Nonlinear surface waves in one-dimensional photonic crystals containing left-handed metamaterials

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The nonlinear TE-polarized surface waves localized at the interface between a nonlinear uniform material and a semi-infinite one-dimensional photonic crystal are investigated. We assume that the nonlinear material is right handed and the photonic crystal is made of alternate left-handed metamaterial and right-handed material. We study the intensity-dependent properties of the nonlinear surface waves for both self-focusing and defocusing Kerr-like nonlinearity. Unlike the linear regime, it is shown that in the case of self-focusing nonlinearity the dispersion curve has two branches corresponding to two types of surface mode structures (one structure with a hump at the interface and the other one with two humps). While, in the case of defocusing nonlinearity the dispersion curve has a single branch. We also show that the existence region of the surface waves is intensity dependent (independent) in the case of self-focusing (defocusing) nonlinear regime and the presence of left-handed materials and nonlinear medium provides a possibility for forward or backward waves selection in the given structure by adjusting the intensity parameter at the interface.

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I. INTRODUCTION

Recently, the study of metamaterials (i.e., materials with negative permittivity and negative permeability) has attracted a great deal of interest due to supreme prospects for applications in both physics and engineering [1–4]. Such materials are usually known as left-handed materials (LHMs) since the electric and magnetic fields form the left-handed set of vectors with the wave vector [5–7].

Although LHMs were first introduced theoretically by Veselago in 1967 [8] a practical way to make a left-handed metamaterial was theorized first by Pendry [9] and experimentally confirmed by Smith *et al.* [10]. It has been demonstrated that LHMs exhibit interesting properties such as cloaking [11,12], negative refraction, reverse Doppler effect, subwavelength imaging and extraordinary waveguiding properties [13]. In particular, an interface between conventional dielectric and LHM is able to support either TE- or TM-polarized guided modes [14], which is impossible for an interface between conventional dielectric. On the other hand, surface waves (SWs) that have been studied in different branches of physics are a special type of waves localized at an interface between two different media and propagate along the interface and decay in the transverse direction. Since SWs have the field maximum at or near the interface, they are a very sensitive and convenient tool for studying physical properties of surfaces. In periodic systems, staggered SWs are often referred to as Tamm states, which were first found in solid-state physics as localized electronic states at the edge of truncated periodic potential. In optics, the periodic structures must be manufactured artificially in order to manipulate dispersion properties of light in a similar way as the properties of electrons are controlled in crystals. Such periodic dielectric structures are known as photonic crystals (PCs). The linear SWs at the interface of a uniform material and a semi-infinite one-dimensional PC have been studied in Refs. [15–17].

Since the using of LHMs in PCs has given rise to many new interesting phenomena for linear as well as nonlinear wave propagation [18,19], in this paper we study the properties of the nonlinear TE-polarized SWs at the interface of nonlinear uniform right-handed materials (RHMs) and a semi-infinite one-dimensional PC, containing alternate LH and RH layers. We show that the dispersion curve of the surface modes has two branches for self-focusing nonlinear material and has a single branch for defocusing nonlinear material. In particular, we demonstrate that the existence region of surface modes depends on the intensity of SW at the interface in the case of self-focusing nonlinearity. But this is not the case for defocusing nonlinearity. This paper is organized as follows. In Sec. II, we introduce the model of the system under consideration. In Sec. III, the intensity-dependent properties of nonlinear SWs are studied. Finally, Sec. IV concludes with brief comments.

II. MODEL

In this section, we study TE-polarized nonlinear SWs propagating along the interface of a nonlinear uniform medium (x axis) and a one-dimensional PC containing alternate LH and RH layers. In the presented study we consider the operating frequency to be in the microwave range suitable for LHMs, so the thicknesses of PC layers is in the order of cm.

In the chosen coordinate system the layers have normal vector along the z axis (see Fig. 1). We assume that the width of terminate layer of the periodic structure ($d_c=d_s+d_t$) is

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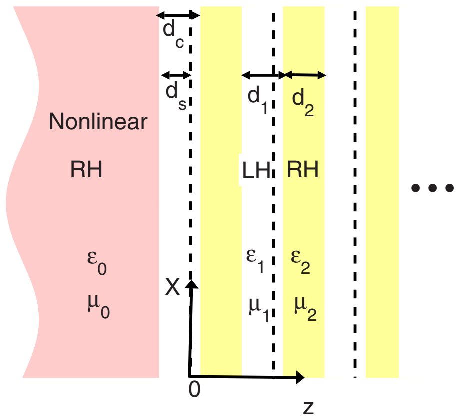

FIG. 1. (Color online) Geometry of the problem. In our calculations we take the following values: $d_1=1.7$ cm, $d_2=0.4$ cm, $d_c=1.1d_1$, $\varepsilon_0=3.24$, $\mu_0=1$, $\varepsilon_1=-6.25$, $\mu_1=-1$, $\varepsilon_2=8.41$, and $\mu_2=1$.

different from the width of other layers of the structure (d_1,d_2) and it consist of two sublayers with the lengths d_s and d_t, respectively. The first sublayer extends from $z=-d_s$ to $z=0$. Then the periodic array that forms the PC consist of the cells each made of three uniform layers of width d_t, d_2, and d_1-d_t whose respective indices of refraction are n_1, n_2, and n_1. The nonlinear medium is characterized by a nonlinear dielectric permittivity [20,21]:

$$\varepsilon_{NL}=\varepsilon_0+\alpha|E|^2, \tag{1}$$

and the magnetic permeability μ_0. Here ε_0 is the linear part of the relative dielectric permittivity, and parameter α describes Kerr-type nonlinearity. In one-dimensional PC each layer is characterized by linear dielectric permittivity ε_i and magnetic permeability μ_i $(i=1,2)$. For simplicity, we assume that the contacting media are lossless, homogeneous, and isotropic. The propagation of monochromatic waves with frequency ω is governed by the scalar wave equation, which for the case of TE-polarized wave is written as

$$\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial x^2}+\frac{\omega^2}{c^2}\varepsilon(z)\mu(z)-\frac{1}{\mu(z)}\frac{\partial\mu(z)}{\partial z}\frac{\partial}{\partial z}\right)E_y=0, \tag{2}$$

where the functions $\varepsilon(z)$ and $\mu(z)$ are dielectric permittivity and magnetic permeability in a bulk medium, respectively; ω is the angular wave frequency, and c is the speed of light in vacuum. We take the electric field of the surface modes as

$$E_y=\psi(z)\exp[-i(\omega t-k\beta x)]+\text{c.c.} \tag{3}$$

Here $k=\omega/c$ and β is the normalized wave-number component along the interface. In the nonlinear medium with $\mu(z)=\mu_0$, Eq. (2) is written as

$$\left[\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial x^2}+\left(\frac{\omega^2}{c^2}\right)(\varepsilon_0\mu_0+\mu_0\alpha|E_y|^2)\right]E_y=0. \tag{4}$$

The solitonlike solutions of Eq. (4) are given as the following equations for the self-focusing $(\alpha>0)$ and defocusing $(\alpha<0)$ nonlinear media, respectively [22],

$$\psi=\eta\sqrt{\frac{2}{\alpha\mu_0}}\,\text{sech}[q_0(z-z_0)], \tag{5}$$

$$\psi=\pm\eta\sqrt{\frac{-2}{\alpha\mu_0}}\,\text{csch}[q_0(z-z_0)]. \tag{6}$$

Here $q_0=k\sqrt{\beta^2-n_0^2}$, $\eta=q_0/k$, $n_0=\sqrt{\epsilon_0\mu_0}$, and z_0 is the center of the sech function in relation (5) which, should be chosen to satisfy the continuity of the tangential components of the electric and magnetic field at the interface. Equations (5) and (6) are valid for $\alpha\mu_0>0$ and $\alpha\mu_0<0$, respectively. Note that solution (6) at $z=z_0$ diverges. Nevertheless, it can be used for constructing SWs provided that the singularity point is located outside the corresponding medium [23,24]. In the right-side periodic structure, the waves are the Bloch modes,

$$\psi(z)=\phi(z)\exp(ik_b z), \tag{7}$$

where k_b is the Bloch wave number, and $\phi(z)$ is the Bloch function which is periodic with the period of the photonic structure [25]. In periodic structure the waves will be decaying provided that k_b is complex; and this condition defines the spectral gaps of an infinite photonic crystal. For the calculation of the Bloch modes, we use the transfer matrix method.

To find dispersion properties of SWs, we use Eqs. (5) and (6) for the nonlinear medium and Eq. (7) for the periodic structure. Then we satisfy the boundary conditions of the electric and the magnetic fields at the interface between nonlinear medium and periodic structure to arrive at the following dispersion relations for self-focusing and defocusing Kerr-like nonlinearity, respectively:

$$\pm\frac{q_0}{k_{1z}}\frac{\mu_1}{\mu_0}\sqrt{1-\tilde{I}/\eta^2}=-i\frac{\lambda-M_{11}-\tilde{M}}{\lambda-M_{11}+M}, \tag{8}$$

$$\frac{q_0}{k_{1z}}\frac{\mu_1}{\mu_0}\sqrt{1+\tilde{I}/\eta^2}=-i\frac{\lambda-M_{11}-\tilde{M}}{\lambda-M_{11}+M}. \tag{9}$$

Here $\tilde{I}=I_s/I_c$ is the normalized intensity of SW at the interface where $I_c=\frac{1}{|\alpha\mu_0|c}\sqrt{\frac{\epsilon_0}{\mu_0}}$, $I_s=\frac{1}{2}\frac{n_0}{\mu_0 c}|E_0|^2$, and E_0 is the electric field amplitude at the interface. $k_{1z}=k\sqrt{n_1^2-\beta^2}$ and λ is the eigenvalue of the transfer matrix in the photonic band gap, M_{11}, M_{12} are elements of transfer matrix M and $\tilde{M}=\exp(-2ik_{1z}d_s)M_{12}$ [16].

Equation (8) (with the plus sign) and Eq. (9) reduce to the linear dispersion relation when $\tilde{I}\to 0$ [16]. While Eq. (8) with the minus sign do not have a linear corresponding. These equations determine the dispersion relation $k=k(\beta)$ of the SWs in the first band gap for $\alpha>0$ and $\alpha<0$, respectively. In order to avoid singularity in the case of $\alpha<0$, we used $z_0>-ds$ to obtain Eq. (9). According to Eqs. (8) and (9), dispersion property of the nonlinear SWs depends on normalized intensity $\tilde{I}$ of SWs.

III. RESULTS AND DISCUSSION

We presented the dispersion property of the SWs for the case of self-focusing medium $(\alpha>0)$ in the first band gap on the plane (k,β) in Fig. 2, for a typical value of the cap layer with thickness $d_c=1.1d_1$. In this figure dispersion properties

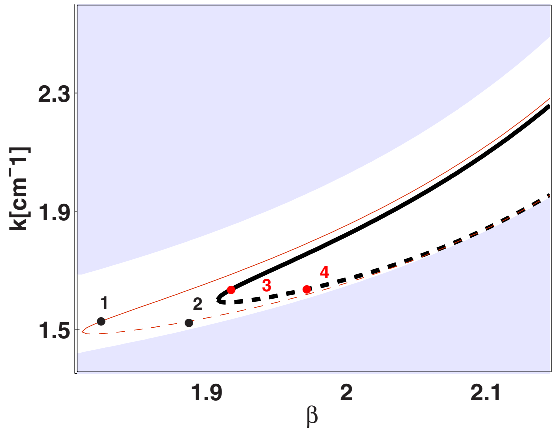

FIG. 2. (Color online) Dispersion property of the SWs in the first band gap for two different dimensionless intensity $\tilde{I}=0.01$ (thin lines) and $\tilde{I}=0.1$ (thick lines). Here unshaded regions show the first band gap of photonic crystal. The other parameters are the same as Fig. 1.

of nonlinear surface modes are demonstrated for different values of the dimensionless intensity of light $\tilde{I}$ at the surface of photonic crystal. Unlike the linear regime [16], Fig. 2 shows that the dispersion curves consist of two branches. Inspection of these branches shows that they are corresponding to two types of SWs with different transverse profile of the electric field. The lower branch corresponds to the modes with one-humped structure at the interface separating nonlinear right-handed medium and photonic crystal and it will be approach to linear dispersion property when $\tilde{I}\to 0$ [16]. The upper branch corresponds to the modes with two-humped structure, resulting from nonlinearity of medium. Furthermore, it is seen that the increasing of intensity $\tilde{I}$ leads to a limitation on the existence region of the SWs with respect to β.

To show the modes with different structures, in Fig. 3 we plotted the transverse profile of some typical surface modes (points 1–4 in Fig. 2) as a function of coordinate z. As one can see, Figs. 3(a) and 3(c) show that the modes in the upper branch of dispersion curve (solid lines in Fig. 2) have two-humps around the interface. While the modes in the lower branch (dashed lines in Fig. 2) have a one-humped structure [see Figs. 3(b) and 3(d)].

We know that in the presence of LHM, the direction of the total energy flow of the SWs can be forward or backward [15,16]. So, it is interesting to study the total energy flow of the nonlinear SWs. The time-averaged Poynting vector of the SW is directed along the x axis ($s_z=0$). The energy flow in the right-handed media and PCs is an integral of the Poynting vector over the corresponding spatial region. To demonstrate backward and forward nonlinear surface modes, we plotted the total energy flow in the modes as a function of the wave number β (see Fig. 4). Our calculations indicated that the mode 1 is a forward wave while, the modes 2, 3, and 4 are backward waves as demonstrated in Fig. 4. The modes 1 and 2 selected from the upper branch of the dispersion curves in Fig. 2 result from nonlinearity of the medium and there is no linearity corresponding to them. So, presence of LHM

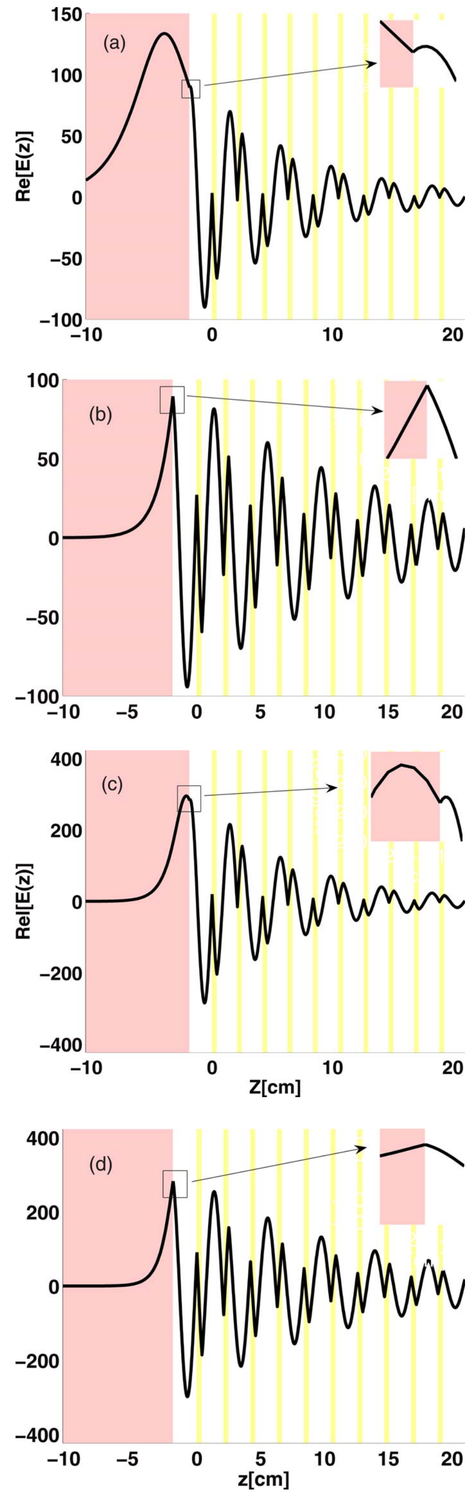

FIG. 3. (Color online) The transverse profile of the SWs vs coordinate z for (a) $\beta=1.825$, $k=1.526\ \mathrm{cm}^{-1}$, (b) $\beta=1.886$, $k=1.526\ \mathrm{cm}^{-1}$, (c) $\beta=1.918$, $k=1.635\ \mathrm{cm}^{-1}$, and (d) $\beta=1.971$, $k=1.635\ \mathrm{cm}^{-1}$ corresponds to the points 1, 2, 3, and 4 in Fig. 1. The insets show blow-up regions of profiles at the surface of PC.

and nonlinear medium provides a possibility for forward or backward waves selection in the given structure by adjusting the intensity parameter at the interface (see solid lines in Fig. 4). Furthermore, Fig. 4(b) shows that by increasing the in-

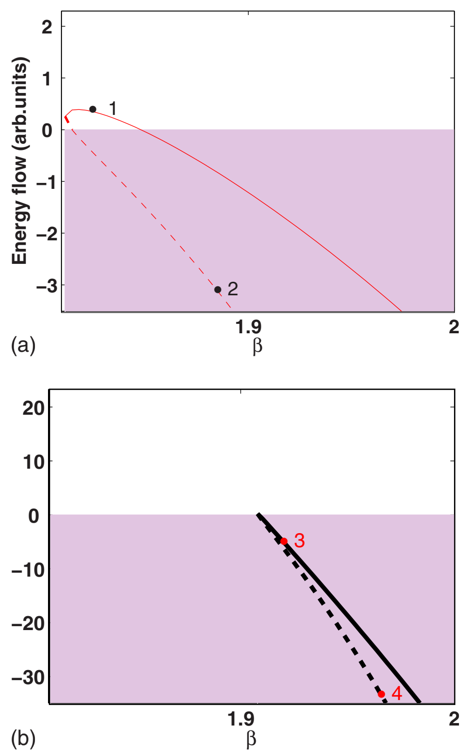

FIG. 4. (Color online) Total energy flow of the SWs in the first band gap vs β for one-humped (dashed line) and two-humped (solid line) structures in the case of $\alpha>0$ for (a) $\tilde{I}=0.01$ and (b) $\tilde{I}=0.1$. The other parameters are the same as Fig. 1.

tensity $\tilde{I}$, the existence regions of two-humped modes with forward energy flow can be canceled, so that all of the SWs become backward.

Since the Kerr-like medium can show both self-focusing and defocusing nonlinearity, in the following we study the dispersion property of the nonlinear SWs for the case of defocusing nonlinearity, $\alpha<0$, (see Fig. 5). In Fig. 5 we display the dispersion curve for two different values of dimensionless intensity $\tilde{I}=0.01$ and $\tilde{I}=0.1$. Contrary to the case of self-focusing nonlinearity ($\alpha>0$), Fig. 5 shows that the dispersion curve has only a single branch without limitation on the existence region of the SWs with respect to β. Inspection of this single branch indicates that the transverse profile of the electric field has a one-humped type of structure at the surface of PC (see Fig. 6). Figure 6(a) and 6(b) correspond to points 1 and 2 in Fig. 5, respectively. So, we see that in higher $\tilde{I}$ we lose the localization of modes for a given structure. As one can see from Fig. 5 this is due the dispersion curve approaching to the band edge in higher $\tilde{I}$. In Fig. 7 we plotted the total energy flow in the modes as a function of the wave number β. We see that all of the modes in the case of defocusing nonlinearity are backward waves.

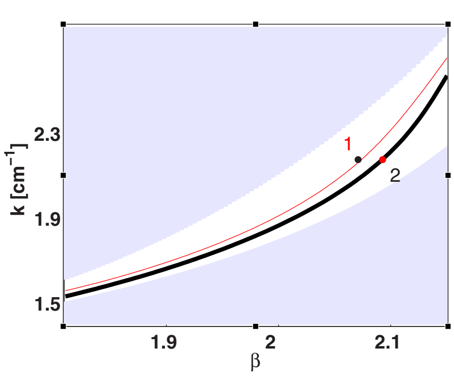

FIG. 5. (Color online) Dispersion property of the SWs in the first spectral gap for two different intensity $\tilde{I}=0.01$ (thin lines) and $\tilde{I}=0.1$ (thick lines). Here the unshaded regions show the first band gap of photonic crystal. Here $d_1=2.5$ cm, $d_2=1.0$ cm, $d_c=0.1d_1$, and $\alpha<0$. The other parameters are the same as Fig. 1.

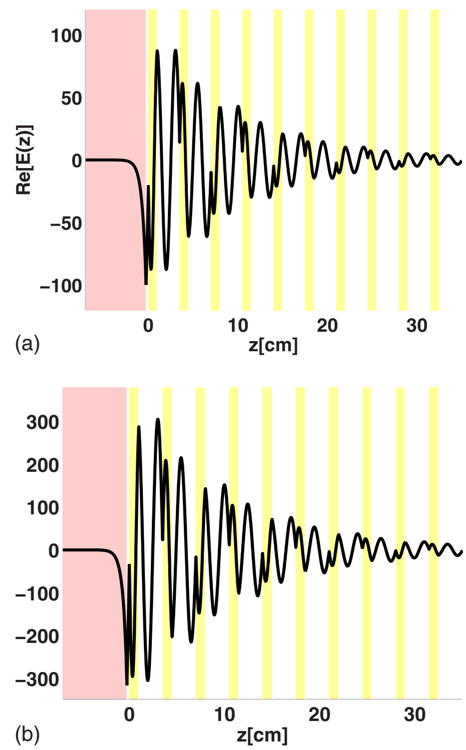

FIG. 6. (Color online) The transverse profile of the SWs vs coordinate z for (a) $\beta=2.075$, $k=2.169$ cm^{-1} and (b) $\beta=2.091$, $k=2.169$ cm^{-1} corresponding to the points 1 and 2 in Fig. 5.

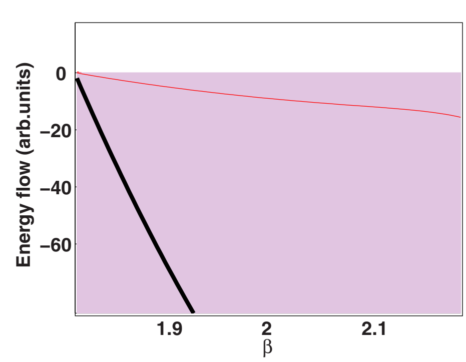

FIG. 7. (Color online) Total energy flow of the SWs in the first band gap vs β in the case of $\alpha<0$ for (a) $\tilde{I}=0.01$ and (b) $\tilde{I}=0.1$. The other parameters are the same as Fig. 5.

IV. CONCLUSION

Briefly, we studied the nonlinear TE-polarized SWs localized at the interface between a nonlinear uniform RHM and a semi-infinite one-dimensional PC made of alternate LH and RH layers. The intensity-dependent properties of the nonlinear SWs for both cases of a self-focusing and a defocusing Kerr-like media were studied. It was shown that in the case of self-focusing Kerr-like medium there are two types of modes with different transverse profile. While, in the case of defocusing Kerr-like medium there is only one type of mode. We also revealed that the existence region of the SWs depends on the intensity of light at the interface in the case of self-focusing nonlinearity. There is no limitation on the existence region of SWs for the defocusing case.

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