Possible entanglement detection with the naked eye

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The human eye can detect optical signals containing only a few photons. We investigate the possibility to demonstrate entanglement with such biological detectors. While one person could not detect entanglement by simply observing photons, we discuss the possibility for several observers to demonstrate entanglement in a Bell-type experiment, in which standard detectors are replaced by human eyes. Using a toy model for biological detectors that captures their main characteristic, namely, a detection threshold, we show that Bell inequalities can be violated, thus demonstrating entanglement. Remarkably, when the response function of the detector is close to a step function, quantum nonlocality can be demonstrated without any further assumptions. For smoother response functions, as for the human eye, postselection is required.

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I. INTRODUCTION

The human eye is an extraordinary light sensitive detector. It can easily stand a comparison to today's best man-made detectors [1]. Already back in the 1940s, experiments on the sensibility of the human eye to weak optical signals were conducted [2], leading to the conclusion that rod photoreceptors can detect a very small number of photons, typically less than ten during an integration time of about 300 ms [3]. To date, this prediction has been confirmed by many experiments [1]. Though most specialists still disagree on the exact number of photons required to trigger a neural response, it seems to be now commonly accepted that there is a threshold number of incident photons, below which no neural signal is sent to the brain. This assumption is supported by the good agreement between theoretical models and experimental data from behavioral experiments. Our visual system works basically as follows: first, a photon is absorbed by the rod, which then amplifies the signal with some very efficient chemical reactions; then, some postprocessing (basically a thresholding) is performed on the signals incoming from a group of 20–100 rods [4]; finally, a neural signal is eventually sent to the brain. The role of the threshold is possibly to maintain a very low dark noise in the visual process, in particular, to get rid of electrical noise originating from the individual rods [4,5].

In quantum information, experiments carried out on photons are now routinely performed for demonstrating fascinating quantum features, such as entanglement and quantum nonlocality [6,7]. In this context, and considering the amazing performances of the human eye, it is quite intriguing to ask whether one could demonstrate entanglement without the help of man-made detectors, but using only naked eyes. It should be reminded at this point that entanglement is usually demonstrated in Bell-type experiments (where correlations between two distant parties are measured), and not via single shot measurements. Therefore one person cannot expect to see entanglement directly. Nevertheless, one could perform a Bell experiment in which man-made photon detectors are replaced by human eyes (see Fig. 1), or more generally, by biological detectors. In case the collected data would lead to the violation of a Bell's inequality [8], one could argue that entanglement has been "seen." Let us stress that, though such an experiment would probably not lead to a better understanding of quantum nonlocality itself, it would definitely be fascinating.

The main difference between man-made photon counters and the human eye is a detection threshold. To test whether a detector is able to detect single photons, one usually checks that the response of this detector to very low intensities is linear. Indeed, this is not the case for the eye, where the efficiency of detection plotted as a function of the number of incoming photons is a typical S-shaped curve (see [1]). In this paper we report a preliminary theoretical study of Bell tests with threshold detectors. Our goal is to provide a good understanding of Bell experiments with a toy model for the detector that captures the main characteristic of the human eye.

The presentation is organized as follows. After a general description of the scenario we consider (Sec. II), we first focus on detectors with a perfect threshold, i.e., no detection below the threshold and perfect detection above (Sec. III). We show that, even for a Poissonian source, the threshold is not a restriction for demonstrating quantum nonlocality; in other words, Bell inequalities can be violated (in the strict sense) with such detectors. Then we smoothed the threshold, in order to make our detector model closer to the human eye (Sec. IV). We show that, except for close-to-perfect thresholds, one must then perform postselection in order to obtain a violation of a Bell inequality (Sec. V).

II. GENERAL FRAMEWORK

Let us consider a typical Bell test scenario. A source sends pairs of entangled particles (each pair being in state ρ) to two

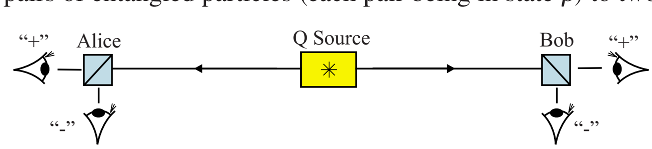

FIG. 1. (Color online) Bell experiments with human detectors.

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distant observers, Alice and Bob, who perform measurements on their respective particles. Here, Alice and Bob choose between two different measurement settings A_1, A_2 and B_1, B_2, each of these measurements giving a binary result $\alpha, \beta \in \{+, -\}$. In this case the relevant Bell inequality is the famous Clauser-Horne-Shimony-Holt (CHSH) inequality [9], which we will express here in the Clauser-Horne (CH) [10] form,

$$\mathrm{CH} \equiv P_{++}(A_1B_1) + P_{++}(A_1B_2) + P_{++}(A_2B_1) - P_{++}(A_2B_2) - P_{+}(A_1) - P_{+}(B_1) \leqslant 0, \qquad (1)$$

where $P_{+}(A_iB_j) \equiv P(++|A_iB_j)$ is the probability that $\alpha=\beta=+$ when Alice (Bob) has performed measurement $A_i(B_j)$. Note that under the hypothesis of no signaling, the CH and CHSH inequalities are strictly equivalent [11].

III. PERFECT THRESHOLD

Now let us bring the threshold detector into the picture. We start by considering a detector with a perfect threshold at N photons; optical signals containing at least N photons are always detected (note that our detector is not photon number resolving), while signals with less than N photons are never detected. The response function of our detector is simply a step function, the step occurring at N photons.

First, it is clear that the number of emitted pairs M has to be larger or equal than the threshold N, otherwise the detectors will never fire. At this point it should be reminded that Bell inequalities are usually considered in a situation where the source emits a single pair of entangled particles at a time. Nevertheless, the violation of Bell inequalities can also be studied in the multipair scenario [12–14]; of particular interest are experimental situations where single entangled pairs cannot be individually created or measured, for instance, in many-body systems [14]. Nevertheless, such studies require a careful analysis, in particular, when postselection is performed, as we shall see in Sec. V.

To gain some intuition, let us start with the simplest situation $M=N$: the source emits exactly the threshold number of pairs. In this case a detector clicks whenever all photons take the same output of the polarizing beam splitter. Thus the probabilities entering the CH inequality are simply given by

$$P_{+}(A_i) = p_{+}(A_i)^N, \quad P_{++}(A_iB_j) = p_{++}(A_iB_j)^N, \qquad (2)$$

where $p_{++}(A_iB_j) = \mathrm{tr}([A_i^{+} \otimes B_j^{+}]\rho)$ is the quantum joint probability for a single pair to give a click in the "+" detector on Alice's and on Bob's side, and similarly for the marginal probability $p_{+}(A_i)$. It should be stressed that, though the detectors are supposed to be perfectly efficient, there are many inconclusive events "$\varnothing$" (not giving any click in the + detector or in the − detector), because of the threshold. Since we consider only the outcome + in the CH inequality, one may relabel the outcomes in the following way: $+ \to +$ and $-, \varnothing \to 0$. Then, the experiment still provides binary outcomes, + or 0, but no events have been discarded.

Inserting probabilities (2) into the CH inequality, we compute numerically the maximal amount of violation for pure entangled states of two qubits $|\psi\rangle = \cos\theta|00\rangle + \sin\theta|11\rangle$. The optimization is performed over the four measurement settings. The results are presented in Fig. 2(a), for different values of the threshold N. Surprisingly, the inequality can be violated for any N; this can be shown analytically for the maximally entangled state (see [15]). For large values of N, this is quite astonishing, since the probabilities (2) are very small; most events do not lead to a click in the + detector. Let us stress that no particular assumptions (such as fair-sampling) are required here, since no events have been discarded. Another astonishing feature is that, for increasing values of N, the state that achieves the largest violation is less and less entangled. Note that in general, the relation between entanglement and nonlocality is not well understood, but hints suggest that a partially entangled state contains more nonlocality than maximally entangled ones [16–18].

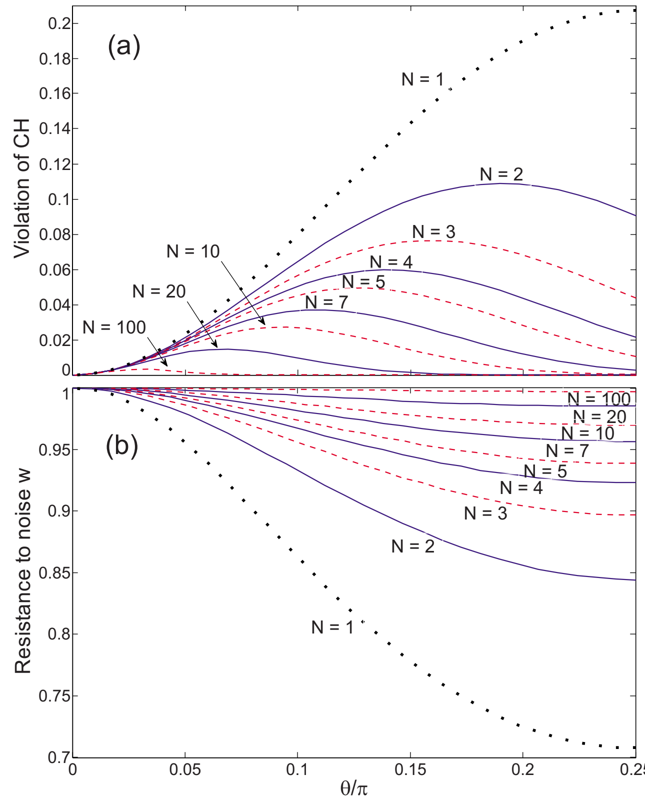

FIG. 2. (Color online) Violation of the CH inequality (a) and resistance to noise w (b) vs the degree of entanglement of the state, for different threshold values N. Remarkably the inequality is violated for any value of the threshold N.

Next we compute the resistance to noise, defined as the maximal amount $(1-w)$ of white noise that can be added to the state $|\psi\rangle$ such that the global state $\rho = w|\psi\rangle\langle\psi| + (1-w)\frac{1}{4}$ still violates the Bell inequality. The optimization is performed as above. We find that the more entangled the state is, the more robust it is, though for $N \geqslant 2$ the maximal violation is not obtained for the maximally entangled state (see Fig. 2). So the close relation that exists, in the standard case $N=1$, between the amount of violation and the resistance to noise, does not hold here anymore [19]. Indeed, in the perspective of experiments, the resistance to noise is the relevant figure of merit.

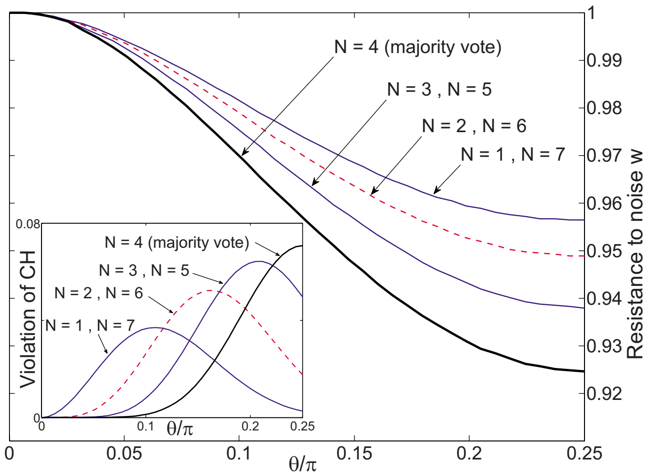

FIG. 3. (Color online) Violation of the CH inequality (inset) and the resistance to noise w (figure) vs the degree of entanglement of the state, for different thresholds N. The source emits a fixed number of pairs $M=7$. The optimal threshold, $N=4$, corresponds to a majority vote (see the text).

Now let us consider the case where the source emits $M \geqslant N$ entangled pairs, and the detector is characterized by a response function $\Theta(x)$, where x is the number of incident photons. The probabilities (2) now read

$$P_+^{(M)} = \sum_{n_+ + n_- = M} \Theta(n_+) M! \frac{p_+^{n_+} p_-^{n_-}}{n_+!\, n_-!},$$

$$P_{++}^{(M)} = \sum_{\Sigma n_{\alpha\beta}=M} \Theta(n_+^A)\Theta(n_+^B) M! \prod_{\alpha,\beta=\pm} \left(\frac{p_{\alpha\beta}^{n_{\alpha\beta}}}{n_{\alpha\beta}!} \right) \tag{3}$$

where the indices $n_{\alpha,\beta}$ represent the numbers of pairs that take the outputs α on Alice's side and β on Bob's side, and $n_+^A \equiv n_{++} + n_{+-}$ while $n_+^B \equiv n_{++} + n_{-+}$.

For now, we still consider detectors with a perfect threshold, i.e., $\Theta(x<N)=0$ and $\Theta(x \geqslant N)=1$. Again, the amount of violation of the CH inequality as well as the resistance to noise can be computed numerically. We have performed optimization for $N \leqslant 10$ and found that the CH inequality can still be violated but that the resistance to noise decreases for increasing values of M. In Fig. 3 we present the results in a slightly different way: for a fixed number of emitted pairs ($M=7$), we compute the violation of the CH inequality and the resistance to noise for different thresholds N. The optimal threshold is found to be $N=\lfloor \frac{M+1}{2} \rfloor$. Note that if we had photon counting detectors, then this threshold would simply correspond to a majority vote [14]: if $n_+ \geqslant n_-$, then the result is $+$, otherwise it is $-$. It should also be pointed out that detectors with threshold N and $M-N+1$ are equivalent, which can be seen by inverting the outputs $+$ and $-$ [20].

Next we consider a Poissonian source. The probability of emitting M pairs is $p_M = e^{-\mu} \frac{\mu^M}{M!}$, where μ is the mean number of emitted pairs. Again we compute the probabilities entering the CH inequality as follows:

$$P_+^{(\mu)} = \sum_M p_M P_+^{(M)},$$

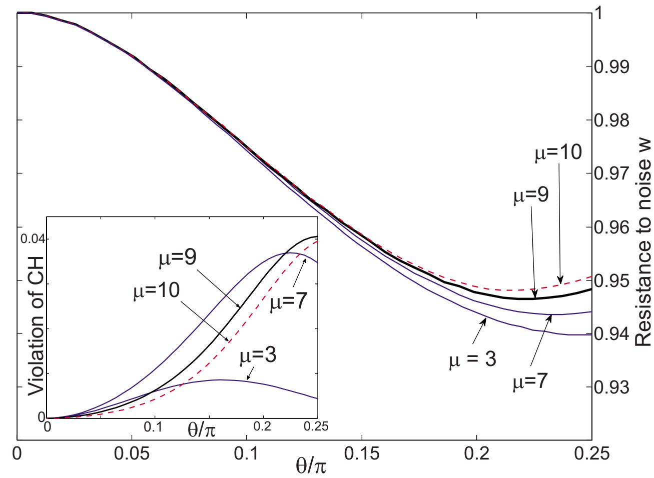

FIG. 4. (Color online) Violation of the CH inequality (inset) and resistance to noise w (figure) vs the degree of entanglement of the state, for a Poissonian source. The threshold is fixed to $N=5$, while the mean number of emitted pairs μ is varied.

$$P_{++}^{(\mu)} = \sum_M p_M P_{++}^{(M)}, \tag{4}$$

with $P_+^{(M)}$ and $P_{++}^{(M)}$ defined in Eqs. (3). Numerical optimizations show that the CH inequality can be violated. Figure 4 shows the results for a detector with a perfect threshold at $N=5$. The largest violation is obtained for $\mu \approx 9.05 \approx 2N-1$, so basically when the threshold corresponds to a majority vote on the mean number of pairs, $N \approx \frac{\mu+1}{2}$. The resistance to noise has a very different dependance on μ (see Fig. 4). Smaller values of μ are more robust against noise. Intuitively this can be understood as follows. The term with $M=N$ pairs is the most robust against noise, as discussed previously. For small values of μ, more weight is given to this term (compared to terms with more pairs), thus leading to a stronger resistance to noise.

IV. SMOOTH THRESHOLD

We just showed that a threshold is not a limiting factor for demonstrating quantum nonlocality, and consequently, entanglement. However, the response function of real biological detectors, such as the human eye, is not a perfect threshold but a smooth curve. Typically, for a number of photons near the threshold, the efficiency is low; for instance, ~20% for 60 incoming photons (see [1] for details), here the threshold being the minimum number of photons that can be detected with a strictly positive probability. Let us also stress that the efficiency of the human eye does strongly depend on the number of incoming photons. Therefore the probability of seeing cannot be characterized by a single parameter (for instance, the efficiency for a single photon) as is the case for linear detectors. One must consider the eye's (S-shaped) response function.

We have checked that, for smooth thresholds, the demonstration of quantum nonlocality in the strict sense is compromised, except if the response function is close to a step function. Therefore, in the case of a response function with a smooth threshold, for instance, in the case of the human eye, postselection must be performed.

V. POSTSELECTION

We postselect only the events leading to a conclusive result on both sides, i.e., when one detector on Alice's side and one detector on Bob's side fire. In this case probabilities must be renormalized such that $\bar{P}(\alpha\beta|ij)=P(\alpha\beta|A_iB_j)/\Sigma_{\alpha,\beta=\pm}P(\alpha\beta|A_iB_j)$. Since we postselect coincidences, it is now more convenient to express the CHSH inequality in its standard form (in which only correlation terms appear)

$$S \equiv |E(A_1,B_1)+E(A_1,B_2)+E(A_2,B_1)-E(A_2,B_2)| \leq L, \tag{5}$$

where $E(A_i,B_j)=\Sigma_{\alpha,\beta=\pm}\alpha\beta\bar{P}(\alpha\beta|A_iB_j)$, and L is the local bound (for a single pair $L=2$). Let us stress already that because of the postselection, the local bound L for multipairs will be modified (see below).

The detector now has a smooth threshold at N photons: below the threshold the efficiency is zero $\Theta(x<N)=0$; at the threshold the efficiency is limited to $0<\Theta(x=N)<1$, and the efficiency above the threshold is, for now, arbitrary. We start again with the case where the source sends exactly N pairs. The source is supposed to send multiple copies of the same state ρ. For the singlet state ($\rho=|\psi^-\rangle\langle\psi^-|$), one has that $p_{\psi_-}(\alpha\beta|\vec{a}\vec{b})=(1-\alpha\beta\vec{a}\cdot\vec{b})/4$; here measurement settings are written as vectors on the Bloch sphere. This leads to

$$E^{(N)}(\vec{a},\vec{b})=\frac{(1-\vec{a}\cdot\vec{b})^N-(1+\vec{a}\cdot\vec{b})^N}{(1-\vec{a}\cdot\vec{b})^N+(1+\vec{a}\cdot\vec{b})^N}. \tag{6}$$

These correlations are stronger than those of quantum physics for a single pair. This is a consequence of the postselection we performed. Note also, that our postselection depends (in general) on the measurement settings, therefore the local bound L of the CHSH inequality must be modified accordingly. Inserting the correlators (6) into the CHSH inequality, one gets an expression that is maximized by the usual optimal settings, i.e., $A_1=\sigma_z$, $A_2=\sigma_x$, $B_1=(\sigma_z+\sigma_x)/\sqrt{2}$, and $B_2=(\sigma_z-\sigma_x)/\sqrt{2}$. In this case one gets

$$S_{\psi_-}^{(N)}=4\frac{(1+1/\sqrt{2})^N-(1-1/\sqrt{2})^N}{(1+1/\sqrt{2})^N+(1-1/\sqrt{2})^N}. \tag{7}$$

For $N\geq 2$, $S_{\psi_-}^{(N)}$ exceeds the Tsirelson bound $(2\sqrt{2})$ [21]: For example, $S_{\psi_-}^{(2)}=8\sqrt{2}/3\approx 3.77$. In fact Eq. (7) tends to the algebraic limit of the CHSH inequality, $\lim_{N\to\infty}S_{\psi_-}^{(N)}=4$.

Thus we find that the violation of the CHSH inequality for the singlet state increases with the threshold N. However, in order to conclude for the presence of entanglement one still has to find the bound for separable states. For $N=1$ this bound is indeed equal to the local limit of the inequality ($L=2$). In the case $N\geq 2$, the local bound will be increased because of the postselection, as intuition suggests. Next we compute this local bound, which is indeed also a bound for any separable state of the form $\rho^{\otimes N}$.

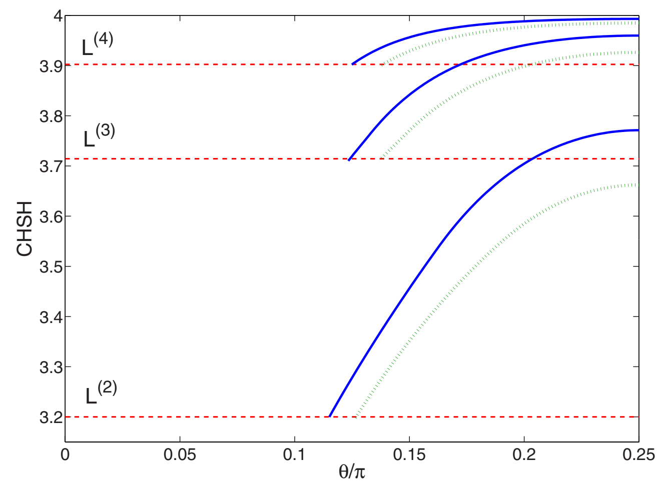

FIG. 5. (Color online) Violation of the CHSH inequality for different thresholds N. The local bound is a function of N (see the text). Violations are obtained for a source emitting exactly N pairs (solid blue lines) as well as for a Poissonian source with $\mu=0.1$ (dotted green lines). Note also that the set of pure entangled states that violate the CHSH inequality becomes smaller for increasing N. The settings are optimized for all states.

We proceed as follows. We perform a numerical optimization over any local probability distribution for two binary settings on each side. This probability distribution is of the form of N copies of a (two-input–two-output) local probability distribution, because of our hypothesis. The largest value of $S^{(N)}$ is obtained for the following probability distribution [22]:

$$p(\alpha=\beta|ij)=3/8, \quad p(\alpha\neq\beta|ij)=1/8, \quad \text{if } i=1 \text{ or } j=1,$$

$$p(\alpha=\beta|ij)=1/8, \quad p(\alpha\neq\beta|ij)=3/8, \quad \text{if } i=j=2, \tag{8}$$

leading to the local bound

$$L^{(N)}=4\left[\frac{3^N-1}{3^N+1}\right]. \tag{9}$$

One can check that $S_{\psi_-}^{(N)}>L^{(N)}$ for any N (see Fig. 5).

Remarkably, probability distribution (8) is obtained quantum mechanically by performing the optimal measurements (mentioned above) on the Werner state [23] [$\rho_w=w|\psi_-\rangle\langle\psi_-|+(1-w)\mathbb{1}/4$] for $w=\frac{1}{\sqrt{2}}$, i.e., when ρ_w ceases to violate the CHSH inequality. Thus, the resistance to noise for the singlet state is independent of N. It would be interesting to see if a strictly lower bound exists for separable states.

Note, however, that the bound (9) is valid only under the assumption that the source sends multiple copies of the same state ρ. In case this assumption breaks, the local bound reaches the algebraic limit of CHSH inequality ($L=4$), thus removing any hope of demonstrating entanglement. More precisely, there is a local model giving $L=4$ for all $N\geq 2$. Whether this bound can be reached by a separable two-qubit state is unclear. We stress that this was not the case for perfect thresholds; there no assumption had to be made on the source.

Curiously, no violation is obtained when the source sends a fixed number of pairs larger than the threshold ($M > N$) [24]. However, for a Poissonian source, the CHSH inequality can be violated for small values of μ, the mean number of emitted pairs (see Fig. 5). Intuitively, if $\mu \ll N$, the term with N pairs is dominant. When $\mu \to 0$, the curve $M = N$ is recovered. Note that for a Poissonian source, the local bound must be defined carefully, since the number of emitted pairs varies. However, when $\mu \ll N$ it is reasonable to consider the local bound $L^{(N)}$.

VI. CONCLUSION

Amazed by the performances of the human eye, which can detect a few photons, we investigated whether biological detectors might replace man-made detectors in Bell-type experiments. We showed that the main characteristic of these detectors, namely, a detection threshold, is not a restriction for violating Bell inequalities. In particular, we showed that close to perfect threshold detectors can be used to test quantum nonlocality without the need of any supplementary assumption, such as fair sampling. For detectors with a smoother response function, one must perform postselection, but Bell inequalities can still be violated, thus highlighting the presence of entanglement under reasonable assumptions. These results represent a first encouraging step, since there is apparently no fundamental restriction to detect entanglement with threshold detectors. Nevertheless, the next crucial step will be to estimate the feasibility of such an experiment with realistic parameters.

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[20] For a threshold $M - N + 1$, having a click in detector $-$ implies that $n_- \geq M - N + 1$. We suppose that in case both detectors fire, the result $+$ is outputted. Thus, to get output $-$, one must add the constraint that $n_+ < M - N + 1$, which implies that $n_- \geq N$, since $n_+ + n_- = M$. So models with thresholds N and $M - N + 1$ are made equivalent by inverting the outputs $+$ and $-$. This also shows that the double click events play no role. If in case of a double click, the outcome $+$ is given, then considering the output $-$ provides a model without the double click.
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