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Backaction noise produced via cavity-aided nondemolition measurement of an atomic clock state

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We use a quantum nondemolition measurement to probe the collective pseudospin of an atomic ensemble in a high-finesse optical cavity. We analyze the backaction antisqueezing produced by the measurement process to show that our protocol could create conditional spin squeezing in the atomic ensemble.

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The application of recently developed techniques in cavity quantum electrodynamics to many-atom ensembles is a promising means toward the realization of novel experimental systems. The performance of many protocols in quantum information processing [1–3] that have been recently demonstrated in free-space atomic systems is limited by the optical depth of the atomic sample, which can be enhanced by placing atomic samples inside optical cavities, as in [4]. Similarly, cavity enhancement can facilitate the preparation of many-atom correlated states, called squeezed states in analogy to squeezed light, which can be used to surpass the usual projection noise limit for atom-based sensors [6,5].

Atomic spin-squeezed states have previously been produced by quantum-state transfer from squeezed light [7] or by entangling two-photon transitions [8,9], but much recent work has focused on generating spin squeezing via quantum nondemolition (QND) measurement [10–13]. A phase-shift measurement protocol has been used to entangle the collective spin states of two atomic ensembles [14], and a QND scheme has been used to measure the state of a superconducting charge qubit coupled to a microwave resonator and to characterize the resulting measurement backaction [15]. A QND measurement using Rydberg atoms to number-squeeze an intracavity photon state has also been demonstrated [16]. Recently, the heating effects of quantum-measurement backaction on an ultracold atomic gas have been studied [17].

In this article, we report the implementation of a cavity-aided continuous QND measurement of the pseudospin of a laser-cooled ^{87}Rb ensemble prepared in a superposition of magnetic-field-insensitive hyperfine clock states. We present a quantitative model for the interaction between the atoms and the probe light and consider the predicted performance. We then describe our experimental implementation, including considerations of inhomogeneous coupling and the dephasing it causes, and provide evidence, based on measurements of the backaction-induced antisqueezing, that our protocol implements our model.

An ensemble of N two-level atoms coupled to a radiation field can be described, in analogy with a spin-$\frac{1}{2}$ system, by a collective pseudospin Bloch vector of length $J=N/2$, with J_z corresponding to the population difference between the two states [5]. An off-resonant probe laser passing through such an ensemble acquires a phase shift due to the atomic index of refraction, which depends on J_z, without affecting J_z, as long as spontaneous emission remains negligible. A subsequent observation of the probe's phase projects J_z onto the value that corresponds to the observed phase shift. The fractional uncertainty in the probe phase is limited by photon shot noise, which thus sets the limit on the uncertainty in J_z, which can be far lower than the projection noise limit for an uncorrelated state. In a cavity, the squeezing factor, which measures the reduction in the projection variance, is enhanced by a factor proportional to the square root of the cavity finesse [18], allowing much stronger squeezing than possible in a free-space configuration.

Quantitatively, assuming identical coupling to the cavity for every atom and ignoring the spontaneous scattering of probe photons into free space and the loss of photons through the cavity mirrors, the Hamiltonian for the interaction of intracavity probe light with the atomic state is given by (see, e.g., [19])

$$H=\hbar n g^2\left[\frac{N}{2}\left(\frac{1}{\Delta_2}+\frac{1}{\Delta_1}\right)+J_z\left(\frac{1}{\Delta_2}-\frac{1}{\Delta_1}\right)\right], \tag{1}$$

where g is the atom-cavity coupling constant, $n=a^\dagger a$ and $N=J_{11}+J_{22}$ are the photon and atom number operators, $J_z=\frac{1}{2}(J_{22}-J_{11})$ is the collective spin projection operator, and $\Delta_{1,2}$ are the detunings from the lower and upper clock states, respectively. Physically, the atoms experience an ac Stark shift due to the presence of intracavity light and the light field experiences a phase shift due to the atomic index of refraction. The term proportional to N only adds an overall phase shift to the light field, so only the term proportional to J_z is relevant for the clock performance analysis.

In our model, the initial state of the system corresponds to a coherent atomic state $|J,J_x\rangle$ and a coherent light state for the intracavity probe field. The interaction leads to the imprinting on the probe field of a phase shift $\Delta\phi=J_z\Omega t$ for $\Omega=g^2(1/\Delta_2-1/\Delta_1)$, where t is the interaction time, while the collective atomic pseudospin vector precesses in the equatorial plane of the Bloch sphere at a rate proportional to n. Since the probe beam's state is a superposition of number eigenstates (Fock states), each of which causes the atomic pseudospin vector to precess at a different rate, the uncertainty in the atomic pseudospin vector's in-plane component (J_y in the frame that rotates with the atomic state, so it remains polarized along x) grows as it precesses. This growth in ΔJ_y corresponds to the quantum backaction of the measurement of its conjugate variable J_z.

Since the Hamiltonian entangles the collective atomic pseudospin J_z with the phase of the probe field, a measurement of the probe's phase projects the atomic state onto a stochastically determined state of J_z. For $n\Omega^2t^2 \ll 1$, the resulting uncertainty in J_z, conditioned on the outcome of the phase measurement, is given by $(\Delta J_z)^2 = (N/4)(1+Nn\Omega^2t^2/2)^{-1}$, compared to the usual projection noise of $(\Delta J_z)^2 = (N/4)$.

For interaction times greater than the cavity photon lifetime, the above picture is modified by the leakage of photons out of the cavity. We treat this in a simplified way, by breaking up the interaction time into intervals corresponding to the photon lifetime τ_{cav}, where, after each interval, the probe state is measured, and the resulting atomic state then interacts with a new coherent photon state. The repeated projection out of the intracavity photon state destroys the atom-light coherence between interaction intervals, causing the pseudospin variances to evolve linearly in time. The squeezing for short times, until the antisqueezed uncertainty begins to wrap around the Bloch sphere, is then given by $(\Delta J_z)^2 = (N/4)(1+N\bar{n}\Omega^2 t\tau_{cav}/\sqrt{2})^{-1}$, where $\bar{n}$ now denotes the time-averaged intracavity photon number, which depends not only on the input power, but also on t and τ_{cav} (in contrast to [20], where all the time dependence is included explicitly), and the corresponding antisqueezing is $(\Delta J_y)^2 = (N/4)(1+N\bar{n}\Omega^2 t\tau_{cav}/\sqrt{2})$.

Atomic spontaneous emission into free space reduces the correlation between the measured probe phase (to which the atoms that have undergone spontaneous emission contribute) and the collective pseudospin of the atoms used for the clock (to which they do not contribute). If the spontaneous emission rate is known, the average phase shift due to the atoms that have scattered can be subtracted out, but the stochastic fluctuations in that phase shift, given by the shot noise on the number of spontaneous emissions, cannot be accounted for and will degrade the conditional squeezing [21].

Our experiment uses a hemispherical optical cavity, previously described in [22], with length $L=10$ cm, finesse $F = 205\,000$, free spectral range (FSR) of 1.505 GHz, half width at half maximum (HWHM) linewidth $\kappa/(2\pi) = 3.7$ kHz, giving $\tau_{cav}=21.5$ μs, and TEM_{00} mode size of 310 μm at the atomic position, corresponding to a maximum atom-cavity coupling $g/(2\pi)=53$ kHz for the $|F=2,m_F=2\rangle \rightarrow |F'=3,m_F=3\rangle$ cycling transition, to probe the collective pseudospin state of a cloud of ^{87}Rb atoms cooled in a magneto-optical trap (MOT) located at the center of the cavity. The atoms are released from the MOT, further cooled by optical molasses, prepared in an equal superposition of the $|F=1,m_F=0\rangle$ and $|F=2,m_F=0\rangle$ clock states, and probed by a standing wave of intracavity light.

The modulation scheme we use to couple laser light into the cavity and perform our measurements is a modified version of the one described in [23], and is shown in Fig. 1. We use a far-detuned resonant sideband ("locking beam") to stabilize the laser to the cavity resonance via a Pound-Drever-Hall lock. The collective atomic pseudospin is measured via the phase shift produced by the atomic index of refraction on a near-detuned resonant sideband ("probe"). The probe is tuned 1.5 GHz to the red of the $5^2S_{1/2}$, $F=2 \rightarrow 5^2P_{3/2}$, $F' = 3$ atomic transition with a cavity input power of 1.2–2.5 nW, while the locking beam, 18.06 GHz farther to the red, has an input power of 2.5 nW.

When we turn on the probe, the atom cloud has a $1/e$ radius of 390 μm and is falling through the cavity mode at 2.9 cm/s while expanding at 3.7 cm/s due to its temperature of 14.5 μK. While the atoms' passage through the cavity limits the interrogation time to several ms, a more stringent limit is imposed by inhomogeneous broadening due to the presence of intracavity light. The intracavity standing waves impose an ac Stark shift, which varies depending on atom position within the cavity mode, with an average of $U/k_B \sim 1$ μK (0.2 μK) for a 2.5-nW probe beam and ~ 0.08 μK (0.06 μK) for a 2.5-nW locking beam, for atoms in the $|F=2,m_F=0\rangle$ ($|F=1,m_F=0\rangle$) state. For the locking light, which is always on, the $1/e$ Rabi oscillation decay time for atoms in the cavity is around 1 ms. For atoms prepared in an equal superposition of the two clock states, which is most sensitive to inhomogeneous broadening, the probe light dephases the collective spin vector of the sample in several tens of μs.

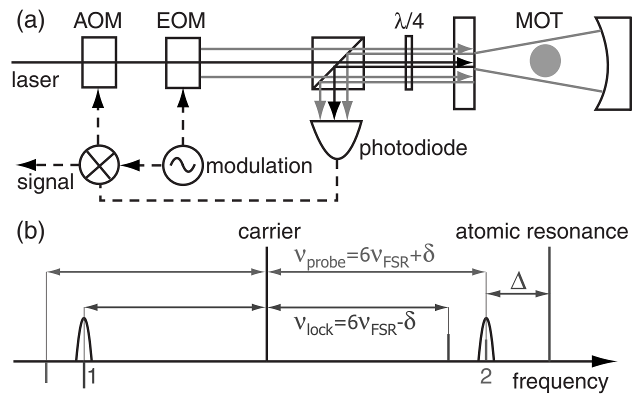

FIG. 1. Schematic of experiment (a) and modulation method (b). Two sidebands resonant with the cavity are modulated onto a nonresonant laser by an electro-optical modulator (EOM). A photodiode monitors the beat notes between the carrier and the sidebands reflected from the cavity. The demodulated beat note between the carrier and the locking sideband (1) is used to drive a double-pass acousto-optical modulator (AOM) that keeps the locking sideband resonant with the cavity, while the beat note between the carrier and the probe sideband (2) provides the experimental measurement of the probe phase shift.

There are two distinct time scales for light-induced inhomogeneous broadening, corresponding to the longitudinal and transverse motions of the atoms in the cavity mode. The atoms are unconfined by the light, and a typical atom's thermal velocity causes it to travel the 390-nm distance between adjacent nodes in the longitudinal standing wave in 10 μs, so measurements over time scales longer than this should average over the longitudinal inhomogeneities in the ac Stark shift. In the transverse direction, however, the ac Stark shift varies by just a few percent in 100 μs of atomic motion, so its effects can be countered on that timescale by using spin echo.

The correlated atomic state is generated and measured in a three-pulse sequence illustrated in Figs. 2(a)–2(c). The probe light is turned on for a time τ_{sq}, then turned off for a time τ_{off} required for the probe light to leak out form the cavity, after which a microwave π pulse that lasts for a time τ_π is used to prepare the spin echo. The probe light is turned on for τ_{sq}, which rephases the atomic spins, then once again

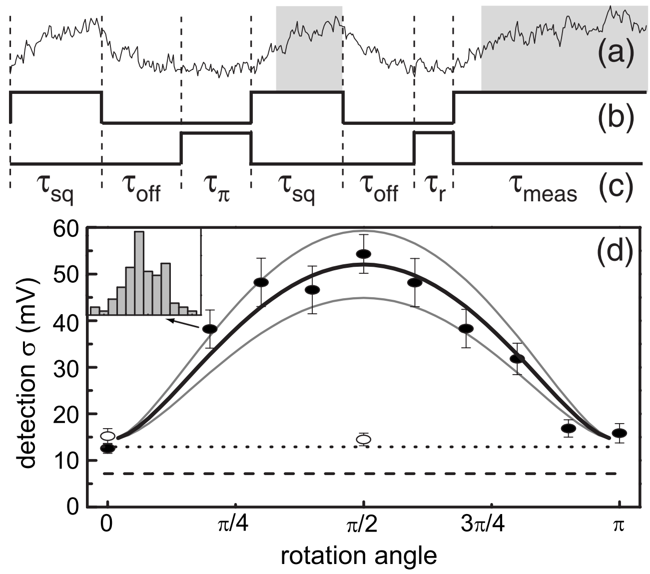

FIG. 2. Measurement protocol and results. The traces show a typical probe signal (a) along with the control sequences for the probe laser (b) and the microwave state rotation (c). The results obtained for the fluctuations in the difference between the means of the two shaded regions in (a), chosen to exclude the initial cavity buildup, as we vary the duration τ_r of the final microwave pulse are shown as solid circles in (d), with each data point corresponding to 58–174 shots, with statistical error bars. The inset in (d) shows a histogram of the difference of means for 87 shots at a final rotation of 0.63 rad, the width of which corresponds to the indicated data point. The solid curve is a theory calculation using our model; the gray curves account for uncertainties in the experimental parameters, dominated by uncertainty in intracavity probe power; the dashed line is the expected projection noise; and the dotted line is the measured noise floor in the absence of atoms (detector dark noise is negligible). The open circles indicate the results obtained for inhomogeneous broadening in the absence of spin echo (see text). The parameters for this measurement are $\tau_{sq}=\tau_{off}=60$ μs, $\tau_{\pi}=50$ μs, $N\approx 57000$, probe power $\approx$2.5 nW, and locking power $\approx$2.5 nW.

extinguished for τ_{off}, and after a possible final microwave rotation, which takes a time τ_r, the probe light is turned on permanently and the atomic state is measured for a time τ_{meas}. During the two preparation pulses, spontaneous emission for all data presented here, measured by observing atom depumping over time, is $\leqslant 6\%$; for the final, destructive detection pulse, spontaneous emission is 20%–40%. We use shot-to-shot fluctuations in the difference in the cavity shift between the second squeezing pulse and the final measurement to quantify the conditional projection noise for our protocol.

To properly calculate the expected projection noise, we must account for spatially varying atom-cavity coupling. For a Gaussian atom cloud with radius r_a and a TEM_{00} standing-wave cavity mode with spot size r_c, the mean atom-cavity coupling $\bar{\Omega}$ is less than the maximal on-axis, antinode value Ω_{max} by a factor of $2[(r_a/r_c)^2+1]$. The variable that the probe phase measurement couples to is then not ΩJ_z, but $\bar{\Omega} J_z$, with an uncorrelated projection noise variance given by the sum of the individual atom variances. Since the inhomogeneous transverse coupling of different atoms does not average out on the time scale of our measurement, this modified collective variance scales not as the square of the mean coupling, but rather as the mean of the squared coupling: $(\Delta\bar{\Omega}J_z)^2=\Sigma_{i=1}^{N}(\Delta\Omega_i j_{zi})^2=(N/16)\Omega_{max}^2[2(r_a/r_c)^2+1]^{-1}$, where i is an index over individual atoms [the unmodified value is $(\Delta\Omega J_z)^2=(N/4)\Omega_{max}^2$]. To take advantage of the squeezing produced by our protocol, subsequent measurements also need to couple to $\bar{\Omega}J_z$, which suggests that interferometer readout should be performed using the cavity shift [24].

By applying a microwave pulse before the final detection pulse, it is possible to rotate the uncertainty ellipse around its center and use the atomic shift to measure its width in an arbitrary direction. A $\pi/2$ pulse rotates the antisqueezed (J_y) component of the collective atomic spin onto a population difference (J_z), which results in maximal noise on the shift, while a smaller rotation produces a correspondingly smaller effect. The results of such a series of measurements are shown in Fig. 2(d).

To characterize the coherence of our atomic states, we vary the phase of the final microwave pulse while keeping its duration fixed at $\tau_{\pi/2}$ by applying a phase offset to the microwave oscillator that generated the pulse. We thus scan the collective pseudospin rotation axis in the equatorial plane of the Bloch sphere, which produces an oscillation in the atomic populations, which we read out via the mean values of the final probe measurement. For the final state produced by our measurement, the contrast in this oscillation is about 73% of the full contrast obtained under the same circumstances in the absence of probe squeezing pulses, which means that the length of the collective spin vector on the Bloch sphere is reduced to 73% of its initial length due to the relative dephasing of the individual spins by the squeezing pulses and the projection noise is reduced by the same amount. We have also confirmed that the lock light has no effect on atomic coherence by comparing the contrast measured after the spin echo sequence without squeezing pulses to the initial contrast measured immediately after the first $\pi/2$ pulse creates the clock-state superposition.

It is important to distinguish between inhomogeneous broadening due to the spatially varying probe light intensity, which leads to dephasing between the pseudospin states of different atoms, and the dephasing of the collective pseudospin—i.e., antisqueezing. To study the two effects independently, we use the fact that spin echo counteracts the majority of the inhomogeneous dephasing, making it possible to produce, without spin echo, the same amount of inhomogeneous dephasing contrast reduction with a much shorter probe pulse (and, consequently, much less antisqueezing). We find that a 20-μs probe pulse without spin echo produces the same amount of inhomogeneous dephasing (as measured by microwave oscillation contrast) as two sequential 60-μs probe pulses with spin echo. However, the 20-μs pulse does not produce a measurable increase in noise [see open circles in Fig. 2(d)].

We measured the antisqueezing as a function of atom number for two different probe intensities. The results, along with theoretical calculations are shown in Fig. 3. The ob-

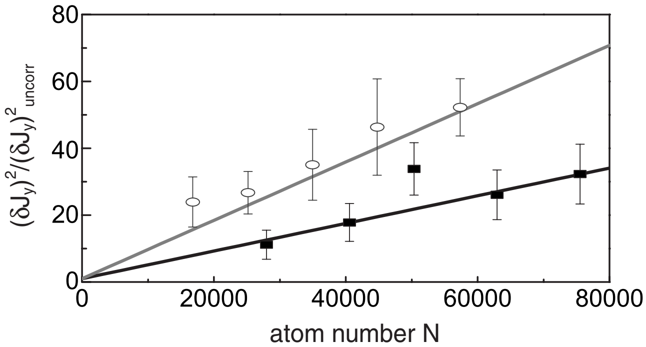

FIG. 3. Antisqueezing of the variance of the in-plane component of the atomic pseudospin (conjugate of J_z) as a function of atom number and probe intensity, obtained by rotating the spin state after the two squeezing pulses (see Fig. 1) by $\pi/2$ and measuring the probe phase shift. The black squares and white circles correspond to probe input powers of $\approx$1.2 nW and $\approx$2.5 nW, respectively, with statistical error bars. The lines correspond to theory calculations using our model; there is an overall scaling uncertainty of $\pm$30% from probe power measurements. Fitting the slopes for the two data sets as free parameters gives $9.7(4)\times 10^{-4}$ and $4.5(5)\times 10^{-4}$, with a ratio of 2.2(3), in good agreement with the ratio of the input powers (2.1).

served linear scaling with atom number N and with probe intensity, and therefore average photon number $\bar{n}$, confirms the validity of our measurement protocol. Variance due to classical intracavity intensity fluctuations would scale quadratically with $\bar{n}$ (we independently measure these fluctuations to be below 0.08% between the two squeezing pulses).

While we are not able to observe squeezing for 60-μs squeezing pulses, using the same protocol with longer pulses, which destroy Rabi oscillation coherence, allows us to resolve J_z to 3.8 dB in the variance below the projection noise for an uncorrelated state with spontaneous emission loss of $\leqslant$30%. Technical measurement noise and residual variations in atom-cavity coupling limit our ability to observe squeezing while preserving coherence. If the probe could be measured at the photon shot-noise limit for the second squeezing pulse and taking into account the atomic loss dictated only by the spontaneous emission and not by inhomogeneous broadening, the antisqueezing shown in Fig. 3 would, for a minimum-uncertainty state, correspond to spin squeezing of up to 10 dB in the variance. Significantly better performance could be achieved by increasing the atom number (a typical ^{87}Rb MOT with 10^8 atoms should give us measurement samples of 10^7 atoms, a hundredfold increase) and by improving the spatial overlap between the cavity mode and the atom cloud, by either confining or cooling the atoms.

Our two-squeezing-pulse scheme has an implicit atom number measurement (required for a practical clock or interferometer), since J_z before the π pulse becomes $-J_z$ after, so averaging the measurements from the two pulses gives a phase shift that depends only on the total atom number. Combined with experimental improvements that allow direct measurement of squeezing, our protocol should significantly reduce the projection noise for atomic metrology.

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