Decoherence and disentanglement for two qubits in a common squeezed reservoir

Maritza Hernandez and Miguel Orszag
Facultad de Física, Pontificia Universidad Católica de Chile, Casilla 306, Santiago, Chile
(Received 12 May 2008; published 31 October 2008)

We study the relation between the sudden death and revival of the entanglement of two qubits in a common squeezed bath and the decoherence. By getting "close" to the decoherence free subspace, we show that the death time shows a decrease and subsequent increase until the sudden death disappears and the concurrence goes to its steady-state value asymptotically.

DOI: 10.1103/PhysRevA.78.042114 PACS number(s): 03.65.Yz, 03.67.Pp, 03.67.Mn, 05.70.−a

In one-party quantum systems, coherence is inevitably destroyed by the action of the environment, a phenomenon that is local and occurs asymptotically in time. On the other hand, there could be multiparty systems, with nonlocal quantum correlations, often referred to as quantum entanglement. Entanglement between separate quantum systems is one of the key features in quantum mechanics. The nonlocality and coherence of the quantum entangled states makes them very important, not only for their fundamental properties but also for their different applications such as quantum teleportation [1], quantum cryptography [2], and dense coding [3].

If the environment would act on the various parties the same way it acts on single systems, one would expect that a measure of entanglement, say the concurrence, would also decay exponentially in time. However, this is not always the case. Recently Yu and Eberly [4–6] showed that under certain conditions, the dynamics could be completely different and the quantum entanglement of a bipartite qubit system may vanish in a finite time. They called this effect "entanglement sudden death" (ESD). In a previous work, Diósi [7] demonstrated that ESD occurs in two state quantum systems, using Werner's criteria for separability. This effect has also been observed experimentally [8].

Different schemes have been derived to remove the effects produced by the environment, for example, quantum error correction, decoherence free subspace (DFS), and quantum Zeno effect. In fact, there are some proposals related to the use of DFS as the memory space for storing the quantum information [9].

Here, we consider two two-level atoms that interact with a common squeezed reservoir, and we will focus on the evolution of the entanglement between them, using as a basis the DFS states, as defined in Refs. [11,12].

This model includes others as special cases (atoms in thermal bath at zero temperature) and has the advantage of having a known DFS, a two-dimensional plane, that will be our starting point in connecting decoherence with sudden death and revival of the entanglement.

In particular, we show that for states that belong initially to the DFS plane, the phenomenon of entanglement sudden death (ESD) never occurs. However, if our initial state is away from the DFS the sudden death shows up, followed by sudden revival [10]. When the ESD occurs, the entanglement of the quantum state undergoes a process of decay from a positive value to zero within a finite time that we will call the death time. In our analysis we find a curious effect. For $N=0$ (no squeezing), the death time increases as we get closer to the DFS, which is the normal behavior one would expect since we have a decrease in the interaction with the reservoir, up to a critical "distance" where the death time becomes infinite and the sudden death effect disappears. On the other hand, as we increase N (nonzero squeezing), as we get near the DFS, the death time decreases, showing an increased disentanglement, down to zero, for a first critical distance, and subsequently, as we get closer, it increases "the normal way" and goes to infinity for a second critical distance. This effect is related to the nature of the reservoir-atom interaction and will be discussed at the end.

The master equation, in the interaction picture, for a two-level system in a broadband squeezed vacuum bath is given by [13]

$$\begin{aligned}\frac{\partial\rho}{\partial t}&=\frac{1}{2}\gamma(N+1)(2\sigma\rho\sigma^\dagger-\sigma^\dagger\sigma\rho-\rho\sigma^\dagger\sigma)\\&+\frac{1}{2}\gamma N(2\sigma^\dagger\rho\sigma-\sigma\sigma^\dagger\rho-\rho\sigma\sigma^\dagger)\\&-\frac{1}{2}\gamma Me^{i\psi}(2\sigma^\dagger\rho\sigma^\dagger-\sigma^\dagger\sigma^\dagger\rho-\rho\sigma^\dagger\sigma^\dagger)\\&-\frac{1}{2}\gamma Me^{-i\psi}(2\sigma\rho\sigma-\sigma\sigma\rho-\rho\sigma\sigma),\end{aligned} \tag{1}$$

where γ is the spontaneous emission rate and $N=\sinh^2|r|$, $M=\sqrt{N(N+1)}$, $r=|r|e^{i\psi}$, and ψ are the squeeze parameters of the bath and $\sigma^\dagger,\sigma$ are the usual Pauli raising and lowering matrices.

It is simple to show that the above master equation can also be written in the Lindblad form with a single Lindblad operator S,

$$\frac{\partial\rho}{\partial t}=\frac{1}{2}\gamma(2S\rho S^\dagger-S^\dagger S\rho-\rho S^\dagger S), \tag{2}$$

with

$$S=\sqrt{N+1}(\sigma)-\sqrt{N}e^{i\Psi}(\sigma^\dagger). \tag{3}$$

For a two two-level system in a common bath, the master equation has the same structure, but now the S operator becomes

$$\begin{aligned}S&=\sqrt{N+1}(\sigma_1+\sigma_2)-\sqrt{N}e^{i\Psi}(\sigma_1^\dagger+\sigma_2^\dagger)\\&=\cosh(r)(\sigma_1+\sigma_2)-\sinh(r)e^{i\Psi}(\sigma_1^\dagger+\sigma_2^\dagger).\end{aligned} \tag{4}$$

The decoherence free subspace for this model is found in Ref. [11] and consists of the eigenstates of S with zero eigenvalue. The states defined in this way form a two-dimensional plane in Hilbert space. Two orthogonal vectors in this plane are

$$|\phi_1\rangle = \frac{1}{\sqrt{N^2+M^2}}(N|++\rangle + Me^{-i\Psi}|--\rangle), \tag{5}$$

$$|\phi_2\rangle = \frac{1}{\sqrt{2}}(|-+\rangle - |+-\rangle). \tag{6}$$

We can also define the states $|\phi_3\rangle$ and $|\phi_4\rangle$ orthogonal to the $\{|\phi_1\rangle,|\phi_2\rangle\}$ plane:

$$|\phi_3\rangle = \frac{1}{\sqrt{2}}(|-+\rangle + |+-\rangle), \tag{7}$$

$$|\phi_4\rangle = \frac{1}{\sqrt{N^2+M^2}}(M|++\rangle - Ne^{-i\Psi}|--\rangle). \tag{8}$$

We solved analytically the master equation by using the $\{|\phi_1\rangle,|\phi_2\rangle,|\phi_3\rangle,|\phi_4\rangle\}$ basis. The various components of the time-dependent density matrix depend on the initial state as well as the squeezing parameters. For simplicity, we assumed $\gamma=1$ and $\psi=0$.

A popular measure of entanglement is the concurrence. This measure was proposed by [14] and is defined as

$$C(\rho) = \max\{0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}\}, \tag{9}$$

where λ_i are the eigenvalues (λ_1 being the largest one) of a non-Hermitian matrix $\rho\tilde{\rho}$, and $\tilde{\rho}$ is defined as

$$\tilde{\rho} = (\sigma_y^1 \otimes \sigma_y^2)\rho^*(\sigma_y^1 \otimes \sigma_y^2), \tag{10}$$

ρ^* being the complex conjugate of ρ (in the standard basis) and σ_y is the usual Pauli matrix. The concurrence C varies from $C=0$ for an unentangled state to $C=1$ for a maximally entangled state. For the class of initial states considered below, the density matrix written in the standard basis is in the form

$$\rho(t) = \begin{pmatrix} \rho'_{11} & 0 & 0 & \rho'_{14} \\ 0 & \rho'_{22} & \rho'_{23} & 0 \\ 0 & \rho'_{32} & \rho'_{33} & 0 \\ \rho'_{41} & 0 & 0 & \rho'_{44} \end{pmatrix}, \tag{11}$$

one easily finds [15] that the concurrence is given by $C(\rho(t))=\max\{0,C_a(\rho),C_b(\rho)\}$, where

$$C_a(\rho) = 2(\sqrt{\rho'_{23}\rho'_{32}} - \sqrt{\rho'_{11}\rho'_{44}}), \tag{12}$$

$$C_b(\rho) = 2(\sqrt{\rho'_{14}\rho'_{41}} - \sqrt{\rho'_{22}\rho'_{33}}), \tag{13}$$

where ρ'_{ij} are the density matrix elements in the standard basis.

In order to study the relation between decoherence and disentanglement, we consider as initial states superpositions of the form

$$|\Psi_1\rangle = \varepsilon|\phi_1\rangle + \sqrt{1-\varepsilon^2}|\phi_4\rangle, \tag{14}$$

$$|\Psi_2\rangle = \varepsilon|\phi_2\rangle + \sqrt{1-\varepsilon^2}|\phi_3\rangle, \tag{15}$$

where ε is a variable amplitude of one of the states belonging to the DFS. We would like to study the effect of varying ε on the sudden death and revival times.

For both $|\Psi_1\rangle$ and $|\Psi_2\rangle$ as initial states, the solution of the master equation written in the standard basis has the form shown in Eq. (11). We can also write C_a and C_b in terms of the density matrix $\rho(t)$ in the $\{\varphi_i\}$ basis as

$$C_a(\rho) = |\rho_{33}-\rho_{22}| - 2\sqrt{\frac{N(\rho_{11}+\rho_{44})+\rho_{44}+2\rho_{14}\sqrt{N(N+1)}}{2N+1}} \times \sqrt{\frac{N(\rho_{11}+\rho_{44})+\rho_{11}-2\rho_{14}\sqrt{N(N+1)}}{2N+1}}, \tag{16}$$

$$C_b(\rho) = \frac{2}{2N+1}|\sqrt{N(N+1)}(\rho_{11}-\rho_{44})+\rho_{14}| - \sqrt{(\rho_{22}-2\rho_{23}+\rho_{33})(\rho_{22}+2\rho_{23}+\rho_{33})}. \tag{17}$$

If we consider first the initial state $|\Psi_1\rangle$, in which case $\rho_{22}=\rho_{23}=0$, the concurrence is given by

$$C(\rho_1(t)) = \max\{0,C_b(\rho_1(t))\}, \tag{18}$$

since $C_a<0$. On the other hand, for the initial state $|\Psi_2\rangle$ the concurrence is

$$C(\rho_2(t)) = \max\{0,C_a(\rho_2(t)),C_b(\rho_2(t))\}, \tag{19}$$

where $\rho_i(0)=|\Psi_i\rangle\langle\Psi_i|$ for $i=1,2$.

In both cases, we vary ε between 0 and 1 for a fixed value of the parameter N. We observe that for the interval $0\leqslant\varepsilon<\varepsilon_c$ the initial entanglement decays to zero in a finite time, t_d. After a finite period of time during which the concurrence stays null, it revives at a time t_r reaching asymptotically its steady-state value. We find that this death and revival cycle happens once for the initial state $|\Psi_1\rangle$. However, when the initial state is $|\Psi_2\rangle$, this cycle may occur twice [16].

For certain values of $\varepsilon=\varepsilon_c$, for the initial state $|\Psi_1\rangle$, $t_d=t_r$ and the entanglement dies and revives simultaneously and eventually goes to its steady-state value. This revival is a curious effect only possible when the two atoms are coupled to a common bath (that is, the distance between them is smaller that the bath correlation length). For the initial state $|\Psi_2\rangle$, the critical value of ε is $\varepsilon_c=\frac{1}{\sqrt{2}}$, so that the initial state is $|\Psi_2\rangle=|-+\rangle$, and contrary to the $|\Psi_1\rangle$ case, it is independent of N.

When $\varepsilon_c<\varepsilon\leqslant1$, that is, when we get "near" the DFS, the whole phenomenon of sudden death and revival disappears for both initial conditions, and the system shows no disentanglement.

Figures 1 and 2 show the dynamic behavior of the entanglement in terms of the concurrence for the initial states $|\Psi_1\rangle$ and $|\Psi_2\rangle$, respectively. In both cases we use $N=0.1$.

In the particular case of $|\Psi_1\rangle=\varepsilon|\phi_1\rangle+\sqrt{1-\varepsilon^2}|\phi_4\rangle$ and $N=0$, the critical value is $\varepsilon_c=0.345$, and the death and revival times are solutions of the equation

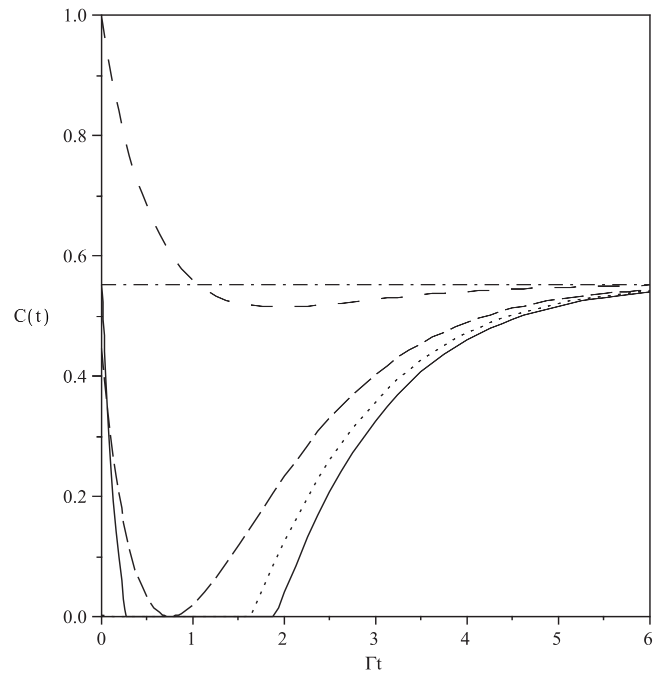

FIG. 1. Time evolution of the concurrence for $|\Psi_1(t)\rangle$ as initial state and $N=0.1$: $\varepsilon=0$ (solid line), $\varepsilon=0.29$ (dotted line), $\varepsilon_c=0.5$ (dashed line), $\varepsilon=0.9$ (space dashed line), and $\varepsilon=1$ (dashed-dotted line).

$$te^{-t} = \frac{\varepsilon}{\sqrt{1-\varepsilon^2}}, \tag{20}$$

and for $N=0$ and $|\Psi_2\rangle=\varepsilon|\phi_2\rangle+\sqrt{1-\varepsilon^2}|\phi_3\rangle$, the critical value is $\varepsilon_c=\frac{1}{\sqrt{2}}$ and the corresponding equation for the death and revival times is

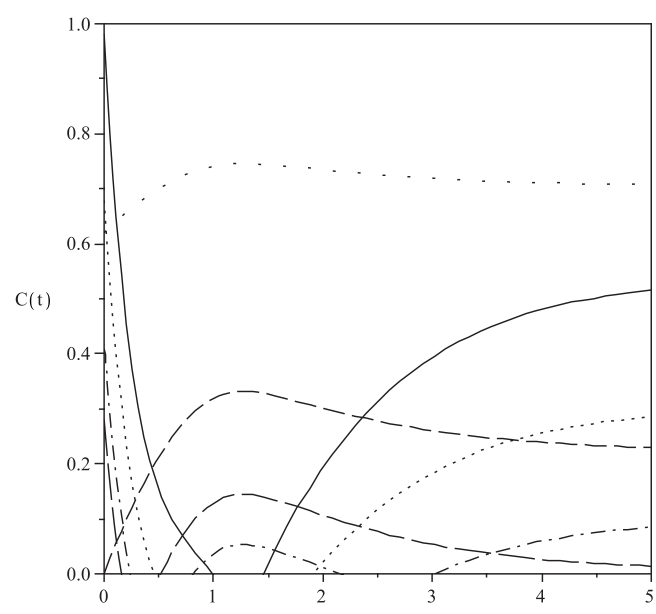

FIG. 2. Time evolution of the concurrence for $|\Psi_2(t)\rangle$ as the initial state and $N=0.1$: $\varepsilon=0.1$ (solid line), $\varepsilon=0.4$ (dotted line), $\varepsilon=0.54$ (dashed-dotted line), $\varepsilon=0.6$ (long dashed line), $\varepsilon_c=0.707$ (dashed line), and $\varepsilon=0.9$ (space dotted line).

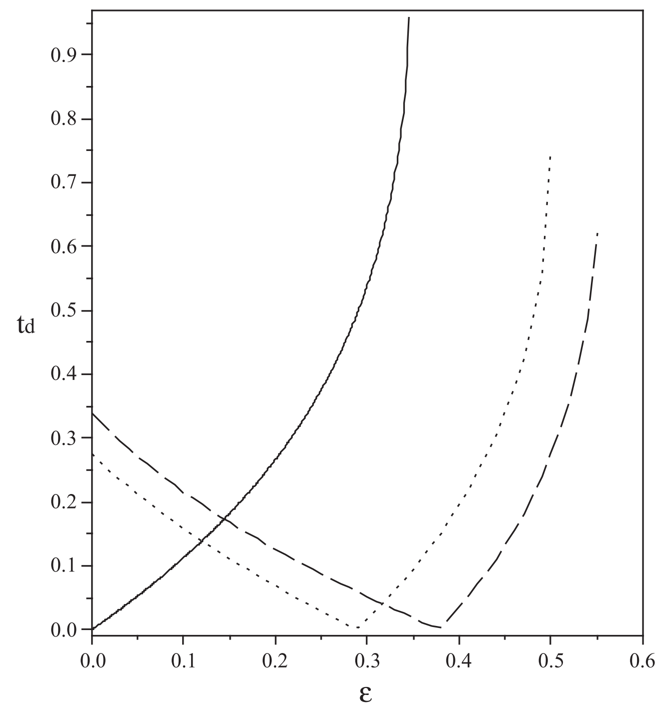

FIG. 3. Death time versus ε, with initial $|\Psi_1\rangle$ for $N=0$ (solid line), $N=0.1$ (dotted line), and $N=0.2$ (dashed line).

$$t = \frac{1}{2}\ln\left(\frac{1-\varepsilon^2}{\varepsilon^2}\right). \tag{21}$$

Another way of seeing the same effect is shown in Fig. 3, where we plot, in the $|\Psi_1\rangle$ case, the sudden death and revival times versus ε, for various values of N. In the case $N=0$, we notice a steady increase of the death time up to ε_c, where the death time becomes infinite. On the other hand, for $N=0.1$ and 0.2, we see that the effect of the squeezed reservoir is to increase the disentanglement, and the death time shows an initial decrease up to the value $\varepsilon=\frac{\sqrt{N(2N+1)}}{2N+1}$, and for larger values, it shows a steady increase, similar to the $N=0$ case.

The physical explanation of the above effect is the following one: the squeezed vacuum reservoir has only nonzero components for an even number of photons, so the interaction between the qubits and the reservoir goes by pairs of photons. Now, for a very small N, the average photon number is also small, so the predominant interaction with the reservoir will be with the doubly excited state via two photon spontaneous emission. Let us write $|\Psi_1\rangle$ in terms of the standard basis

$$|\Psi_1\rangle = k_1|+\,+\rangle + k_2|-\,-\rangle, \tag{22}$$

with

$$k_1 = \frac{\varepsilon N + M\sqrt{1-\varepsilon^2}}{\sqrt{N^2+M^2}}, \quad k_2 = \frac{\varepsilon M - N\sqrt{1-\varepsilon^2}}{\sqrt{N^2+M^2}}. \tag{23}$$

In Fig. 4 for $N=0.1$, we see that initially k_1 increases with ε, thus favoring the coupling with the reservoir, or equivalently, producing a decrease in the death time. This is up to $\varepsilon=0.288$, where the curve shows a maxima. This is also

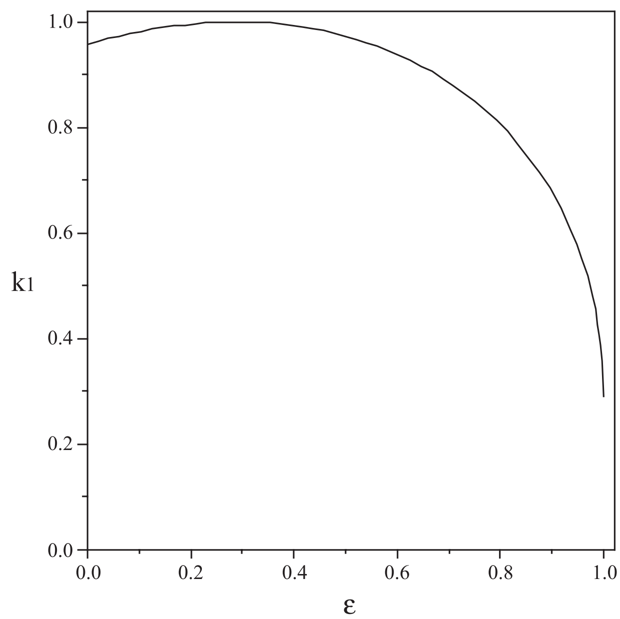

FIG. 4. k_1 versus ε for $N=0.1$.

precisely the point where the death time in Fig. 3 starts to increase as a function of ε. Beyond this point, k_1 starts to decrease and therefore our system is slowly decoupling from the bath and therefore the death time shows a steady increase. Of course, if one is in the DFS, the entanglement does not decay at all.

Finally, a few words about the model. Since we consider two atoms in a common bath, they will have to be quite near, at a distance no bigger than the correlation length of the bath. This implies that one cannot avoid an interaction between the particles. This interaction between the two-level systems can, in principle, affect the DFS. For example, one can consider a dipole-dipole Van der Waals coupling of the form $H_D=\hbar\Omega(\sigma_1\sigma_2^\dagger+\sigma_1^\dagger\sigma_2)$, with $\Omega=|\mathbf{d}|^2\frac{(1-3\cos^2\theta)}{R^3}$, where R is the modulus of the distance between the atoms and θ the angle between the separation vector and $\mathbf{d}$ (dipole matrix). It is simple to verify that $(\sigma_1\sigma_2^\dagger+\sigma_1^\dagger\sigma_2)|\phi_1\rangle=0$ and $(\sigma_1\sigma_2^\dagger+\sigma_1^\dagger\sigma_2)|\phi_2\rangle=-|\phi_2\rangle$. As we can see, for an initial state within the DFS, with this type of coupling the state remains within the DFS. A similar conclusion is reached with an Ising-type Hamiltonian. Thus this DFS is robust against these types of interactions.

In summary, we found a simple quantum system where we establish a direct connection between the local decoherence property and the nonlocal entanglement between two qubits sharing a common squeezed reservoir.

We showed how the phenomena of entanglement sudden death and revival depend on the distance from the DFS. If the initial state belongs to the DFS, this never occurs. In the case $N=0$ (no squeezing), the death time increases as we get closer to the DFS, up to a critical distance where the death time becomes infinite and the sudden death effect disappears. On the other hand, when we increase N (nonzero squeezing) to a relatively small value, that is, for a reservoir with a small average photon number, as we get near the DFS, the death time decreases, showing an increased disentanglement, down to zero, for a first critical distance, and subsequently, as we get closer, it increases the normal way and goes to infinity for a second critical distance. This effect is related to the nature of the squeezed reservoir that for small N interacts more efficiently with the atoms when both of them are excited. These results could be easily extended to mixed states in the DFS with a variable component outside the subspace. Also, the sudden death times versus ε could be measured using an all optical experimental setup [17].

M.H. was supported by a Conicyt grant. M.O was supported by Fondecyt Grant No. 1051062. The authors thank Professor Sascha Wallentowitz for useful discussions.

[1] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70**, 1895 (1993).
[2] A. K. Ekert, Phys. Rev. Lett. **67**, 661 (1991); D. Deutsch, A. K. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, *ibid.* **77**, 2818 (1996).
[3] C. H. Bennett and S. J. Wiesner, Phys. Rev. Lett. **69**, 2881 (1992).
[4] T. Yu and J. H. Eberly, Phys. Rev. Lett. **93**, 140404 (2004).
[5] T. Yu and J. H. Eberly, Phys. Rev. Lett. **97**, 140403 (2006).
[6] T. Yu and J. H. Eberly, Opt. Commun. **264**, 393 (2006).
[7] L. Diósi, Lect. Notes Phys. **622**, 157 (2003).
[8] M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Wallborn, P. H. Souto Ribeiro, and L. Davidovich, e-print arXiv:quant-ph/0701184; M. F. Santos, P. Milman, L. Davidovich, and N. Zagury, Phys. Rev. A **73**, 040305(R) (2006).
[9] G. M. Palma, K. A. Suominen and A. K. Ekert, Proc. R. Soc. London, Ser. A **452**, 567 (1996).
[10] Z. Ficek and R. Tanas, Phys. Rev. A **74**, 024304 (2006).
[11] D. Mundarain and M. Orszag, Phys. Rev. A **75**, 040303(R) (2007).
[12] D. Mundarain, M. Orszag, and J. Stephany, Phys. Rev. A **74**, 052107 (2006).
[13] M. Orszag, *Quantum Optics*, 2nd ed. (Springer-Verlag, Berlin, 2007).
[14] W. K. Wootters, Phys. Rev. Lett. **80**, 2245 (1998).
[15] M. Ikram, F. L. Li, and M. S. Zubairy, Phys. Rev. A **75**, 062336 (2007).
[16] T. Yu and J. H. Eberly, J. Mod. Opt. **57**, 2289 (2007).
[17] M. P. Almeida, F. de Melo, M. Hor-Meyll, A. Salles, S. P. Walborn, P. H. Souto Ribeiro, and L. Davidovich, Science **316**, 579 (2007).