Efficient conversion between photons and between photon and atom by stimulated emission

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Quantum-state conversion among different information-carrying media is essential in a quantum network. It is found that the many schemes of quantum-state conversion can be unified under one simple process—that is, the parametric process of three-wave mixing. These schemes include quantum-state storage in atomic media, where a photonic state is converted to an atomic Raman state and vice versa, the frequency up- and down-conversion in photonic states, and photonic-state conversion to Brillouin phononic states and vice versa. The efficiency of conversion can in principle reach 100%, and the conversion process is noiseless. When the same idea is applied to a four-wave mixing process, unit conversion efficiency between a two-photon state and a single-photon state can be achieved. When applied to a two-photon entangled state in parametric down-conversion, entangled states of multiple photons can be generated.

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I. INTRODUCTION

Photons are generally believed to be good quantum-information carriers for transmission and are dubbed the term "flying qubits," whereas atoms are best for storing and processing the quantum information. Therefore, a quantum network usually consists of nodes made of atoms and connected by photons [1]. In the network, quantum information is constantly transferred between photons and atoms. In addition, another type of information transfer is also occurring—that is, the transfer between photons of different wavelengths. This is because quantum information is sensitive to losses and the wavelength at which optical communication system has low losses ($\sim$1.56 μm) does not usually match the wavelength to which atoms are coupled ($\sim$0.8 μm).

Quantum-information transfer between atoms and photons was proposed [1,2] and realized [3] in cavity QED systems thanks to the strong coupling between atom and photon in the systems. Such systems usually require precise control of the atoms and become very complicated. Significantly simplified schemes were proposed by Duan, Lukin, Cirac, and Zoller (the DLCZ scheme) [4] and realized [5–7] in a Raman system for the efficient information transfer from atomic states to photonic states in the reading process (but not from photon to atom).

For the process of converting photonic states to atomic states, Fleischhauser and Lukin [8] proposed a method based on the electromagnetically induced transparency (EIT) effect to achieve photon stoppage (storage) in an atomic medium. Subsequent experiments realized photon storage in an atomic vapor cell [9,10] and Bose-Einstein condensate (BEC) [11] and in solid materials [12]. In these investigations, however, the emphasis is on photon stoppage. Atoms are simply the medium for information storage. Although atomic states are important in this effect, no study was done of them. As mentioned before, another role for atoms as nodes in a quantum network is to process the stored information quantum mechanically. Thus it is necessary to know what kind of states the atomic system is in.

For the photonic-state transfer between different wavelengths, the frequency up-conversion process was realized many years ago [13–16], but none was reported for frequency down-conversion. This is so because current research focus is on counting photons at optical communication wavelengths ($\sim$1.56 μm) for quantum cryptography [16]—detectors at 1.56 μm are just far more noisy than those at shorter wavelengths around 0.8 μm. But the main reason perhaps is the strong belief that the parametric frequency down-conversion process is always quantum mechanically noisy due to spontaneous emission [14,17]. Nevertheless, efficient frequency down-conversion is essential in a quantum network for converting photons emitted by atoms ($\sim$0.8 μm) to photons transmitted in optical fibers ($\sim$1.56 μm).

Recently, it was discovered that a parametric down-conversion process can be greatly enhanced by stimulated emission due to constructive multiphoton interference [18]. The enhancement factor is proportional to the intensity of the stimulating field in the linear nondepleted regime. And it is accompanied by spontaneous emission, often treated as noise. However, when the stimulating field becomes so strong that it comes the regime when the pump field starts to be depleted, the situation is completely different, and as we will see in the following, the process becomes noise free and direct transfer between the pump and idler fields becomes possible.

In this paper, we will consider the parametric amplification process with a large injected signal field but a small pump field so that the regime of depletion is reached and show that the pump photon may be converted to the idler photon with 100% efficiency under certain conditions, realizing efficient photonic frequency down-conversion. When the same idea is applied to other three-wave mixing processes, a similar effect occurs between two of the three waves. In particular, for a Raman process, we find that the transfer between photon and atom can be achieved with unit efficiency. Although this process was studied extensively as

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the phenomenon of light storage and stoppage in EIT, our emphasis is on the atomic state and its manipulation by incoming photons. Similarly for a Brillouin scattering process, we may achieve 100% conversion efficiency between photons and phonons. Furthermore, applying the same idea to a four-wave mixing process, we can achieve efficient conversion between a single-photon state and a two-photon state.

The paper is organized as follows: we will first discuss the enhancement effect in stimulated emission in Sec. II. Then we apply the idea to the general process of three-wave mixing by considering frequency up- and down-conversion in Sec. III. Next, we deal with a photon-atom system of the Raman process and consider the conversion between photonic quantum states and atomic states. We will also discuss the Brillouin process and deal with photon-phonon conversion in the same section. In Sec. IV, we generalize to a four-wave mixing process and discuss the conversion between a one-photon state and a two-photon state.

II. ENHANCEMENT DUE TO STIMULATED EMISSION IN PARAMETRIC DOWN-CONVERSION

We start with a parametric down-conversion process described by the Hamiltonian [19]

$$\hat{H}_{PDC} = i\hbar\,\eta A_p \hat{a}_s^\dagger \hat{a}_i^\dagger - i\hbar\,\eta^* \hat{a}_s \hat{a}_i \hat{A}_p^*, \tag{1}$$

where the pump field denoted by A_p is a strong classical field and s and i stand for signal and idler fields for historic reasons. Energy conservation gives the frequency relation $\omega_p = \omega_s + \omega_i$. This leads to the parametric amplifier when the pump is undepleted [17]:

$$\hat{a}_s^{(out)} = \hat{a}_s \cosh|\eta A_p|\tau - e^{j\varphi_p}\hat{a}_i^\dagger \sinh|\eta A_p|\tau,$$

$$\hat{a}_i^{(out)} = \hat{a}_i \cosh|\eta A_p|\tau - e^{j\varphi_p}\hat{a}_s^\dagger \sinh|\eta A_p|\tau, \tag{2}$$

with an amplitude gain given by $G=\cosh|\eta A_p|\tau$, where τ is the interaction time and $e^{j\varphi_p} \equiv \eta A_p/|\eta A_p|$. The amplification $G>1$ is from the stimulated emission. This process is often used in nonlinear optics to achieve frequency down-conversion: $\omega_p \to \omega_s$ with ω_s tunable.

However, the appearance of the $\hat{a}^\dagger$ terms in Eq. (2) leads to spontaneous quantum noise [14,17]. Therefore, this process is not suitable for frequency down-conversion of quantum fields. Nevertheless, we may gain some insight into quantum-state transfer from this process by considering the low-gain case with $G \sim 1$ or $|\eta A_p \tau| \ll 1$. Under this condition, the evolution operator from the Hamiltonian in Eq. (1) has the approximate form of

$$\hat{U} = e^{\eta A_p \tau \hat{a}_s^\dagger \hat{a}_i^\dagger - \text{H.c.}} \approx 1 + (g\hat{a}_s^\dagger \hat{a}_i^\dagger + \text{H.c.}), \tag{3}$$

with $g \equiv \eta A_p \tau$. This operator is used to evaluate the state evolution of the system in the Schrödinger picture:

$$|\Phi\rangle_{out} = \hat{U}|\Phi\rangle_{in}. \tag{4}$$

With a vacuum input of $|\Phi\rangle_{in} = |0\rangle$, we have the output state as

$$|\Phi\rangle_{out}^{(0)} = \hat{U}|0\rangle \approx |0\rangle + g|1_s\rangle \otimes |1_i\rangle. \tag{5}$$

The last term gives the spontaneous emission with a probability of $R=|g|^2$. When the input is an N-photon state in the signal field $|N_s\rangle \otimes |0_i\rangle$, we have

$$\begin{aligned}|\Phi\rangle_{out}^{(N)} &\approx |N\rangle_s|0\rangle_i + g(\hat{a}_s^\dagger|N\rangle_s) \otimes (\hat{a}_i^\dagger|0\rangle_i)\\ &= |N\rangle_s|0\rangle_i + g\sqrt{N+1}|N+1\rangle_s \otimes |1\rangle_i.\end{aligned} \tag{6}$$

The photon emission probability from the amplifier is $N+1$ times that of the spontaneous emission. Each fold of increase is attributed to the stimulated emission from one individual photon in the input N-photon state. This emission enhancement effect due to stimulated emission was observed by Lamas-Linares *et al.* [20] for the one-photon case and by Sun *et al.* [18] for the two-photon case and is interpreted as a multiphoton interference effect.

Although the above exercise does not avoid the quantum noise from spontaneous emission, it indicates that the down-conversion probability can be greatly enhanced by injecting a strong signal field with the enhancement factor N as the number of injected photons.

Now let us see what will happen if we can increase N indefinitely so that the overall down-conversion probability $(N+1)|g|^2 \to 1$. In this case, we can no longer treat g as a constant because the amplifier enters a regime when the pump field starts to be depleted. So a significant amount of pump photons are converted to the signal and idler photons. Among these conversions, because $N \gg 1$, only a negligible portion $(1/N)$ is the spontaneous emission, while the majority of the pump photons are coherently converted to the signal and idler photons via stimulated emission. Because the spontaneous emission is negligible, we may achieve a noiseless frequency down-conversion of photons in this process. We will show this in the following.

III. EFFICIENT CONVERSION BETWEEN WAVES IN THREE-WAVE MIXING

The parametric down-conversion process is in essence a three-wave mixing process and can be described by the Hamiltonian [21]

$$\hat{H}_{III} = i\hbar\,\eta \hat{a}_p \hat{a}_s^\dagger \hat{a}_i^\dagger - i\hbar\,\eta^* \hat{a}_s \hat{a}_i \hat{a}_p^\dagger. \tag{7}$$

Here we use operators to replace field amplitudes. With a strong pump field, it becomes a parametric down-conversion process described in Eq. (1). But when the signal field is strong while the pump and idler fields are weak, the three-wave mixing process becomes a frequency converter and is described by the Hamiltonian [13]

$$\hat{H}_F = i\hbar\,\eta \hat{a}_p \hat{a}_i^\dagger A_s^* - i\hbar\,\eta^* \hat{A}_s a_i \hat{a}_p^\dagger, \tag{8}$$

where the signal field is treated classically and the operators are replaced by c numbers. It converts an idler photon $(\hat{a}_i)$ to a pump photon $(\hat{a}_p)$ or vice versa. This can be easily seen in the evolution of the system [13,22]:

$$\hat{a}_p^{(out)} = \hat{a}_p \cos|\eta A_s|\tau + e^{j\varphi_s}\hat{a}_i \sin|\eta A_s|\tau,$$

$$\hat{a}_i^{(out)} = \hat{a}_i \cos|\eta A_s|\tau - e^{j\varphi_s}\hat{a}_p \sin|\eta A_s|\tau, \tag{9}$$

where $e^{j\varphi_s} = \eta A_s^*/|\eta A_s|$. Notice that the input-output relation in Eqs. (9) is the same as the one for a lossless beam splitter if we set its transmissivity $t=\cos|\eta A_s|\tau$ and reflectivity $r = e^{j\varphi_s}\sin|\eta A_s|\tau$.

When $|\eta A_s|\tau=\pi/2$ and $\varphi_s=0$, we have $t=0$, $r=1$ and achieve a complete conversion: $\hat{a}_i \to \hat{a}_p^{(out)}$, $\hat{a}_p \to -\hat{a}_i^{(out)}$ with a unit conversion efficiency. This conversion is a field conversion. Thus, the complete quantum states are converted from p to i and vice versa. As can be seen, this conversion process adds no quantum noise as long as $r=1$. Otherwise, vacuum noise from unused input port will be added, just like the case of a detector with nonunit quantum efficiency.

The frequency up-conversion of $\hat{a}_i \to \hat{a}_p^{(out)}$ is easily understood via the sum frequency process: if there are enough signal photons, the existence of an idler photon will quickly fuse it with one of the abundant signal photons to produce a photon of higher energy (the pump photon). The pump photon may also convert back to an idler photon if there are too many signal photons. The frequency down-conversion process $\hat{a}_p \to -\hat{a}_i^{(out)}$ can be understood as a result of stimulated emission, as discussed in the previous section.

Although the frequency up-conversion process from idler to pump was demonstrated [13,14] as a noise-free process and used to convert long-wavelength photons to short-wavelength photons for photon counting in quantum cryptography [16], the down-conversion process from pump photon to idler photon has never been realized. It is so perhaps because the process was often mistaken as the amplification process in Eq. (2) with unavoidable quantum noise injection from the idler [14]. However, as we see from Eq. (9), when the unit efficiency is achieved with $\sin|\eta A_s|\tau=1$, no input idler field contribution to the idler output. Thus, it should be noise free.

IV. APPLICATION TO RAMAN AND BRILLOUIN PROCESSES: EFFICIENT PHOTON-ATOM AND PHOTON-PHONON CONVERSIONS

The above three-wave mixing process mixes optical fields of different frequencies. However, the idea can be generalized to mixing of arbitrary waves. In particular, generalization to a Raman process, as we will see in the following, results in efficient transfer from photonic state to atomic state and vice versa. It is well known by now [4] that in a collective Raman process (Fig. 1) with an ensemble of N_a atoms, a pair of lower-level metastable states $|g\rangle$ and $|m\rangle$ are coupled through an excited state $|e\rangle$ by the pump field $\hat{a}_p$ and the Stokes field $\hat{a}_S$, respectively. After adiabatic elimination of the upper excited state $|e\rangle$, the effective Hamiltonian of the subsystem of $|g\rangle$ and $|m\rangle$ and the fields has the form of [4]

$$\hat{H}_R = i\hbar\eta\hat{a}_p\hat{a}_S^\dagger\hat{S}^\dagger - i\hbar\eta\hat{a}_S\hat{S}\hat{a}_p^\dagger, \tag{10}$$

where $\eta = g_{eg}g_{em}^*\sqrt{N_a}/\Delta$ with Δ as the detuning of light fields from the upper excited state in Fig. 1 and g_{eg}, g_{em} as the coupling coefficients between light fields and respective atomic states. $\hat{S} \equiv (1/\sqrt{N_a})\Sigma_i|g\rangle_i\langle m|$ is the collective atomic spin field, which is coupled to the Stokes and the pump fields via the Hamiltonian in Eq. (10).

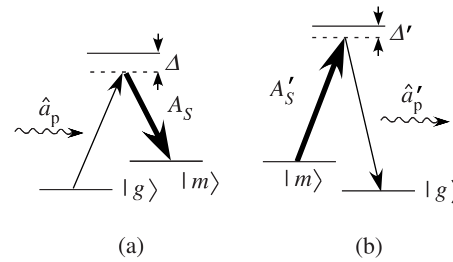

FIG. 1. Raman process with a strong Stokes for efficient conversion between photon and atom. (a) Conversion from photon to atom. (b) The reverse process of photon retrieval.

When the pump field is strong and the Stokes field is initially in vacuum and the atomic state in ground state $|g\rangle$, the process becomes spontaneous Raman scattering and is described by the Hamiltonian [4]

$$\hat{H}_{R1} = i\hbar\eta A_p\hat{a}_S^\dagger\hat{S}^\dagger - i\hbar\eta A_p^*\hat{a}_S\hat{S}, \tag{11}$$

which produces an atom-light entangled state of [6]

$$\begin{aligned}|\psi\rangle &\approx |0\rangle_m|0\rangle_S + \sqrt{p_c}\hat{S}^\dagger|0\rangle_m\hat{a}_S^\dagger|0\rangle_S + o(p_c)\\ &= |0\rangle_m|0\rangle_S + \sqrt{p_c}|1\rangle_m|\hat{1}\rangle_S + o(p_c),\end{aligned} \tag{12}$$

with a small excitation probability p_c. Here $|1\rangle_m \equiv (1/\sqrt{N_a})\Sigma_i|m\rangle_i$ is the collective single-atom excited state of the metastable state $|m\rangle$. With an injection at the Stokes field, it becomes a Raman amplifier for the Stokes $\hat{a}_S$ field, but with intrinsic quantum noise from spontaneous Raman scattering [23].

On the other hand, when there is a strong Stokes field at input while the pump field is weak, we have

$$\hat{H}_{R2} = i\hbar\eta A_S^*\hat{a}_p\hat{S}^\dagger - i\hbar\eta A_S\hat{a}_p^\dagger\hat{S} \tag{13}$$

and the evolution of the fields is given by

$$\begin{aligned}\hat{a}_p^{(out)} &= \hat{a}_p\cos|\eta A_S|\tau + e^{j\varphi_S}\hat{S}\sin|\eta A_S|\tau,\\ \hat{S}^{(out)} &= \hat{S}\cos|\eta A_S|\tau - e^{j\varphi_S}\hat{a}_p\sin|\eta A_S|\tau.\end{aligned} \tag{14}$$

Here $e^{j\varphi_S} \equiv \eta A_S^*/|\eta A_S|$. Note that Eq. (14) is similar to Eq. (10) of Ref. [8], but in a different language. Similar to Eq. (9), when $|\eta A_S|\tau=\pi/2$ and $\varphi_S=0$, we have a complete conversion from light to atom and vice versa: $\hat{S}^{(out)}=-\hat{a}_p$, $\hat{a}_p^{(out)}=\hat{S}$.

In the transfer from light to atom with $\hat{S}^{(out)}=-\hat{a}_p$, a photon in the $\hat{a}_p$ field is annihilated and is transferred to one of the atoms in the ensemble in the metastable state $|1\rangle_m$: $\hat{S}^\dagger|0\rangle_m=(1/\sqrt{N_a})\Sigma_i|m\rangle_i$. In the reverse process, the atomic excitation can be converted back to a photon, which may have a different frequency from the original one if the strong A_S' field is in a different frequency. Through the whole process, light is stored in the atomic ensemble and retrieved. If the retrieved photon has the same frequency as the original one, we have the phenomenon of light stoppage, inasmuch the same way as the phenomenon of light stoppage with the EIT effect [8–12]. Notice the difference between the two phenomena: the EIT effect requires resonance between light fields and atomic states ($\Delta, \Delta'=0$). Furthermore, the control light needs to be the same field in EIT; i.e., the retrieval field is the same one as the initial writing field, while in the Raman scheme discussed here, they may be different. The input and output quantum fields may also be different, thus realizing a frequency converter.

The practical implementation of the above frequency conversion scheme is very similar to the experimental realization of the DLCZ scheme [5–7], but with, of course, different parameters. We will need a quantum field for conversion in addition to the write field. The read part is exactly the same. The storage time depends on the decoherence time of the $|m\rangle$ state of the atomic ensemble.

In the above discussion, the retrieved and converted light [Fig. 1(b)] is a confirmation of the existence of the atomic state of $\hat{S}^{\dagger}|0\rangle_m=(1/\sqrt{N_a})\Sigma_i|m\rangle_i$, which is the result of the first conversion process from the photonic state to the atomic state [Fig. 1(a)].

It is interesting to note that when we discuss the quantum-state conversion from the photon field of $\hat{a}_p$ to the spin wave $\hat{S}$, $\hat{a}_p$ satisfies the bosonic commutation relation $[\hat{a}_p,\hat{a}_p^{\dagger}]=1$, whereas $\hat{S}$ does not: $[\hat{S},\hat{S}^{\dagger}]=\Sigma_i(|g\rangle_i\langle g|-|m\rangle_i\langle m|)/N_a\neq 1$. So how can we transfer the quantum state between the two different waves? In fact, if the atomic system is prepared in the ground state $|g\rangle$ and the number of photons for conversion is much smaller than N_a so that $\langle\Sigma_i|m\rangle_i\langle m|\rangle\ll N_a$ and $\langle\Sigma_i|e\rangle_i\langle e|\rangle\ll N_a$, we may make the approximation $[\hat{S},\hat{S}^{\dagger}]\approx\Sigma_i(|g\rangle_i\langle g|+|m\rangle_i\langle m|+|e\rangle_i\langle e|)/N_a=1$. Therefore, the spin wave field $\hat{S}$ can be treated as a bosonic field and be exchanged with a photonic field.

We used a simplified and idealized model for the Raman process considered here. In practice, there may be other processes that take place concurrently during laser-matter interaction, such as four-wave mixing. However, the Raman process is considered to be the dominant one as long as the two-photon Raman condition $\omega_p-\omega_S=\omega_{mg}$ is satisfied and there is a large detuning $\Delta\gg\Gamma_e$ from the intermediate state $|e\rangle$. There is another process that may influence the one under consideration—that is, the spontaneous Raman scattering for the strong Stokes field; i.e., the strong Stokes field may itself act as a pump and produce another Stokes field spontaneously. But this spontaneous process takes little energy if any from the strong field and thus has a small effect on the main Raman process. We do not consider the propagation effect here, either. For example, there will be a temporal walk-off problem between the Stokes pulse and the pump pulse due to dispersion. But if we make the pulse duration long enough, such an effect is negligible.

The three-wave mixing scheme discussed in the previous section can also be applied to the Brillouin scattering process, where light is scattered by a sound wave [22]. So one of the waves in three-wave mixing is a sound wave or phonon wave. The related Hamiltonian has the form of

$$\hat{H}_B=i\hbar\eta\hat{a}_p\hat{a}_s^{\dagger}\hat{b}_{pn}^{\dagger}-i\hbar\eta^*\hat{a}_s\hat{b}_{pn}\hat{a}_p^{\dagger}, \tag{15}$$

where $\hat{b}_{pn}$ is the phonon annihilation operator for the sound wave. $\hat{a}_s$ and $\hat{a}_p$ represent the scattered and incoming light fields, respectively. Energy conservation requires $\omega_p=\omega_s+\Omega$, where Ω is the frequency of the phonon.

There are two situations in this case. The first one is similar to Raman scattering process discussed above. Here we can convert photon waves to phonon waves via stimulated Brillouin scattering [22] when there is a strong stimulating field at ω_s. Like before, when the intensity of the stimulating field is strong enough we can achieve photon-to-phonon conversion $\hat{a}_p\to\hat{b}_{pn}$ with unit efficiency and vice versa. This was recently demonstrated by Zhu *et al.* [24] in light storage in an optical fiber. Note the conversion is a quantum field conversion. So it applies to photonic quantum-state storage.

The second situation realizes a frequency conversion process of light, $\hat{a}_p\leftrightarrow\hat{a}_s$ or $\omega_p\leftrightarrow\omega_s$, when the sound wave is strong. This situation is actually just the acoustic optical modulation (AOM) process commonly used for frequency shift in optics [25]. Because the conversion is at the level of quantum fields, we have just shown that the AOM process preserves the quantum nature of the fields.

V. APPLICATION TO FOUR-WAVE MIXING: EFFICIENT CONVERSION BETWEEN A SINGLE-PHOTON STATE AND A TWO-PHOTON STATE

Next, we apply the same idea to a four-wave mixing process and demonstrate how we can convert a two-photon state to a single-photon state and vice versa with unit efficiency. The Hamiltonian for near-degenerate four-wave mixing is given by [22]

$$\hat{H}_{IV}=\chi^{(3)}\hat{a}_p^2\hat{a}_s^{\dagger}\hat{a}_i^{\dagger}+\chi^{(3)*}\hat{a}_s\hat{a}_i\hat{a}_p^{\dagger 2}, \tag{16}$$

where $\hat{a}_p$ is traditionally the strong pump field and may be treated as classical waves of amplitude A_p. Then we have the traditional four-wave mixing Hamiltonian

$$\hat{H}_{FWM}=i\hbar\eta\hat{a}_s^{\dagger}\hat{a}_i^{\dagger}-i\hbar\eta^*\hat{a}_s\hat{a}_i, \tag{17}$$

with $\eta\equiv\chi^{(3)}A_p^2/i\hbar$. The four-wave mixing Hamiltonian discussed above has been realized in a dispersion-shifted fiber system [26] and a photonic crystal fiber system [27].

Now similar to Eq. (8), we let the signal field be strong and the Hamiltonian becomes (see Fig. 2)

$$\hat{H}_H=\chi^{(3)*}A_s\hat{a}_i\hat{a}_p^{\dagger 2}+\text{H.c.} \tag{18}$$

This is the Hamiltonian for harmonic generation. But unlike traditional harmonic generation, the coupling coefficient is enhanced by the A_s field. This enhancement is due to stimulated emission. Moreover, the frequencies of the fields are related by $\omega_i=2\omega_p-\omega_s$ and can be arbitrary depending on that of the strong signal field (in harmonic generation, we have $\omega_i=2\omega_p$, instead).

Let us now show that, by the Hamiltonian in Eq. (18), we may achieve high conversion efficiency from two-photon events in the $\hat{a}_p$ field to single-photon events in the $\hat{a}_i$ field, in analogy to the efficient conversion between photons from the Hamiltonian in Eq. (8) and between photon and atom from the Hamiltonian in Eq. (13). To demonstrate this, we apply the Hamiltonian in Eq. (18) to an input state of $|0\rangle_i|2\rangle_p$. The

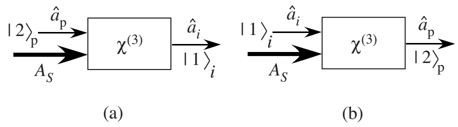

FIG. 2. Near-degenerate four-wave mixing for a (a) two-photon annihilator and (b) photon doubler.

unitary operator from the Hamiltonian in Eq. (18) is given by

$$\hat{U}_H = \exp[\zeta \hat{a}_i \hat{a}_p^{\dagger 2} - \text{H.c.}], \tag{19}$$

with $\zeta \equiv \chi^{(3)*} A_s \tau / i\hbar$. The output state is then

$$|\psi^{(out)}\rangle = \exp[\zeta \hat{a}_i \hat{a}_p^{\dagger 2} - \text{H.c.}]|0\rangle_i |2\rangle_p. \tag{20}$$

By using the relations

$$[\zeta \hat{a}_i \hat{a}_p^{\dagger 2} - \text{H.c.}]|0\rangle_i |2\rangle_p = -\sqrt{2}\zeta^* |1\rangle_i |0\rangle_p,$$

$$[\zeta \hat{a}_i \hat{a}_p^{\dagger 2} - \text{H.c.}]^2 |0\rangle_i |2\rangle_p = -2|\zeta|^2 |0\rangle_i |2\rangle_p, \tag{21}$$

we obtain

$$|\psi^{(out)}\rangle = \cos\,\theta |0\rangle_i |2\rangle_p - e^{-j\phi} \sin\,\theta |1\rangle_i |0\rangle_p, \tag{22}$$

with $\theta \equiv \sqrt{2}|\zeta|$ and $e^{-j\phi} \equiv \zeta^*/|\zeta|$. When $\theta = \pi/2$, we achieve the unit conversion efficiency from $|0\rangle_i |2\rangle_p$ to $|1\rangle_i |0\rangle_p$. Note that the two photons in the $\hat{a}_p$ field are completely annihilated. Thus we dub this process a "two-photon annihilator." Likewise, for the input state of $|1\rangle_i |0\rangle_p$, we can easily show that

$$|\psi^{(out)\prime}\rangle = \exp[\zeta \hat{a}_i \hat{a}_p^{\dagger 2} - \text{H.c.}]|1\rangle_i |0\rangle_p$$
$$= \cos\,\theta |1\rangle_i |0\rangle_p + e^{j\phi} \sin\,\theta |0\rangle_i |2\rangle_p. \tag{23}$$

The unit conversion efficiency is achievable for transferring $|1\rangle_i |0\rangle_p$ back to $|0\rangle_i |2\rangle_p$. This is a perfect "photon doubler" from one photon in $\hat{a}_i$ to two photons in $\hat{a}_p$ (see Fig. 2).

The two-photon annihilator and photon number doubler described above have many interesting applications. For example, when a weak coherent state of the form $|\alpha\rangle \approx |0\rangle + \alpha|1\rangle + (\alpha^2/\sqrt{2})|2\rangle$ passes through the two-photon annihilator, the two-photon term will vanish, leading to an antibunched photon state. This idea was proposed before with a two-photon absorber [28,29], but because of the low two-photon absorption efficiency, the antibunching effect is not significant. An improved scheme based on the two-photon interference effect was recently used to cancel out the two-photon term in a weak coherent state [30]. But that scheme is extremely sensitive to the mode match between participating quantum fields due to the interference effect. Although the current scheme also depends on mode match for a large coupling coefficient $\chi^{(3)}$, the coefficient can always be compensated from A_s because $\theta \propto \chi^{(3)*} A_s$. The two-photon annihilator can also be used to generate two-photon holes by taking out the photons from a coherent state [31]. The correlated two-photon hole state may have interesting properties for applications in quantum information.

For a three-photon state input to the mode $\hat{a}_p$ in the system with the Hamiltonian in Eq. (18), we can show that the output state has the form of

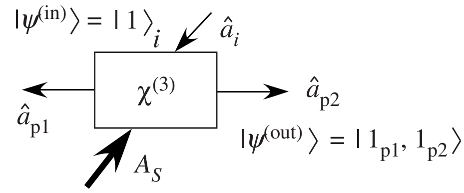

FIG. 3. Nondegenerate four-wave mixing for a nondegenerate photon doubler.

$$|\psi^{(out)}(3)\rangle = \cos\,\theta' |0\rangle_i |3\rangle_p - e^{-j\phi} \sin\,\theta' |1\rangle_i |1\rangle_p, \tag{24}$$

where $\theta' = \sqrt{6}|\zeta|$. Unit conversion efficiency can be achieved when $\theta' = \pi/2$, but notice the different forms of θ and θ'. This process can be used to take out the three-photon state from a coherent state in order to reduce the probability of multiphoton events for applications in quantum cryptography [32].

For input states with other photon numbers, there are more than two possibilities in the output state. So it becomes complicated and will not be discussed here.

The two-photon doubler, on the other hand, can be used to create multiphoton entangled states from an easily available two-photon entangled state. To achieve this, we need to use nondegenerate four-wave mixing with a Hamiltonian of

$$\hat{H}_{IV}^{(N)} = \chi^{(3)} \hat{a}_{p1} \hat{a}_{p2} \hat{a}_s^\dagger \hat{a}_i^\dagger + \chi^{(3)*} \hat{a}_s \hat{a}_i \hat{a}_{p1}^\dagger a_{p2}^\dagger, \tag{25}$$

where there are two pump fields. This usually occurs in a phase conjugate mirror [33] where the two pump fields propagate in opposite directions and the incoming signal and the outgoing idler (conjugate counterpart) fields also travel in opposite directions. In our current photon converter scheme, we will have a strong signal field and the input is a single-photon state at the idler field. Like before, a nondegenerate photon doubler Hamiltonian of the form

$$\hat{H}_D^{(N)} = \chi^{(3)*} A_s \hat{a}_i \hat{a}_{p1}^\dagger \hat{a}_{p2}^\dagger + \text{H.c.} \tag{26}$$

will lead to a nondegenerate photon doubler where the input idler photon is converted to two distinguishable photons traveling in opposite directions (Fig. 3)—i.e., $|1\rangle_i \to |1\rangle_{p1}|1\rangle_{p2}$.

If the input state is a nondegenerate two-photon entangled state $|2,\xi\rangle \approx |0\rangle + \xi|1\rangle_a|1\rangle_b$ from parametric down-conversion ($|\xi| \ll 1$), by using the nondegenerate photon doubler on the two photons, respectively, we can create a four-photon entangle state of $|4,\xi\rangle \approx |0\rangle + \xi|1\rangle_{a1}|1\rangle_{a2}|1\rangle_{b1}|1\rangle_{b2}$ (see Fig. 4). Cascading the process further down the line, entangled states with six and more even-number photons can be generated with high efficiency. This state will be extremely useful for

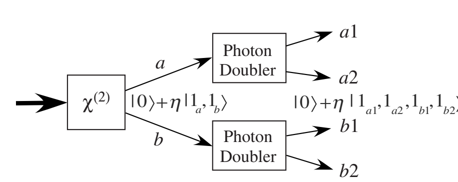

FIG. 4. Nondegenerate photon doubler for the generation of multiphoton entangled states from parametric down-conversion.

multibit quantum-information processing. Furthermore, by using the Raman process discussed before to convert the multiphoton entangled states to atomic states, we should be able to create entangled states of multiple atoms.

VI. SUMMARY

In this paper, we discussed quantum-state conversion in various three-wave mixing processes, including optical waves, Raman atomic spin waves, and sound waves. It was shown that the quantum state can be converted from one medium to another with unit efficiency under certain conditions, thus realizing quantum-information transfer among different media for communication and storage. When the same idea is applied to a four-wave mixing process, we can achieve unit conversion efficiency between a single-photon state and a two-photon state. For a nondegenerate four-wave mixing process, this is an efficient method for creating multiphoton entangled states from the popular two-photon state in parametric down-conversion.

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