Quantum reflection: The invisible quantum barrier

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We construct an invisible quantum barrier which represents the phenomenon of quantum reflection using available data on atom-wall and Bose-Einstein-condensate–wall reflection. We use the Abel equation to invert the data. The resulting invisible quantum barrier is double valued in both axes. We study this invisible barrier in the case of atom and Bose-Einstein condensate (BEC) reflection from a solid silicon surface. A time-dependent, one-spatial-dimension Gross-Pitaevskii equation is solved for the BEC case. We found that the BEC behaves very similarly to the single atom except for size effects, which manifest themselves in a maximum in the reflectivity at small distances from the wall. The effect of the atom-atom interaction on the BEC reflection and correspondingly on the invisible barrier is found to be appreciable at low velocities and comparable to the finite-size effect. The trapping of an ultracold atom or BEC between two walls is discussed.

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I. INTRODUCTION

Quantum reflection is an intriguing phenomenon which has been under experimental scrutiny in the last few years. Several papers have been written on the subject. In 1983, Nayak, Edwards, and Masuhara [1] measured the scattering of ^{4}He atoms grazing a liquid-^{4}He surface and found that the reflectivity approaches unity as the velocity tends to zero. Berkhout *et al.* [2] measured the quantum reflection of H atoms from a concave spherical mirror. Yu *et al.* [3] reconsidered the experiment of [1] and measured the sticking probability. More recently, [4] looked at the phenomenon of quantum reflection of very slow metastable neon atoms from a solid silicon surface and a BK7 glass surface. More data were collected in the following years. A sodium Bose-Einstein condensate (BEC) containing 3×10^5 atoms reflected at normal incidence from a silicon surface at incident velocities of 1–8 mm/s was reported by [5]. The extension of this study to quantum reflection of a ^{23}Na BEC containing 10^6 atoms at velocities below 2.5 mm/s on a dilute silicon surface demonstrated the effect of atom-atom interaction and an extended density profile as the reflectivity saturated at 0.6 rather than 1.0 [6]. The observed reflectivity is explained as quantum reflection caused by the attractive Casimir-Polder (CP) atom-wall potential [7]. The significant reduction in the reflectivity of the BEC at very low velocities allegedly [8] arises from the modulation in the BEC density profile which ensues owing to the formation of standing waves resulting from the superposition of incident and reflected matter waves. As a result, dynamical excitations, such as solitons and vortex rings, are created in the BEC which fragment it and disperse its atoms. Such work on quantum reflection of BECs is important for the understanding of Bose-Einstein condensate stability near surfaces [9]. Several recent theoretical works on the general foundation of quantum reflection have been published [10–13].

The observed reflectivity is explained as quantum reflection caused by the *attractive* Casimir-Polder potential [7]. The reflection is accordingly classically forbidden. The Wentzel-Kramers-Brillouin (WKB) approximation would not work in such a case since the variation of the local de Broglie wavelength is comparable to the wavelength itself, regardless of the sign of the interaction. This, however, does not preclude the use of adequate perturbation methods to calculate the reflectivity very approximately [10]. On the other hand, reflection implies tunneling and one can thus invert the data on reflectivity $R(E)$ [or tunneling, $T(E)=1-R(E)$] using WKB-type formulas [14], to obtain the width of what might be called the invisible quantum barrier. Such a procedure would avoid the use of ill-defined concepts such as the quantum potential [10]. The purpose of this paper is to use the inversion procedure to obtain the invisible barrier responsible for quantum reflection. We apply this method to the available data on quantum reflection of atoms [4] and of BECs [5,6]. We also consider the resulting invisible barrier in the context of the trapping of a BEC between two walls.

The paper is organized as follows. In Sec. II, the phenomenon of quantum reflection is reviewed and the Casimir-Polder potential is discussed. In Sec. III, the invisible barrier is derived using the Abel equation for existing data on atom-wall and BEC-wall reflection. In Sec. IV, the invisible barrier is obtained for a BEC using a one-dimensional Gross-Pitaevskii equation. The free space trapping of BEC between two walls is also considered. The trapping occurs within the confines of two invisible barriers which sit at an appreciable distance from the walls that create them. Finally, in Sec. V, several concluding remarks are made.

II. QUANTUM REFLECTION

The phenomenon of quantum reflection occurs whenever the local de Broglie wavelength of the moving particle is comparable to the distance over which the potential varies rapidly, regardless of whether the potential is repulsive or attractive. For an attractive potential, such as the one between an atom and a wall (the Casimir-Polder interaction), the quantum reflection is a classically forbidden process, just like tunneling through a barrier. In this section we give a short account of quantum reflection, in connection with the CP interaction.

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The CP atom-wall interaction, which has the same physical vacuum fluctuation origin as the famous Casimir force between parallel walls separated by a distance L, $V_C(r) = -(\hbar c \pi^2/240)(1/L^4)$, is derived in [7] and can be written as an integral (see, e.g., [15,16]),

$$V_{\mathrm{CP}}(r) = -\frac{1}{4\pi\alpha_{\mathrm{fs}}r^4}\int_0^\infty dx\, \alpha\left(\frac{ix}{\alpha_{\mathrm{fs}}r}\right)\exp(-2x)(2x^2+2x+1), \tag{1}$$

where $\alpha(i\omega)$ is the dynamic electric dipole polarizability of the atom evaluated at the imaginary frequency $i\omega$ and α_{fs} is the fine-structure constant. The quantity $\alpha(i\omega)$ can be evaluated using sum rules and it can be written as

$$\alpha(i\omega) = S_n \frac{f_n}{(E_n - E_0)^2 + \omega^2}, \tag{2}$$

where the sum S_n runs over the discrete dipole states and the continuum scattering states. In the above, the dipole oscillator strength f_n is given by

$$f_n = \frac{2m_e}{3\hbar^2}(E_n - E_0)\left|\left\langle 0 \left| \sum_0^N \mathbf{r}_i \right| n \right\rangle\right|^2. \tag{3}$$

In the above equation N represents the number of electrons. Further, the quantity $S_n f_n$ is just the dipole sum rule $(m_e/\hbar^2)N$ [16].

The asymptotic form of the CP potential is invariably written as

$$V_{\mathrm{CP}}(r) = \frac{1}{r^3}\frac{-C_4}{(r + 3\lambda/2\pi^2)}, \tag{4}$$

where C_4 has the value, e.g., for sodium atoms incident on a silicon surface and $\lambda = 590$ nm [5,6].

The result of the quantum reflection measurements cited above show a strikingly robust quantum reflection at very low velocities. The presence of such quantum reflection raises a question about transmission to the solid wall, where the atoms are adsorbed. The transmission or tunneling implies the presence of an invisible quantum barrier "sitting" at a distance close to the wall. Can one determine this barrier from the available data on quantum reflection? How does a BEC behave compared to a single atom? The answer to these queries is the thrust of the present paper.

Once we know the reflection coefficient, the transmission one is simply the complement to unity, viz., $T(E) = 1 - R(E)$. In the following we use the generalized WKB form for the tunneling probability $T(E)$ [17],

$$T(E) = \frac{1}{1 + \exp[2\sqrt{2m/\hbar^2}\int_{r_1}^{r_2} dr\sqrt{V(r) - E}]}. \tag{5}$$

Here, r_1 and r_2 are the inner and outer turning points which are the roots of the equation $V(r) = E$. Further, the above form of the tunneling probability is exactly 1/2 at the top of the barrier as the exact solution of the Schrödinger equation requires.

Knowing $T(E)$ from the data, one can invert it to obtain the barrier. This procedure is intimately related to the solution of Abel's theorem and the corresponding classical mechanics problem [18]. The equation that does this is given by

$$r_2 - r_1 = \frac{-1}{\pi}\frac{\hbar}{\sqrt{2m}}\int_V^B dE \frac{R'(E)}{R(E)T(E)}\frac{1}{\sqrt{E - V(r)}}, \tag{6}$$

where B is the height of the barrier, generally defined through $T(B) = R(B) = \frac{1}{2}$, which results from $\int_{r_1}^{r_2} dr \sqrt{V(r) - B} = 0$. This last relation defines theoretically the height of the barrier, while the former supplies an "experimental" definition of this height.

The above equation, which we shall refer to as the Abel formula, has been used in a variety of forms to investigate inversion in the WKB-type description of bound systems [14]. It has also been used to extract the barrier responsible for tunneling into the classically forbidden region in the sub-barrier fusion of nuclei [19]. The potential that results from the inversion of the data is a multivalued function. Of course, by its nature, Eq. (6) dictates that the thickness function $r_2 - r_1$ is determined by $R(E)$, or equivalently $T(E)$, for $V < E < B$, namely, in the energy region where $R(E) \geqslant 1/2$.

III. INVISIBLE QUANTUM BARRIER

Cole and Good [14] have suggested using the Abel formula to invert the semiclassical reflectivity and tunneling probability to obtain the thickness of the barrier in cases of reflection in the classically allowed region. Here we have used the Abel formula to obtain what we may call the invisible quantum barrier responsible for quantum reflection (QR), reflection in the classically forbidden region, of single sodium atoms [4] from a silicon surface, and for the QR of a Bose-Einstein condensate composed of 3×10^5 sodium atoms in a trap [5] and of 10^6 such atoms [6]. Although the interaction between a BEC and a surface is less well understood than that of a single atom with a surface, we take the practical view of using the same type of CP interaction as was done in [5,6,8]. When using Eq. (6), we have fitted the data points with an analytical function of the velocity and used this function in the inversion. Of course the inversion alluded to above would only supply the barrier value as a function of its thickness, $V_{\mathrm{inv}}(x_1 - x_2)$. Of course this procedure is applicable at energies where the reflectivity is larger than $\frac{1}{2}$.

For energies greater than the height of the barrier, $R(E) < \frac{1}{2}$, one can use the parabolic barrier approximation valid near the position R_B of the height B of the barrier, $V_{\mathrm{inv}}(r) \approx B - \frac{1}{2}|(d^2V_{\mathrm{inv}}/dr^2)|_{r=R_B}(r - R_B)^2$, which gives $R(E) = \frac{1}{1+\exp[(2\pi/\hbar\omega)(E-B)]}$, where $\hbar\omega$ measures the curvature of the assumed inverted parabolic barrier, $\omega^2 = (1/m)(d^2V_{\mathrm{inv}}/dr^2)|_{r=R_B}$, where m is the mass of the atom. This procedure is quite common in the theory of nuclear fission [20]. By fitting the data points for $R(E) \leqslant \frac{1}{2}$ we easily obtain the values of the height [which can also be read off from the condition $R(E) = \frac{1}{2}$] and, more importantly, the curvature of the invisible barrier. The height and curvature parameter of the barrier for the data on Ne atom reflection off a silicon wall [4] are, respectively, $B = 1.03$ nK and

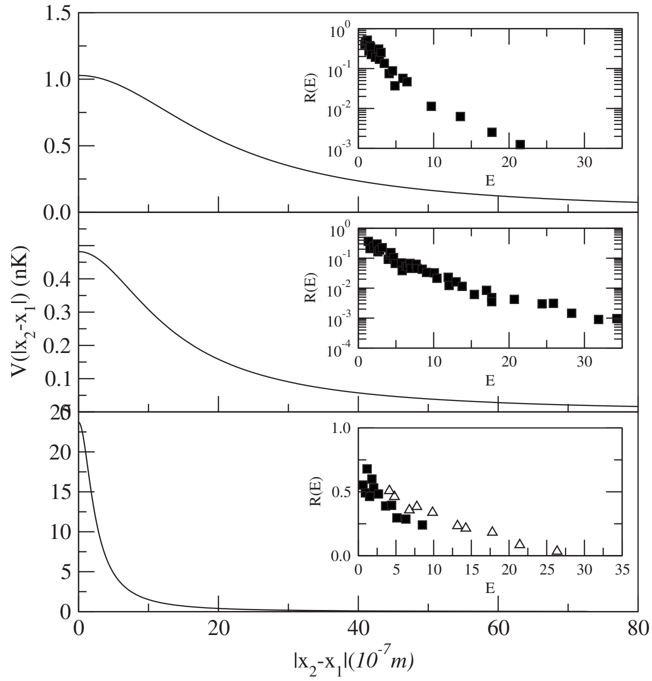

FIG. 1. Invisible barrier vs its thickness for the data of [4,6]. The experimental reflectivities were adjusted with a simple function and used in Eq. (6). Insets: Reflectivity data versus the normal incident velocity on the Si(1,0,0) surface [4] (top), on the BK7 glass surface [4] (middle), and reflectivity of sodium BEC data versus incident velocity (bottom). Data points [6] correspond to a sodium BEC confined in a magnetic trap with trap frequencies $2\pi \times (2.0, 2.5, 8.2)$ Hz (full squares) and $2\pi \times (4.2, 5.0, 8.2)$ Hz (open triangles). See text for details.

$[d^2V(r)/dr^2]|_{r=R_B}=0.011$ nK/10^{-14} m^2, and on a BK7 glass wall [4], $B=0.05$ nK and $[d^2V(r)/dr^2]|_{r=R_B}$ $=0.015$ nK/10^{-14} m^2. In the case of the Na BEC reflection data of [6] we find for these parameters the values B $=23.7$ nK and $[d^2V(r)/dr^2]|_{r=R_B}=17.0$ nK/10^{-14} m^2. Clearly the barrier for BEC reflection is more than an order of magnitude higher and thinner than that for single-atom reflection.

The Abel formula was employed using the available data for $R(E) > \frac{1}{2}$ and extrapolating these to lower energies using an appropriate fitting function. The fit formula that we have employed is of the general form $R(E)=\exp(-CE^D)$. Such a functional form reproduces the overall trend of the data of [4] and to a large extent those of the BEC reflectivity of [6]. In the case of reflection of neon atoms from a silicon wall of [4], we find $C=0.81$ and $D=0.71$, whereas for reflection of neon atoms from a BK7 glass wall, $C=0.92$ and $D=0.29$. In the case of the sodium BEC reflection from a silicon wall [6], we obtained $C=0.23$ and $D=0.39$. This latter case could account for the small reflectivity (higher velocities) and misses altogether the observed saturation at smaller velocities. The inversion alluded to above would supply the barrier value only as a function of its thickness, $V_{\mathrm{inv}}(x_1-x_2)$. Such potentials are shown in Fig. 1. The barrier heights and curvatures obtained from the inversion procedure are B $=1.03$ nK and $[d^2V(r)/dr^2]|_{r=R_B}=0.0086$ nK/10^{-14} m^2 for neon on a silicon wall [4], $B=0.048$ nK and

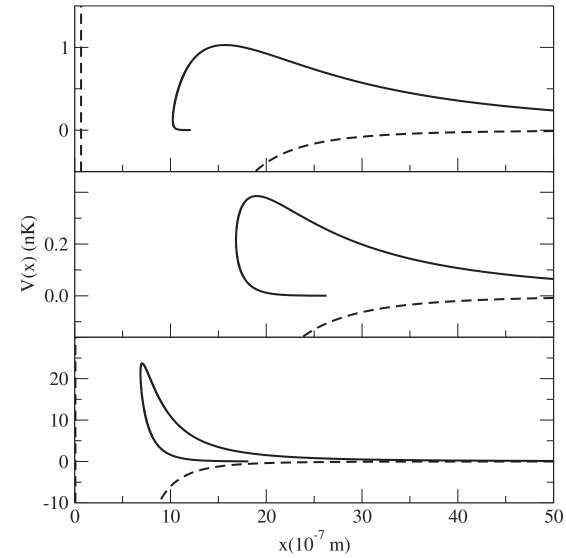

FIG. 2. The invisible barrier vs distance x for Ne reflection from a silicon wall (at $x=0.0$), from a BK7 glass wall [4], and for Na BEC reflection from a silicon wall [5,6]. The dashed curve indicates the Casimir-Polder potential. See text for details.

$[d^2V(r)/dr^2]|_{r=R_B}=0.0108$ nK/10^{-14} m^2 for neon on a BK7 glass wall [4], and $B=23.71$ nK and $[d^2V(r)/dr^2]|_{r=RB}$ $=17.64$ nK/10^{-14} m^2 for sodium BEC reflection from a silicon wall [6]. The values of the curvatures are quite consistent with those obtained using the parabolic approximation above.

The result of the calculation of the BEC V_{inv} using the time-dependent Gross-Pitaevskii equation is given below. Before we discuss these results, it would be useful to obtain the invisible barrier as a function of the distance between the reflected entity and the silicon wall. For this purpose we need another equation involving the two turning points x_1 and x_2. One possible suggestion is to relate their product to the square of the virtual turning point $r(E)$ related to the Casimir-Polder potential V_{CP}, namely, $E=V_{\mathrm{CP}}(r(E))$, and $r(E)^2=x_1x_2$. Through this relation we get the real turning points x_1 and x_2 separately, and accordingly we can construct the invisible quantum barrier as a function of the distance from the wall. This procedure is not unique, as a different relation involving the turning points and the Casimir-Polder potential would yield a different shape for the invisible barrier. Not having available another plausible relation, we proceed and use the product relation above. The result of such a calculation is presented in Fig 2. We see clearly that the barrier is a double-valued function on both axes. This feature may be a reflection of the use of an energy-dependent relation between the turning points and $r(E)$. It would be certainly of value to explore other relations, such as x_1+x_2 $=2r(E)$.

IV. INVISIBLE QUANTUM BARRIER FOR A BEC

We now turn to the study of quantum reflection of a one-dimensional Bose-Einstein condensate from the silicon surface. Several theoretical studies of one-dimensional (1D)

BECs have been published (see, e.g., [21,22]). It has been argued that by lowering the dimension to 1D, the transition temperature for BEC formation is increased. These systems are subtle and exhibit features not encountered in 2D or 3D. Bearing in mind the intrinsic differences between the 1D and 3D BECs, we shall, nevertheless, discuss the quantum reflection of a 1D BEC for the purpose of simplicity. Would the condensate suffer reflection just like a single atom? In a way, to answer this question is in line with a broader one connected with the quantum behavior of the motion of mesoscopic systems in general. One is reminded here of the pioneering work of Arndt *et al.* [23,24] on the interference pattern of C_{60} and other heavy molecules as they pass through a grating. As in [6], we consider N ^{23}Na atoms Bose-Einstein condensed into the ground state in an anisotropic trap, $\Omega_\rho \gg \Omega_x$, where Ω_ρ is the trap frequency in the transverse directions and Ω_x the frequency in the x direction. Neither the quantum reflection nor the interaction between atoms will excite motion in the transverse direction. With these assumptions, the condensate is described by the one-dimensional Gross-Pitaevskii (GP) equation [25]. In fact, a one-dimensional BEC has been formed and studied in [26–29]. The 1D GP equation we shall solve here is [25]

$$i\hbar\frac{\partial\psi}{\partial t} = \left(-\frac{\hbar^2}{2M}\frac{d^2x}{dx^2} + V(\mathbf{x}) + g|\psi|^2\right)\psi, \tag{7}$$

where $g=Ng_{1D}$ with N the number of atoms and $g_{1D}=2\hbar\Omega_\rho a$, where a is the s-wave scattering length, which can be tuned through the Feshbach resonance [30]. The condition of validity of the above formula for g_{1D} is $a \ll \sqrt{\hbar/m\Omega_\rho}$ [31]. Notice that the number of atoms appears in the strength of the interaction g as the wave function has been normalized to unity,

$$\int |\psi|^2 dx = 1. \tag{8}$$

The external potentials include the harmonic trap in the x direction and the Casimir-Polder potential induced by a silicon surface [5,6], $V(\mathbf{x})=V_{\mathrm{trap}}+V_{\mathrm{CP}}$. The harmonic trap potential is $V_{\mathrm{trap}}=\hbar\Omega_x^2 x^2/2$. In this paper, the trap frequency is set to be $\Omega_x=2\pi\times 3.5$ Hz, in the range of values of recent experiments [5,6,26]. The Casimir-Polder potential has the form of Eq. (4). Similarly to [8], the model Casimir-Polder potential is separated into two regions. If $x<d-\delta$, we have the normal Casimir-Polder potential. Here $\delta=0.15$ μm is a small offset to the silicon surface. When $x>d-\delta$, we use a complex potential $V_c=V_{\mathrm{CP}}(\delta)-i(x-\delta)V_i$ modeling adsorption at the surface with $V_i=1.6\times 10^{-26}$ J m^{-1}.

Recently, experiments on the reflection of condensates were realized in three-dimensional traps with the number of condensed atoms reaching 10^6 [5,6]. However, the number of atoms that can be condensed in a one-dimensional trap is reduced due to thermal and quantum fluctuations. We use the number of atoms in the BEC of a recent experiment [26], $N_{1D}=1.5\times 10^4$. First we "prepare" a ground state of the condensate in the harmonic trap. The center of the trap is shifted to the Si surface suddenly by a distance Δx. Therefore the condensate is suddenly put at a high potential: $\Delta V=M\Omega_x^2\Delta x^2/2$. This induces the condensate to travel toward the surface (see Fig. 3). At the surface, the incident velocity is approximated by $v_x \approx \Omega_x \Delta x$. We use a fourth-order Runge-Kutta method in the interaction picture to perform the time propagation of the solution of the Gross-Pitaevskii equation. After one complete reflection, we define the reflection probability as the number of left-traveling atoms divided by the number of incident atoms. Unlike a single atom reflected by the surface, the condensate has a finite spatial extension. This yields a minimal but nonzero velocity of the incident condensate. In addition, during the reflection process, atoms are lost continuously. The nonlinear term in the Gross-Pitaevskii equation is also reduced accordingly since it is proportional to the total number of atoms in the condensate.

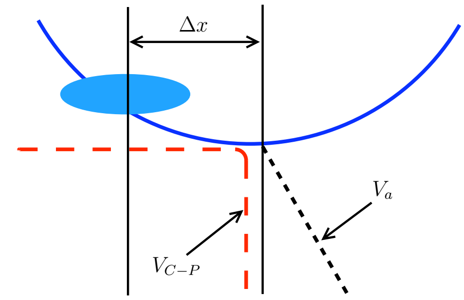

FIG. 3. (Color online) Schematic diagram of the potentials. The blue curve is the harmonic trap. A ground state condensate was created in the trap. The red curve is the Casimir-Polder potential. In the numerical calculation, we set a finite offset to the surface (see text). The black curve indicates the imaginary part of the absorbing or adsorbing potential. If atoms reach this region, they will be adsorbed at the surface or inelastically scattered away from the trap.

The simulation is presented in Fig. 4. When the velocity is reduced, the reflection probability approaches unity. However, due to the facts mentioned above, the velocity cannot reach zero. So, in principle, we get a less-than-unity maximum reflection, similarly to the experimental results of [6]. With the reflectivity calculated, we can now obtain the invisible barrier following the procedure detailed above.

We use Eq. (6) to get the barrier at different thicknesses. The corresponding $V_{\mathrm{inv}}(r)$ is shown in Fig. 6 and it resembles

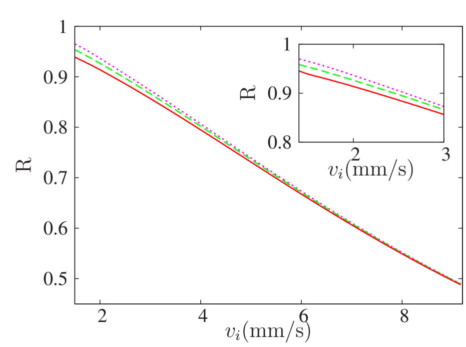

FIG. 4. (Color online) Quantum reflection of one-dimensional sodium Bose-Einstein condensate from a silicon surface. Velocity is measured in mm/s. Three values of the atom-atom interaction strength were considered: $g_s=0.0$ (dashed line), 50.0 (dash-dotted line), and 200.0 (solid line). See text for details.

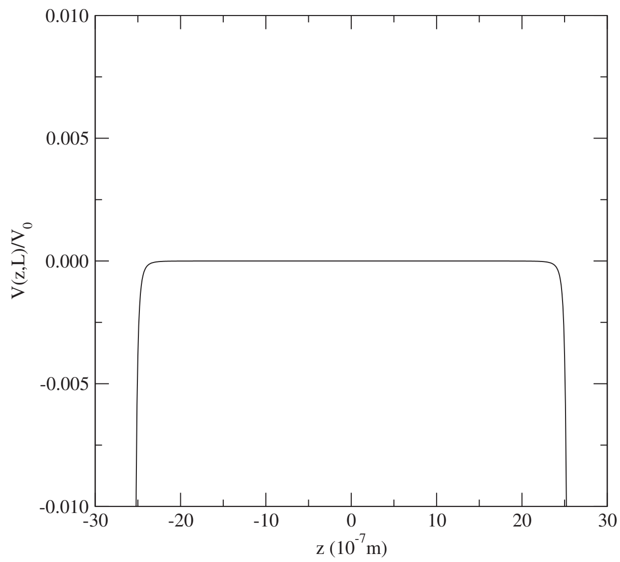

FIG. 5. Profile of the wall-atom-wall potential according to Eq. (14). The walls are situated at $(-26,+26)\times 10^{-7}$ m.

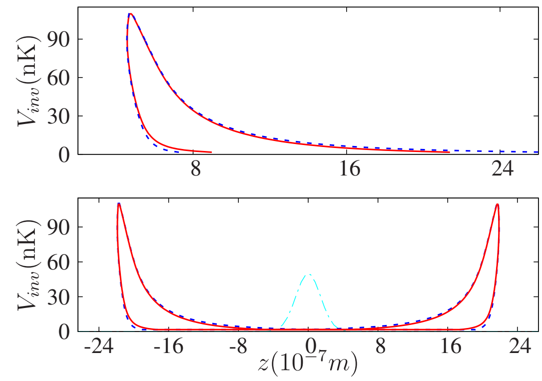

FIG. 6. (Color online) Top: Invisible barrier of a silicon surface (at $x=0.0$) for a sodium Bose condensate for two values of the interaction strength, $g_s=0.0$ (dashed) and 200.00 (full). Here the distance is in meters and energy in nanokelvins. Bottom: The same but with two silicon walls situated at $x=(-26,+26)\times 10^{-7}$ m, schematically showing the quantum reflection trap. The Gaussian form at the center is the BEC. See text for details.

very well the one obtained from the data of [6], Fig. 2. This attests to the consistency of our calculation. More importantly, it shows clearly that a BEC behaves very much like a single atom, albeit with a finite size. The size effect is felt when the BEC comes very close to the wall, as has already been mentioned. We have also investigated the effect of the strength of the atom-atom interaction g on the reflectivity of the BEC. In the numerical simulation, we scale the energy and length with the ground state harmonic oscillator energy $E_0=\hbar\Omega_x$ and harmonic length $l_0=\sqrt{\hbar/m\Omega_x}$. In this way, we solve the equation with scaled interaction strength $g_s=0.0$ (ideal gas), $g_s=50$, and $g_s=200.00$. The resulting reflectivity and the corresponding $V_{\text{inv}}(r)$ show appreciable sensitivity to the value of g, and accordingly to the scattering length a (Fig. 6), which is comparable to the finite-size effect alluded to above. We have also calculated the parameters of the barrier using the inverted parabola approximation of Hill-Wheeler. We find, for the height $B=111$ nK and the curvature $[d^2V(r)/dr^2]|_{r=R_B}=195$ $(g=0.0)$ and 141 $(g=200)$ nK$/10^{-14}$ m^2. These values are to be compared to the ones extracted from the data, mentioned above. The calculated ones are a factor of 5–6 larger than the ones obtained from the data. This discrepancy might be traced to the several limitations inherent in our 1D GP simulation.

It has been recently suggested that one could trap cold atoms and BECs by quantum reflection from two walls [32,33]. Our findings above give a specific mechanism for such trapping. The cold atom or BEC will be in the confines of the two invisible barriers that such walls will generate. As long as the velocity of the atom or the BEC does not exceed the top of the invisible barrier, it will be trapped in free space without touching the walls, in accordance with the conclusions of [32,33]. The wall-atom-wall (WAW) CP potential can be written as [16]

$$V_{\text{WAW}}(z,L)=\frac{1}{L^4}\left(\frac{1}{360}-\frac{3-2\cos^2(\pi z/L)}{8\cos^4(\pi z/L)}\right), \tag{9}$$

where z is the distance of the atom from the midpoint between the walls, taken to be separated by L. The profile of the above WAW potential is shown in Fig. 5. For the purpose of completeness we calculate the corresponding double invisible barrier when the BEC is placed between two walls.

A schematic picture of the confining double barrier in the case of the sodium BEC is shown in Fig. 6. One sees clearly that the double barrier defines the confining space to be the interval $(-22,+22)\times 10^{-7}$ m, while the actual walls are situated at $(-26,+26)\times 10^{-7}$ m. The space between the barrier and the wall is completely inaccessible. This, however, depends on the energy, as the invisible barrier is energy dependent. In the case of the sodium BEC [6], the extracted critical velocity that the BEC must have in order to reach this forbidden region corresponds to an energy of about 24.0 nK. Our 1D GP calculation gave a height which is five times greater. This is due to the inherent limitation of the 1D simulations. We should also warn the reader that our confining boxlike potential was constructed using the plausible condition on the turning points $x_1x_2=r(E)^2$, with $r(E)$ being the virtual turning point associated with the Casimir-Polder potential. Another condition would give a different confining region.

V. CONCLUSIONS

In conclusion, we have inverted the recent data on reflectivity of neon atoms and of sodium Bose-Einstein condensates to obtain the invisible quantum barrier. We have found that this object is double valued in both axes. The height and curvature of the invisible barrier were determined from the data. The value of the height we obtain from the data of [6] is $B=24$ nK, while for the neon atoms of [4] it is 1.0 nK. The former is high enough to make the trapping of BEC quite feasible at energies smaller than B. Such trapping could be potentially of great value in the fabrication of atom laser devices, as the BEC will maintain its integrity without suffering from absorption-adsorption at the walls. Further, wehave verified that a BEC, though mesoscopic and containingmillions of atoms, behaves, to a large extent, like a single, albeit large, atom when reflected from a solid wall. This finding corroborates those of [5,6] on the persistence of quantum behavior of large mesoscopic objects and is in line with the results of the recent grating diffraction experiments on C_{60} and other large molecules [23,24].

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