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Crescent vortex solitons in strongly nonlocal nonlinear media

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We demonstrate that strongly nonlocal nonlinear media can support two-dimensional solitons in the form of rotating crescent vortices, which are generated by a superposition of two ordinary concentric vortices with equal powers and topological charges differing by 1 (similar objects were recently found in the local Gross-Pitaevskii equation with rotation and an anharmonic trapping potential). These solitons demonstrate additional swinging motion under the action of a kick. Nested crescent solitons, which feature synchronous rotation, are generated by the superposition of several pairs of vortices. Power imbalance between the underlying vortices gives rise to unsteady crescent solitons.

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I. INTRODUCTION

Nonlocal nonlinear response is featured by many physical media. It has been found that nonlocality can prevent the collapse of self-focusing beams in media with cubic nonlinearity [1,2], suppress azimuthal instabilities of vortex solitons [3,4], and stabilize soliton clusters of the Laguerre and Hermite types [5], azimuthons [6], and multipole solitons [7]. Nonlinear media with strong nonlocality, whose spatial range is much larger than the beam's size, support the so-called "accessible solitons," which are described by quasilinear solutions tantamount to wave functions of the two-dimensional harmonic oscillator [8]. Experiments have revealed fundamental [9] and vortex optical solitons [3,10] supported by the strong nonlocality, as well as a possibility of steering solitons in such settings [11]. In addition, theoretical analysis predicts the stabilization of other self-trapped modes [12], partially coherent "accessible" solitons [13], and stable spinning "bearing-shaped" solitons in strongly nonlocal nonlinear media [14]. A review of solitons in nonlocal optical media can be found in Ref. [15].

In local and nonlocal settings alike, optical vortex solitons have drawn much attention, both as objects of fundamental interest and due to their potential applications to all-optical data processing, as well as for guiding and trapping of atoms. A well-known problem is that vortex solitons are frequently subject to the azimuthal instability [16]. Two-dimensional (2D) bright vortex solitons can be stabilized by the cubic-quintic (CQ) nonlinearity; see the review [17]. Stable spinning spatiotemporal solitons with topological charge $m=1$ have been identified in both the CQ model [18] and its counterpart with the quadratic-cubic nonlinearity, where the cubic term is self-defocusing [19]. Bright vortex solitons with $m>1$ may also be stable in these models [20].

A specific type of localized state, in the form of *crescent vortex solitons* (CVSs), was found in the 2D Gross-Pitaevskii equation (GPE) combining the self-attractive local cubic nonlinearity, rotation around a central pivot, and a radial anharmonic (in fact, quadratic-quartic) potential [21]. When a simple vortex with topological charge m becomes unstable in this model, it is replaced by (meta)stable states built as a mixture of three vortices with charges m and $m\pm1$.

In this work, we demonstrate that the existence of stable CVS states is not specific to the above-mentioned complex setting, as they can also be found in a much simpler model of a strongly nonlocal medium with the self-focusing nonlinearity, viz., the above-mentioned "accessible-solitons" model, where they are generated by a superposition of two concentric vortices with topological charges $(m-1,m)$ and equal powers. These CVSs demonstrate stable spinning motion, and they may move along more complex ("swinging") trajectories under the action of an initial kick. Multiple CVSs can be generated too, by the superposition of several pairs of concentric vortices.

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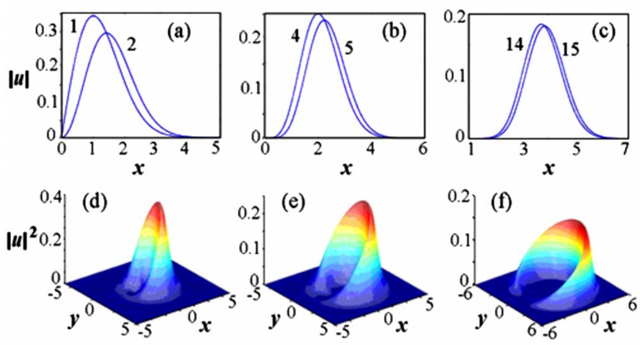

FIG. 1. (Color online) (a)–(c) Profiles of vortex solitons with different topological charges $m=1$, 2, 4, 5, 14, and 15, for equal widths $r_0=1$. (d)–(f) CVSs generated by superimposed pairs of concentric vortices from panels (a)–(c).

II. MODEL

The "accessible-solitons" model is based on the following version of the 2D nonlocal nonlinear Schrödinger equation for the evolution of the field amplitude, $u(x,y)$, along the transmission distance, z,

$$iu_z+\frac{1}{2}(u_{xx}+u_{yy})-Pr^2u=0, \quad P=\int\int|u(x,y)|^2dxdy, \tag{1}$$

where P is the total power of the beam, and $r=\sqrt{x^2+y^2}$ is the radial coordinate. This equation may describe, e.g., an optical medium with thermal nonlinearity [15], assuming that local properties of the medium are modified along r as per Eq. (1).

Because P is an obvious dynamical invariant of Eq. (1), this equation by itself is linear. However, the model as a whole is not a truly linear one, as the superposition principle for a combination of two solutions *does not* hold, as the superposition changes the value of P. In fact, the results presented below provide for a direct illustration to the latter feature.

A family of exact vortex-soliton solutions to Eq. (1) with integer topological charge m can be easily found in polar coordinates (r,φ),

$$u(x,y,z)=Ar^{|m|}\exp(-r^2/2r_0^2)\exp(ikz+im\varphi),$$
$$k=-(1+|m|)/r_0{}^2, \tag{2}$$

where the radius of the beam and its amplitude are determined by the total power,

$$r_0=(2P)^{-1/4}, \quad A=[2^{(1+|m|)/4}/\sqrt{\pi|m|!}]P^{(3+|m|)/4}. \tag{3}$$

The maximum of the local power in solution (2) is located at $r=\sqrt{|m|}\cdot r_0$. A set of profiles of the vortex solitons with large values of the topological charge, which are used below to construct the CVSs, are displayed in the top part of Fig. 1 (note that stable vortex solitons with a large topological charge have been observed in an experiment performed in a nonlocal optical medium [10]).

III. CRESCENT VORTEX SOLITONS

The CVS state in the model based on Eq. (1) can be constructed as a superposition of two vortices with equal widths r_0 (i.e., equal powers), but different topological charges m_1, m_2,

$$u(x,y)=[A_1r^{|m_1|}\exp(im_1\varphi)+A_2r^{|m_2|}\exp(im_2\varphi)]$$
$$\times\exp(-r^2/2r_0^2). \tag{4}$$

As said above, the superposition of two solutions to Eq. (1) does not generate a new exact solution, because P changes. In particular, the total power of the initial superposition of two vortices with $m_1\neq m_2$ is $P=P_1+P_2$, while in the case of two vortices with equal charges but different powers the result of the superposition is more complicated: $P=P_1+P_2+2^{|m|+2}(P_1P_2)^{(3+|m|)/4}(\sqrt{P_1}+\sqrt{P_2})^{-(|m|+1)}$ (in the case of $P_1=P_2$, this expression naturally reduces to $P=4P_1$).

Thus, expression (4) does *not* immediately yield a new exact solution, but rather an initial state, from which a solution should be found by solving Eq. (1), with $P=P_1+P_2$. The analysis can be performed via the expansion of the initial state over the full set of eigenstates of this linear equation, but this approach is cumbersome, while the direct numerical solution of Eq. (1) readily yields the result. In this way, it has been found that robust CVSs can be generated by the superimposed vortex pairs with $|m_2-m_1|=1$. Typical examples of

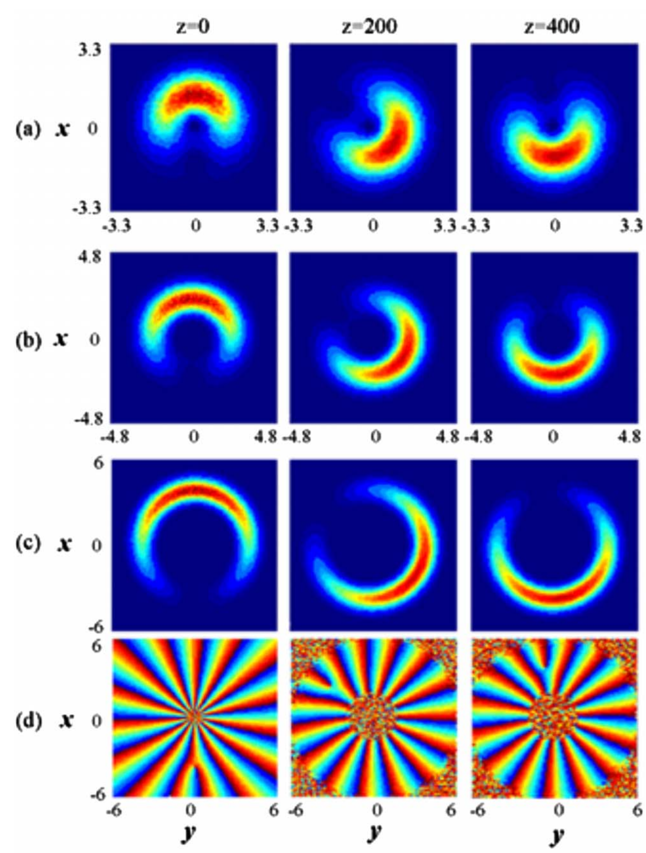

FIG. 2. (Color online) (a)–(c) Counterclockwise-spinning CVSs whose profiles are shown in Figs. 1(d)–1(f). (d) Phase patterns of the solitons from row (c).

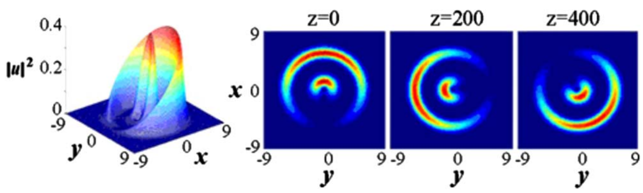

FIG. 3. (Color online) Counterclockwise-spinning double CVSs generated by the superposition of two pairs of concentric vortices, with ($m_1=1$, $m_2=2$), ($m_3=30$, $m_4=31$), and equal widths, $r_1=r_2=r_3=r_4=1.024$.

the so obtained patterns are displayed in the bottom part of Fig. 1, where pairs of concentric vortices are used, with equal widths $r_0=1$, and charges $m_1=1$, $m_2=2$ (d), $m_1=4$, $m_2=5$ (e), and $m_1=14$, $m_2=15$ (f).

As shown in Fig. 2, the resulting CVSs rotate counterclockwise, which corresponds to the sign of the total angular momentum of the vortex pairs that generate these states. The angular velocity of the rotation can be found as the group velocity of the phase circulation following from Eqs. (2),

$$\Omega = -dk/dm = \mathrm{sgn}(m)/r_0^{\,2} \tag{5}$$

[recall r_0 is given by Eq. (3)]. This simple result is valid due to the quasilinear character of the model, while in local and quasilocal ones the rotation of solitons (alias spiraling, in terms of the spatial-domain interpretation in optics) may be essentially affected by the nonlinearity, as in the case of dipole-mode states [22], azimuthons [23], and gap-soliton vortices supported by a 2D optical lattice [24]. The radial size of the CVS may be identified as a "point of the intersection" of the vortices building the crescent, u_1 and u_2, i.e., as a root of equation $|u_1|=|u_2|$, which yields $r_{\mathrm{int}}=\sqrt{|m_2/m_1|}r_0$.

As mentioned above, qualitatively similar structures were recently found in the GPE, which includes the local self-attraction, rotation (i.e., the Coriolis term), and a combination of quadratic and quartic trapping radial potentials [21].

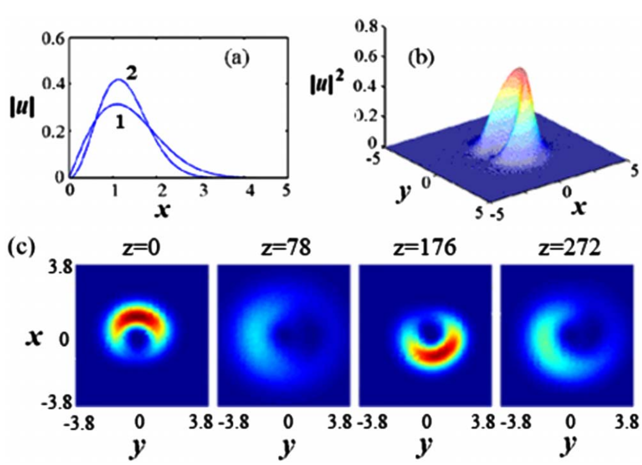

FIG. 4. (Color online) (a) Profiles of vortices with topological charges $m=1$ and 2 and unequal widths, $r_1=1.1$ and $r_2=0.8$. (b) The CVS generated by the initial superposition of these vortices. (c) Unsteady rotation of the CVS.

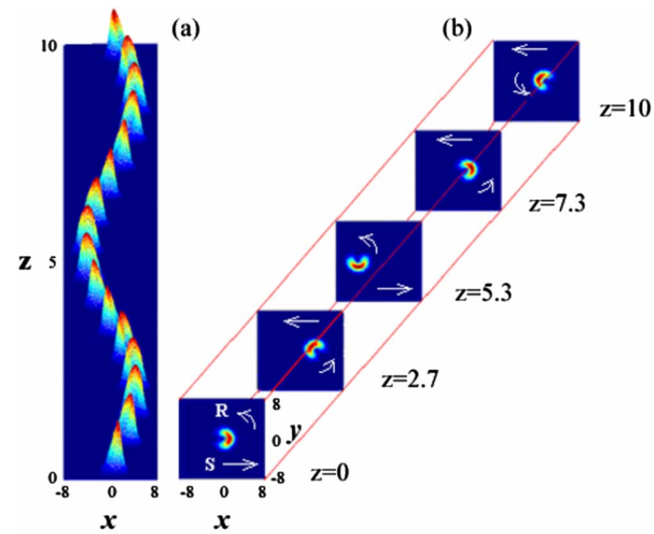

FIG. 5. (Color online) (a) The superposition of the swinging and rotary motions of a kicked CVS from Fig. 2(a). (b) Contour maps of the soliton in plane (x,y) correspond to (a), with circular (R) and straight (S) arrows indicating directions of the rotary and swinging motions.

In that case, crescent-shaped patterns were formed by a superposition of three vortices, with topological charges m and $m\pm1$ (in the typical case, the charges were 2, 3, and 4). The "crescents" cannot be stable in the absence of the quartic term in the confining potential. With the increase of the strength of the local nonlinearity, they gradually shrink into "center-of-mass" states, which seem to be 2D solitons set at a distance from the rotation pivot [21]. The results presented here demonstrate that the CVSs are, as a matter of fact, more generic dynamical states, which can be built in less sophisticated settings—in particular, with the quadratic trap only.

Multiple *nested* CVSs can be constructed by superimposing several pairs of vortex beams. In particular, double CVSs, with the larger one embracing its smaller counterpart, are formed by four vortices; see an example in Fig. 3.

Within the framework of the linear equation with fixed P, the CVSs are obviously stable. A relevant issue is their stability against deviation from the equality of the widths (i.e., powers) of the vortices with topological charges m and $m-1$, whose superposition builds the "crescent soliton." An example is shown in Fig. 4, where the CVS is generated by vortices with $m_1=1$, $m_2=2$ and essentially unequal widths, $r_1=1.1$ and $r_2=0.8$. It is seen that the rotating crescent does not keep a stationary shape in this case; instead, it periodically shrinks and expands. Actually, the CVS transforms itself into a crescent breather, which remains a robust localized object [note that the evolution interval displayed in Fig. 4(c) corresponds to about 50 rotation cycles]. If a double CVS is formed by two pairs of vortices, with a difference in the width between them ($r_1=r_2\neq r_3=r_4$), the inner and outer crescents display asynchronous rotation (not shown here), in accordance with Eq. (5).

More general motion of the CVS can be generated by applying a kick to it, i.e., replacing $u(x,y)$ by $u(x,y)\exp(i\alpha x)$, with real momentum α. As the kick does not alter the soliton's power, this problem is a linear one. However, the solution formally available in the analytical form (the decomposition over the full set of eigenstates of the 2D harmonic oscillator at $z=0$, and the inverse transformation at the final point) leads to very cumbersome expressions. On the other hand, simulations of Eq. (1) readily lead to the result illustrated by Fig. 5: in compliance with the Ehrenfest theorem, the kicked CVS demonstrates a combination of the underlying rotary motion and kicked-induced oscillations (swinging) in the direction of x, in the effective parabolic potential (a nontrivial feature of the motion is that the CVS keeps its integrity). This robust dynamical regime resembles the motion of a 2D gap soliton in the GPE combining the local repulsive nonlinearity, optical-lattice potential, and an inverted parabolic one [24].

IV. CONCLUSION

We have demonstrated that the quasilinear model of "accessible solitons" is the simplest setting which supports 2D localized objects in the form of crescent vortex solitons (CVSs). In this model, CVSs are generated by the superposition of two concentric vortices with topological charges m and $m-1$ and equal powers. The solitons exhibit robust circulations around the center, which may be combined with swinging motion, after the application of a kick. Multiple nested CVSs are generated by the superposition of several pairs of concentric vortices. The deviation of the model from the linearity is manifested by the unsteady dynamics of CVSs, which are created by vortex pairs with imbalanced powers.

While finding the crescent solitons in the present model is facilitated by its quasilinear form, it is plausible that similar patterns may be common to isotropic media with a finite-range nonlocal nonlinearity. It may also be interesting to extend the work into the three-dimensional geometry. Recently, stable three-dimensional clusters of localized states were predicted in the model of "accessible solitons" [25].

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