Single-atom-aided probe of the decoherence of a Bose-Einstein condensate

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We study a two-level atom coupled to a Bose-Einstein condensate. We show that the rules governing the decoherence of mesoscopic superpositions involving different classical-like states of the condensate can be probed using this system. This scheme is applicable irrespective of whether the condensate is initially in a coherent, thermal, or more generally, in any mixture of coherent states. The effects of atom loss and finite temperature to the decoherence can therefore be studied. We also discuss the various noise sources causing the decoherence.

PACS number(s): 03.75.Gg, 03.65.Ta, 03.67.−a

I. INTRODUCTION

Decoherence is a process of losing quantum superpositions due to entanglement between the system and its environment [1]. Studies of the decoherence are pivotal to understanding the emergence of the classical world from an underlying quantum substrate [1,2]. This is because if decoherence did not suppress quantum superpositions in macroscopic systems, then one would end with situations such as the Schrödinger's cat [3], which are not observed in practice. By now, superpositions and decoherence of microscopic systems have been observed in several experiments such as cavity QED [4,5] and trapped ions [6,7]. However, in view of the fundamental implications for the quantum to classical transition, it is necessary to gradually extend such experimentation to superpositions in macroscopic systems, perhaps tackling mesoscopic systems at first. In this context, several years ago, Leggett and co-workers proposed the possibility of observing superpositions of macroscopically distinct flux states in superconducting systems [8,9], an idea which has only recently been experimentally realized [10]. Another class of experiments, clearly probing quantum superpositions and their decoherence in the mesoscopic domain, is the diffraction of large molecules [11,12]. In order to move to mesoscopic systems with slightly larger masses and investigating their decoherence, there exist a number of proposals for using nanoscale movable mirrors coupled to quantized light in cavities [13], or coupled to a Cooper pair box [14] or to photons in an interferometer [15]. The above three schemes [13–15] rely on the common idea of coupling a microscopic (coherent) system to a mesoscopic or macroscopic (decoherent) system to probe the decoherence of the latter. In this paper, we formulate a scheme based on the same general principle for a completely different mesoscopic system, namely, a Bose-Einstein condensate. While in the earlier work involving nanoscale mirrors it is a superposition of spatially separated coherent states whose decoherence is studied, in the current paper it is the decoherence of a superposition of coherent states with different phases (or relative phases in the case of two mode condensates) which will be studied.

Recently, a general scheme has been proposed [16] to probe the decoherence of a mesoscopic harmonic oscillator with a qubit whose state couples to the position of the oscil-lator. It provides a simultaneous method to generate superpositions and probe the decoherence of a mesoscopic system. The basic assumption in this scheme is just to maintain the coherence of the qubit during the whole period of the experiment, while the oscillator is allowed to decohere (it is this decoherence that the scheme aims to detect). When a qubit and the oscillator start in appropriate states, they become entangled and a superposition in their joint system is created. The special form of the coupling entails that after a certain period of time evolution the qubit naturally disentangles from the oscillator and the oscillator is brought back to its original state. The evidence of the decoherence of the oscillator is then imprinted on the partial coherence of the qubit. This enables us to determine the decoherence rate of the mesoscopic oscillator by measuring the qubit's state. Perhaps the strongest positive feature of this type of scheme is the fact that the mesoscopic oscillator will not have to be cooled to a pure or nearly pure state in order for the scheme to be successful in probing its decoherence [13–16]; in fact it can be in a thermal state at arbitrary temperature. This happens because of the special nature of the qubit-oscillator coupling and the fact that the mesoscopic system is never directly probed. Naturally an important question is whether such a scheme can be formulated for other kinds of qubits (the qubits used earlier have been photon number or path [13,15] or superconducting [14]) such as atomic hyperfine levels which are more coherent, or oscillators which are more coherent than nanoscale mirrors but at the same time still mesoscopic in some sense. More fundamentally, would such a scheme work, particularly would it still be applicable to thermal states of the oscillator if the qubit was coupled to a different variable of the oscillator instead of its position? The system studied in this paper shows that the answer to both the above questions is affirmative.

Atomic Bose-Einstein condensates (BEC's) are mesoscopic systems with a low dissipation rate. They are promising candidates for observing decoherence. In fact, experimental studies have rapidly developed since the Bose-Einstein condensation of atomic gases was observed in a magnetic trap in 1995 [17]. For example, the quantum phase transition from a superfluid phase to a Mott-insulator phase has been observed using atoms in optical lattices [18]. The trapping of multicomponent BEC's of ^{87}Rb [19] have been realized. The control of scattering lengths using Feshbach
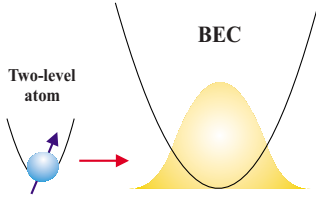
FIG. 1. (Color online) A single two-level atom is coupled to a Bose-Einstein condensate via coherent collisions. The single atom is trapped in a state-dependent potential such that the upper state $|e\rangle$ interacts with the trapped condensate merely.

resonances have also been reported [20–23]. These demonstrate the sophisticated techniques available to manipulate ultracold atoms precisely. Notably, schemes for deterministic single atom preparation in the ground state in a trap potential have been proposed [24,25]. In fact, a single atomic qubit has been demonstrated to be confined in an optical dipole trap [26] and to transport using a submicrometer optical tweezer [27]. This paves the way for studying the dynamics of a single atom coupled to a superfluid BEC [28]. In particular, schemes for cooling a single atom [29] and probing the phase fluctuations of BEC [30] have been suggested with precisely such an atom-BEC coupled system. In this paper, we will present a foundational application of such a system in probing the decoherence of a BEC (Fig. 1).

Naturally, because of the low dissipation rates of BECs, there have been quite a few proposals for preparing them directly in nonclassical states such as Schrödinger cat states [31–33] even without an additional system such as a single atom. If realized, these proposals will also enable one to detect the decoherence that such states of a BEC experience. However, the nonclassical states of these papers, being superpositions of states differing by large atom numbers in a given mode, are highly decoherent and their preparation is extremely hard (may even require one to engineer the environment [32] and may be destroyed even due to scattering a single photon [33]). Moreover, to prepare such states one needs to have a BEC in its ground state and thus whether the detection of decoherence will work for a thermal state is unclear. Most importantly, it is a superposition involving the most "classical-like" states of a harmonic oscillator, namely, its coherent states, whose decoherence is the most relevant for studying the transition from the quantum world to the classical world (as among all the available quantum states of a harmonic oscillator, the coherent states are the closest to classical states because they have equal and minimum uncertainty in all variables). The scheme of this paper is more ideally placed to detecting decoherence of superpositions involving distinct coherent states as opposed to the Schrödinger cat states of a mode occupation number [31–33].

In this paper, we study a two-level atom coupled to an atomic BEC via coherent collisions. A trapped BEC behaves like a harmonic oscillator [34–36] such that we can attempt to formulate a scheme similar to Refs. [13,16] to probe the decoherence of a BEC with a single atom. In this way, the aim is to test the decoherence of superpositions of a mesoscopic oscillator. The heuristic formula for the decoherence rate of a superposition of two coherent states is given by ($\hbar = 1$) [13,16]

$$\tilde{\Gamma} = 2\Gamma\left(\bar{n} + \frac{1}{2}\right)D^2, \qquad (1)$$

where Γ is the characteristic damping constant, D is the separation of two coherence states in phase space, $\bar{n} = [\exp(\omega/k_B T) - 1]^{-1}$, ω is the frequency of the oscillator, k_B is the Boltzmann constant, and T is the temperature.

We will show that our scheme can be applied to detect the decoherence rate of an initial coherent state and even a thermal mixed state. Finite temperature effects on the decoherence of a BEC can thus be studied. Particle loss of the condensates gives rise to the decoherence [39]. Atom loss is caused by inelastic collisions between atoms, and it is the dominant source of decoherence in the BECs [40,41].

This paper is organized as follows: In Sec II, we introduce the coupled atom-BEC system. The coupling of a single two-level atom to a single BEC and a two-component BEC are examined. Both cases can be shown to map to a qubit-oscillator model. In Sec III, we describe our scheme to probe the decoherence of condensates when they are initially in coherent and thermal states. In Sec IV, we discuss the main decoherence sources of the BECs and the limitations of tuning the scattering length with Feshbach resonances in Sec V.

II. SYSTEM

We consider a single atom with two hyperfine spin states trapped in the motional ground state of a potential. The Hamiltonian of this two-level atom is written as

$$H_s = \omega_0(|e\rangle\langle e| - |g\rangle\langle g|), \qquad (2)$$

where $2\omega_0$ is the energy splitting, and $|e\rangle$ and $|g\rangle$ are the upper and lower states, respectively. This spin-half system can be expressed in terms of a Pauli operator: $\sigma_z = |e\rangle\langle e| - |g\rangle\langle g|$, $\sigma_+ = |e\rangle\langle g|$, and $\sigma_- = |g\rangle\langle e|$. Thus, the Hamiltonian can be cast in the form as

$$H_s = \omega_0\sigma_z. \qquad (3)$$

We study a single atom coupling to a single BEC and a two-component BEC, respectively. We discuss these two cases in the following two sections.

A. Case I: A BEC

We first consider the single atom coupled to a condensate. The Hamiltonian of a BEC confined in a trapping potential is given by

$$H_1 = \int dr^3 \Psi_1^\dagger(r)\left[-\frac{1}{2m_1}\nabla^2 + V_1(r)\right.$$
$$\left. + \frac{U_{11}}{2}\Psi_1^\dagger(r)\Psi_1(r)\right]\Psi_1(r), \qquad (4)$$

where $\Psi_1(r)$ is the annihilator field operator at the position r, $V_1(r)$ is the external trapping potential, U_{11} is the self-interaction strength, and m_1 is the mass of an atom. The

condensate is assumed to be trapped in a deep potential such that the BEC can be well described within the single-mode approximation [34,35], i.e., $\Psi_1(r) \approx a\psi_1(r)$. Here a and $\psi_1(r)$ are the annihilator operator and the mode function of the condensate, respectively. Then, the Hamiltonian is written as

$$H_1 = \omega_1 a^\dagger a + \kappa_1 (a^\dagger a)^2, \tag{5}$$

where ω_1 and κ_1 are the energy frequency and the self-interaction strength, respectively.

The single two-level atom interacts with the condensates via coherent collisions. The Hamiltonian describes such a coupling as

$$H_1^I = 2\kappa_{1e}|e\rangle\langle e|a^\dagger a \tag{6}$$

$$= \kappa_{1e}(\sigma_z + 1)a^\dagger a, \tag{7}$$

where $\kappa_{1e} = \pi a_{1e} \int dr^3 |\psi_1^*(r)\psi_e(r)|^2 / m$ and $\psi_e(r)$ is the wave function of the single atom and a_{1e} is the s-wave scattering length between the atom at the upper state $|e\rangle$ and the condensate. We consider this single atom trapped in a state-dependent potential [42] so that only the upper state $|e\rangle$ interacts with the condensate [30]. Besides, we assume that coherent collisions between the atom and the BEC will not further excite the motional state of the single atom. Otherwise, it will give rise to additional noise and affect our detection scheme.

The size of the ground state wave function of the atom and the BEC are roughly equal to the trap size. We denote the "volume" of the trap as V. The interaction parameter κ can be found as $\pi a_{1e}/mV$. In general, this interaction strength is weak. Nevertheless, the scattering length a_{1e} can be greatly increased by tuning an external magnetic field near a Feshbach resonance so that the interaction strength is greatly enhanced [20–23]. This is a very useful tool for controlling the atom-BEC coupling.

B. Case II: Two-component BEC

Next, we consider the single atom to be coupled to a two-component BEC. The Hamiltonian of a two-component condensate is given by

$$H_2 = \int dr^3 \Psi_\alpha^\dagger(r)\left[-\frac{1}{2m_\alpha}\nabla^2 + V_\alpha(r) + \frac{U_{\alpha\alpha}}{2}\Psi_\alpha^\dagger(r)\Psi_\alpha(r) + \frac{U_{\alpha\beta}}{2}\Psi_\beta^\dagger(r)\Psi_\beta(r)\right]\Psi_\alpha(r), \tag{8}$$

where $\Psi_\alpha(r)$ is the annihilator field operator for the component α, $V_\alpha(r)$ is the trapping potential, $U_{\alpha\alpha}$ and $U_{\alpha\beta}$ are the intracomponent and intercomponent interactions, respectively, and $\alpha, \beta = 1, 2$.

As before, we adopt the single-mode approximation on the two-component condensates in which they are confined in a deep potential [34,36]. We write $\Psi_\alpha(r) \approx \eta\psi_\alpha(r)$, where $\eta = a, b$ and $\psi_\alpha(r)$ are the annihilation operator and the mode function for the component α, respectively. Thus, the Hamiltonian can be written as

$$H_2 = \omega_1 a^\dagger a + \omega_2 b^\dagger b + \kappa_1(a^\dagger a)^2 + \kappa_2(b^\dagger b)^2 + \kappa_{12}a^\dagger a b^\dagger b, \tag{9}$$

where the energy frequency is ω_α, the self-interaction strength is κ_α, and the intercomponent interaction strength is κ_{12}. It is convenient to express the Hamiltonian in terms of the angular momentum operators as

$$H_2 = \tilde{\omega}J_z + \tilde{\kappa}J_z^2, \tag{10}$$

where $J_x = (a^\dagger b + b^\dagger a)/2$, $J_y = (a^\dagger b - b^\dagger a)/2i$, and $J_z = (a^\dagger a - b^\dagger b)/2$. The parameters $\tilde{\omega}$ and $\tilde{\kappa}$ are $(\omega_1 - \omega_2)/2$ and $\kappa_1 + \kappa_2 - \kappa_{12}$, respectively.

We consider the single atom coupled to the two-component BEC via coherent collisions. The Hamiltonian for such an atom-BEC coupling has the form

$$H_2^I = 2|e\rangle\langle e|(\kappa_{1e}a^\dagger a + \kappa_{2e}b^\dagger b), \tag{11}$$

where $\kappa_{\alpha e} = \pi a_{\alpha e} \int dr^3 |\psi_\alpha^*(r)\psi_e(r)|^2 / m_\alpha$ and $a_{\alpha e}$ is the s-wave scattering length between the atom in state $|e\rangle$ and the component α. We can rewrite the Hamiltonian in terms of the angular momentum operators as

$$H_2^I = (\kappa_{1e} - \kappa_{2e})(\sigma_z + 1)J_z + (\kappa_{1e} - \kappa_{2e})N, \tag{12}$$

where N is the total number of atoms. The constant term will be omitted in the subsequent discussion. To strengthen the atom-BEC coupling, we can increase the scattering length between the atom and one of the components by adjusting a magnetic field approaching a Feshbach resonance. For example, we can modulate the magnetic field to increase the scattering length between the excited state $|e\rangle$ and the component $\alpha = 2$.

In fact, the collective excitations of the two mode condensates behave like a harmonic oscillator. This can be shown by taking the leading approximation based on the Holstein-Primakoff transformation (HPT) [43] to map the angular momentum operators into the harmonic oscillators: $J_+ \approx \sqrt{N}c^\dagger$, $J_- \approx \sqrt{N}c$, and $J_z = c^\dagger c - N/2$. This approximation is valid as long as the excitations are very small [36], i.e., $\langle c^\dagger c \rangle/N \ll 1$. Therefore, the effective atom-BEC Hamiltonian can be readily obtained as follows:

$$\tilde{H}_2^I = (\kappa_{1e} - \kappa_{2e})(\sigma_z + 1)c^\dagger c. \tag{13}$$

We assume the interaction strength κ_{1e} and κ_{2e} are unequal to each other, i.e., $\kappa_{1e} \neq \kappa_{2e}$. The state of collective excitations of the BEC can be approximated as a coherent state $|\alpha\rangle$ [44] and $|\alpha|^2$ is the mean excitations of the two-component condensate. The amount of mean excitations can be adjusted by using a two-photon Rabi pulse [19].

III. SCHEME TO PROBE DECOHERENCE

As discussed above, both the single and two-component BECs can be described as harmonic oscillators. In other words, the single atom (qubit) is effectively coupled to a harmonic oscillator. In fact, the Hamiltonian in both cases is of the same form and can be written as

$$H = \omega_0 \sigma_z + \omega d^\dagger d + \kappa (d^\dagger d)^2 + \chi(\sigma_z + 1) d^\dagger d, \qquad (14)$$

where d and ω are the annihilation operator and the frequency of the harmonic oscillator, respectively, κ is the strength of the nonlinearities, and χ is the qubit-oscillator coupling strength.

We consider the interaction strength χ is much stronger than the strength κ so that the effect of nonlinearities arising from particle interactions can be ignored in the quantum dynamics. The strength κ is roughly equal to 100 Hz [34] for the trap frequency around 1 kHz. The interaction strength χ can be enhanced to several times κ by approaching the Feshbach resonance with a magnetic field. Therefore, the system is equivalent to a qubit-harmonic oscillator system without any nonlinearity.

We present a scheme to detect the decoherence of the BEC. The dominant decoherence source of the BECs is the atom loss due to three-body inelastic collisions [40]. The master equation for the condensate is given by [45–47]

$$\dot{\rho} = \frac{\gamma_3}{6} [2d^3 \rho d^{\dagger 3} - d^{\dagger 3} d^3 \rho - \rho d^{\dagger 3} d^3], \qquad (15)$$

where $\gamma_3 = K_3 \int dr^3 |\psi(r)|^6$, K_3 is the three-body coefficient, and d and $\psi(r)$ are the destruction operator and the mode function of the condensate mode, respectively. Remarkably, in the limit of a large number of atoms, the master equation (15) can be well approximated to the one-body master equation but with a new dissipation parameter Γ [46] as follows:

$$\dot{\rho} = \Gamma[2d\rho d^\dagger - d^\dagger d\rho - \rho d^\dagger d], \qquad (16)$$

where $\Gamma = 3N^2 \gamma_3 / 2$. For the case of a two-component BEC, we assume the atom loss mainly coming from one of the components, say $\alpha = 2$. We note that losing one atom in the component $\alpha = 2$ results in one loss in the relative population $\langle J_z \rangle$. Thus, the process of atom loss in the two-component BEC can be described by the master equation (16) in the large atom number limit.

The master equation (16) can be solved exactly and its solution is best expressed in the coherent state basis. In our paper we will require the time evolution of density operator terms of the form $|\alpha\rangle\langle\alpha e^{i\theta}|$, where $|\alpha\rangle$ and $|\alpha e^{i\theta}\rangle$ are two coherent states differing by a rotation in phase space, under the master equation (16). Up to a normalization constant, the time evolution of the above term is of the form [38,39]

$$\begin{aligned}
\tilde{\rho}(t) \propto &|\alpha e^{-\Gamma t/2}\rangle\langle\alpha e^{-\Gamma t/2}| + |\alpha e^{i\theta-\Gamma t/2}\rangle\langle\alpha e^{i\theta-\Gamma t/2}| \\
&+ e^{-|\alpha|^2(1-e^{i\theta})(1-e^{-\Gamma t})}(|\alpha e^{-\Gamma t/2}\rangle\langle\alpha e^{i\theta-\Gamma t/2}| \\
&+ e^{-|\alpha|^2(1-e^{-i\theta})(1-e^{-\Gamma t})}|\alpha e^{i\theta-\Gamma t/2}\rangle\langle\alpha e^{-\Gamma t/2}|). \quad (17)
\end{aligned}$$

For the short times, i.e., $\Gamma t \ll 1$, one can approximate $(1 - e^{\Gamma t})$ and $|\alpha e^{-\Gamma t/2}\rangle$ as Γt and $|\alpha\rangle$, respectively. This is a good approximation for an underdamped oscillator if the detection time scale χ^{-1} is much shorter than the dissipation time scale Γ^{-1}. Note that the decoherence time scale $(\Gamma|\alpha|^2)^{-1}$ can still be comparable to χ^{-1}, so that the decoherence can be detected.

In addition, we assume that the two-level atom has very long coherence times so that it can act as a faithful microscopic probe to detect the decoherence. In fact, the long coherence times of the atomic condensates with two hyperfine states have been measured using Ramsey spectroscopy [48].

A. Initial coherent state

Our scheme is very simple in which it involves a few procedures only. First, we perform a unitary transformation of the two-level atom to create an equal superposition of the states $|g\rangle$ and $|e\rangle$ whereas the BEC is prepared as a coherent state. Such the unitary transformation can be easily made by a Rabi pulse [42,48]. Initially, a separable state of the two-level atom and the harmonic oscillator is considered as

$$|\Psi(0)\rangle = |\psi\rangle_Q \otimes |\alpha\rangle, \qquad (18)$$

where $|\psi\rangle_Q = (|e\rangle + |g\rangle)/\sqrt{2}$ and $|\alpha\rangle$ is a coherent state. To manifest the evolution of the atom-BEC system, we first consider the case without the decoherence setting in. The atom becomes entangled with the harmonic oscillator just after switching on the atom-BEC interaction. The state can be written as

$$|\Psi(t)\rangle = \frac{1}{\sqrt{2}}(e^{-i\omega_0 t}|e\rangle \otimes |e^{-2i\chi t}\alpha(t)\rangle + e^{i\omega_0 t}|g\rangle \otimes |\alpha(t)\rangle),$$
$$(19)$$

where $|\alpha(t)\rangle \approx e^{-i\omega d^\dagger dt}|\alpha\rangle$ for $\chi \gg \kappa$. The atom-BEC interaction gives rise to a rotation of coherent state in phase space. Thus, the phase shifts of the condensate are acquired according to the states $|g\rangle$ and $|e\rangle$, respectively. As a result, a superposition of two coherent states is generated.

The "distance" $D(t)$ in phase space between the two states can be defined as $2|\alpha|\sin \chi t$ [4]. The quantity D can indicate the distance of the superpositions. However, the atom completely disentangles with the BEC at time $t' = \pi/\chi$ and the state reads

$$|\Psi(t')\rangle = \frac{1}{\sqrt{2}}(e^{i\omega_0 t'}|g\rangle + e^{-i\omega_0 t'}|e\rangle) \otimes |\alpha(t')\rangle. \qquad (20)$$

Now we consider the system in the presence of the decoherence. We begin the investigation of the decoherence of the harmonic oscillator in the underdamped case. We must describe the system with the density matrix because the system will evolve to a statistical mixture. Initially, the density matrix is

$$\rho_c(0) = |\psi\rangle\langle\psi|_Q \otimes |\alpha\rangle\langle\alpha|. \qquad (21)$$

At time $t = t'/2$, the density matrix evolves as

$$\begin{aligned}
\rho_c(t) = \frac{1}{2}[&|g,\alpha_0\rangle\langle g,\alpha_0| + |e,\alpha_1\rangle\langle e,\alpha_1| \\
&+ e^{-\bar{\Gamma}/2}(e^{2i\omega_0 t}|g,\alpha_0\rangle\langle e,\alpha_1| + e^{-2i\omega_0 t}|e,\alpha_1\rangle\langle g,\alpha_0|)],
\end{aligned}$$
$$(22)$$

where $|g,\alpha_0\rangle = |g\rangle|\alpha_0\rangle$, $|e,\alpha_1\rangle = |e\rangle|\alpha_1\rangle$, and $|\alpha_0\rangle$ and $|\alpha_1\rangle$ are $|\alpha(t)\rangle$ and $|e^{-2i\chi t}\alpha(t)\rangle$, respectively.

The decoherence factor $e^{-\bar{\Gamma}/2}$ appears in the terms $|g, \alpha_0\rangle\langle e, \alpha_1|$ and $|e, \alpha_1\rangle\langle g, \alpha_0|$. At the end of detection times $t = t'$, the density matrix reads

$$\rho_c(t') = [|g\rangle\langle g| + |e\rangle\langle e| + e^{-\bar{\Gamma}}(e^{2i\omega_0 t'}|g\rangle\langle e| + e^{-2i\omega_0 t'}|e\rangle\langle g|)]$$

$$\otimes \frac{1}{2}|\alpha(t')\rangle\langle\alpha(t')|. \tag{23}$$

The atom disentangles with the condensate and the condensate is brought back to its original state. Although Eqs. (22) and (23) look similar to Eqs. (2) and (3) in [16], we should point out that we study the decoherence here with completely different mechanisms in [16]. Here we investigate the detection of the decoherence due to three-body atom loss of BECs. In Ref. [16], the probing of the decoherence of the Caldeira-Leggett model was studied [49].

The atom-BEC coupling can be effectively switched off by tuning the external magnetic field and then we can slowly move out the atom from the BEC. Then, the state of the atom is measured and the probability of a single atom at the state $|e\rangle\langle e|$ can be found as

$$P(|e\rangle\langle e|) = \frac{1 + e^{-\bar{\Gamma}}\cos 2(\omega_0 t' + \delta)}{2}, \tag{24}$$

if the initial state of the atom $|\psi(\delta)\rangle_Q = (|g\rangle + e^{i\delta}|e\rangle)/\sqrt{2}$ is considered, where δ is the phase shift between the states $|g\rangle$ and $|e\rangle$. The partial coherence factor $e^{-\bar{\Gamma}}$ appears in the probability of the state $|e\rangle\langle e|$. The measurement of the visibility of the fringes as a function of δ leads us to determining the factor $e^{-\bar{\Gamma}}$.

Since the instantaneous superposition of states decoheres as $\Gamma D^2(t)$, the average value of the decoherence rate $\langle\Gamma\rangle$ can be evaluated as

$$\langle\Gamma\rangle = \frac{4\Gamma\chi|\alpha|^2}{\pi}\int_0^{\pi/\chi} dt \sin^2 \chi t. \tag{25}$$

The average rate $\langle\Gamma\rangle$ can be obtained as $2\Gamma|\alpha|^2$. Therefore, the decoherence factor $\bar{\Gamma} = \langle\Gamma\rangle\pi/\chi$ after the completion of the probing process is given by $2\pi\Gamma|\alpha|^2/\chi$. We can probe the decoherence factor $\bar{\Gamma}$ by measuring of the probability of the excited state of the single atom. Hence, the damping rate Γ of the BEC can be determined.

It is noted that our scheme can be applied to find out the effective size of superpositions where the superposed components have a large overlap [50]. We can first create a superposition of two coherent states with a small distance $D(\tilde{t}) = 2|\alpha|\sin \chi\tilde{t}$ for a small parameter $\chi\tilde{t}$. This can be achieved by just turning on the atom-BEC interaction for short times $\tilde{t}$. The superpositions decohere with a rate $4\Gamma|\alpha|^2 \sin^2 \chi\tilde{t}$, which is proportional to $N \sin^2 \chi\tilde{t}$ [50]. The condensate is then brought back to the original state shortly with a fast ramp speed. This can ensure that the decoherence mainly comes from the superpositions with large overlap. We can determine this decoherence rate by our detection scheme.

B. Initial thermal state

The temperature of the BEC is nearly absolute zero; indeed its temperature ranges from 100 nK to 500 pK [17,51]. The state of the BEC can be well described as a thermal state if the finite temperature is taken into account. Our detection can be used for probing the decoherence with the initial thermal state. This enables us to study the decoherence due to the finite temperature effect. Initially, the density matrix is of the form

$$\rho_{\text{th}}(0) = |\psi\rangle\langle\psi|_Q \otimes \int d^2\alpha\, p(\alpha)|\alpha\rangle\langle\alpha|, \tag{26}$$

where $p(\alpha)$ are probabilities $\exp(-|\alpha|^2/\bar{n})/\pi\bar{n}$. The evolution of the density matrix at time $t = t'/2$ is

$$\rho_{\text{th}}(t) = \int d^2\alpha \frac{p(\alpha)}{2}[|g, \alpha_0\rangle\langle g, \alpha_0| + |e, \alpha_1\rangle\langle e, \alpha_1| + e^{-\bar{\Gamma}'_\alpha/2}$$

$$\times (e^{2i\omega_0 t}|g, \alpha_0\rangle\langle e, \alpha_1| + e^{-2i\omega_0 t}|e, \alpha_1\rangle\langle g, \alpha_0|)], \tag{27}$$

where $e^{-\bar{\Gamma}'_\alpha}$ is the decoherence factor at time t for each α. Finally, the atom disentangles with the condensate and the density matrix is found to be

$$\rho_{\text{th}}(t') = \frac{1}{2}[|g\rangle\langle g| + |e\rangle\langle e| + e^{-\bar{\Gamma}'}(e^{2i\omega_0 t'}|g\rangle\langle e|$$

$$+ e^{-2i\omega_0 t'}|e\rangle\langle g|)] \otimes \int d^2\alpha\, p(\alpha)|\alpha(t')\rangle\langle\alpha(t')|. \tag{28}$$

The decoherence factor $e^{-\bar{\Gamma}} = \int d^2\alpha\, p(\alpha)e^{-\bar{\Gamma}'_\alpha}$ sums up all contributions from the decoherence of the different possible coherent states $|\alpha\rangle$. Similarly, the decoherence factor $e^{-\bar{\Gamma}}$ can be detected through the measurement of the visibility of the atom. We have shown that our scheme can be used to probe the decoherence of the condensates with the initial coherent and thermal states. In fact, as Eq. (28) is valid for any distribution $p(\alpha)$, the scheme is valid for any mixture of coherent states being the initial state of the condensate. So, for example, if the amplitude of the coherent state was known but its phase was completely unknown, our method of probing decoherence would still be applicable.

IV. DECOHERENCE SOURCES

The main decoherence sources of the BEC are three-body inelastic collisions and collisions with the background gas [40]. We sort out several noise sources of the BECs and discuss them as follows.

A. Atom loss

Background gases and spontaneous light scattering. The background gases and spontaneous emission contribute the one-body loss and thus induce the decoherence of the BECs [41]. The loss rate Γ_1 is of the form $K_1 N$, where K_1 is the one-body loss coefficient. However, such a decoherence effect is weak in the current experiment circumstance [41].

Two-body and three-body inelastic collisions. The two-body inelastic collisions are very rare in the atomic condensates and its loss rate is $K_2 N^2/V$, where K_2 is the two-body coefficient and V is the volume of the trapped BEC. The inelastic collisions mainly occur due to the three-body collisions [22,40,41]. The rate of decoherence Γ is $K_3 N^3/V^2$, where K_3 is the three-body coefficient. The three-body coefficient K_3 is about $\sim 10^{-29}$ cm^6 s^{-1} [22,40]. It is noted that inelastic collisions are greatly enhanced near Feshbach resonances [41]. However, we assume this effect is minimal to our detection scheme if the probing times are very short.

B. Phase damping

Phase damping describes a process of the loss of the coherence without losing energy. Elastic collisions between the BEC and the surrounding gases can cause phase damping [37,52,53]. The elastic scattering with vacuum noises can also contribute to the dephasing. However, our scheme is not applicable in detecting the phase damping for the coherent states with the same magnitude $|\alpha|^2$ but with the different phases. The decoherence factor cannot be imprinted on the atom. In fact, the dephasing rate Γ_p depends on the temperature of the gases and therefore its rate is very low in the current experiments of the BEC [37,52,53]. The decoherence time scale of the phase damping is much longer than that of the atom loss. Thus, the dephasing effect is negligible compared to the atom loss.

V. DISCUSSION

Our scheme involves the active control of the scattering length using a magnetic field approaching Feshbach resonances. We can greatly increase the scattering length a with the Feshbach resonance and therefore speed up the process of creating superpositions. The scattering length a is varied as a function of external magnetic field B [41,54].

$$a = a_{bg}\left(1 + \frac{\Delta}{B_0 - B}\right), \tag{29}$$

where a_{bg} is the off-resonant scattering length, B_0 is the resonant magnetic field, and Δ is the width of the Feshbach resonance.

Nevertheless, the change of scattering length a will be accompanied with an increasing three-body inelastic collision rate. The rate of three-body collisions loss increases to 20(60) times for the low magnetic field of sodium atoms [41]. This limits the use of Feshbach resonance to create a superposition state. In the worst case, the single atom may be lost due to the formation of molecules [54]. Then, our scheme is no longer applicable—of course, if we lose the atom from our trap, we no longer continue with that run of the experiment and simply restart the experiment with a fresh atomic qubit. It is quite possible, though, to maintain the coherence of the single atom and create superpositions with a fast ramp speed [22]. Thus, our scheme can be applied to the situation that the detection rate is much shorter than the decay rate.

VI. CONCLUSION

In summary, we have studied the coupling of a single atom to the single and two-component BECs, respectively. We have presented a scheme to create mesoscopic superpositions involving distinct classical-like (or coherent) states of the BEC and probe their decoherence. The probing of the decoherence is applicable to both initial coherent and thermal states of the BEC. Only the state of the atom state needs to be directly measured in this experiment to probe the decoherence of the BEC. The various noise sources leading to the decoherence of the condensates are also discussed. This allows us to investigate the major decoherence source due to atom loss in detail.

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