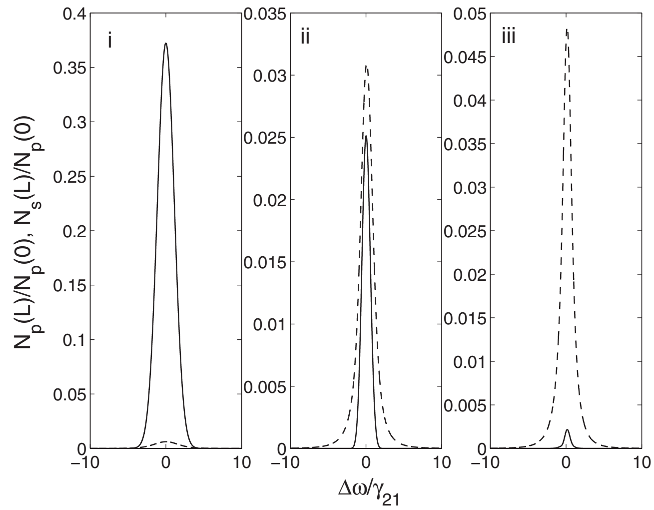

FIG. 3. Calculated probe (solid line) and Stokes (dashed line) transmitted signals [Eq. (22a)] as functions of frequency detuning $\Delta\omega = \omega_p - \omega_{31} = (2\omega_{32} - \omega_{31}) - \omega_s$ for the following dimensionless parameters: $G=0.2$, $\gamma_{21}\gamma_{31}/\Omega^2=0.4$, (i) $\phi=0.2$, (ii) $\phi=0.8$, (iii) $\phi=2$, that correspond, for example, to the following set of real parameters of Λ systems, formed by hyperfine splitting of the ground state of D1 line in the vapor of Na atoms ($3S_{1/2} \to 3P_{1/2}$ transition, corresponding wavelength $\lambda=589.6$ nm, $\omega_{21}=1772$ MHz, $\Omega=12.4$ MHz, $\gamma_{31}=160$ MHz, $\gamma_{21}=400$ kHz, $L=7.5$ cm, (i) $N_1=7\times10^9\ \mathrm{cm}^{-3}$, (ii) $N_1=2\times10^{10}\ \mathrm{cm}^{-3}$, (iii) $N_1=5\times10^{10}\ \mathrm{cm}^{-3}$), similar to that realized in the experiment [26].

$$\frac{\langle\Delta\omega\rangle_{\mathrm{EIT}}}{\langle\delta\omega\rangle_{\mathrm{PIT}}} \approx \frac{1}{\sqrt{\frac{\gamma_{31}}{\omega_{21}}\phi}} \gg 1,$$

i.e., the PIT band turns out to be much narrower. Dispacement of the PIT line center with respect to the EIT line center is of the order of the PIT linewidth. In the instability band,

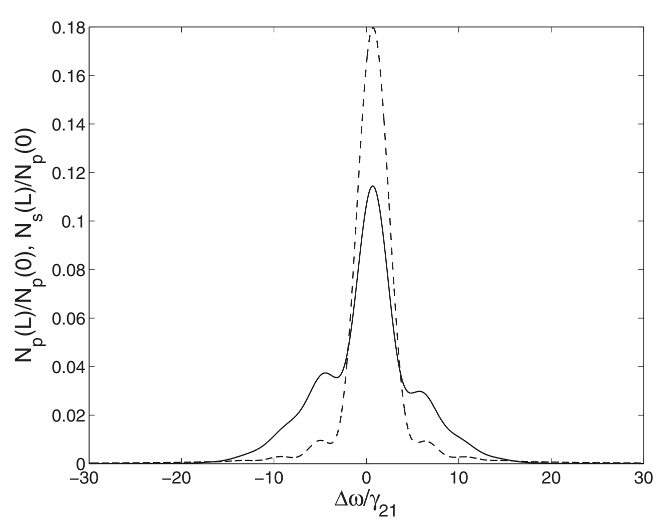

FIG. 4. The oscillations of the frequency dependence of the transmitted probe (solid line) and Stokes (dashed line) radiation intensity, calculated for parameters $G=0.5$, $\gamma_{21}\gamma_{31}/\Omega^2=0.005$, $\phi=1.5$.

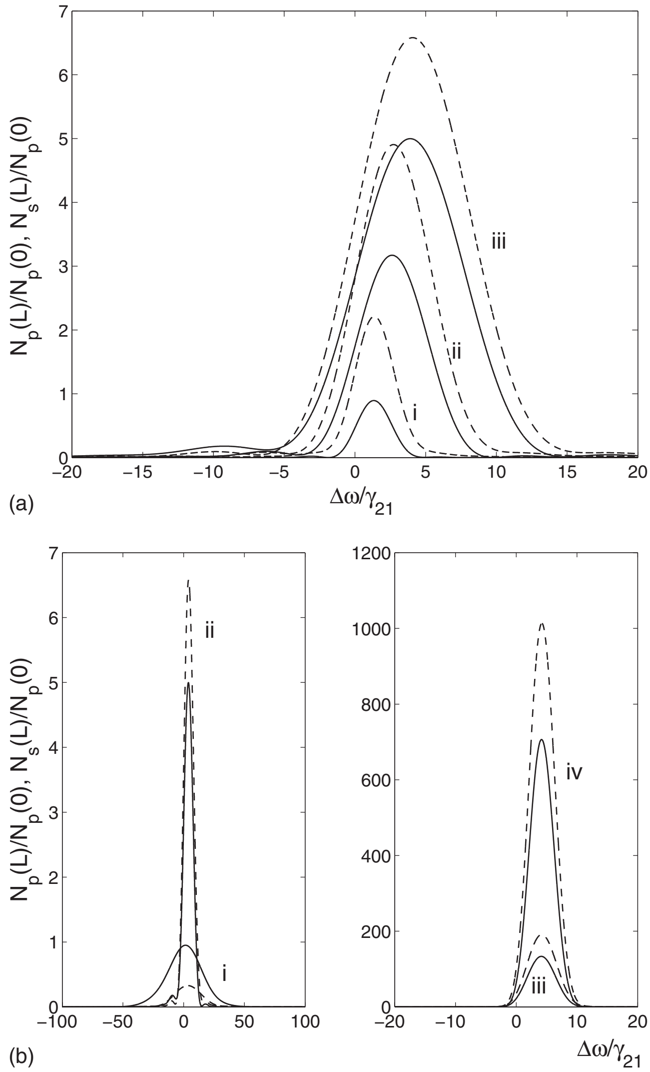

FIG. 5. The regime of parametric instability. The calculated probe (solid line) and Stokes (dashed line) signals. The parameters are chosen with aim to be realized in the experiment in Rb vapor within the $D1$ manifold ($\lambda = 794$ nm, $\omega_{21} = 6.83$ GHz, $\gamma_{31} = 100$ MHz, $\gamma_{21} = 15$ kHz). (a) The dependence of amplification coefficient on the drive-field Rabi-frequency for the fixed product of cavity length and atomic concentration ($LN_1 = 5\ \mathrm{cm} \times 2.6 \times 10^{11}\ \mathrm{cm}^{-3}$ that corresponds to $\phi = 4$), (i) $\Omega = 10$ MHz (corresponding parameter $G = 1$), (ii) $\Omega = 14$ MHz ($G = 2$), (iii) $\Omega = 17.5$ MHz ($G = 3$). (b) The development of instability with the cavity length or dense growth (i) $LN_1 = 5\ \mathrm{cm} \times 2.6 \times 10^{11}\ \mathrm{cm}^{-3}$, (ii) $LN_1 = 15\ \mathrm{cm} \times 2.6 \times 10^{11}\ \mathrm{cm}^{-3}$, (iii) $LN_1 = 30\ \mathrm{cm} \times 2.6 \times 10^{11}\ \mathrm{cm}^{-3}$, (iv) $LN_1 = 38\ \mathrm{cm} \times 2.6 \times 10^{11}\ \mathrm{cm}^{-3}$; $G = 3$ (the power of drive field is 30 mW with spot size of 5 mm).

the P and S components have intensities of the same order of magnitude ($Q_{O,X}|_{\delta\omega=0} \approx \mp i e^{-2i\Theta}$). The group velocity of both partial modes is given by $V_{O,X}^{\mathrm{gr}} \approx 2V_{\mathrm{EIT}}^{\mathrm{gr}}$, i.e., the probe's group delay characteristic of EIT is preserved. In the instability region (19h) the interference oscillations are certainly insignificant. Outside the limits of region (19h), the S wave has significantly stronger power-law "tails" of the transparency line than the P wave. In particular, under the condition $\Omega^2/\gamma_{31} \gg |\delta\omega| \gg \Delta\omega_{\mathrm{PIT}}$ one can make use of Eqs. (22c), where

$$\frac{1}{4|\sigma|^2} \approx \left(\frac{\Omega^2}{\omega_{21}\delta\omega}\right)^2. \tag{22e}$$

For $\tau_{\mathrm{EIT}}^{\min} \gg 1$, the X mode dominates in the transmitted radiation outside the limits of the instability region as well. Under the condition $\tau_{\mathrm{EIT}}^{\min} \leqslant 1$, the intensities of the O and X modes in the vicinity of the boundaries of the instability region are of the same order of magnitude; in this case, as is shown in the previous section, the effects stipulated by the inhomogeneity of the D-wave intensity and not described by Eqs. (22) can also be manifested.

The calculated [Eq. (22a)] signals corresponding to the described instability for the different parameters appropriate for possible experiment in Rb vapor are presented in Fig. 5. The dependence of the amplification coefficient on the drive-field intensity (parameter G) can be traced from Fig. 5(a). The modification of the output Probe and Stokes signal with a rise in cavity length and/or atom concentration (parameter ϕ) is shown in Fig. 5(b).

VI. COMPARISON WITH EXPERIMENTS

The induced transparency with the effect of S-wave generation has been experimentally investigated in Λ systems, formed by hyperfine splitting of the ground state of the $D1$ line in the vapor of Rb^{87} atoms $[S_{1/2}(F=2) \rightarrow P_{1/2}(F=2)$ transition] [24,25] and Na atoms ($3S_{1/2} \rightarrow 3P_{1/2}$ transition) [26]. The theory developed in the present paper corresponds qualitatively to the experimental data [24–26]. Nevertheless there is no strict quantitative correspondence. Evidently, it is caused by a number of simplifications used in theory construction: (1) three-level model, (2) only the homogeneous broadening of spectrum lines taking into account (in all mentioned experiments the inhomogeneous broadening if D_1 line was essentially larger than the homogeneous one, the time-of-flight broadening for the light beams was about inverse lifetime of the ground state), (3) approximation of well resolved transitions (which is failed, in particular, for the conditions of experiments in Ref. [26]), (4) linear approximation for P and S waves, (5) approximation of predetermined populations, (6) strict one-photon resonance for the drive field, (7) neglect of drive field depletion and others.[6]

In collating up the results of our analytical calculations with experimental data, we considered the effective population of the ground state N_1, the relaxation rate at HF transition γ_{31} and in some case drive power as adjustable parameters, keeping in mind just mentioned idealizations. The conformity between analytical dependences and experimental curves was achieved for magnitudes N_1 several times less

[6]The numerical model free of corresponding approximation was used in Ref. [24].

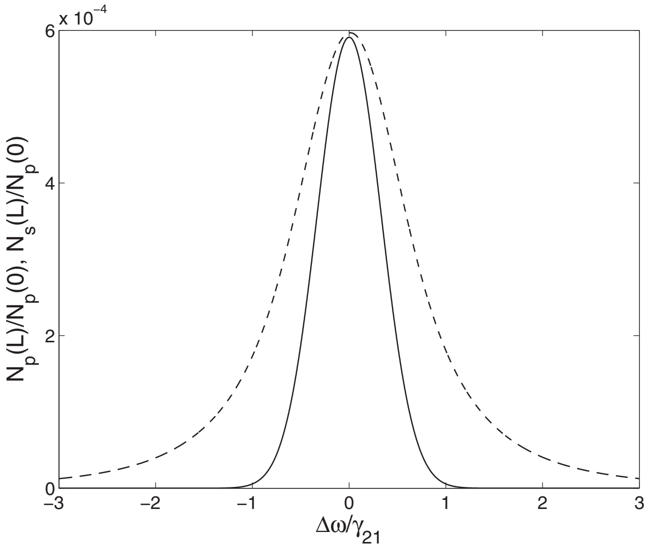

FIG. 6. Calculated probe (solid line) and Stokes (dashed line) signals in the regime of equal maximum intensities, demonstrating the difference between P and S linewidths. Dimensionless parameters: $G=0.025$, $\gamma_{21}\gamma_{31}/\Omega^2=0.6$, $\phi=0.19$.

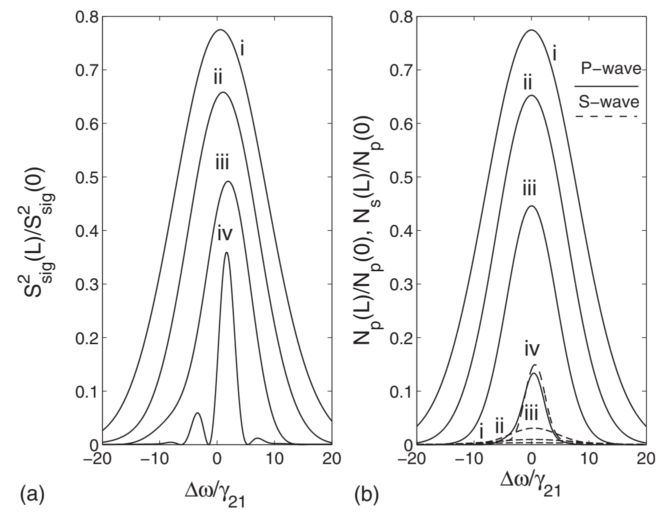

FIG. 7. Calculated signal as function of probe and Stokes detuning $\Delta\omega=\omega_p-\omega_{31}=(2\omega_{32}-\omega_{31})-\omega_s$ fulfilled for parameters similar to the experiment in Rb vapor [24]: ($\lambda=794$ nm, $\omega_{21}=6.83$ GHz, $\Omega=7$ MHz, $\gamma_{31}=100$ MHz, $\gamma_{21}=15$ kHz, $L=5$ cm, (i) $N_1=2.4\times10^{10}$ cm^{-3}, (ii) $N_1=4\times10^{10}$ cm^{-3}, (iii) $N_1=8\times10^{10}$ cm^{-3}, (iv) $N_1=2.4\times10^{11}$ cm^{-3}). (a) The quantity $S^2_{sig}\propto|E_dE_s^*+E_d^*E_p|^2$ such as measured by the detection scheme. The probe and Stokes intensities are presented in (b).

(from 5 to 2.5) than measured atomic concentrations[7] and magnitudes γ_{31} approaching to the constant of inhomogeneous broadening in order of magnitude, the adjustable drive power is maximally twice less than experimental value.

The data in Fig. 3 qualitatively correspond to the conditions of experiment [26], where conversion from standard EIT regime to the regime of practically total transfer of P wave to S wave with a rise in atomic dense was investigated. The experimental parameters: cavity length $L=7,5$ cm, drive radiation power 12 mW and spot size 2 mm, the atomic density varied from $3,5\times10^{10}$ to $2,6\times10^{11}$ cm^{-3}. The results of simulations fulfilled on the basis of the presented theory agree with the experimental conclusions of Ref. [26], with the difference that specialities of line shapes for transmitted P and S radiation were not investigated in Ref. [26].

The frequency dependences, followed from Eqs. (22a) and presented in Fig. 6, demonstrate the regime discovered in experiments [25], in which the medium is transparent for the S wave in frequency window wider than that for the P wave. To make the effect more obvious we chose the calculation parameters (in particular parameter ϕ) so that the intensities of the transmitted P and S waves were equal in maximum. The authors of Ref. [25] explored a mix of Rb^{87} and N_2 buffer gas at the pressure of 3 Torr at temperature 70–90 °C; the power of drive field was 100–500 μW, beam diameter was 5 mm; they did not investigate theoretically predicted dependence of the frequency bands and intensities of P and S lines on the optical depth of the medium.

A density dependent spectral narrowing of EIT window and novel, even narrower, resonances superimposed on the EIT line were observed in dense Rb vapor in Ref. [24]. The experimental parameters: cavity length $L=5$ cm, D-wave power 5–10 mW, spot size 5 mm, atomic densities varied from 6×10^{10} to 6×10^{11} cm^{-3}. It is essential that in those experiments the heterodyne detection scheme measured the beat signal at frequency $\omega_l=\omega_{21}+\Delta\omega$ with power $S^2_{sig}\propto|E_dE_s^*+E_d^*E_p|^2$, instead of P and S waves intensities separately. As atomic density increases the frequency dependence $S^2_{sig}(\Delta\omega)$ transforms from the function with one maximum to the oscillating curve. This indicates the conversion from the regime with dominating P wave to the regime with P and S waves of similar intensities. Graphics followed from Eqs. (22a) calculated for parameters similar to the experimental parameters and confirming this conclusion are introduced in Fig. 7(b). The dependences $S^2_{sig}(\Delta\omega)$, calculated using Eqs. (20a), (20g), and (21c) are presented in Fig. 7(a). These dependences correspond qualitatively to the experimental curves [24]: for sufficiently large atomic density the frequency line becomes asymmetrical with oscillations at the periphery.

VII. CONCLUSION

The analytical solution we obtained in this paper for the problem of propagation of radiation in a three-level medium demonstrates the possibility of a very nontrivial behavior of the interacting waves due to collective effects. The following main interrelated processes can be distinguished: the nonlinear coupling between a high-power D wave and S wave leads to the occurrence of a dissipative instability of the S wave;

[7]This discrepancy is probably caused by "extraction" of a part of atoms from resonant interaction with the field due to Doppler frequency difference. The maximum discrepancy—up to 5 times—took place for data [26], where inhomogeneous broadening exceeded spacing between lower levels in three-level systems.

the nonlinear coupling between the P wave and the undamped (unstable) S wave, which is stipulated by a high-power D wave, ensures transport of the probe radiation through a layer of Λ atoms due to collective effects even in the case where the layer is opaque within the framework of the standard EIT regime; the coupled P and S waves generate normal biharmonic modes, which, in many respects, are similar to the normal waves in anisotropic media. The approach developed here can be extended and modified further for an analysis of nonlinear problems with allowance for the influence of the finite intensity of the signal and Stokes waves, for taking into account the influence of the inhomogeneous broadening, etc.

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