Enhancement of on-site interactions of tunneling ultracold atoms in optical potentials using radio-frequency dressing

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We show how it is possible to more than double the on-site interaction energy of neutral atoms in optical potentials by the technique of radio-frequency (rf) dressing, while maintaining interwell dynamics. We calculate Bose-Hubbard parameters for rf-dressed optical lattices and arrays of rf-dressed dipole traps. We show that decreasing the distance between wells, by the interpolation of wells confining different m_F states, increases the interaction energy more than decreasing the height of the classically forbidden region between existing wells. The schemes we propose have negligible Landau-Zener losses caused by atomic motion; this was a dominant effect in the first experimental demonstration of the modification of an optical potential by radio-frequency dressing.

The study of complex nonlinear quantum systems is a major area of research in the fields of atomic and condensed-matter physics. A subject of current interest is the effect of the nonlinear interaction term on the behavior of ultracold atoms confined in optical lattices. The dynamics of ultracold bosonic atoms in such a system has been shown to be described by the Bose-Hubbard Hamiltonian [1]

$$H = -J\sum_{\langle i,j \rangle} a_i^\dagger a_j + (U/2)\sum_i \hat{n}_i(\hat{n}_i - 1), \qquad (1)$$

where i and j denote lattice sites of a homogeneous lattice. The ground state of the Bose-Hubbard Hamiltonian passes from superfluid to Mott insulator as the parameters controlling U and J are varied; this behavior has been demonstrated experimentally [2].

The on-site interaction energy, the Hubbard U, is the dominant parameter characterizing interactions between ultracold atoms in optical lattices. As such, it plays a pivotal role in phase transitions [2] and entanglement [3]. The ability to generate complex entangled states has drawn interest to these systems for the purposes of quantum computing [4] and quantum simulation [5]. Typically, the magnitude of U controls the purity of the resulting many-particle state or the speed at which this entanglement may be generated [6]. It is likely that there are a significant number of "impurity" atoms present in the Mott-insulator states currently being made [3,7]; for applications such as quantum computing and simulation, it is desirable to reduce the number of these impurities as much as possible. Furthermore, by increasing the on-site interaction relative to the tunneling energy, it may be possible to push optical lattice systems into new regimes—for example, where the on-site interaction energy is greater than the band gap [8].

In this paper we study ways to increase the on-site interaction energy U, while still maintaining slow interwell dynamics, using the technique of radio-frequency (rf) dressing of optical potentials. We study both optical lattices and arrays of highly focused laser dipole spots, both seen as prom-

ising systems for quantum information processing [5,9]. We restrict our study to dressing the optical potential along a single direction of the lattice. The atoms in these potentials will have far lower Landau-Zener loss rates than in the recent experiment [10] and low collisional loss rates. For a number of specific experimental configurations we calculate the enhancement factor for U at a particular interwell tunneling rate.

Radio-frequency dressing has been used in a variety of experiments to dress magnetic potentials [11–13]. Dressing of optical potentials has received much less attention, although recently both theoretical [14] and experimental [10] studies have been carried out. A dominant feature of the recent experiment [10] was high loss rates, attributed to nonadiabatic Landau-Zener transitions [14]. We should mention other methods which may be adapted to increase on-site interactions in a lattice include tunable scattering lengths and Feshbach resonances [15,16], and other methods proposed to give sub-half-wavelength structure to optical lattices—for example, Raman processes [17,18].

The phase transition from the superfluid state ceases to be adiabatic when the inverse time scale $1/\tau$ becomes of the same order of magnitude as the frequencies of the lowest-lying excitations, which are $\sim U$ in the limit $U \gg J$; increasing U will decrease the number fluctuations in the final state. Furthermore, when finite-temperature effects are taken into account, it has been shown that increasing U increases the purity of the final state [19], as one would expect from thermodynamic considerations. Both temperature and nonadiabatic defects are exponentially suppressed by increasing U, as discussed below.

We treat the case with the spin-dependent potential only along one direction of the lattice; dressing a spin-dependent potential of more than one dimension tends to lead to structures with extended potential minima [14]. The methods we describe may be used with or without additional tunneling in the perpendicular directions (along spin-independent potentials). Note that a low-dimensional lattice can already have a larger U than a higher-dimensional lattice due to tight confinement being possible along the perpendicular directions.

In this work we confine our analysis to bosons, specifically to ^{87}Rb. We choose to work in the $F=1$ lower hyperfine

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state with three magnetic substates. With B_{rf} and B_0 perpendicular (B_0 being the static magnetic field), there will be equal intensities of σ^+ and σ^- rf dressing fields. We assume a state-independent (π polarized or with large frequency detuning) optical lattice in the y and z directions, meaning that the difference in energy between magnetic substates is independent of y and z. We can then write, for these three substates,

$$H = \begin{pmatrix} V_{+1}(\mathbf{x}) + \delta_{+1} & \Omega_{rf}/2 & 0 \\ \Omega_{rf}/2 & V_0(\mathbf{x}) & \Omega_{rf}/2 \\ 0 & \Omega_{rf}/2 & V_{-1}(\mathbf{x}) + \delta_{-1} \end{pmatrix}, \qquad (2)$$

with $\delta_{\pm 1} = \Delta E_{\pm 1,0}/\hbar \pm \omega_{rf}$. The optical dipole potentials $V_{mF}(\mathbf{r})$ can be shown to be [20]

$$V_{m_F}(\mathbf{r}) = (\pi c^2 \Gamma/2\omega^3)$$
$$\times \left[(1 - Pg_F m_F/\Delta_{D_1}) + (2 + Pg_F m_F/\Delta_{D_2})\right] I(\mathbf{r}),$$
$$(3)$$

where Δ_{D_1} and Δ_{D_2} are the frequency differences from the D lines, $\Gamma = (\Gamma_{D_1} + \Gamma_{D_2})/2$, $\omega = (\omega_{D_1} + 2\omega_{D_2})/3$, and $P = \pm 1, 0$ for optical $\sigma^\pm, \pi$ polarization.

Using a Born-Oppenheimer-type approximation [21], the internal and external degrees of freedom of the atom can be decoupled as long as the kinetic energy of the atoms is much less than the energy spacing of the dressed levels. This is the case for typical Ω_{rf}, although Landau-Zener losses may occur as a result of this condition being weakly violated, as discussed below. Under this approximation, the eigenvalues of the above matrix give the potential energy of the atoms in the corresponding dressed eigenstate. The rf coupling between the magnetic substates gives rise to avoided crossings, as shown in Fig. 1.

We calculate the one-dimensional Wannier functions for the lowest eigenstate of the Hamiltonian [Eq. (2)], corresponding to the local ground state of the atoms. We diagonalize the Hamiltonian of a single potential period and so find the eigenfunctions of the dressed potential by using Bloch's theorem. We find the maximally localized Wannier functions at the lattice sites by recursively rephasing the eigenstates [22] before summing over quasimomentum. From the lowest-band Wannier functions we calculate the Bose-Hubbard parameters

$$U = 2a_s \hbar \sqrt{\omega_{\perp 1} \omega_{\perp 2}} \int |w_0(x)|^4 dx, \qquad (4)$$

$$J = \frac{1}{N_q} \sum_q E_q e^{iqx_r}, \qquad (5)$$

with $\omega_{\perp 1,2}$ the trapping frequencies in the perpendicular directions, q the quasimomentum, E_q the energy of the eigenstate with quasimomentum q, x_r the separation between neighboring lattice sites, and a_s the scattering length in the dressed state (for ^{87}Rb, approximately equal to the undressed scattering length as $a_s \approx a_t$ [23]). The photon scattering rate is calculated from the Wannier functions and details of the dressed potential.
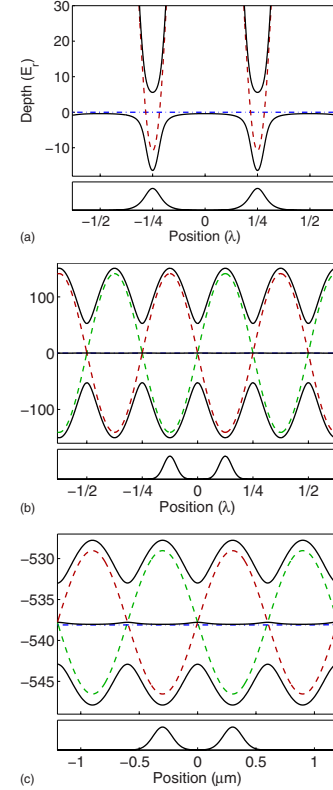

FIG. 1. (Color online) The types of rf-dressed potentials considered in this paper. Dressed potentials are solid black lines; undressed potentials are dashed lines [$m_F = +1$ and -1 (green and red dashed lines), 0 (blue dash-dotted lines)]. The Wannier functions for the lowest state are displayed below the potentials for two of the wells. The $m_F = +1$ state of (a) is at an energy substantially higher than the states shown. The optical lattices of (a) and (b) have σ^+ polarization. The recoil energy is defined as $E_r = h^2/2m\lambda^2$. Parameters: (a) $\lambda = 790.06$ nm, $B_0 = 300$ G, $\nu_{rf} = 213.65$ MHz, $\Omega_{rf}/2\pi = 70$ kHz, $P = 32$ mW, $w_0 = 50$ μm, $\nu_{\perp 1,2} = 30$ kHz; (b) $\lambda = 790.06$ nm, $B_0 = 4$ G, $\nu_{rf} = 3.295$ MHz, $\Omega_{rf}/2\pi = 270$ kHz, $P = 35$ mW, $w_0 = 50$ μm, $\nu_{\perp 1,2} = 30$ kHz; (c) $\lambda = 800$ nm, $B_0 = 4$ G, $\nu_{rf} = 2.78$ MHz, $\Omega_{rf}/2\pi = 25$ kHz, $P = 25$ μW, $w_0 = 1$ μm, $\nu_{\perp 1,2} = 30$ kHz, $\Delta x = 0.6 w_0$.

We compare the on-site interaction energy between dressed and undressed potentials for a certain value of Hubbard J. We choose $J/h = J_{lim}/h = 25$ Hz so that the tunneling time $\tau_{tun} = h/2zJ$ is $\tau_{tun} = 10$ ms (z being the coordination number). We choose this value as this is around the limit where slow interwell dynamics still occurs on a typical experimental time scale; as J is decreased further, the phase transition will go from being an adiabatic to a nonadiabatic process. Interwell dynamics then ceases, with the final purity of the Mott insulator depending on U_{lim}, the value of U when $J = J_{lim}$.

The first potential we consider is given in Fig. 1(a), the "truncated sinusoidal" potential. This is an optical lattice at the ^{87}Rb, $F = 1$, $m_F = 0$ tune-out wavelength of 790.06 nm [24]. The magnetic field magnitude and direction are chosen so that the $m_F = +1$ state is detuned well above the other two states by the nonlinear Zeeman effect. Experimentally plau-
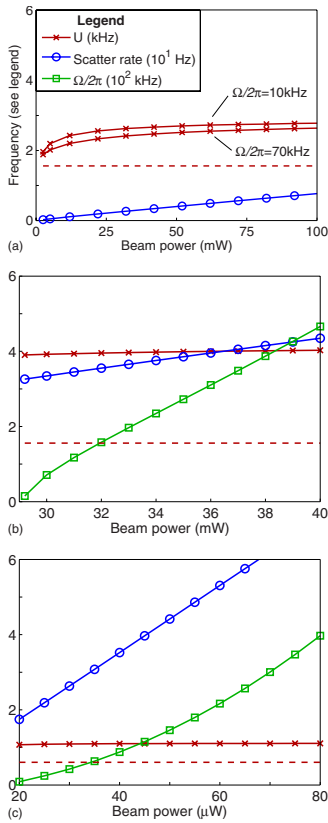
FIG. 2. (Color online) Calculated properties of atoms in the ground state of the dressed potentials. The optical power P is the independent variable; dressing parameters ν_{rf} and Ω_{rf} are chosen subject to the conditions that the neighboring wells are the same depth and that the tunneling parameter J/h is 25 Hz. These conditions specify ν_{rf} and Ω_{rf} for (b) and (c). For (a) the conditions only specify ν_{rf}, so Ω_{rf} can be chosen arbitrarily; two choices are shown. The dashed horizontal line gives the undressed U for comparison, a single value calculated for $J/h=25$ Hz in the undressed σ^+ or σ^- potential. The scattering rates are from the dressed lattice beams only. Aside from P, ν_{rf}, and Ω_{rf}, the parameters of the potentials are as given in Fig. 1.

sible parameters are chosen. The lowest parts of the $m_F=-1$ potential intersect the flat $m_F=0$ potential. Once dressed, the lowest adiabatic potential differs greatly from a sinusoidal shape, with "pockets" of strong confinement separated by an almost flat potential. There is typically only a single bound state at each site. The enhancement of U_{lim}, for the chosen value of J_{lim}, in this potential is given in Fig. 2(a) for $\Omega_{rf}=2\pi \times 70$ kHz and $2\pi \times 10$ kHz; the maximum increases we find are around 70% and 80%, respectively. The enhancement factor is only weakly dependent on Ω_{rf} for a large range of rf power.

The second potential we consider is given in Fig. 1(b), the "interpolated sinusoidal" potential. Optically, it is very similar to the truncated sinusoid, but at a much lower magnetic field of a few gauss, so the $m_F=\pm 1$ states are approximately symmetrical around the $m_F=0$ state. In this case the rf dressing doubles the number of wells in the lattice. The enhancement of U_{lim} in this potential is given in Fig. 2(b); the typical increase is around 150%.

The last dressed potential [Fig. 1(c)] we consider is based on an array of independently addressable dipole traps.[1] We choose neighboring dipole traps to have the same intensity but opposite σ polarization. Due to the rf dressing, the barrier between the potential minima can be considerable even when neighboring Gaussian spots are optically unresolvable. The enhancement of U_{lim} is shown in Fig. 2(c); the maximum enhancement due to the dressing is around 80%. Note that, for potentials (b) and (c), the atomic spin adiabatically flips when a single atom tunnels from one site to its neighbor, enabling sublattice addressability and readout.

The results of these calculations show that the dressed U_{lim} is largely independent of the optical power, but is dependent on the dressing scheme. The results for the truncated sinusoidal potential show that radically altering the shape of the periodic potential, to confine more tightly while still allowing tunneling, but without changing the distance between wells, can only increase U_{lim} by a modest amount. However, U_{lim} becomes substantially larger if atoms are confined in the interpolated sinusoidal potential, with neighboring wells separated by $\lambda/4$. The interpolation method would therefore seem the most promising way to increase U_{lim} by rf dressing an optical lattice.

The results show that the on-site interaction energy of atoms confined in a focused Gaussian array can also be increased by the interpolation technique. The typical values of U_{lim} in this case are not greatly less, for our chosen parameters, than for atoms in an undressed three-dimensional (3D) counterpropagating lattice. This raises the prospect of site-addressable highly number squeezed states in a hybrid focused Gaussian two-dimensional optical lattice apparatus, with a comparable on-site interaction energy per atom to that observed in a Mott insulator in a 3D optical lattice [2].

Even a modest increase in U_{lim} could be useful, though, as the number of imperfections is likely to be a strong function of U_{lim}. Estimates of the thermal excitations would be $n_{th} \approx Ae^{-U/kT}$ (the $J \ll U$, $kT \ll U$ limit of the Bose-Einstein distribution) and of nonadiabatic excitations $n_{na} \approx Be^{-CU^2}$; the latter from the form of the Landau-Zener avoided the crossing transition probability [25] in the limit $J \ll U \approx \epsilon_{01}$, where ϵ_{01} is the energy difference between the ground and first excited states of the many-body system.

We briefly consider how atoms may be loaded into the potentials (when $J>U$). Adiabatic loading of the truncated sinusoidal potential can be seen to be accomplished by simply ramping the rf frequency, once atoms are trapped in the bare lattice. Adiabatic loading of the interpolated sinusoidal potential and Gaussian array may be accomplished by ramping the laser intensity (with the rf already on), but the mechanism is less obvious. In effect, during the final stages of the ramping of the lattice intensities, the atoms adiabatically delocalize between the two sets of potential minima as long as the rate of change of the offset between the two sublattices, ϵ_i, is much less than the other Hamiltonian parameters.

It is important to note that spin-changing collisions and

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[1]To use Wannier functions we assume that the region of interest is far enough away from the edge of the array that we can neglect the finite array size.

other two-body collisional loss rates are greatly suppressed for atoms trapped in the lowest eigenstate [11,26,27]. This is because a collision which changes the internal states of these atoms needs an input of energy which is significantly greater than the energy available from kinetic or potential energy (of order J or U). The other loss mechanism is Landau-Zener losses arising from atomic motion at the avoided crossings, which have dominated the only rf dressed optical potential experiment carried out to date [10]. However, Landau-Zener losses become negligible when the atoms are trapped in the lowest eigenstate, as the transition from the lower to the upper dressed level needs energy which is required to come from the kinetic energy of the atoms; this kinetic energy $\sim J$ is typically very much less than Ω_{rf}, so Landau-Zener losses can be expected to be negligible for atoms in the lowest trapped state [28].

A remaining loss mechanism, specific to the truncated lattice, is the coupling of weakly bound states to the continuum. This can only occur when the highest-energy state in the lowest band becomes greater than the binding energy of the site. When $n_i U$ and J are significantly less than the binding energy of the site, as is the case in the examples given above (assuming around one atom per site), this loss mechanism is, to a large extent, suppressed; however, with higher numbers of atoms per site there is a chance that some atoms will be ejected from the lattice.

The calculations shown do not specify whether tunneling is proceeding in the perpendicular (spin-independent) directions; J along the dressed lattice direction is independent of J in the other directions. The lattice parameters in the other directions only influence U through a multiplicative factor in Eq. (4). Therefore, when there is tunneling in all directions in such a lattice, the enhancement factor for U arising from dressing along a single direction will be the same whether or not there is also tunneling in the perpendicular directions. Thus this technique may be used in a 3D lattice (composed of one-dimensional (1D) spin-dependent and two-dimensional (2D) spin-independent lattices) to enhance U_{lim} by the same factor as in purely 1D lattices.

If spin-dependent potentials are present along more than one direction, the 2D or 3D rf-dressed potential cannot be simply expressed as a sum of 1D potentials; one consequence is that the potential minima may occur along lines or surfaces rather than at points. A short discussion may be found in Ref. [14]; we do not consider these cases further in this Rapid Communication.

A practical complication with this scheme, in common with other rf-dressed optical potential schemes, lies in the use of magnetic-field-sensitive transitions. The typical magnetic field drift in a laboratory environment is of order ~ 1 mG [29]; this would manifest as a sublattice-dependent energy offset of order ~ 1 kHz. Clearly we need this site-dependent offset to be less than U; practically, this would mean using a magnetic-shielding technique such as mu-metal cladding, which can decrease these ambient fields by a factor of around 100 [30].

In conclusion, we show that substantial enhancement of on-site interactions may be achieved by rf dressing optical potentials. We study three cases of interest and show that it is possible to enhance nonlinear parameters by more than a factor of 2, with negligible Landau-Zener losses, which should greatly improve the purity of the resulting Mott insulator state. We find that decreasing the distance between neighboring wells has a greater effect on the limiting on-site interaction than modifying the form of the potential between existing wells. In summary, the techniques described have the potential to make complex quantum states with neutral atoms purer and faster.

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