Modification of the absorption spectrum by stochastic-noise-induced quantum interference

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The absorption spectrum is considered for a two-level atom coherently driven by a laser field and incoherently perturbed by stochastic noise. In the noise-dominated regime where incoherent perturbation far exceeds coherent excitation, the amplifying and dispersionlike components in the standard absorption spectrum [B. R. Mollow, Phys. Rev. A **5**, 2217 (1972); F. Y. Wu *et al.*, Phys. Rev. Lett **38**, 1077 (1977)] develop into an absorption peak and a narrow dip, respectively. The modified absorption spectrum is attributed to the quantum interference induced by stochastic noise.

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I. INTRODUCTION

The interaction of a two-level atom with an intense laser field is of fundamental importance in quantum optics and laser spectroscopy. The absorption properties of two-level atoms driven by a strong monochromatic field and probed by a weak field have been extensively studied theoretically [1,2] and experimentally [3,4]. When the driving field is exactly resonant with the atomic transition frequency, the absorption spectrum displays a symmetric structure, in which the Rabi sidebands exhibit a dispersionlike profile, and one of them displays absorption feature, while the other shows amplification characteristic. However, for an off-resonance driving field, for example, when the carrier frequency of the driving field is larger than the atomic frequency, it differs, the central line exhibits a dispersionlike feature, while one of the Lorentzian sidebands displays absorption, the other exhibits amplification, i.e., the atom emits energy into the probe field, rather than absorbs from it.

Quantum interference in multilevel atoms, which leads to coherent population trapping (CPT) [5], electromagnetically induced transparency (EIT) [6], highly monochromatic fluorescence [7,8], total inhibition of fluorescence [9,10], and spectral line narrowing [9,11–13], etc., is an effective mechanism to control and modify the absorption spectrum. An external probe signal field passing through the gas of coherently driven three-level atoms may be absorbed or amplified, as in the standard two-level case, but under specific operating conditions, amplification or absorption can occur over the entire frequency domain where the atomic response is appreciable [11,12]. The absorption spectrum is composed of two inner sidebands, either of which is the coherent superposition of two Lorentzians, though the resultant sidebands are not Lorentzian functions as frequency. Ficek and Freedhoff [14] studied the absorption by a two-level atom in a bichromatic field with one strong and one weak component, and predicted the spectrum displayed emission-dispersion-absorption features. Carreño *et al.* [15] reported the feasible control of absorption and gain of the probe field over a wide frequency region by employing vacuum-induced coherence (VIC) and a broadband squeezed vacuum.

In general, incoherent perturbation can lead to the disappearance of quantum interference. However, under appropriate conditions, even incoherent perturbation can induce quantum interference as well. Quantum interference stemming from collisions brings about pressure-induced extra resonances [16]. Hole-burning and dispersive profiles centered in the fluorescence spectrum are traced to quantum interference induced by the fluctuations in the phase of a driving laser [17]. Karpati *et al.* [18] showed that stochastic noise led to phase correlation, from which quantum interference between the related transitions arose.

Here attention is paid to how stochastic noise affects the standard absorption spectrum of a two-level atom [1,3]. For this purpose, the absorption spectrum is considered for a coherently driven and incoherently perturbed two-level atom. When stochastic noise is predominant over the Rabi oscillations of the driving field, the amplification sideband in the absorption spectrum reverts to an absorption one, and the central dispersionlike profile is reduced to a narrow dip. The modification of the absorption spectrum is attributable to stochastic noise-induced quantum interference in the dressed-atom picture [19,20].

II. MODEL AND EQUATIONS

A two-level atom, which has a ground state $|1\rangle$ and an excited state $|2\rangle$, is driven by a strong laser field $\mathbf{E}(t) = \frac{1}{2}\mathbf{E}e^{-i\omega_L t} + \text{c.c.}$ with carrier frequency ω_L and amplitude $\mathbf{E}$, and spontaneously decay into all the vacuum modes of the electromagnetic field. Here an assumption is made that the atomic resonance transition frequency ω_0 is subject to fluctuations due to stochastic perturbation. In this model, the stochastic noise is responsible for the fluctuations of the atomic resonance frequency, and such fluctuations may result from, for example, elastic, dephasing collisions between the atoms in a gas or between photons and atoms in a solid [21], furthermore, the collisions between a single two-level atom and other atoms cause a phase randomization of the atomic dipole, namely, phase decay [22]. In the frame rotating with frequency ω_L and within the dipole approximation and the rotating wave approximation, the Hamiltonian of a coherently driven and incoherently perturbed two-level atom is written as

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$$H=\hbar[\omega_0(t)-\omega_L]\sigma_{22}-\frac{\hbar}{2}(\Omega\sigma_{21}+\Omega^*\sigma_{12}), \quad (1)$$

where $\omega_0(t)=\omega_0+\delta\omega_0(t)$ is the fluctuating atomic transition frequency, $\Omega=\mathbf{d}\cdot\mathbf{E}/\hbar$ is the Rabi frequency of the laser field with $\mathbf{d}$ being the atomic transition electric dipole moment, $\sigma_{jk}=|j\rangle\langle k|$ $(j,k=1,2)$ are the dipole operators for $j\neq k$ and the projection operators for $j=k$.

It is also assumed that the fluctuations in the transition frequency obey the Gaussian statistic distribution and take the form [21]

$$\langle\delta\omega_0(t)\rangle=0,\quad \langle\delta\omega_0(t)\delta\omega_0(t')\rangle=2\gamma_{ph}\delta(t-t'), \quad (2)$$

where γ_{ph} characterizes the magnitude of the stochastic noise. If the noise stems from collisions, γ_{ph} is the collision rate between the atoms.

The master equation for the reduced density matrix of the atom is equivalent to that for spontaneous emission, phase decay plus a coherent part corresponding to the coherent driving term, which is cast into the required form [22,23]

$$\dot{\rho}=-\frac{i}{\hbar}[\langle H\rangle,\rho]+(\mathcal{L}\rho)_{sp}+(\mathcal{L}\rho)_{st}, \quad (3)$$

where $\langle H\rangle$ stands for the mean atomic Hamiltonian obtained by averaging over the stochastic noise of Eq. (1) and is given by $\langle H\rangle=\hbar\Delta\sigma_{22}-\frac{\hbar}{2}(\Omega\sigma_{21}+\Omega^*\sigma_{12})$, with $\Delta=\omega_0-\omega_L$ being the detuning between the atomic transition and the laser field. Note that the ensemble average affects only the $\delta\omega_0(t)$ factor [21]. $(\mathcal{L}\rho)_{sp}$ describes the atomic spontaneous decay from the level $|2\rangle$ to $|1\rangle$ with the rate γ and is written in the form [22]

$$(\mathcal{L}\rho)_{sp}=\frac{\gamma}{2}(2\sigma_{12}\rho\sigma_{21}-\sigma_{21}\sigma_{12}\rho-\rho\sigma_{21}\sigma_{12}), \quad (4)$$

whereas $(\mathcal{L}\rho)_{st}$ represents the dephasing process. The pure dephasing or phase decay, in practice, stems from the elastic collisions between a single atom and the other atoms or molecules [22], and is necessarily governed by a certain master equation for the reduced density operator in the Markovian approximation according to the generalized reservoir theory [21]. Walls and Milburn applied the generalized reservoir theory to the damping of a two-level atom by a high-temperature thermal reservoir and presented the resulting equation for the dephasing as follows [22]:

$$(\mathcal{L}\rho)_{st}=\frac{\gamma_{ph}}{4}(2\sigma_z\rho\sigma_z-\sigma_z\sigma_z\rho-\rho\sigma_z\sigma_z), \quad (5)$$

with $\sigma_z=\sigma_{22}-\sigma_{11}$ being the population inversion operator.

According to the master equation (3), a set of equations of motion for the reduced density-matrix elements can be written in a compact form

$$\frac{d}{dt}\mathbf{X}(t)=\mathbf{Q}\mathbf{X}(t)+\mathbf{R}, \quad (6)$$

where the coupling matrix

$$\mathbf{Q}=\begin{pmatrix}-\left(\frac{1}{2}\gamma+\gamma_{ph}+i\Delta\right) & 0 & -i\Omega\\ 0 & -\left(\frac{1}{2}\gamma+\gamma_{ph}-i\Delta\right) & i\Omega^*\\ -\frac{i}{2}\Omega^* & \frac{i}{2}\Omega & -\gamma\end{pmatrix}, \quad (7)$$

the column vector $\mathbf{X}(t)=(\langle\sigma_{12}\rangle,\langle\sigma_{21}\rangle,\langle\sigma_{22}\rangle)^T$, and $\mathbf{R}=(\frac{i}{2}\Omega,-\frac{i}{2}\Omega^*,0)^T$. The steady-state solution $(t\to\infty)$ of Eq. (6) is $\mathbf{X}(\infty)=-\mathbf{Q}^{-1}\mathbf{R}$.

III. RESULTS AND DISCUSSION

In order to calculate the steady-state absorption spectrum, it is necessary to introduce a third tunable probe beam with frequency ω_p and amplitude $\mathbf{E}_p$. The probe field is applied to the laser-driven transition and used to monitor the stationary absorption of the atomic transition. Above all, a substantial assumption is made that the probe field is sufficiently weak not to perturb the dressed-atom evolution [19]. Thus according to the linear-response theory [1], the steady-state absorption spectrum of the weak probe field can be presented in terms of the Fourier transform of the average value of the two-time commutator of the atomic dipole operators as [14]

$$A(\omega_p)=A_0\,\mathrm{Re}\int_0^\infty d\tau e^{i(\omega_p-\omega_L)\tau}\lim_{t\to\infty}\langle[\sigma_{12}(t+\tau),\sigma_{21}(t)]\rangle, \quad (8)$$

where $A_0=2\omega_p\gamma u(\mathbf{r})|\mathbf{d}\cdot\mathbf{E}_p|^2/\hbar$, and $u(\mathbf{r})=\frac{3}{8\pi}\sin^2\beta$ is a normalization constant, β is the angle between the observation direction $\mathbf{r}$ and the atomic transition dipole moment $\mathbf{d}$.

On the basis of the quantum regression theorem [24] and Fourier transform, the steady-state absorption spectrum is cast into the required form

$$A(\omega_p)=A_0\,\mathrm{Re}\{M_{22}[1-2X_3(\infty)]+M_{23}X_1(\infty)\}, \quad (9)$$

where $\mathbf{M}=[i(\omega-\omega_L)\mathbf{I}-\mathbf{Q}]^{-1}$ with $\mathbf{I}$ being an identity matrix.

In the numerical calculation, the frequencies Δ, Ω and $\omega_p-\omega_L$ are normalized to the decay rate γ, and the absorption spectrum is scaled in units of A_0. Figure 1 presents the steady-state absorption spectrum as a function of $\omega_p-\omega_L$ for $\Delta=0$, $\Omega=5$, and $\gamma_{ph}=0$ (dotted line), 1 (solid line) (a), 5 (b), 20 (c). When the noise vanishes, i.e., $\gamma_{ph}=0$, the absorption profile is the well-known absorption spectrum theoretically predicted by Mollow [1], which is in marked contrast to the conventional absorption spectrum with real and positive values. The negative values in the spectrum indicate not absorption but amplification, namely, the atom emits energy into the probe field, rather than absorbs from it. But when the strength γ_{ph} of the noise is much less than the Rabi frequency Ω, the spectrum still exhibits amplification, as shown in Fig. 1(a). When the strength γ_{ph} increases to 5, the spectrum evolves into an absorption doublet, accompanied with the broadened sidebands, comparing to the standard Mollow absorption spectrum [1], therefore, a dip is centered in the spectrum, as displayed in Fig. 1(b). When the strength is adjusted to 20, the two absorption peaks get broader, and the

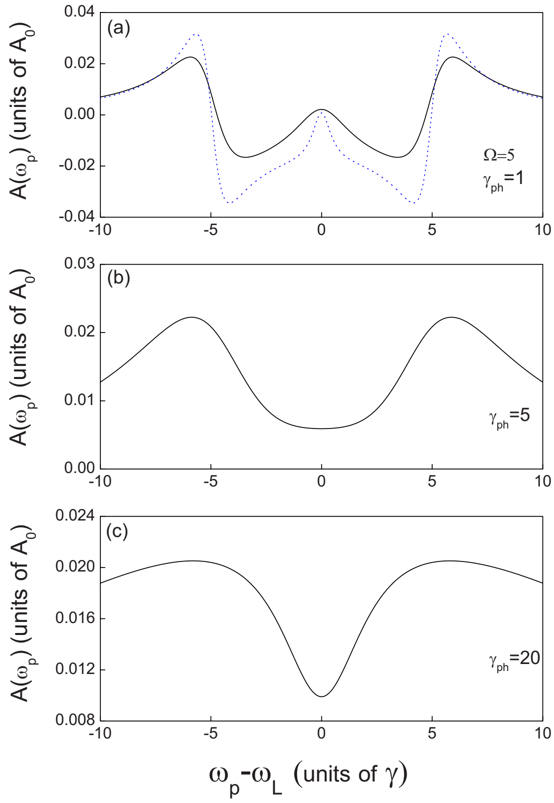

FIG. 1. (Color online) The steady-state absorption spectrum versus $\omega_p-\omega_L$ for $\Delta=0$, $\Omega=5$, and $\gamma_{ph}=0$ (dotted line), 1 (solid line) (a), 5 (b), 20 (c).

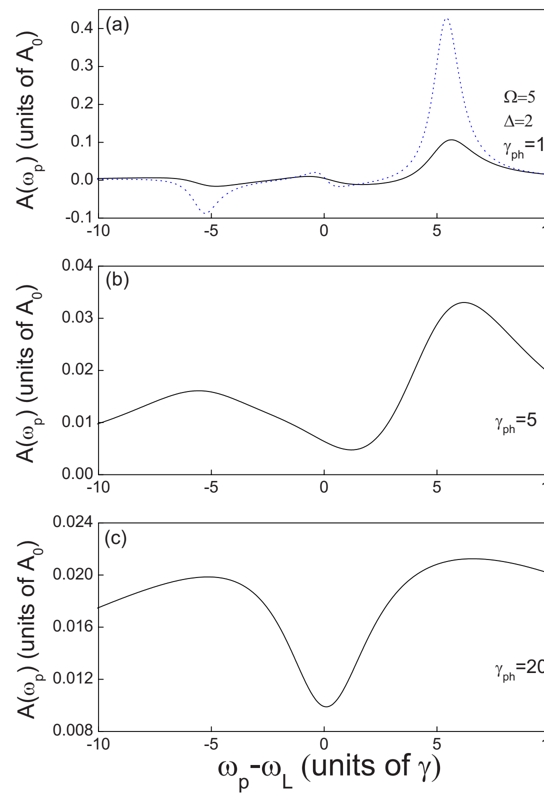

FIG. 2. (Color online) The steady-state absorption spectrum versus $\omega_p-\omega_L$ for $\Delta=2$, $\Omega=5$, and $\gamma_{ph}=0$ (dotted line), 1 (solid line) (a), 5 (b), 20 (c).

dip becomes narrower, as shown in Fig. 1(c).

Figure 2 presents the steady-state absorption spectrum as a function of $\omega_p-\omega_L$ for $\Delta=2$, $\Omega=5$, and $\gamma_{ph}=0$ (dotted line), 1 (solid line) (a), 5 (b), 20 (c). When the noise is absent, i.e., $\gamma_{ph}=0$, the profile is the absorption spectrum experimentally observed by Wu and his co-workers [3], where the negative values demonstrate gain, but with the increase in strength of the noise, the amplification sideband develops into an absorption peak, and the central dispersive profile reverts to a narrow dip.

The dressed-atom picture [19,20] provides useful insight into the dynamics of this system. The eigenvectors of the Hamiltonian $\langle H\rangle$ in Eq. (3) are written in the form

$$|+,N\rangle=\cos\theta|2\rangle|N-1\rangle-\sin\theta|1\rangle|N\rangle,$$

$$|-,N\rangle=\sin\theta|2\rangle|N-1\rangle+\cos\theta|1\rangle|N\rangle, \qquad (10)$$

and the eigenenergies are given by

$$E_{+,N}=\hbar(N\omega_L+\Omega_+),\quad E_{-,N}=\hbar(N\omega_L+\Omega_-), \qquad (11)$$

with

$$\Omega_+=\frac{\Delta+G}{2}>0,\quad \Omega_-=\frac{\Delta-G}{2}<0, \qquad (12)$$

where $\theta=\frac{1}{2}\arctan(\frac{\Omega}{\Delta})$ $(-\frac{\pi}{2}\leqslant\theta\leqslant\frac{\pi}{2})$, the product (undressed) state $|i\rangle|N\rangle$ $(N\gg1)$ is the state with the atom in the state $|i\rangle$ and N photons in the driving laser mode, $\Omega=g\sqrt{N}$ is the Rabi frequency of the laser field, and $G=\sqrt{\Delta^2+\Omega^2}$ is referred to as the generalized Rabi frequency. Thus the states (10) form an infinite ladder of doublets with interdoublet separated by ω_L and intradoublet spaced by G.

(i) In the dressed-atom basis the system is no longer a two-level system. It is a multilevel system with three different spontaneous transition frequencies, ω_L and $\omega_L\pm G$, and four nonvanishing dipole moments $\mathbf{d}_{ij,N}=\langle N+1,i|\mathbf{d}|j,N\rangle$ connecting the dressed states between the neighboring manifolds,

$$\mathbf{d}_{++,N}=\langle+,N+1|\mathbf{d}|+,N\rangle=-\mathbf{d}_{21}\sin\theta\cos\theta,$$

$$\mathbf{d}_{--,N}=\langle-,N+1|\mathbf{d}|-,N\rangle=\mathbf{d}_{21}\sin\theta\cos\theta,$$

$$\mathbf{d}_{+-,N}=\langle+,N+1|\mathbf{d}|-,N\rangle=\mathbf{d}_{21}\cos^2\theta,$$

$$\mathbf{d}_{-+,N}=\langle-,N+1|\mathbf{d}|+,N\rangle=-\mathbf{d}_{21}\sin^2\theta, \qquad (13)$$

which are parallel or antiparallel to each other.

(ii) Interaction between the atom and the vacuum modes of the electromagnetic field results in a spontaneous emission cascade by the dressed atom down its energy manifold ladder. The rate of a transition between any two dressed states is proportional to the absolute square of the dipole transition moment connecting them

$$\gamma_{ij} = \gamma|\langle i,N+1|\sigma_{21}|j,N\rangle|^2. \tag{14}$$

In light of Eqs. (10) and (14), the transition rates between the states $|i,N+1\rangle$ and $|j,N\rangle$ are listed as follows:

$$\gamma_{++} = \gamma_{--} = \gamma \sin^2\theta \cos^2\theta,$$

$$\gamma_{+-} = \gamma \cos^4\theta,$$

$$\gamma_{-+} = \gamma \sin^4\theta. \tag{15}$$

(iii) The effect of incoherent perturbation on the pure states can be determined from the Lindblad form (5) of the master equation (3). The action of the operator $\frac{1}{2}\sqrt{\gamma_{ph}}\sigma_z$ corresponds to an event generated by the stochastic noise. On resonance, the operator leads to the transitions between the dressed states $|+,N\rangle$ and $|-,N\rangle$,

$$\tfrac{1}{2}\sqrt{\gamma_{ph}}\sigma_z|+,N\rangle = \tfrac{1}{2}\sqrt{\gamma_{ph}}|-,N\rangle,$$

$$\tfrac{1}{2}\sqrt{\gamma_{ph}}\sigma_z|-,N\rangle = \tfrac{1}{2}\sqrt{\gamma_{ph}}|+,N\rangle. \tag{16}$$

In general, for the off-resonance case, in order to show the effect of the stochastic noise, the time-dependent Hamiltonian in the Langevin equation is transformed into the semiclassical dressed-state representation,

$$H = E_+|+,t\rangle\langle+,t| + E_-|-,t\rangle\langle-,t| = \mathcal{H}_{\rm free} + \mathcal{H}_{\rm ini},$$

$$\mathcal{H}_{\rm free} = E_+|+\rangle\langle+| + E_-|-\rangle\langle-|,$$

$$\mathcal{H}_{\rm ini} = -\frac{1}{2}\hbar\,\delta\omega_0(t)\frac{\Omega}{G}(|+\rangle\langle-| + |-\rangle\langle+|), \tag{17}$$

where $\mathcal{H}_{\rm free}$ is the diagonalized mean atomic Hamiltonian in the dressed-atom picture, and $\mathcal{H}_{\rm ini}$ indicates the transitions between the dressed states $|\pm\rangle$ resulting from the stochastic noise, with $|+,t\rangle \approx |+\rangle - \frac{\Omega}{2G^2}\delta\omega_0(t)|-\rangle$ and $|-,t\rangle \approx |-\rangle + \frac{\Omega}{2G^2}\delta\omega_0(t)|+\rangle$.

(iv) The reduced populations and coherence are defined as

$$\rho_{ii} = \sum_N \langle i,N|\rho|i,N\rangle, \quad \rho_{ij} = \sum_N \rho_{i,N;j,N-1}. \tag{18}$$

For the high intensity limit $\Omega \gg \gamma$, the coupling terms between matrix elements related to different frequencies may be omitted to $O(\gamma/\Omega)$. As a consequence, the equations for the diagonal and off-diagonal elements are decoupled from each other and are rewritten in the form

$$\dot{\rho}_{++} = -\Gamma_1\rho_{++} + \Gamma_0,$$

$$\dot{\rho}_{+-} = -(i\Omega_R + \Gamma_2)\rho_{+-} + \Gamma_3\rho_{-+},$$

$$\dot{\rho}_{-+} = -(-i\Omega_R + \Gamma_2)\rho_{-+} + \Gamma_3\rho_{+-}, \tag{19}$$

where

$$\Gamma_0 = \gamma \sin^4\theta + 2\gamma_{ph}\sin^2\theta\cos^2\theta,$$

$$\Gamma_1 = \gamma(\cos^4\theta + \sin^4\theta) + \gamma_{ph}\sin^2\theta(4\cos^2\theta - 1),$$

$$\Gamma_2 = \tfrac{1}{2}\gamma(1 + 2\sin^2\theta\cos^2\theta),$$

$$\Gamma_3 = (\gamma_{ph} - \gamma)\sin^2\theta\cos^2\theta. \tag{20}$$

If the Rabi frequency Ω is much larger than Γ_2 and Γ_3, the terms associated with different resonant frequencies, $\Gamma_3\rho_{-+}$ and $\Gamma_3\rho_{+-}$ in Eq. (19), are both negligibly small in the secular approximation [19]. As a result, the transitions $|+,N+1\rangle \leftrightarrow |-,N\rangle$ and $|-,N+1\rangle \leftrightarrow |+,N\rangle$ are independent. Otherwise, they are correlated [25], i.e., as the atom decays from $|+,N+1\rangle$ to $|-,N\rangle$, it drives the other transition from $|-,N\rangle$ to $|+,N+1\rangle$, and vice versa. It is revealed that the photons absorbing from these detected transitions are indistinguishable, thus quantum interference between these transition channels dominates.

In the noise-dominated regime, i.e., $\gamma_{ph} > \Omega$, among several possible absorption channels there are two pairs: $|-,N\rangle \to |+,N\rangle \to |-,N+1\rangle$ and $|-,N\rangle \to |+,N+1\rangle \to |-,N+1\rangle$ ($|+,N\rangle \to |-,N+1\rangle \to |+,N+1\rangle$ and $|+,N\rangle \to |-,N\rangle \to |+,N+1\rangle$) that differ exclusively by time order between collisional mixing and photon absorptions [26,27]. Photons absorbing along these channels are indistinguishable, so their interference is possible. For the sake of simplicity, two new operators are introduced, that is, $A_{ij}^{(+)} = |i,N+1\rangle\langle j,N|$, $A_{ji}^{(-)} = |j,N\rangle\langle i,N+1|$. Besides, there are two pairs of cross dampings between the possible transitions in the dressed-state basis. For the transitions $|-,N+1\rangle \to |-,N\rangle$ and $|-,N+1\rangle \to |+,N\rangle$, in terms of Eqs. (13) and (15), the cross damping is expressed as

$$\mathcal{L}_{--}^{-+}\rho = -\gamma|\sin^3\theta|\cos^2\theta[A_{--}^{(-)}\rho A_{-+}^{(+)} + A_{+-}^{(-)}\rho A_{--}^{(+)}], \tag{21}$$

for the transitions $|-,N+1\rangle \to |-,N\rangle$ and $|-,N\rangle \to |+,N+1\rangle$, the spontaneous emission-induced coherence is written in the form

$$\mathcal{L}_{+-}^{--}\rho = \frac{1}{2}\gamma\sin^2\theta\cos^3\theta \times \begin{bmatrix} 2A_{-+}^{(-)}\rho A_{--}^{(+)} - A_{--}^{(+)}A_{-+}^{(-)}\rho - \rho A_{--}^{(+)}A_{-+}^{(-)} \\ + 2A_{--}^{(-)}\rho A_{+-}^{(+)} - A_{+-}^{(+)}A_{--}^{(-)}\rho - \rho A_{+-}^{(+)}A_{--}^{(-)} \end{bmatrix}, \tag{22}$$

for the transitions $|+,N+1\rangle \to |+,N\rangle$ and $|+,N\rangle \to |-,N+1\rangle$, the cross coupling is of the form

$$\mathcal{L}_{++}^{+-}\rho = \frac{1}{2}\gamma|\sin^3\theta|\cos^2\theta \times \begin{bmatrix} 2A_{++}^{(-)}\rho A_{-+}^{(+)} - A_{-+}^{(+)}A_{++}^{(-)}\rho - \rho A_{-+}^{(+)}A_{++}^{(-)} \\ + 2A_{+-}^{(-)}\rho A_{++}^{(+)} - A_{++}^{(+)}A_{+-}^{(-)}\rho - \rho A_{++}^{(+)}A_{+-}^{(-)} \end{bmatrix}, \tag{23}$$

and for the transitions $|+,N+1\rangle \to |-,N\rangle$ and $|+,N+1\rangle \to |+,N\rangle$, the vacuum-induced quantum interference takes the form

$$\mathcal{L}_{-+}^{++}\rho = -\gamma\sin^2\theta\cos^3\theta[A_{-+}^{(-)}\rho A_{++}^{(+)} + A_{++}^{(-)}\rho A_{+-}^{(+)}]. \tag{24}$$

In the final analysis, stochastic noise plays several significant roles as follows:

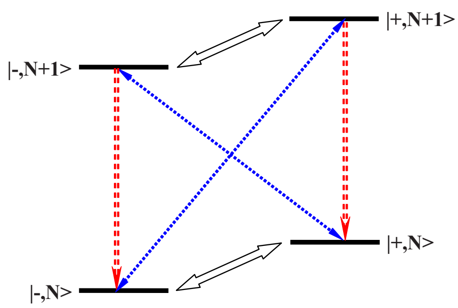

FIG. 3. (Color online) Different dressed-state transition channels for a coherently driven and stochastically perturbed two-level atom.

(a) Stochastic noise leads to the cross correlations between those transitions from different dressed states within the neighboring manifolds and produce multiple indistinguishable absorption routes, which makes quantum interference possible.

(b) Stochastic noise results in the transition between the different dressed states within any one manifold, and multiple closed loop configurations which are interlaced and overlapped, emerge due to the transitions within the same manifold and the adjacent manifolds.

(c) Multiple cross couplings between the transitions from the dressed states within the neighboring manifolds take place, and they are interrelated owing to multiple common transitions.

(d) The composite contributions of multiple indistinguishable absorption routes and cross couplings give rise to the modified absorption spectrum.

Therefore, it is the quantum interference induced by stochastic noise that plays a key role in the modification of the absorption spectrum of a coherently driven two-level atom.

IV. CONCLUSION

In conclusion, on the basis of the master equation for the reduced density matrix and from the quantum regression theorem, the absorption spectrum of a coherently driven and incoherently perturbed two-level atom is studied (Fig. 3). In the case of dominant incoherent perturbation, the amplification sideband is switched to the absorption one, and the central dispersive line shape is altered to a narrow dip.

In general, stochastic noise results in the cancellation of quantum interference effect. On the contrary, for the present system, stochastic noise leads to the transitions from the different dressed states within a given manifold, whereas the noise-induced transitions and spontaneous transitions produce multiple interlaced closed loop configurations. On the other hand, in the noise-dominated regime, stochastic noise gives rise to multiple indistinguishable absorption channels and interrelated cross couplings. It is not difficult to find that stochastic noise plays a crucial role in the modification of the standard absorption spectrum [1,3]. In essence, the modified absorption spectrum originates from the stochastic noise-induced quantum interference.

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APPENDIX

This model can also describe the system of two-level atoms driven by, e.g., a diode laser field with fluctuating phase. When a two-level atom with transition frequency ω_0 is driven by a diode laser field $\mathbf{E}(t)=\frac{1}{2}\mathbf{E}e^{-i\omega_L t-i\phi(t)}+\text{c.c.}$ with carrier frequency ω_L and amplitude $\mathbf{E}$ and fluctuating phase $\phi(t)$, the fluctuating phase $\phi(t)$ results in a stochastic frequency (Gaussian random force) $\nu(t)=\dot{\phi}(t)$, and the dynamics for the stochastic phase variable $\phi(t)$ is Markovian [28], i.e.,

$$\langle \nu(t)\rangle = 0, \quad \langle \nu(t)\nu(t')\rangle = 2\gamma'_{ph}\delta(t-t'), \tag{A1}$$

where γ'_{ph} is the strength of the frequency fluctuations and physically describes the effective bandwidth of the laser beam due to the phase diffusion if the phase drift is neglected.

In the frame oscillating with frequency ω_L and within the dipole approximation and the rotating wave approximation, the master equation for the reduced density matrix is expressed as

$$\dot{\rho} = -\frac{i}{\hbar}[H',\rho] + (\mathcal{L}\rho)_{sp}, \tag{A2}$$

where the Hamiltonian of the system is written in the form

$$H' = \hbar\Delta\sigma_{22} - \frac{\hbar}{2}(\Omega\sigma_{21}e^{-i\phi(t)} + \Omega^*\sigma_{12}e^{i\phi(t)}). \tag{A3}$$

Correspondingly, the equations of motion for the reduced density-matrix elements are given by

$$\dot{\rho}_{21} = -\left(\frac{1}{2}\gamma + i\Delta\right)\rho_{21} - i\Omega e^{-i\phi(t)}\rho_{22} + \frac{i}{2}\Omega e^{-i\phi(t)},$$

$$\dot{\rho}_{12} = -\left(\frac{1}{2}\gamma - i\Delta\right)\rho_{12} + i\Omega^* e^{i\phi(t)}\rho_{22} - \frac{i}{2}\Omega^* e^{i\phi(t)},$$

$$\dot{\rho}_{22} = -\frac{i}{2}\Omega^* e^{i\phi(t)}\rho_{21} + \frac{i}{2}\Omega e^{-i\phi(t)}\rho_{12} - \gamma\rho_{22}. \tag{A4}$$

After performing an ensemble average of (A4) over the ensemble of $\nu(t)$ and duplicating the procedures suggested in Ref. [28], but not shown here, one can obtain the same equations of motion as Eq. (A2) except for the substitution of γ'_{ph} for γ_{ph}. Obviously, the fluctuating phase leads to phase decay or dephasing process as well.

Therefore, the effect of the stochastic noise (due to collisions) on the phase damping is equivalent to that of a diode laser field with fluctuating phase on the phase decay. For the composite system of the atom plus driving field, the collisions between the atom directly lead to an atomic transition frequency shift, corresponding to phase decay, in an alternative way, the fluctuating phase of the driving field produces a Gaussian random force, which also results in dephasing process. In essence, both the fluctuating atomic transition frequency and Gaussian random force are assumed to obey the Markovian approximation (the system is Markovian), so the stochastic noise and the incoherent field both make the same contribution to the phase decay. Whether the two-level atoms are both driven by a perfectly coherent field and perturbed by collisions or the two-level atoms are driven by a nonzero-bandwidth laser with fluctuating phase, the atomic dynamics is effectively equivalent to the case that the unperturbed two-level atoms interact with a perfectly coherent field and zero-point fluctuations, and are damped by the phase randomization of the atomic dipole. Thereby, in the frame rotating with frequency ω_L and within the dipole approximation and the rotating wave approximation, the time evolution of the system is effectively described by the master equation for the reduced density matrix, which takes the form

$$\dot{\rho} = -\frac{i}{\hbar}[\mathcal{H},\rho] + (\mathcal{L}\rho)_{sp} + (\mathcal{L}\rho)_{st}. \tag{A5}$$

The Hamiltonian $\mathcal{H}$ in Eq. (A5) is referred to as the effective Hamiltonian of the system and is composed of two terms,

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2, \tag{A6}$$

where $\mathcal{H}_1 = \hbar\Delta\sigma_{22}$ is the unperturbed atomic Hamiltonian, and $\mathcal{H}_2 = -\frac{\hbar}{2}(\Omega\sigma_{21} + \Omega^*\sigma_{12})$ describes the coupling between the perfectly coherent driving field and the free atom.

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