Bose-Fermi pair correlations in attractively interacting Bose-Fermi atomic mixtures

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We study static properties of attractively interacting Bose-Fermi mixtures of uniform atomic gases at zero temperature. Using the Green's function formalism, we calculate the boson-fermion scattering amplitude and the fermion self-energy in the medium to lowest order of the hole-line expansion. We study the ground-state energy and pressure as a function of the scattering length for a few values of the boson-fermion mass ratio m_b/m_f and the number ratio N_b/N_f. We find that the attractive contribution to the energy is greatly enhanced for small values of the mass ratio. We study the role of Bose-Fermi pair correlations in the mixture by calculating the pole of the boson-fermion scattering amplitude in the medium. The pole shows a standard quasiparticle dispersion for a Bose-Fermi pair. In addition, we also study the fermion dispersion relation. We find two dispersion branches with the possibility of avoided crossings. This strongly depends on the number ratio N_b/N_f.

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I. INTRODUCTION

Recent developments in the field of cold atomic gases have proven that this system provides an ideal laboratory for the studies of quantum many-body systems [1]. This is due to the experimental facilities which allow control of various parameters characterizing the many-body system, e.g., external potentials including optical lattices, choice of atoms obeying Bose or Fermi statistics and their mixtures, variable particle densities, etc. The use of Feshbach resonances, in particular, makes atomic gases an extremely flexible system as it provides a means to control atomic interactions [2,3]. It was thus possible, for instance, to study the crossover from a Bose-Einstein condensate (BEC) to a BCS system in a two-component Fermi system, which has been under intense investigation for decades [4]. By changing the resonance energies through the external magnetic field, one can in principle change the magnitude and the sign of the scattering length of the interacting particles, keeping track all the way from a resonating fermion pair to a bound composite particle, a bosonic molecule.

The aim of the present paper is a study of pair correlations in a different system, a Bose-Fermi (BF) mixture. Degenerate mixtures of bosons and fermions have been created for several years, and studies of static and dynamic properties have been performed [5]. Among those are studies of attractively interacting BF systems, where one finds a sudden loss of fermions as the BF attractive interaction is effectively increased [6]. A mean-field study of this stability was performed in [7]. However, recently the finding of Feshbach resonances and formation of boson-fermion molecules have been reported [8,9]. It is thus expected that by controlling the BF interaction one may realize an analog of the process found in two-component Fermi systems. What should be expected if one replaces fermion pairs in the BEC-BCS crossover process by BF pairs? Such studies have indeed been performed theoretically [10] (see also [11]). By adopting a Cooper-type two-particle problem on top of the boson-fermion degenerate system, it was shown that a stable correlated BF pair is created even before the threshold for the BF bound state. In contrast to the BCS case, however, the system allows only one correlated BF pair with a given center-of-mass (c.m.) momentum because of the fermionic nature of the composite particle. It is then suggested that, by increasing the BF attractive interaction, one may create BF pairs with different c.m. momentum stepwise, until finally a new Fermi sea of BF pairs is completed.

In Ref. [10] a separable BF interaction was adopted to elucidate the mechanism of the creation of BF pairs. In the present paper we adopt a more standard pseudopotential for the interaction, and calculate the energy and pressure of the system for various values of input parameters. We use the Green's function formalism for this system and calculate perturbatively the relevant diagrams to lowest order of the hole-line expansion. This formalism has been developed in [12] together with the calculation of the energies including the Bose-Bose (BB) interaction. Our formulation is similar to that of [12], but we use the renormalization procedure of [13] in relating the pseudopotential strength to the *S*-wave scattering length. This allows us to formally take the limit $|a| \to \infty$, the unitarity limit [14], which is necessary when one considers a (nearly) bound state of a pair of atoms. We then calculate the poles of the BF pair scattering amplitude in the BF medium, which may be compared with the results of [10]. Studies of the behavior of the poles as a function of input parameters give us suggestions about the role of the BF pair correlations in the static properties of the system.

The content of the paper is as follows. In the next section we present our model based on the Hamiltonian without Bose-Bose interaction. We calculate the BF scattering amplitude in a BF mixture in the ladder approximation, and give formulas for physical quantities in terms of the amplitude. In Sec. III we show numerical results for the ground-state energy and pressure for various choices of the boson and fermion masses and values of the Bose-Fermi interaction. We then study Bose-Fermi pair correlation in Sec. IV by focusing on the pole structure of the boson-fermion scattering amplitude in the mixture. We also calculate the pole of the single-fermion Green's function and study the role of the Bose-Fermi pair and its dispersion. We summarize our results in Sec. V, together with a comment on the effects of the Bose-Bose interaction. Detailed expressions for the scattering amplitude are given in the Appendix.

II. FORMULATION

We consider a uniform system of a polarized Bose-Fermi mixture of atomic gases with attractive boson-fermion interaction. The model Hamiltonian of the system is given by

$$H = T_b + T_f + H_{bf},$$

$$T_b = \int d^3\mathbf{x}\, \phi^\dagger(\mathbf{x})\left(-\frac{\nabla^2}{2m_b} - \mu_b\right)\phi(\mathbf{x}),$$

$$T_f = \int d^3\mathbf{x}\, \psi^\dagger(\mathbf{x})\left(-\frac{\nabla^2}{2m_f} - \mu_f\right)\psi(\mathbf{x}),$$

$$H_{bf} = g_{bf}\int d^3\mathbf{x}\, \phi^\dagger(\mathbf{x})\psi^\dagger(\mathbf{x})\psi(\mathbf{x})\phi(\mathbf{x}), \tag{1}$$

where ψ and ϕ are the boson and fermion field operators, respectively, T_b and T_f denote bosonic and fermionic kinetic energies, while H_{bf} denotes the boson-fermion interaction with strength g_{bf} (<0) of the boson-fermion pseudopotential. Effects of the boson-boson interaction will be mentioned later, while the fermion-fermion interaction is omitted throughout as we consider one-component (polarized) fermions. We will adopt the Bogoliubov approximation in treating the Bose-Einstein condensate, and therefore include in T_b the bosonic chemical potential μ_b. A chemical potential μ_f for the fermions is also introduced.

A. Green's function formalism in the Bose-Fermi mixture

Contrary to the formulation developed in [10,17], which is number conserving, we here adopt for the condensed bosons the conventional Bogoliubov method by separating the zero-momentum mode from the remainder:

$$\phi(\mathbf{x}) = \sqrt{n_0} + \varphi(\mathbf{x}), \tag{2}$$

together with its conjugate.

$$n_0 = N_0/V \tag{3}$$

is the number density of bosons with momentum $\mathbf{k}=\mathbf{0}$. As usual, we omit the fluctuation of the boson number in the condensate. The boson number operator $\hat{N}_b$ is written

$$\hat{N}_b = N_0 + \int d^3\mathbf{x}\, \varphi^\dagger(\mathbf{x})\varphi(\mathbf{x}), \tag{4}$$

and the Hamiltonian takes the form

$$H = H_0 + H_{bf}, \tag{5}$$

where

$$H_0 = \int d^3\mathbf{x}\, \varphi^\dagger(\mathbf{x})\left(-\frac{\nabla^2}{2m_b} - \mu_b\right)\varphi(\mathbf{x}) + \int d^3\mathbf{x}\, \psi^\dagger(\mathbf{x})\left(-\frac{\nabla^2}{2m_f} - \mu_f\right)\psi(\mathbf{x}) - \mu_b N_0 \tag{6}$$

and

$$H_{bf} = n_0 g_{bf}\int d^3\mathbf{x}\, \psi^\dagger(\mathbf{x})\psi(\mathbf{x}) + \sqrt{n_0}g_{bf}\int d^3\mathbf{x}\, \psi^\dagger(\mathbf{x})\psi(\mathbf{x}) \times[\varphi^\dagger(\mathbf{x}) + \varphi(\mathbf{x})] + g_{bf}\int d^3\mathbf{x}\, \psi^\dagger(\mathbf{x})\varphi^\dagger(\mathbf{x})\varphi(\mathbf{x})\psi(\mathbf{x}). \tag{7}$$

Physical quantities can be expressed in terms of Green's functions. We define the boson and fermion Green's functions by

$$iG^f(x-y) = \frac{\langle\Psi_0|T[\psi_H(x)\psi_H^\dagger(y)]|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle}, \tag{8}$$

$$iG^b(x-y) = \frac{\langle\Psi_0|T[\varphi_H(x)\varphi_H^\dagger(y)]|\Psi_0\rangle}{\langle\Psi_0|\Psi_0\rangle}, \tag{9}$$

where $\psi_H(x)$ and $\varphi_H(x)$ are the field operators in the Heisenberg picture, and $|\Psi_0\rangle$ represents the interacting ground state.

The energy of the system can be expressed in terms of the Green's functions. The fermion and boson kinetic energies are calculated according to the standard procedure [15] as

$$\langle T_f\rangle = \left\langle\frac{-\nabla^2}{2m_f}\right\rangle = -iV\int\frac{d^4q}{(2\pi)^4}\epsilon_{\mathbf{q}}^f G^f(q)e^{iq_0\eta}, \tag{10}$$

$$\langle T_b\rangle = \left\langle\frac{-\nabla^2}{2m_b}\right\rangle = iV\int\frac{d^4q}{(2\pi)^4}\epsilon_{\mathbf{q}}^b G^b(q)e^{iq_0\eta}, \tag{11}$$

where the $G(q)$'s are the Fourier transforms of the Green's functions, η is a positive infinitesimal, and we set $\epsilon_{\mathbf{q}}^{b,f} = \mathbf{q}^2/2m_{b,f}$. The different signs in the two expressions come from the ordering of the field operators.

To calculate the interaction energy, we first consider the Heisenberg equation of motion for the fermion field:

$$i\frac{\partial}{\partial t}\psi_H(x) = [\psi_H(x), H] = \left(-\frac{\nabla^2}{2m_f} - \mu_f\right)\psi_H(x) + n_0 g_{bf}\psi_H(x) + \sqrt{n_0}g_{bf}\psi_H(x) \times[\varphi_H^\dagger(x) + \varphi_H(x)] + g_{bf}\psi_H(x)\varphi_H^\dagger(x)\varphi_H(x). \tag{12}$$

Multiplying by $\psi_H^\dagger(x')$ and integrating over $\mathbf{x}$, we obtain

$$\langle H_{bf}\rangle = -i\int d^3\mathbf{x} \lim_{\substack{\mathbf{x}'\to\mathbf{x}\\ t'\to t}} \left(i\frac{\partial}{\partial t}+\frac{\nabla^2}{2m_f}+\mu_f\right)G^f(t\mathbf{x},t'\mathbf{x}')$$
$$= -iV\int \frac{d^4q}{(2\pi)^4}(q_0-\epsilon^f_{\mathbf{q}}+\mu_f)G^f(q)e^{iq_0\eta}. \quad (13)$$

We now introduce fermion and boson self-energies $\Sigma^f(q)$ and $\Sigma^b(q)$ through

$$G^{b,f}(q)=\frac{1}{q_0-\epsilon^{b,f}_{\mathbf{q}}+\mu_{b,f}-\Sigma^{b,f}(q)}. \quad (14)$$

In the integrand of Eq. (13) one may use the relation from Eq. (14)

$$(q_0-\epsilon^f_{\mathbf{q}}+\mu_f)G^f(q)=1+\Sigma^f(q)G^f(q), \quad (15)$$

leading to

$$\langle H_{bf}\rangle = -iV\int \frac{d^4q}{(2\pi)^4}\Sigma^f(q)G^f(q)e^{iq_0\eta}, \quad (16)$$

the first term in the right-hand side in Eq. (15) giving a null contribution to the integral. The total energy E of the system is finally obtained as

$$E=\langle T_f\rangle+\langle T_b\rangle+\langle H_{bf}\rangle$$
$$= -iV\int\frac{d^4q}{(2\pi)^4}[\epsilon^f_{\mathbf{q}}+\Sigma^f(q)]G^f(q)e^{iq_0\eta}$$
$$+iV\int\frac{d^4q}{(2\pi)^4}\epsilon^b_{\mathbf{q}}G^b(q)e^{iq_0\eta}. \quad (17)$$

The thermodynamic potential at zero temperature is given by

$$\Omega(N_f,N_0,\mu_b)=\langle H\rangle=E-\mu_b\langle\hat{N}_b\rangle-\mu_f\langle\hat{N}_f\rangle, \quad (18)$$

where

$$\langle\hat{N}_b\rangle=N_0+iV\int\frac{d^4q}{(2\pi)^4}G^b(q)e^{iq_0\eta} \quad (19)$$

and

$$\langle\hat{N}_f\rangle=-iV\int\frac{d^4q}{(2\pi)^4}G^f(q)e^{iq_0\eta}. \quad (20)$$

The parameter N_0 should be chosen to minimize the thermodynamic potential

$$\frac{\partial\Omega}{\partial N_0}=0, \quad (21)$$

which leads to an explicit expression for μ_b as shown below.

We also will calculate the pressure to discuss the stability of the system. As usual, it is obtained from the thermodynamic relation

$$P=-\frac{\partial E}{\partial V}. \quad (22)$$

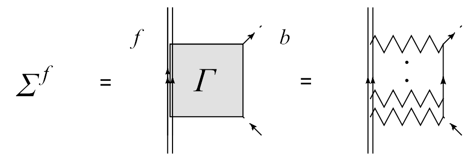

FIG. 1. Fermion self-energy in ladder approximation. The double solid lines represent fermion propagation, while the single solid line represents noncondensed free boson propagation. The arrows denote condensed bosons and are associated with the factor $\sqrt{n_0}$. The zigzag lines represent the boson-fermion interaction g_{bf}.

B. Self-energy in the ladder approximation

To obtain the total energy of the system we calculate the fermion self-energy Σ^f in the ladder approximation. Here the self-energy is expressed in terms of the two-particle scattering amplitude $\Gamma(\mathbf{q},\mathbf{q}',P)$, in the medium of a Bose-Fermi mixture as shown in Fig. 1. The interaction energy is accordingly calculated up to the lowest two-particle correlation diagram, Fig. 2, in the spirit of the hole-line expansion [15].

The scattering amplitude Γ in the present model obeys the integral equation [10,12,15,16]

$$\Gamma(\mathbf{q},\mathbf{q}',P)=g_{bf}+ig_{bf}\int\frac{d^4k}{(2\pi)^4}G^f_0\left(\frac{m_f}{m_f+m_b}P+k\right)$$
$$\times G^b_0\left(\frac{m_b}{m_f+m_b}P-k\right)\Gamma(\mathbf{k},\mathbf{q}',P), \quad (23)$$

where P denotes the four-momentum of the center-of-mass motion of the interacting particles, while $\mathbf{q}$ and $\mathbf{q}'$ are the relative momenta in the final and initial states. The boson and fermion free Green's functions in the medium are given by

$$G^f_0(p)=\frac{\theta(|\mathbf{p}|-k_F)}{p_0-\epsilon^f_{\mathbf{p}}+\mu_f+i\eta}+\frac{\theta(k_F-|\mathbf{p}|)}{p_0-\epsilon^f_{\mathbf{p}}+\mu_f-i\eta}, \quad (24)$$

$$G^b_0(p)=\frac{1}{p_0-\epsilon^b_{\mathbf{p}}+\mu_b+i\eta}, \quad (25)$$

where

$$k_F=\sqrt{2m_f\mu_f}. \quad (26)$$

After the integration over k_0, Eq. (23) becomes

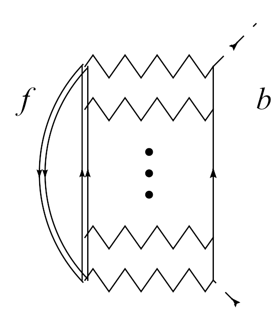

FIG. 2. Ladder diagram contribution to the interaction energy. The downward double solid line indicates a hole propagation. Otherwise as in Fig. 1.

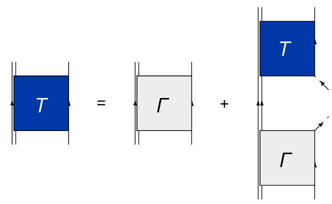

FIG. 3. (Color online) Graphical equation for full BF T matrix.

$$\Gamma(\mathbf{q},\mathbf{q}',P) = g_{bf} + g_{bf} \times \int \frac{d^3\mathbf{k}}{(2\pi)^3} \frac{\theta(|\tilde{\mathbf{P}}_f + \mathbf{k}| - k_F)}{P_0 - \epsilon^f_{\tilde{\mathbf{P}}_f+\mathbf{k}} - \epsilon^b_{\tilde{\mathbf{P}}_b-\mathbf{k}} + \mu_b + \mu_f + i\eta} \times \Gamma(\mathbf{k},\mathbf{q}',P) \tag{27}$$

with $\tilde{\mathbf{P}}_f = m_f/(m_f+m_b)\mathbf{P}$ and $\tilde{\mathbf{P}}_b = m_b/(m_f+m_b)\mathbf{P}$, with respect to the T-matrix equation in [10,17]. We dropped the hole propagation part in accordance with the present approximation. With respect to the T-matrix equation in [10], we notice that there the phase space factor in (27) is replaced by $\theta(|\tilde{\mathbf{P}}_f+\mathbf{k}|-k_F) \to \theta(|\tilde{\mathbf{P}}_f+\mathbf{k}|-k_F)+N_0$. This is natural, because in [10] the shift operation (2) for the bosons was not performed and therefore the free boson occupancy N_0 appears additionally. The two formulations are, however, essentially equivalent. From the structure of Eq. (27), one easily finds that Γ depends only on the variable P, and we hereafter write simply $\Gamma(P)$. Using the above vertex function, we can calculate the proper self-energies for the fermion and the boson as

$$\Sigma^f(p) = n_0 \Gamma(p), \tag{28}$$

$$\Sigma^b(p) = -i \int \frac{d^4p'}{(2\pi)^4} G_0^f(p')\Gamma(p+p'). \tag{29}$$

One should notice that Γ is not the full BF scattering matrix because only the bosons out of the condensate are considered. The full BF T matrix is graphically represented in Fig. 3 and this equation is given by

$$T(P) = \Gamma(P) + \Gamma(P)n_0 G_0^f(P)T(P) = (P_0 - \epsilon^f_{\mathbf{P}} + \mu_f)\Gamma(P)G^f(P). \tag{30}$$

Here we used Eq. (14) and the relation between $\Gamma(P)$ and $\Sigma^f(P)$ which is shown in (28). The pole structure is then the same as that of the fermion Green's function, Eq. (14).

Please note that, as in the usual procedure for the T-matrix approach, all the medium ingredients, except for the chemical potentials, are taken at their noninteracting values. This particularly means that the Fermi occupation numbers stay step functions and all bosons N_b are in the condensate, i.e., $N_0 = N_b$ in Eq. (30). We use this free gas occupation throughout this work. It is a weak-coupling approximation. More realistically, one should calculate the fermion and boson occupation numbers self-consistently. This, however, is a difficult task and left for future work. Nonetheless, to get a feeling for the qualitative behavior, we also will use our formalism for rather strong boson-fermion interactions. Such procedure is familiar from the study of two-component Fermi gases.

For the calculation of Γ, we used some regularization procedures. One knows that the integral in Eq. (27) requires a momentum cutoff, which originates from the use of the zero-range interaction. We can remedy this shortcoming by employing the observable S-wave scattering length a, instead of the pseudopotential coupling constant g_{bf}. We perform this renormalization following the procedure adopted in [13] (see also [18]), slightly different from the one in [12]. The S-wave scattering length is related to the two-particle scattering amplitude Γ_0 in vacuum by the relation

$$\Gamma_0(\mathbf{q}=\mathbf{q}'=\mathbf{0},\ P=0) = \frac{2\pi a}{\nu} \tag{31}$$

where ν is the reduced mass

$$\nu^{-1} = m_f^{-1} + m_b^{-1}, \tag{32}$$

and Γ_0 obeys an equation similar to Eq. (23) with G_0 replaced by the free Green's function in vacuum. By solving the equation for Γ_0 one obtains

$$\frac{2\pi a}{\nu} = \frac{g_{bf}}{1 + g_{bf}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{1}{\epsilon^f_{\mathbf{k}} + \epsilon^b_{\mathbf{k}}}}, \tag{33}$$

where the integral in the denominator involves again an implicit momentum cutoff. Now one may combine the above expression with Eq. (27), and eliminate g_{bf} in favor of the scattering length a, finally obtaining

$$\Gamma(P) = \frac{2\pi a}{\nu}\left(1 - \frac{2\pi a}{\nu} I(P_0,|\mathbf{P}|)\right)^{-1} \tag{34}$$

with

$$I(P_0,|\mathbf{P}|) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left(\frac{\theta(|\tilde{\mathbf{P}}_f+\mathbf{k}|-k_F)}{P_0 - \epsilon^f_{\tilde{\mathbf{P}}_f+\mathbf{k}} - \epsilon^b_{\tilde{\mathbf{P}}_b-\mathbf{k}} + \mu_b + \mu_f + i\eta} + \frac{1}{\epsilon^f_{\mathbf{k}} + \epsilon^b_{\mathbf{k}}}\right). \tag{35}$$

Since the integral in the denominator is convergent at large $|\mathbf{k}|$, we can let the momentum cutoff go to infinity. The expression (34) involves all orders in the scattering length and allows us to formally take the unitarity limit $|a| \to \infty$ in the following section. This limit has been studied for two-component Fermi systems in relation to the BEC to BCS crossover phenomenon. Whether or not a similar phenomenon is expected for Bose-Fermi pairs will be studied in the next section.

III. RESULTS FOR TOTAL ENERGY AND PRESSURE

We calculate the energy of the system in the leading order of the hole-line expansion, that is, we replace the Green's functions in Eq. (17) with the free one $G_0^{f,b}$ in Eq. (25), and obtain

$$E \sim -iV\int \frac{d^4q}{(2\pi)^4}[\epsilon_{\mathbf{q}}^f + \Sigma^f(q)]G_0^f(q)e^{iq_0\eta}$$
$$= \frac{3}{5}E_F N_f\left(\frac{\mu_f}{E_F}\right)^{5/2} + N_0\int \frac{d^3\mathbf{p}}{(2\pi)^3}\theta(k_F - |\mathbf{p}|)\Gamma(\epsilon_{\mathbf{p}}^f,\mathbf{p}), \tag{36}$$

where E_F is the Fermi energy of a noninteracting Fermi gas. Within the same approximations, the thermodynamic potential at zero temperature is given by

$$\Omega = \frac{3}{5}E_F N_f\left(\frac{\mu_f}{E_F}\right)^{5/2} + N_0\int \frac{d^3\mathbf{p}}{(2\pi)^3}\theta(k_F - |\mathbf{p}|)\Gamma(\epsilon_{\mathbf{p}}^f,\mathbf{p}) - \mu_b N_0 - \mu_f N_f. \tag{37}$$

Thus, the equilibrium condition (21) for Ω leads to integral equations for μ_b and μ_f,

$$\mu_b = \int \frac{d^3\mathbf{p}}{(2\pi)^3}\theta(k_F - |\mathbf{p}|)\Gamma(\epsilon_{\mathbf{p}}^f,\mathbf{p}), \tag{38}$$

where Γ depends on μ_b and μ_f. From (20), another equation for μ_b and μ_f is given by

$$N_f = -iV\int \frac{d^4q}{(2\pi)^4}G^f(q)e^{iq_0\eta}. \tag{39}$$

This equation can be rewritten in quasiparticle approximation [10] as

$$N_f = \int \frac{d^3\mathbf{q}}{(2\pi)^3}\sum_i \frac{\theta(-\xi_{\mathbf{q}}^i)}{1 - \left.\frac{\partial\Sigma^f}{\partial q_0}\right|_{q_0=\xi_{\mathbf{q}}^i}}, \tag{40}$$

where $\xi_{\mathbf{q}}^i$ is the pole of the fermion Green's function and i represents the number of solutions.

The total energy of the system is then finally given by

$$E = \frac{3}{5}E_F N_f\left(\frac{\mu_f}{E_F}\right)^{5/2} + \mu_b N_b. \tag{41}$$

Details of the calculation and the analytic expression for Γ are given in the Appendix.

We may rewrite Eqs. (38) and (40) in scaled form as

$$\tilde{\mu}_b = 2\left(1 + \frac{1}{\zeta}\right)\int_0^{\sqrt{\tilde{\mu}_f}} d\tilde{p}\,\tilde{p}^2\tilde{\Gamma}(\tilde{p},\tilde{\mu}_b,\tilde{\mu}_f,\zeta) \tag{42}$$

and

$$1 = \int \frac{d^3\tilde{\mathbf{q}}}{(2\pi)^3}\sum_i \frac{\theta(-\tilde{\xi}_{\mathbf{q}}^i)}{1 - \left.\frac{\partial\Sigma^f}{\partial q_0}\right|_{q_0=\xi_{\mathbf{q}}^i}}, \tag{43}$$

where we introduced dimensionless quantities (with tildes) through

$$\Gamma(p,\mu_b) = \frac{2\pi^2}{\nu k_{F0}}\tilde{\Gamma}(\tilde{p},\tilde{\mu}_b),$$

$$\tilde{a} = k_{F0}a, \quad \tilde{p} = \frac{|\mathbf{p}|}{k_{F0}}, \quad \tilde{\mu}_{b,f} = \frac{\mu_{b,f}}{E_F}, \quad \tilde{\xi}_{\mathbf{q}}^i = \frac{\xi_{\mathbf{q}}^i}{E_F}.$$

Here, k_{F0} represent the Fermi momentum in the noninteracting Fermi gas, which is related to the Fermi energy as

$$E_F = \frac{k_{F0}^2}{2m_f}.$$

The expression shows that the scaled chemical potential $\tilde{\mu}_b$ depends only on the mass ratio

$$\zeta = \frac{m_b}{m_f}$$

and the dimensionless scattering length $\tilde{a}$. We solved Eqs. (42) and (43) for $\tilde{\mu}_b$ and $\tilde{\mu}_f$ numerically as functions of the boson-fermion mass ratio ζ for different values of the interaction strength represented by $\tilde{a}$. In terms of the scaled quantities, the ground-state energy per particle is expressed from Eq. (41) as

$$\frac{E}{N_f} = \frac{3}{5}E_F\gamma, \tag{44}$$

where the dimensionless parameter γ is given by

$$\gamma = \tilde{\mu}_f^{5/2} + \frac{5}{3}\tilde{\mu}_b\frac{N_b}{N_f}. \tag{45}$$

We first show the results for energy and pressure in the unitarity limit, $|a| \to \infty$. In this limit, assuming S-wave scattering and neglecting effective range, we are left with only one length scale k_F^{-1}, or $n_f^{-1/3}$ in terms of the density $n_f = N_f/V$ [14,19] for a given mass ratio ζ. Note that the chemical potentials $\tilde{\mu}_b$ and $\tilde{\mu}_f$ have no k_F dependence in the unitarity limit, and the parameter γ depends only on the number ratio N_b/N_f. Thus the ground-state energy per particle, Eq. (44), is proportional to E_F, and the dependence on the parameters are all absorbed in a simple multiplicative factor γ.

We show in Fig. 4 the chemical potentials μ_b and μ_f and in Fig. 5 the γ parameter as functions of the mass ratio $\zeta = m_b/m_f$, both in the unitarity limit. Note that the results are independent of the magnitude of the individual mass parameters as we show dimensionless quantities scaled with E_F. Figure 4 shows that the boson chemical potential is always negative. This fact reflects the attractive boson-fermion interaction in the unitarity limit, in accordance with Eq. (33), which implies negative g_{bf}. The behavior of γ in Fig. 5 simply follows that of the chemical potential. The results suggest that the attractive interaction becomes more effective for small values of the mass ratio ζ, and the effect is greatly enhanced as the particle number ratio N_b/N_f becomes larger, that is, as the number of bosons increases with respect to the fermions. The dependence on ζ may partly be understood by noting that the relative phase space available for the intermediate states in the two-body scattering in the mixture will be larger for small ζ, i.e., for a relatively larger m_f, because of

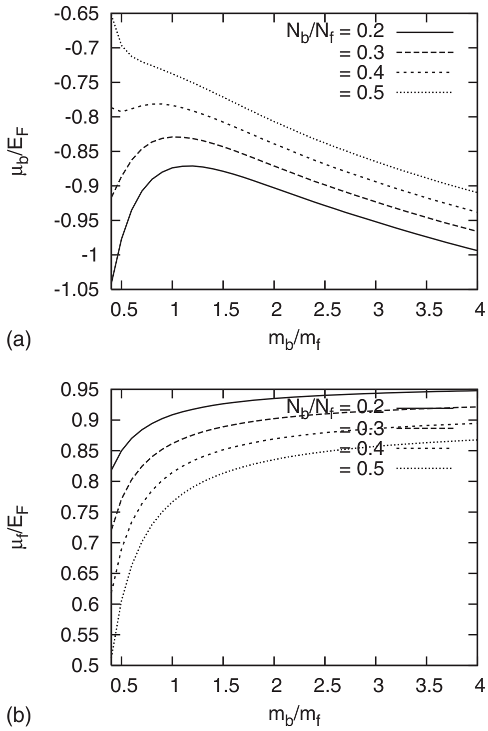

FIG. 4. Scaled chemical potentials $\tilde{\mu}_{b,f}=\mu_{b,f}/E_F$ as a function of the boson-fermion mass ratio $\zeta=m_b/m_f$ in the unitarity limit for $N_b/N_f=0.2,0.3,0.4,0.5$.

the lower Fermi energy and higher level density.

We next consider the pressure to study the stability of the system. Since the total energy takes a universal form and the γ parameter has no volume dependence in the unitarity limit, the pressure is simply given by

$$P=-\frac{\partial E}{\partial V}=\frac{2}{5}\frac{N_f}{V}E_F\gamma \quad \text{(unitarity limit)}. \tag{46}$$

For $\gamma<0$ the pressure becomes negative, and the system collapses. This happens especially for larger values of N_b/N_f, where the pressure always becomes negative irrespective of the mass ratio ζ. In actual experiments, e.g., for the ^{40}K-^{87}Rb mixture which has $\zeta\sim 2.3$, the number ratio is typically $O(1)-O(10^3)$ and the system will collapse in the unitarity limit. This is not in contradiction to recent experimental results [6].

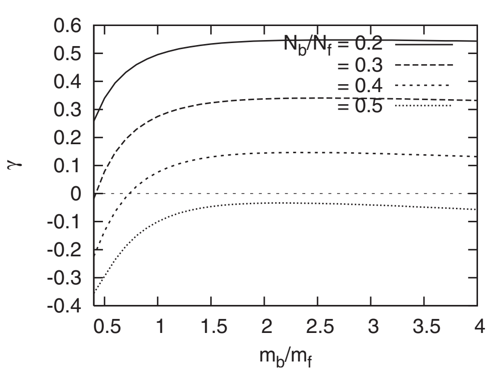

FIG. 5. γ parameter as a function of the boson-fermion mass ratio in the unitarity limit for $N_b/N_f=0.2,0.3,0.4,0.5$ from top to bottom.

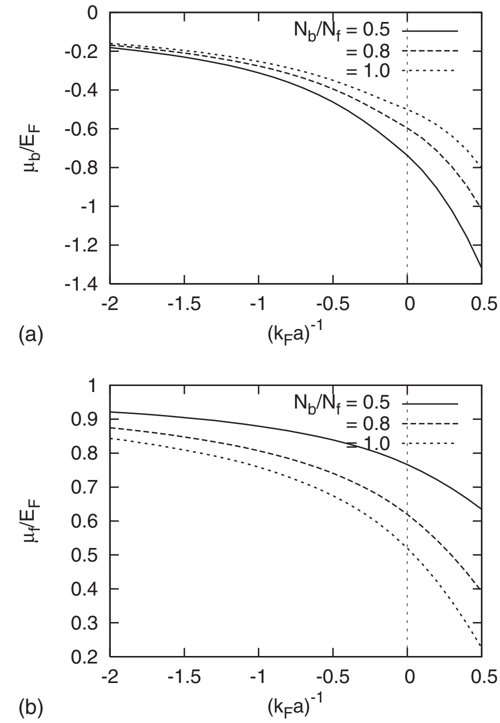

FIG. 6. Scaled chemical potentials $\tilde{\mu}_{b,f}=\mu_{b,f}/E_F$ as a function of $(k_Fa)^{-1}$ for $m_b/m_f=1$ and $N_b/N_f=0.5,0.8,1.0$.

We now study the case with an arbitrary value of the scattering length, keeping in mind that for $a>0$, i.e., in the strong-coupling regime, our T-matrix approximation is not valid, and the corresponding values should be considered as indicative only. First, we show the chemical potential μ_b as a function of $(k_Fa)^{-1}$ in Fig. 6, where the mass ratio ζ and N_b/N_f are set to 1. Then we show the energy and pressure as functions of $(k_Fa)^{-1}$ in Figs. 7–9, for the mass ratio ζ=0.8, 1.0, and 1.2, with different values of $N_b/N_f=0.5,0.8,1.0$. For $a\neq 0$, the pressure is given by

$$P=\frac{2}{5}\frac{N_f}{V}E_F\left(\tilde{\mu}_f^{5/2}+\frac{5}{3}\tilde{\mu}_b\frac{N_b}{N_f}-\frac{15}{4}\tilde{\mu}_f^{3/2}V\frac{\partial\tilde{\mu}_f}{\partial V}-\frac{3}{2}V\frac{\partial\tilde{\mu}_b}{\partial V}\frac{N_b}{N_f}\right), \tag{47}$$

where the third and fourth terms reflect that the chemical potentials, and hence the γ parameter, depend on V. From the results for energies, we see that the system becomes more attractive at smaller values of the mass ratio ζ as in the unitarity limit, although the effect is not large in this parameter range. We find a strong increase of the attraction as the parameter $(k_{F0}a)^{-1}$ passes through zero, the unitarity limit, from negative to positive. This is in accord with the naive picture where positive values of the scattering length imply a newly formed bound state. One should, however, note that even in the present approximation the effects of the medium modify the two-body scattering amplitude, and a simple picture of independent bound pairs does not hold in general.

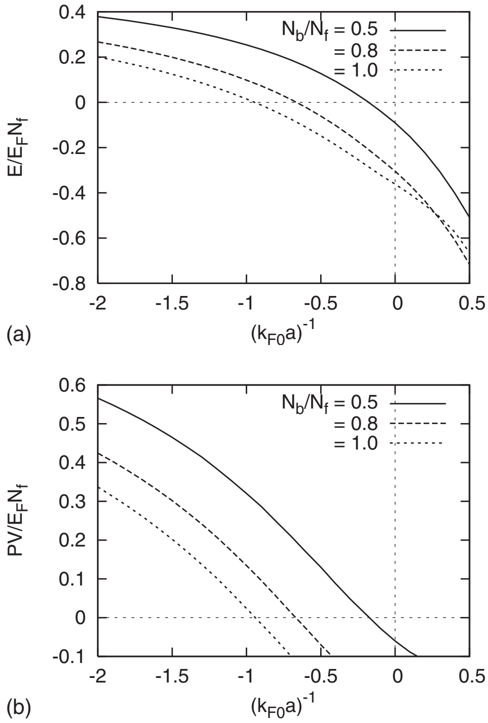

FIG. 7. Scaled energy E/E_FN_f (upper panel) and pressure PV/E_FN_f (lower panel) as a function of $(k_Fa)^{-1}$ for m_b/m_f=0.8. Three curves corresponds to N_b/N_f=0.5, 0.8, 1.0 from top to bottom. The vertical line indicates the unitarity limit a=∞.

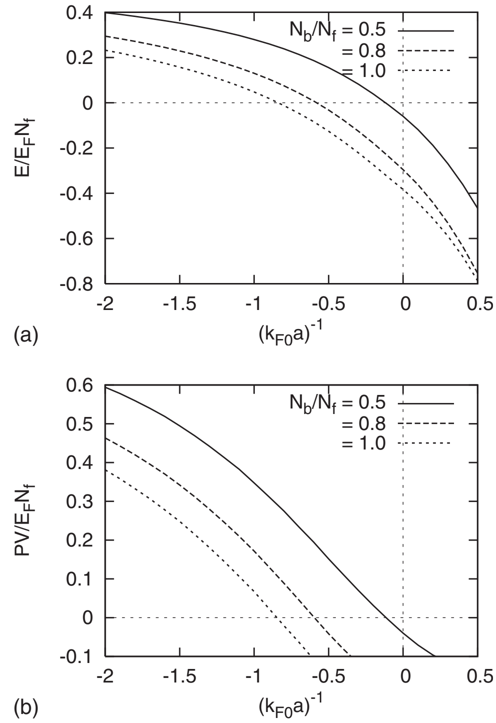

FIG. 8. Same as Fig. 7, but for m_b/m_f=1.0.

IV. BOSE-FERMI PAIR CORRELATION

The results of the previous section indicate that the strong attraction in the mixture will show up especially for positive a, which may eventually lead to a collapse of the system. We now consider another scenario for the attractively interacting mixture, the possibility of a Bose-Fermi pair formation [10,11]. For this purpose we study in this section the behavior of the pole of the Bose-Fermi scattering amplitude $\Gamma(P)$ in the mixture.

In unpolarized (or two-component) Fermi systems, an infinitesimal attraction around the Fermi surface leads to formation of Cooper pairs with a center-of-mass momentum $\mathbf{P}=\mathbf{0}$, causing a transition to the BCS state. In a Bose-Fermi mixture, on the other hand, the difference in the momentum distributions of the two particles and the fermionic character of the Bose-Fermi pair, in particular, predict quite a different scenario for the formation of the pairs in the mixture. It requires consideration of the balance of the kinetic energies of different kinds of particles and the magnitude of the attractive interaction.

In this section, for the study of correlated BF pairs as BF bound states, we will refrain from considering medium-modified chemical potentials μ_f and μ_b and use only their uncorrelated values as input, that is, $\mu_f=E_F$, $\mu_b=0$. In the two-component fermion case, this approximation is believed to describe the pairing physics, at least qualitatively, at zero temperature up to the unitarity limit. We believe that in the BF case also the qualitative trends are described properly. Inclusion of interaction-renormalized chemical potentials μ_f and μ_b, as in Secs. II and III, considerably complicates the discussion. Anyway, for a more quantitative analysis, chemical potentials as well as occupation numbers must be calculated from a self-consistent cycle. This, as mentioned above, will be done in future work.

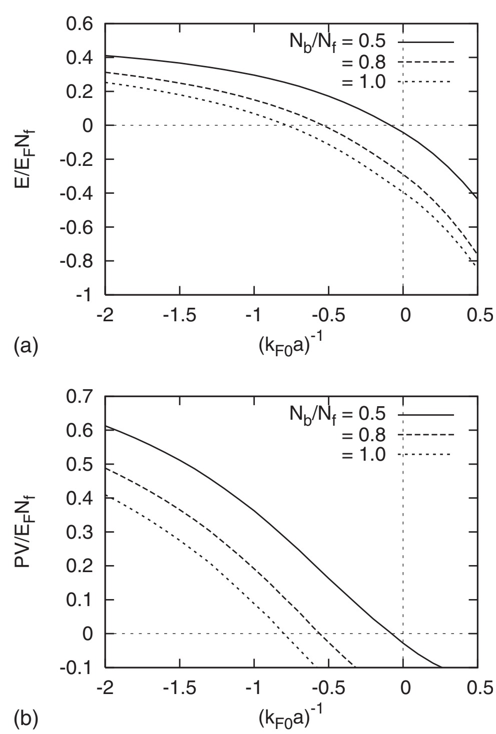

FIG. 9. Same as Fig. 7, but for m_b/m_f=1.2.

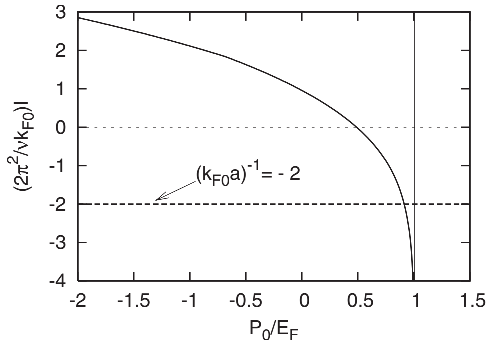

FIG. 10. Behavior of the right-hand side of Eq. (49). The horizontal line is the left-hand side. Here, m_b/m_f=1. The vertical line shows the position of the logarithmic divergence.

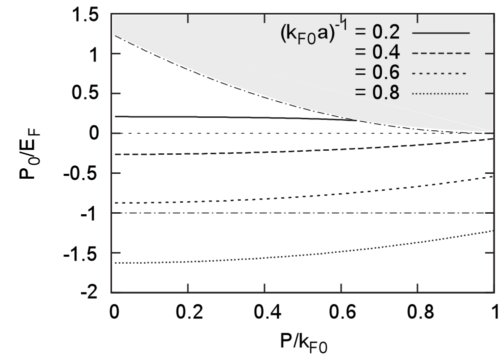

FIG. 11. Behavior of P_0 as a function of center-of-mass momentum $|\mathbf{P}|$. Here, the boson-fermion mass ratio is m_b/m_f=0.8. This result does not depend on the boson-fermion number ratio.

A. Pole behavior of the two-particle scattering amplitude

The investigation here follows essentially the lines of our previous work [10]; the main difference being the use of the more commonly used pseudopotential and a clear discussion of the various BF correlation regimes. We first want to study the pole structure of $\Gamma(P)$, which is important in itself. Later, in Sec. IV B we also will consider the pole structure of the full T matrix given in Eq. (30). This function $\Gamma(P)$ describes the repeated scattering of a fermion and a boson out of the condensate in the background of noninteracting Fermi and Bose gases of zero temperature. Let us consider the pole condition, that is,

$$\frac{\nu}{2\pi a} = I(P_0,|\mathbf{P}|), \tag{48}$$

or in scaled form

$$\frac{\pi}{\tilde{a}} = \tilde{I}(\tilde{P}_0,|\tilde{\mathbf{P}}|), \tag{49}$$

where $\tilde{I}(\tilde{P}_0,|\tilde{\mathbf{P}}|)=(2\pi^2/\nu k_F)I(P_0,|\mathbf{P}|)$. We want to repeat again that here, with respect to (27), we put $\mu_f=E_F$ and $\mu_b=0$, that is, the free gas values. For $|\tilde{\mathbf{P}}|=0$, we show in Fig. 10 the right-hand side of (49) as a function of $\tilde{P}_0$ for $m_b/m_f=1$. We see the development of a logarithmic divergency as P_0 approaches $k_F^2/2m_b-\mu_B$. This stems from the fact that the above dispersion integral has exactly the same structure as the one encountered in the problem of Cooper for a fermion pair in a Fermi sea. We want to point out, however, that the pole corresponds to a composite *fermion*, which has important consequences for the physics. Nevertheless, the fact is that a stable collective BF pair develops for any infinitesimal attraction, i.e., even in the limit $a\to -0$, quite in analogy with the original Cooper pole [15].

First let us consider the case without a medium. So we study the two-particle problem in vacuum. In the limiting procedure with two chemical potentials and k_F set to be zero, Eq. (49) becomes very simple:

$$\frac{1}{a} = \sqrt{-\left(\frac{\nu P}{m_b}\right)^2 + 2\nu\left(-P_0+\frac{P^2}{2m_b}\right)}. \tag{50}$$

This equation has solutions only for $a \geqslant 0$. The dispersion relation of the BF bound state is given by

$$P_0 = -\frac{1}{2\nu a^2} + \frac{P^2}{2m_b}\frac{\nu}{m_f}. \tag{51}$$

We see that, as usual, the bound-state formation starts at the unitarity limit, i.e., $a=\infty$.

Now we discuss the collective pole contained in $\Gamma(P)$. As in the corresponding problem for a two-component Fermi gas, we want to distinguish three energy domains: (i) the region where the pole lies above the sum of the chemical potentials of the two species, (ii) the region where the pole lies below the sum of the chemical potentials but above the threshold for the appearance of a true bound state, and (iii) the bound-state region. We will study the poles in the three domains as a function of center-of-mass momentum, which is an important aspect, since the BF pairs have fermion statistics. Since the definition of our Hamiltonian (1) actually corresponds to the free energy, the three domains mentioned above correspond to the following energy regions of P_0:

$$\text{(i)}\quad P_0 > 0 \quad \text{or } \tilde{P}_0 > 0,$$

$$\text{(ii)}\quad -E_F < P_0 < 0 \quad \text{or } -1 < \tilde{P}_0 < 0,$$

$$\text{(iii)}\quad P_0 < -E_F \quad \text{or } \tilde{P}_0 < -1. \tag{52}$$

In Figs. 11–13 the three domains are separated by the two horizontal lines of $\tilde{P}_0=0,-1$. We call the domain (ii) the "Cooper" region because, once the energy of the stable BF branch goes under the sum of the chemical potentials, that is $\mu_f+\mu_b=E_F$ for the case considered here, one can assume a strong restructuring of the Fermi surface, since the BF pairs may start to replace the original fermions in the buildup of the Fermi sea. This is, of course, only indicative here, since the many-BF-pair case needs a much more extended study. In any case, a variational ground-state wave function for the many-BF-pair case in an analogy to the BCS ansatz does not seem to exist in this case. Nonetheless, it is interesting to see

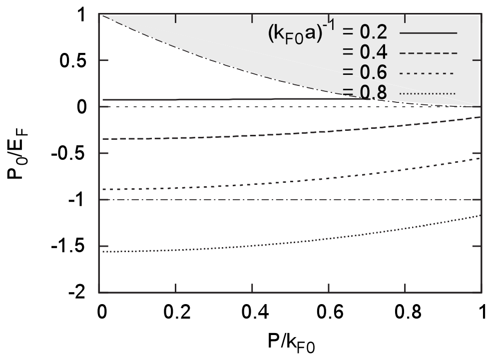

FIG. 12. Same as Fig. 11 but here the boson-fermion mass ratio is m_b/m_f=1.0.

that, contrary to the pure fermion case, a stable BF pair mode exists also in region (i). Indirectly, the mode of region (i) will certainly also influence the ground-state energy via quantum fluctuations. As mentioned already, this stable mode exists even for infinitesimal attraction, although it can sustain only very small center-of-mass momenta before it decays into the continuum (dash-dotted lines in Figs. 11–13). Let us now study more analytically how the borders of the different domains (i)–(iii) depend on the scattering length. We will concentrate on the case where the BF pairs are at rest, i.e., $|\mathbf{P}|=0$. For this case the pole conditions for regions (i) and (ii), i.e., $\tilde{P}_0 \geqslant -1$, is written as

$$\frac{\pi}{\tilde{a}} = 2 + \sqrt{\tilde{A}_0} \ln \left| \frac{1-\sqrt{\tilde{A}_0}}{1+\sqrt{\tilde{A}_0}} \right| \tag{53}$$

with

$$\tilde{A}_0 = \frac{m_b}{m_f + m_b}(1+\tilde{P}_0). \tag{54}$$

Let us first consider the crossing of the $\tilde{P}_0=0$ line. From Eq. (53) we obtain

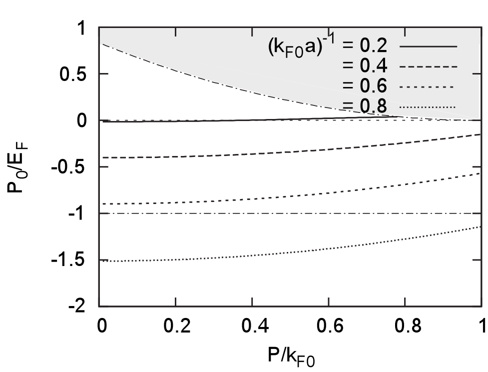

FIG. 13. Same as Fig. 11 but here the boson-fermion mass ratio is m_b/m_f=1.2.

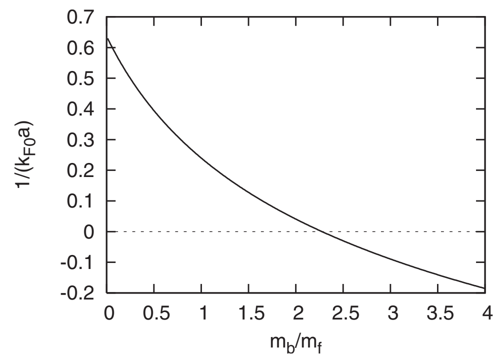

FIG. 14. Mass ratio dependence of the critical value of $1/\tilde{a}$ for the formation of Cooper-type BF pairs.

$$\frac{1}{\tilde{a}} = \frac{1}{\pi}\left(2 + \sqrt{\frac{\zeta}{1+\zeta}} \ln \left| \frac{1-\sqrt{\zeta/(1+\zeta)}}{1+\sqrt{\zeta/(1+\zeta)}} \right| \right), \tag{55}$$

where $\zeta = m_b/m_f$ is the mass ratio of bosons vs fermions. We show this critical scattering length as a function of ζ in Fig. 14. We see that the intensity of the BF attraction becomes smaller and smaller as the fermions become lighter (with respect to the bosons). Or, the other way round, the heavier the bosons are, the less attraction is needed to cross the $\tilde{P}_0=0$ line from above. On the other hand, the transit to the bound state seems to be independent of the mass ratio $\zeta = m_b/m_f$ as can be seem in Figs. 11–13. This is confirmed analytically by putting $\tilde{P}_0=-1$ into (53). One obtains

$$\frac{1}{a} = \frac{2}{\pi} k_{F0} \simeq 0.64 k_{F0}. \tag{56}$$

In vacuum ($k_{F0}=0$) we again obtain the result that the bound state appears at unitarity. One can also define an "in medium" scattering length

$$a_{\mathrm{eff}} = \frac{\nu}{2\pi}\Gamma(P=0) \tag{57}$$

and then one finds from (27) for the bound-state condition

$$\frac{1}{a_{\mathrm{eff}}} = 0, \tag{58}$$

that is, in the medium also the bound state appears at unitarity, i.e., for $a_{\mathrm{eff}}=\infty$.

B. Fermion dispersion and level crossing

As we remarked in Eq. (30) the scattering matrix $\Gamma(P)$ is only a part of the full BF T matrix. The latter contains in addition to the poles of $\Gamma(P)$ the poles where the bosons stay in the condensate and the fermions only scatter elastically off the condensate. These poles are just given by the free fermion dispersion relation $P_0(|\mathbf{P}|) = \mathbf{P}^2/2m_f$. Of course, these two branches of the T matrix are not independent and interact, depending on the values of the system parameters more or less strongly. We want to investigate this here. As we see from (30), the poles of the T matrix are the same as those of the fermion Green's function. Therefore, we have to solve

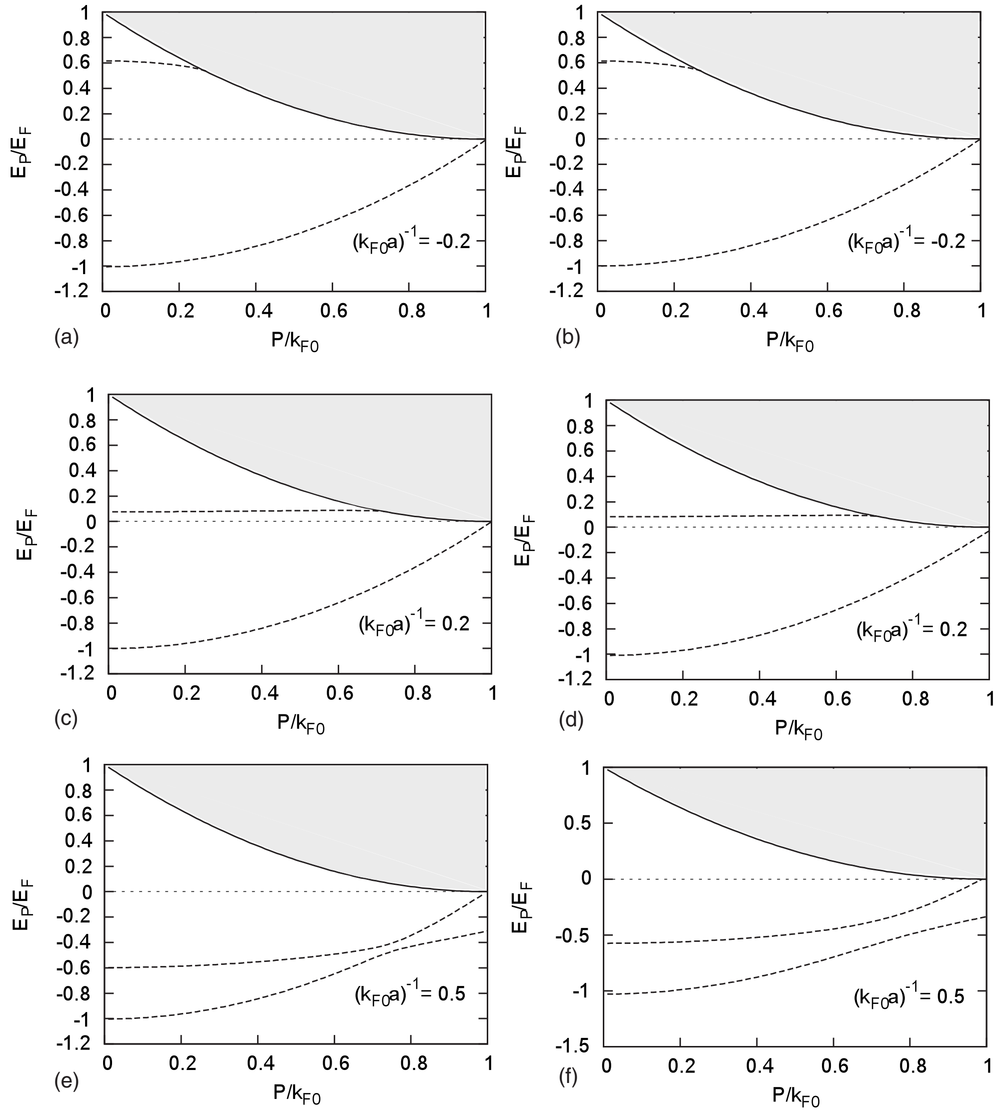

FIG. 15. Fermion dispersion curves. Left column corresponds to $N_b/N_f=0.001$ and right column to $N_b/N_f=0.01$. From top line to bottom, $(k_Fa)^{-1}$ values are -0.2, 0.2, and 0.5. m_b/m_f is fixed to 1.0.

the following implicit equation for the dispersion:

$$E_{\mathbf{P}} = \epsilon^f_{\mathbf{P}} + \Sigma^f(E_{\mathbf{P}},\mathbf{P}). \tag{59}$$

Of course, as before, we keep the chemical potentials at their noninteracting values. With the definition of $\Sigma^f(P)$ in (28) with $\mu_b=0$ and the expression (27) for $\Gamma(P)$, we can easily investigate the poles of $T(P)$, that is, of $G^f(P)$, for various system parameters. We first consider cases, as in [10], where $N_b \ll N_f$.

In Fig. 15, we show on the left panel the case of $N_b/N_f = 1/1000$ and on the right panel the case of $N_b/N_f=1/100$. From top to boom, we have $(k_{F0}a)^{-1}=-0.2, 0.2, 0.5$. Here the mass ratio $m_b/m_f=1$. The gray-colored area is the region where the imaginary part of Σ^f is different from zero. As in our earlier work [10], we see an avoided crossing of the two branches at some finite value of P/k_{F0} for instance, for $(k_{F0}a)^{-1}=0.5$. We also see that the interaction between the two branches becomes stronger as N_b/N_f increases. This level crossing is a very interesting feature. Indeed if we could consider the BF pairs as independent then we just would fill up the new Fermi sea according to the lowest branch. After the avoided crossing, the branches nonetheless exchange their characters and the Fermi sea starts to be composed of BF—"Cooper"—pairs beyond the crossing. Of course, as discussed already above, the BF pairs are not independent and their interaction, at least the one which is due to Bose and Fermi statistics, should be taken into account in a self-consistent way, similarly to what is done in BCS theory for the original Cooper pairs.

In Fig. 16 we show cases close to the $N_b \simeq N_f$ case. For $(k_{F0}a)^{-1}=0.5$ and $N_b/N_f=1/10$, the results are shown on the left panel and for $N_b/N_f=1$ on the right panel. We see that no crossing feature is visible any longer, due to the very strong interaction between the two branches, probably inducing a complete hydridization of the two modes. In a future presentation we intend to investigate in a systematic way the nature of the two branches as a function of the system parameters.

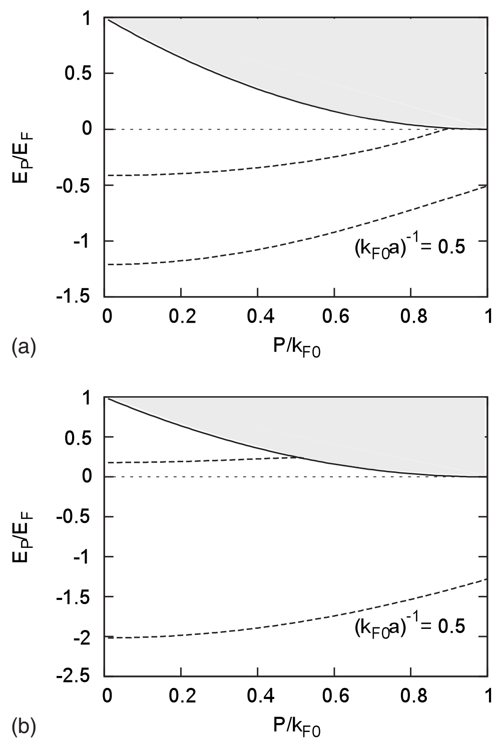

FIG. 16. Fermion dispersion curves. The upper panel corresponds to N_b/N_f=0.1 and the lower one corresponds to N_b/N_f=1.0. $(k_Fa)^{-1}$ values are fixed to 0.5. And m_b/m_f=1.0.

V. SUMMARY, DISCUSSION, AND CONCLUSION

We studied in this paper the static properties of the Bose-Fermi mixture in the lowest order of the hole-line expansion, i.e., in the T-matrix approximation. The interaction parameter is expressed in terms of the scattering length up to infinite order using the renormalization procedure of Ref. [13], so as to allow for the calculation around the unitarity limit. The T-matrix approach is a common approximation often used in the past in Fermi and Bose systems [15]. It has, however, not been investigated very much in Bose-Fermi mixtures and, therefore, one has little experience about its quality in that situation. Recently, however, a study for a one-dimensional system appeared [20], from which exact quantum Monte Carlo results are available for comparison. In Ref. [17] it is shown that the T-matrix approach yields quite reasonable results also in the Bose-Fermi case, even in a one-dimensional case, which is probably the worst situation possible.

With this background in mind, we first studied energy and pressure as functions of the inverse scattering length for several choices of the mass ratio $\zeta = m_b/m_f$ and the number ratio N_b/N_f. The energy of the system becomes strongly attractive as the inverse scattering length changes sign from negative to positive, i.e., around the unitary limit. As one increases the number of bosons with respect to the fermions, a point arrives where the pressure becomes negative, i.e., the system becomes unstable (collapse). The effect is stronger for small values of m_b/m_f. This is not in contradiction with experiments [6].

Next we studied the possibility of stable BF pairs, as in [10]. We, indeed, also found in the present model that even for infinitesimal BF attraction a stable BF mode appears, reminiscent of the Cooper pole in a two-component Fermi gas, since in both cases its origin stems from the presence of a sharp Fermi surface. However, in the BF case the BF pair is a composite fermion, whereas in the original Cooper problem, one has a composite fermion pair, i.e., a bosonlike cluster. In the latter case, many pairs can well be treated by the usual BCS formalism. On the other hand, the case of many BF pairs still has to be worked out. This will be done in future work. For some finite values of the attractive interaction, the formation of truly bound BF pairs occurs.

Up to this point we assumed an ideal case where the Bose-Fermi interaction is dominant, while the Bose-Bose interaction was neglected. To compare with a real atomic gas system, the Bose-Bose interaction cannot be discarded even when the Bose-Fermi interaction is enhanced, e.g., through Feshbach resonances. We checked the effect of the Bose-Bose interaction on the energy of the system up to the first order in $N_b^{1/3}a_{bb}$. We took the Bose-Bose scattering length a_{bb} in the range $-0.3 \leq k_F a_{bb} \leq 0.3$, and the number ratio N_b/N_f =2.0, 1.0, and 0.5, and repeated the calculation of the β parameter and the pressure [21]. The calculation shows that the effect of the Bose-Bose interaction is small within the adopted parameter values: The β value at the unitarity limit, for instance, deviates by less than 10% for $m_b = m_f$ and $N_b = N_f$, and does not change conclusions obtained at $a_{bb}=0$. However, to attain a more realistic expression for the boson part in Eq. (37), one needs to include a general Bose-Bose interaction and this will be presented in the future.

In summary, we suggest that the energy gain in the Bose-Fermi mixture at positive values of a_{bf} is influenced by the formation of resonant Bose-Fermi pairs, and that the center-of-mass momenta of the pairs are dependent on the ratio m_b/m_f due to the statistics of the two kinds of particles. The T-matrix approach used here is valid for weak coupling, qualitatively at least up to unitarity. We nonetheless went with our parameter values of the interaction beyond the unitarity limit because interesting physics like formation of so-called BF Cooper pairs or BF bound states can appear. We hope that the qualitative features remain valid within a more refined self-consistent procedure. This will be investigated in the future.

ACKNOWLEDGMENTS

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APPENDIX: EXPRESSION FOR THE SCATTERING AMPLITUDE

Let us calculate $I(P_0,|\mathbf{P}|)$ (34) of the scattering amplitude $\Gamma(P)$,

$$I(P_0,|\mathbf{P}|) = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left(\frac{\theta(|\tilde{\mathbf{P}}_f+\mathbf{k}|-k_F)}{P_0-\epsilon^f_{\tilde{\mathbf{P}}_f+\mathbf{k}}-\epsilon^b_{\tilde{\mathbf{P}}_b-\mathbf{k}}+\mu_b+i\eta}+\frac{1}{\epsilon^f_{\mathbf{k}}+\epsilon^b_{\mathbf{k}}}\right). \tag{A1}$$

As we are interested in the real part of the pole of $\Gamma(P)$, we hereafter omit $i\eta$ in the denominator. Each term in the integrand shows an ultraviolet divergence, and we formally introduce a cutoff Λ which will be taken to infinity later:

$$I_1(P_0,|\mathbf{P}|) = \frac{1}{(2\pi)^2}\lim_{\Lambda\to\infty}\int_{k_F}^{\Lambda} dk\Bigg[\left(-\frac{m_b}{|\mathbf{P}|}\right)k \times \ln\left|\frac{k^2/2\nu + |\mathbf{P}|k/m_b - P_0 + \mathbf{P}^2/2m_b - \mu_b}{k^2/2\nu - |\mathbf{P}|k/m_b - P_0 + \mathbf{P}^2/2m_b - \mu_b}\right| + 2k^2\frac{1}{k^2/2\nu}\Bigg]. \tag{A2}$$

The divergent term at $\Lambda\to\infty$ coming from the first term in the integrand is canceled out by the second term, and we obtain the finite quantity

$$I(P_0,|\mathbf{P}|) = \frac{1}{(2\pi)^2}\left(\frac{m_b}{2|\mathbf{P}|}\right)\Bigg\{\left[k_F^2 - \left(\frac{\nu|\mathbf{P}|}{m_b}\right)^2 - A\right] \times \ln\left|\frac{(k_F + \nu|\mathbf{P}|/m_b)^2 - A}{(k_F - \nu|\mathbf{P}|/m_b)^2 - A}\right| + \frac{4\nu|\mathbf{P}|}{m_b}k_F - \frac{4\nu|\mathbf{P}|}{m_b}AF(A)\Bigg\}, \tag{A3}$$

where

$$F(A) = \begin{cases} \dfrac{-1}{2\sqrt{A}}\ln\left|\dfrac{(k_F + \nu|\mathbf{P}|/m_b - \sqrt{A})(k_F - \nu|\mathbf{P}|/m_b - \sqrt{A})}{(k_F + \nu|\mathbf{P}|/m_b + \sqrt{A})(k_F - \nu|\mathbf{P}|/m_b + \sqrt{A})}\right| & (A > 0), \\ \dfrac{2k_F}{k_F^2 - (\nu|\mathbf{P}|/m_b)^2} & (A = 0), \\ \dfrac{\pi}{\sqrt{-A}} - \dfrac{1}{\sqrt{-A}}\arctan\left(\dfrac{k_F + \nu|\mathbf{P}|/m_b}{\sqrt{-A}}\right) - \dfrac{1}{\sqrt{-A}}\arctan\left(\dfrac{k_F - \nu|\mathbf{P}|/m_b}{\sqrt{-A}}\right) & (A < 0), \end{cases} \tag{A4}$$

with

$$A = \left(\frac{\nu|\mathbf{P}|}{m_b}\right)^2 - 2\nu\left(-P_0 + \frac{\mathbf{P}^2}{2m_b} - \mu_b\right). \tag{A5}$$

The final expression for $\Gamma(P)$ is given in terms of $I(P)$ by

$$\Gamma(P) = \frac{2\pi a}{\nu}\left(1 - \frac{2\pi a}{\nu}I(P_0,|\mathbf{P}|)\right)^{-1}. \tag{A6}$$

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