Quantum-beat lasers as bright sources of entangled sub-Poissonian light

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We show that a quantum-beat system with incoherent pump or coherent driving produces an entangled sub-shot-noise laser that operates well above threshold. The numerical results and physical analyses are presented using the combination modes of the lasing fields. The relative mode is decoupled from the active medium and thus remains in its vacuum state, while the sum mode operates well above threshold and has sub-shot-noise. The quantum beat and the sum mode intensity noise reduction combine to yield entanglement between two bright beams and sub-Poissonian photon statistics of the respective beams.

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Atomic coherence leads to many novel effects, among them correlated spontaneous emission lasers (CELs) [1–12], lasers without inversion [13–27], and generation of squeezed light from them [7–12,25–27]. In a quantum-beat laser [1–7] (a particular form of CEL), a beam of three-level atoms in V configuration emits photons into two modes. The atomic upper levels are initially prepared in a coherent superposition or are coupled by a coherent field. The normally ordered variances of two Hermitian operators corresponding, respectively, to the relative phase and the relative amplitude vanish. That is, the noise is reduced to the vacuum noise level. In a two-photon CEL [7–12], a beam of three-level atoms in cascade configuration is employed. The top and bottom states are initially prepared in a coherent superposition. The phase noise is reduced below the vacuum noise level by 50%. In a laser without inversion, laser gain is established by using atomic coherence. At the same time, the coherence leads to noise squeezing [25–27]. The fluctuations of the laser intensity are suppressed up to 50% below the shot noise. We also note that noise squeezing is possible even when neither initial coherence nor an external coherent field is used. This is caused by a dynamic noise reduction mechanism [28–31]. Combining the quantum-beat laser operation and the above quantum noise squeezing mechanisms, we predicted that the two lasing modes and the sum mode would exhibit intensity noise squeezing [32,33].

Most recently, it was shown that a two-mode correlated spontaneous emission laser can be used as an entanglement amplifier [34–36]. In a linear theory these authors showed that the amplifier yields continuous variable entanglement for a short time. However, the entanglement tends to vanish as time goes on and the laser intensity increases. Steady state analyses were also presented but no nonlinear effects were included [37]. It was also shown that four-wave mixing as a form of quantum-beat operation can lead to two-mode continuous variable entanglement [38]. Li *et al.* [39] showed that, with the help of the clock transition in a three-level system, the entanglement is enhanced and the mean intracavity photon number is increased compared with the two-level case. Pielawa *et al.* [40] showed that an Einstein-Podolsky-Rosen (EPR) state can be obtained by using an atomic reservoir engineering based on four-wave mixing. It is well known that four-wave mixing is an optical parametric process, in which the pump field is converted into the output signal [41]. The conversion is determined by the third-order susceptibility of the medium. Since the atomic dynamics is saturated by the strong pump field, the third-order susceptibility is very small. In sharp contrast, a laser oscillator as an active device yields, via resonance-stimulated amplification, a bright source of light. The linear gain is determined by the first-order susceptibility [42], which is much larger than the third-order susceptibility. Well above threshold, the laser intensity is much larger than the saturation intensity. It is desirable to devise an entanglement laser as an active device that operates well above threshold and that produces entangled light in a long time regime.

Here we reinterpret the results found in [32,33] to show that it is possible to generate entangled sub-shot-noise lasers using a quantum-beat scheme together with incoherent pump or coherent driving. Entanglement between two modes, reduction of the intensity fluctuations of the respective modes, and lasing well above threshold are compatible with each other. This device produces two bright beams of entangled sub-Poissonian light.

In order to analyze whether the entanglement appears between the two Gaussian state lasing modes $a_{1,2}$, we use a sufficient criterion for continuous variable entanglement proposed in Refs. [43,44]. According to the criterion, the two lasing fields are entangled if the sum of the variances of two EPR-like operators u and v of the two modes satisfies

$$M \equiv \langle(\delta u)^2\rangle + \langle(\delta v)^2\rangle < 2. \tag{1}$$

Here $u = X_1 + X_2$, $v = P_1 - P_2$, and $X_l = (1/\sqrt{2})(a_l e^{-i\phi_l} + a_l^\dagger e^{i\phi_l})$, $P_l = (-i/\sqrt{2})(a_l e^{-i\phi_l} - a_l^\dagger e^{i\phi_l})$ are the quadrature operators for the two modes a_l, where ϕ_l are chosen [45] so that X_l and P_l are the amplitude and phase quadratures corresponding to the field a_l around its steady state value, $l=1,2$.

It is convenient to use a combination mode approach [5] to describe the quantum-beat laser or CEL. The combination modes are defined as

$$A = \frac{1}{\sqrt{2}}(a_1 e^{-i\theta_1} + a_2 e^{-i\theta_2}), \quad B = \frac{1}{\sqrt{2}}(-a_1 e^{-i\theta_1} + a_2 e^{-i\theta_2}), \tag{2}$$

where $\theta_{1,2}$ are two constants, which are determined by the external field parameters. The Hermitian operators corre-

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sponding to the sum amplitude and sum phase are proportional to the real and imaginary parts of the annihilation operator A. Similarly, the real and imaginary parts of the annihilation operator B determine the Hermitian operators corresponding to the relative amplitude and relative phase [5]. Because of these correspondences, the mode A is called the "sum mode" and the mode B the "relative mode."

The biggest advantage of using the combination modes lies in the fact that, for a CEL, the relative mode B is usually decoupled from the active medium and simply undergoes absorption by the vacuum reservoir, while the sum mode A is amplified and runs well above threshold, as will be shown below. This implies null amplitude for the relative mode B, $\langle B\rangle=0$. Consequently, we have $\langle a_1\rangle e^{-i\theta_1}=\langle a_2\rangle e^{-i\theta_2}$. This indicates that the phase difference of modes a_1 and a_2 is locked to $\phi_1-\phi_2=\theta_1-\theta_2$. The choice of the phase ϕ_1 or ϕ_2 may be made by injecting a weak signal such that it hardly changes the steady state amplitudes but leads to locking of the respective phases to fixed values [46].

The operators u and v are expressed in terms of the operators A and B as $u=Ae^{-i(\phi_1-\theta_1)}+A^\dagger e^{i(\phi_1-\theta_1)}$, $v=i(Be^{-i(\phi_1-\theta_1)}-B^\dagger e^{i(\phi_1-\theta_1)})$, which are the amplitude and phase quadratures corresponding to the sum mode A and the relative mode B, respectively. The decoupling from the medium implies that the mode B stays in its vacuum state, i.e., the variance in the quadrature operator v drops to its vacuum noise level [5,32,33], $\langle(\delta v)^2\rangle=1$. For a laser far above threshold, the fluctuation in the amplitude is negligibly small compared with the amplitude itself. Under these circumstances, the variance of quadrature u is related to Mandel factor Q through the relation $\langle(\delta u)^2\rangle=1+Q$, where $Q=\langle:(\delta I)^2:\rangle/\langle I\rangle$ and $I=A^\dagger A$. Here 1 stands for the vacuum noise level, and the Mandel factor Q is the normally ordered normalized variance of the sum mode intensity and measures the deviations from Poissonian statistics. It is well known that Q is zero for a coherent state (Poissonian photon statistics) and turns negative for a nonclassical state of an intracavity field (sub-Poissonian photon statistics). Using the Q factor, we obtain the normally ordered part of the output fluctuation spectrum

$$S(\omega)=2\int_0^\infty d\tau\cos(\omega\tau)\langle:i(t+\tau),$$

$$i(t):/\langle i(t)\rangle=2Q(2\kappa/\lambda)\lambda^2/(\lambda^2+\omega^2).$$

Here $i(t)=\kappa A^\dagger(t)A(t)$ corresponds to the output photon flux operator and the inverse laser intensity correlation time λ is proportional to the differential gain. As is well known, $S(\omega)=0$ corresponds to shot noise and $0>S(\omega)\geqslant-1$ to sub-Poissonian photon statistics.

Using the above relations we obtain the output spectrum

$$M(\omega)=2+S(\omega), \tag{3}$$

which indicates that the two lasing fields are entangled if sub-Poissonian statistics in the sum mode is existent. It should be emphasized that the relation (3) is valid only when the correlated emission takes effect and the system operates sufficiently far above threshold.

Using the variances in the sum and relative modes we obtain the Mandel factors $Q_{1,2}$ for the respective intensities $I_l=a_l^\dagger a_l=\frac{1}{2}A^\dagger A$ $(l=1,2)$ as $Q_{1,2}=Q/2$, which indicates that sub-Poissonian statistics for the sum mode and for the respective modes are compatible with each other. The amount of the noise reduction for the respective modes is half of that for the sum mode. Correspondingly, the normally ordered parts of the output fluctuation spectra for the respective modes are expressed as $S_l(\omega)=S(\omega)/2$. Quadrature squeezing in the respective modes $a_{1,2}$ occurs when sub-Poissonian statistics in the sum mode is existent $[S(\omega)<0]$.

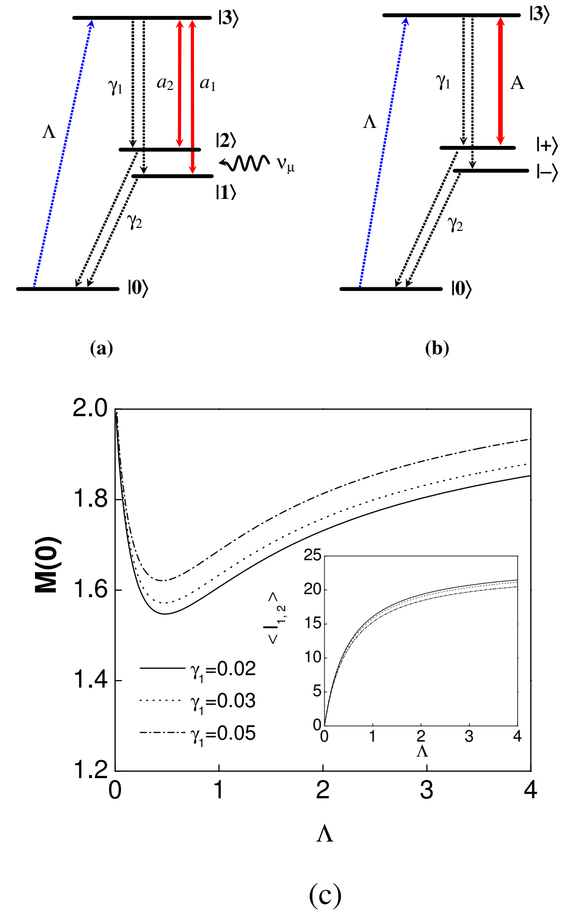

FIG. 1. (Color online) (a) Atom-field coupling scheme for a quantum-beat laser with incoherent pump. (b) Equivalent system in the picture dressed by the microwave field. (c) The zero-frequency output spectrum $M(0)$ as a function of Λ for $\gamma_1=0.02$ (solid), 0.03 (dotted), and 0.05 (dashed) and $C=200$. The steady state intensities $\langle I_{1,2}\rangle$ (in units of γ_2^2/g^2) are shown in the inset. All rates are in units of γ_2.

It can be easily deduced from the above that entanglement between two modes $[M(\omega)<2]$ and squeezing in the respective modes are based on the combined effect of the correlated spontaneous emission $[\langle(\delta v)^2\rangle=1]$ and the sum mode intensity noise reduction $[S(\omega)<0]$. In what follows we give two model systems, for which analyses of the intensity fluctuations were presented using a nonlinear theory in Refs. [32,33].

Entangled sub-shot-noise lasers via a quantum-beat scheme with conventional pump. An ensemble of N four-level atoms [Fig. 1(a)] is placed in a two-mode cavity. For simplicity, we assume that the rates of spontaneous decay from level $|3\rangle$ to levels $|1\rangle$ and $|2\rangle$ are equal to γ_1, and the rates of decay from the levels $|1\rangle$ and $|2\rangle$ to $|0\rangle$ are equal to γ_2. The rate of incoherent pumping from level $|0\rangle$ to $|3\rangle$ is equal to Λ. An external microwave field of frequency ν_μ is resonantly coupled to the $|1\rangle \leftrightarrow |2\rangle$ transition with Rabi frequency $\Omega_\mu e^{-i\theta_\mu}$, where Ω_μ and θ_μ are the real amplitude and phase. Two lasing modes $a_{1,2}$ of frequencies $\omega_{1,2}$ are respectively coupled to the $|1,2\rangle \leftrightarrow |3\rangle$ transitions of resonance frequencies ω_{3l} with equal coupling constant g. We assume that the microwave field is strong, i.e., $\Omega_\mu \gg (g\langle a_l\rangle, \Lambda, \gamma_l)$, $l=1,2$. By tuning the lasing modes such that $\omega_l=\omega_{3l}-\frac{1}{2}\Omega_\mu$ and neglecting the fast-oscillating terms such as $e^{\pm i\Omega_\mu t}$, we write the interaction Hamiltonian as [32]

$$H = i\hbar g \sum_{j=1}^{N} [A^\dagger(|+\rangle\langle 3|)_j - A(|3\rangle\langle +|)_j], \tag{4}$$

where we have used the atomic dressed states $|\pm\rangle = (1/\sqrt{2})(\pm e^{-i\theta_\mu/2}|1\rangle + e^{i\theta_\mu/2}|2\rangle)$, and the sum mode A with $\theta_1=-\theta_2=\frac{1}{2}\theta_\mu$ [Eq. (2)]. It is clear that the mode B is decoupled from the interaction system while the mode A is coupled to the $|+\rangle \leftrightarrow |3\rangle$ transition [Fig. 1(b)].

In our numerical calculation we scale the incoherent pump rate Λ, the atomic decay rate γ_1, and the cavity loss rate κ (the cavity loss rates for the two modes are assumed to be equal to κ) in units of γ_2. The cooperativity parameter is defined as $C=2g^2N/(\gamma_2\kappa)$. In Fig. 1(c) we plot the zero-frequency output spectrum $M(0)$ as a function of Λ for $\gamma_1=0.02$ (solid), 0.03 (dotted), and 0.05 (dashed) and $C=200$. For large cooperativity parameter ($C \gg 1$) the system operates sufficiently far above threshold [29,42]. Plotted in the inset are the intensities $\langle I_1\rangle = \langle I_2\rangle$ in units of γ_2^2/g^2. It is seen from Fig. 1(c) that the output spectrum $M(0)$ first decreases and then increases gradually as the incoherent pump rate increases. At the same time, the intensity rises with the incoherent pump rate. On the other hand, the intensity increases and the output spectrum $M(0)$ decreases as the decay rate γ_1 decreases. In the limiting case ($\gamma_1 \to 0$ and rate matching [28–31]), the spectrum $M(0)$ reaches its minimal value $\frac{3}{2}$. That shows that the entanglement criterion is satisfied when the quantum-beat laser works well above threshold. In addition, the output spectra for the respective modes have their minimal value $S_l(0)=-\frac{1}{4}$, which corresponds to 25% noise reduction. Two factors are responsible for entanglement and squeezing. The first is the correlated spontaneous emission. Only the sum mode A is coupled to the medium while the relative mode B is decoupled from the system. The second is the dynamical noise reduction. For $\gamma_1 \ll \gamma_2, \Lambda$, the population in state $|-\rangle$ is negligible. Then the four-level system in Fig. 1(b) is reduced to a three-level system ($|0\rangle$, $|+\rangle$, and $|3\rangle$). In the reduced system, the succession of the transition $|+\rangle \leftrightarrow |3\rangle$ and the two-step incoherent process $|+\rangle \to |0\rangle \to |3\rangle$ recycles the active laser electron from $|+\rangle$ through $|0\rangle$ to $|3\rangle$, which regularizes the laser electrons. The above two factors combine to cause entanglement and squeezing.

Entangled sub-shot-noise lasers via a quantum-beat scheme with coherent driving. Placed in the two-mode cavity is an ensemble of N four-level atoms as shown in Fig. 2(a). The rates of decay from levels $|1\rangle$ and $|2\rangle$ to $|3\rangle$ ($|0\rangle$) are equal to γ_1 (γ_2). The rates of incoherent pump from level $|0\rangle$ to $|1\rangle$ and $|2\rangle$ are equal to Λ. Two coherent fields of frequencies $\nu_{1,2}$ are applied to the transitions $|1,2\rangle \leftrightarrow |3\rangle$ with Rabi frequencies $\Omega_{1,2}$, respectively. A microwave field of frequency ν_μ is resonantly coupled to the $|1\rangle \leftrightarrow |2\rangle$ transition with Rabi frequency $\Omega_\mu e^{-i\theta_\mu}$. The two lasing modes $a_{1,2}$ of frequencies $\omega_{1,2}$ are respectively generated from the $|0\rangle \leftrightarrow |1,2\rangle$ transitions. By tuning the coherent driving fields $\nu_l=\omega_{l3}+\frac{1}{2}\Omega_\mu$ and setting the cavity fields $\omega_l=\omega_{3l}+\frac{1}{2}\Omega_\mu$, we write the interaction Hamiltonian as [33]

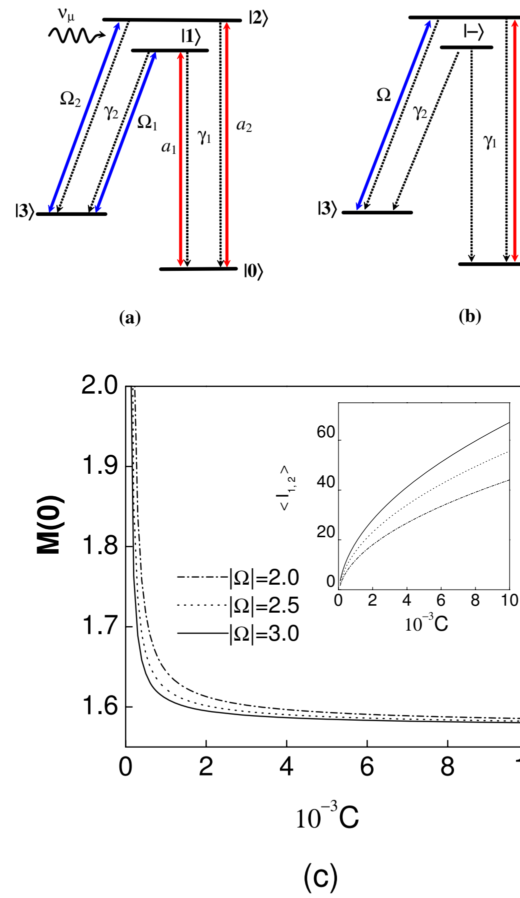

FIG. 2. (Color online) (a) Atom-field coupling scheme for a quantum-beat laser with coherent driving. (b) Equivalent system in the picture dressed by the microwave field. (c) The zero-frequency output spectrum $M(0)$ versus the cooperativity parameter C for $|\Omega|=2.0$ (dashed), 2.5 (dotted), and 3.0 (solid) and $\gamma_1=4.0$, $\Lambda=0.5$. Shown in the inset are the steady state intensities $\langle I_{1,2}\rangle$ (in units of γ_2^2/g^2). Rabi frequency and all rates are in units of γ_2.

$$V = i\hbar g \sum_{j=1}^{N} [A^\dagger(|+\rangle\langle 0|)_j - A(|+\rangle\langle 0|)_j] + \frac{\hbar}{2}\sum_{j=1}^{N} [\Omega(|3\rangle\langle +|)_j + \Omega^*(|+\rangle\langle 3|)_j], \tag{5}$$

where we have used $\Omega=(1/\sqrt{2})(\Omega_1 e^{-i\theta_\mu/2}+\Omega_2 e^{i\theta_\mu/2})$ and the sum mode A with $\theta_1=-\theta_2=-\frac{1}{2}\theta$ [Eq. (2)]. We have also assumed equal coupling constant g for the two cavity modes. Equation (5) shows that the B mode is no longer mediated into the interaction, and both the mode A and the effective driving field Ω are resonantly coupled to the active medium [Fig. 2(b)].

Plotted in Fig. 2(c) is the zero-frequency output spectrum $M(0)$ as a function of the cooperativity parameter $C = 2g^2N/(\gamma_2\kappa)$ (equal cavity loss rate κ for two modes) for $|\Omega|=2.0$ (dashed), 2.5 (dotted), and 3.0 (solid), $\gamma_1=4.0$, and $\Lambda=0.5$. Such a choice of the parameters ($\Lambda<\gamma_1$) indicates that the system operates without population inversion [25,33]. We have scaled the parameters γ_1, Λ, Ω, and κ in units of γ_2. The intensities $\langle I_1\rangle=\langle I_2\rangle$ in units of γ_2^2/g^2 are plotted in the inset. The output spectrum $M(0)$ decreases as the cooperativity parameter C or the effective Rabi frequency $|\Omega|$ increases. For the limiting case ($\gamma_2 \ll \gamma_1$ and laser operation well above threshold [24,25,33]), the spectrum approaches $M(0)=\frac{3}{2}$. The output intensity fluctuation of each mode is also 25% below the shot-noise limit. Through the incoherent and coherent channels $|0\rangle\to|\pm\rangle\to|3\rangle$ and $|3\rangle\to|+\rangle\to|0\rangle$, the electrons are recycled and the sum mode intensity fluctuations are suppressed. Also, it is the combination of the correlated spontaneous emission and the sum mode intensity noise reduction that gives rise to entanglement and squeezing.

In conclusion, we have shown that it is possible to generate entangled sub-Poissonian light from a quantum-beat laser with conventional pump or coherent driving. The three characteristic features are compatible with each other. (i) The quantum-beat lasers as active devices operate well above threshold and provide bright sources. (ii) Each of the two bright beams exhibits sub-Poissonian photon statistics. (iii) Continuous variable entanglement occurs between these two bright beams.

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