

Resonant transmission of electromagnetic fields through subwavelength zero- ϵ slits

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We theoretically investigate the transmission of electromagnetic radiation through a metal plate with a zero- ϵ metamaterial slit, where the permittivity tends towards zero over a given bandwidth. Our analytic results demonstrate that the transmission coefficient can be substantial for a broad range of slit geometries, including subwavelength widths that are many wavelengths long. This resonant effect has features quite unlike the Fabry-Perot-like resonances that have been observed in conductors with deep channels. We further reveal that ultranarrow zero- ϵ channels can have significantly *greater* transmission compared to slits with no wave impedance difference across them.

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With the current state of the art in nanofabrication technologies, and the recent observation of resonant optical transmission through small metal holes [1], there has been a renewed interest in electromagnetic wave transmission and diffraction in metallic nanostructures. Although diffraction effects involving optical slits and gratings has a long history, scientific curiosity coupled with the potential for device applications has spurred considerable research activity involving the manipulation and confinement of electromagnetic waves in nanoscale resonant structures. With the concurrent advent of metamaterials, or composite structures with tailored electromagnetic properties, the interplay of classical transmission systems with these new composite media has become an important issue. By engineering metallic nanoapertures, gratings, or channels to incorporate metamaterials, it is anticipated that various enhanced or exotic transmission characteristics will ensue.

Lately, there has been interest in zero- ϵ metamaterials, which are structures that exhibit an effective permittivity of zero (or nearly so) in the passband, akin to the property of some precious metals [2] near their plasma frequency. It was shown that waveguide devices containing zero- ϵ inclusions could potentially be more efficient via the reduction of unwanted reflections if one of the physical dimensions was made smaller [3]. This can possibly negate the ill effects of impedance mismatch. A multilayer structure having nonlinear electric and magnetic responses, and a refractive index close to zero, was shown to effectively shield [4] electromagnetic fields. Experimental work has verified [5,6] electromagnetic tunneling through zero- ϵ metamaterials at microwave frequencies. It was shown [7] that the effective magnetic permeability μ can also be resonantly tuned to vanish, creating a matched metamaterial with an effective zero index of refraction. These media have been linked to applications in miniaturized resonators, highly directive antennas [8], delay lines with zero-phase difference I/O , and transformers that convert small-curvature wave fronts into output beams with planarlike wave fronts [9].

A fundamental system in which to investigate zero- ϵ diffraction and transmission effects is a subwavelength channel through a metal film. If the slit is filled with air, it has been established that for light polarized perpendicular to the slit in an optically thick perfect metal of length l , Fabry-Perot-like waveguide modes arise when l is approximately a half inte-

ger number of wavelengths: $l \approx n\lambda/2$. These harmonic modes, which follow from geometrical arguments, result in transmission peaks when the waves coherently superimpose over the given path length. As the geometrical parameters vary, the resonant wavelength can shift [10,11] in metals with slit perforations, as observed with microwaves [12]. For ultrashort incident pulses, the Fabry-Perot-like modes can be resonantly activated [13] through the Fourier components of the wave packet inside the slit, leading in some cases to enhanced transmission. The resonant enhancement of a nanometer scale electromagnetic pulse was also shown to be spatially and temporally localized [14] in the near field. For perfect metals with holes rather than slits, transmission is strictly limited to incident wavelengths that are less than twice the diameter of the openings.

In this Rapid Communication we reveal some unexpected and exotic transmission behavior of light through a subwavelength zero- ϵ slit. Our theoretical framework demonstrates that significant transmission can occur in these structures for a considerable range of ultranarrow widths that are sufficiently deep. The inverse relationship between the slit length and width that achieves maximal transmission is shown to be highly nontrivial. We show that zero- ϵ metamaterials with inherently large intrinsic impedances can have greater transmission than matched zero index slits, where μ is also vanishingly small. We also take advantage of the subwavelength geometry (that is, the slit width is smaller than about half the incident wavelength) to invoke the single mode approximation, which has been shown to yield valuable physical insight into resonance phenomena for perfect [15] and real [16] metals. Our analytic results thus permit one to efficiently map out the relevant sector of parameter space deemed appropriate for any future experimental endeavors.

We consider a planar perfect metallic structure that is translationally invariant in the x - z plane and has channel length l normal to the plane with width p (along x). The incident beam is TM polarized, so that the magnetic field \mathbf{H} is directed along z . The wave vector, \mathbf{k} , forms an angle θ with the normal to the plane. For this configuration, the z component of the magnetic field, H_z , must satisfy the scalar Helmholtz equation,

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \epsilon \mu k_0^2 H_z = 0, \quad (1)$$

where $k_0 = \omega/c$. Our focus is primarily metamaterial slits very near the characteristic plasma frequency and, as in recent works, represents the frequency dispersive electrical response by an effective Drude model, $\epsilon = 1 - \omega_p^2 / [\omega(\omega + i\Gamma)]$, where Γ is correlated to the mean-free path in the filling material. Other than when discussing absorption loss, we generally take Γ equal to zero in order to isolate the electromagnetic effects inherent to vanishing constitutive relations.

We solve Eq. (1) in a given region and then match each solution at the corresponding interfaces using the appropriate boundary conditions. To construct the solution in the ‘‘continuum’’ regions of free space surrounding the metal sheet, we Fourier transform Eq. (1) along x and use separation of variables. This renders an expression written in terms of plane wave expansions,

$$H_{z,1} = e^{i(k_{0x}x - k_{0y}y)} + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\alpha R(\alpha) e^{i\alpha x} e^{i\beta y}, \quad (2)$$

$$H_{z,3} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} d\alpha T(\alpha) e^{i\alpha x} e^{-i\beta y}, \quad (3)$$

where $k_{0x} = k_0 \sin \theta$, $k_{0y} = k_0 \cos \theta$, and $\beta = \sqrt{k_0^2 - \alpha^2}$. Here the subscripts 1 and 3 denote the entrance and exit regions, respectively. The unknown coefficients $R(\alpha)$ and $T(\alpha)$ are determined below. The general eigenmode expansion for the field within the slit region is [10]

$$H_{z,2} = \sum_{m=0}^{\infty} \cos[k_m(x - p/2)] [a_m e^{i\tau_m y} + b_m e^{-i\tau_m y}], \quad (4)$$

where $k_m \equiv m\pi/p$, and $\tau_m \equiv \sqrt{\epsilon \mu k_0^2 - k_m^2}$. The summation is over a single index m reflecting the interconnection among the wave numbers consistent with the wave equation. The electric field is obtained directly via application of Maxwell’s equation: $\mathbf{E} = i/(\epsilon k_0) \nabla \times \mathbf{H}$. It is generally valid to retain only the lowest order mode in the expansion (4) for the ultranarrow channels considered in this Rapid Communication. Moreover, as ϵ goes to zero, the summation results in higher order evanescent fields that have a negligible contribution to the time-averaged energy flux. Thus for the electric field we have

$$E_{x,2} = \eta (-ae^{i\tau y} + be^{-i\tau y}) \zeta(x), \quad (5)$$

where $\tau \equiv \tau_0$, $\eta = \sqrt{\mu/\epsilon}$ is the intrinsic impedance, and the function $\zeta(x)$ is unity in the slit and vanishes elsewhere.

Matching the electric field at the entrance and exit interfaces yields the R and T coefficients:

$$R(\alpha) = \frac{\sqrt{2\pi} k_{0y} \delta(k_{0x} - \alpha)}{\beta} + \frac{w}{\beta} (a - b) \Phi(\alpha p), \quad (6a)$$

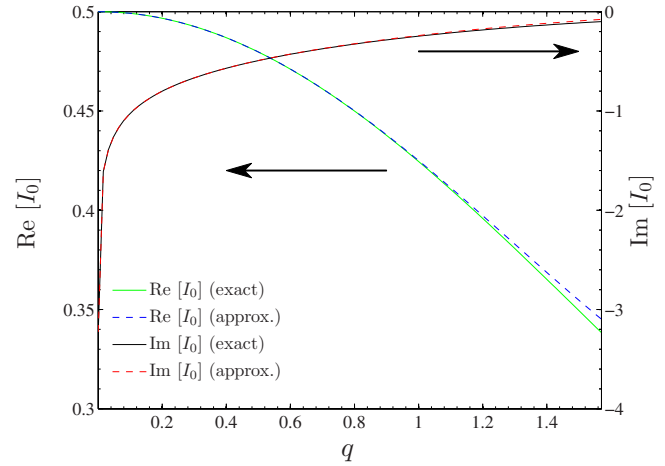


FIG. 1. (Color online) Real and imaginary components of I_0 as a function of the dimensionless width $q \equiv \pi p/\lambda$. The approximate solutions and exact integral [Eq. (8)] have good agreement over the relevant range of widths. As q vanishes, the imaginary component has a slow divergence [see Eq. (9)].

$$T(\alpha) = \frac{w}{\beta} (-ae^{-i\tau l} + be^{i\tau l}) e^{-i\beta l} \Phi(\alpha p), \quad (6b)$$

where we define $\Phi(x) \equiv \sqrt{2/\pi} \sin(x/2)/x$. A convenient technique [15] that utilizes the boundary conditions to determine the remaining unknown a and b coefficients is the use of the following relationships at the appropriate openings: $\langle H_{z,1}(x, 0), \zeta(x) \rangle = \langle H_{z,2}(x, 0), \zeta(x) \rangle$, and $\langle H_{z,2}(x, -l), \zeta(x) \rangle = \langle H_{z,3}(x, -l), \zeta(x) \rangle$. After some tedious algebra, we have for b ,

$$b = \frac{2\sqrt{2\pi}\Phi(k_{0x}p)(1 + wI_0)}{(1 + wI_0)^2 - e^{2i\xi}(1 - wI_0)^2}, \quad (7)$$

where $w \equiv k_0 p \eta$, $\xi \equiv \tau l$, and $a = be^{2i\xi}(wI_0 - 1)/(wI_0 + 1)$. The complex valued integral I_0 is a function solely of q :

$$I_0 = \frac{1}{\pi q^2} \int_0^{\infty} du \frac{\sin^2(uq)}{u^2 \sqrt{1 - u^2}}, \quad (8)$$

where $q \equiv k_0 p/2$ is the dimensionless measure characterizing the slit width. The series expansion for the imaginary component of I_0 has the following leading terms:

$$\text{Im}[I_0] \approx -\frac{1}{6\pi} \left[(q^2 - 6)(\gamma + \ln q) + 9 - \frac{19}{12}q^2 \right], \quad (9)$$

where γ is Euler’s constant. Similarly for the real part of I_0 , we have (to fourth order), $\text{Re}[I_0] \approx \frac{1}{2} - \frac{1}{12}q^2 + \frac{1}{120}q^4$. It will be seen shortly that I_0 is a very important quantity in the determination of the transmission. The accuracy of these expansions is demonstrated graphically in Fig. 1, where the truncated expansions are plotted alongside the numerically integrated I_0 [Eq. (8)]. Clearly, the approximations for I_0 are satisfactory for the small q of interest here, deviating only slightly for the larger q , as would be expected for a power series centered about the origin.

Next, we insert the calculated a and b coefficients into Eq. (5) to determine the electric field at the bottom of the slit,

$$E_x^{\text{bot}} = \frac{2i \sin(q \sin \theta) \eta}{q \sin \theta [\sin \xi + 4I_0 q \eta_2 (i \cos \xi + I_0 q \eta_2 \sin \xi)]}, \quad (10)$$

which is valid for arbitrary μ and ϵ . The magnetic field is easily obtained via the relationship $H_z^{\text{bot}} = 2qI_0 E_x^{\text{bot}}$. The transmission coefficient, τ , is defined as the time-averaged Poynting vector at the bottom of the slit, $\langle S_y \rangle_{y=-l}$, integrated over the exit opening, and divided by the Poynting vector of the incident plane wave:

$$\tau = \frac{16q^2 \text{Re}[I_0] |\eta|^2 \mathcal{F}(\theta)}{|\sin \xi + 4I_0 q \eta (i \cos \xi + I_0 q \eta \sin \xi)|^2}, \quad (11)$$

which is dimensionless after incorporating a normalization factor, k_0 . The angular dependence is encapsulated entirely by the expression $\mathcal{F}(\theta) = \text{sinc}^2(q \sin \theta) / \cos \theta$, which depends weakly on q for subwavelength slits, and is close to unity for source waves near normal incidence. For small q , we can further write (for $\sin \xi \neq 0$): $\tau \approx 8(\mu/\epsilon)q^2 \text{csc}^2(k_0 l \sqrt{\epsilon\mu})$, otherwise if we consider channels with some integer multiple of the Fabry-Perot length, $l = \lambda / (2\sqrt{\epsilon\mu})$, Eq. (11) approximately reduces to $\tau \approx \frac{1}{2}|I_0|^{-2}$, which clearly only depends on the ratio of the slit width to the wavelength [see Eq. (8)].

Having derived the general expression for transmission of electromagnetic fields through a subwavelength channel containing, to this point, conventional material parameters, we now examine the effects of taking the limit of vanishing ϵ . It is relatively straightforward to show that Eq. (10) for the electric field at the slit exit now reduces to the succinct form

$$E_x^{\text{bot}} = \sqrt{\frac{\pi}{2}} \frac{\Phi(2q \sin \theta)}{qI_0(1 - iI_0 k_0 l q \mu)}, \quad (12)$$

giving a transmission of

$$\tau = \frac{\text{Re}[I_0]}{|I_0|^2 |1 - iI_0 k_0 l q \mu|^2}, \quad (13)$$

where $H_z = 2qI_0 E_x^{\text{bot}}$, and is spatially constant in the slit. It can be deduced from Faraday's law, $\mathbf{H} = -i/(\mu k_0) \nabla \times \mathbf{E}$, that E_x must therefore be a linear function of the coordinate y within the slit: $E_x = (y/l) \delta E_x + E_x^{\text{top}}$, where $\delta E_x \equiv E_x^{\text{top}} - E_x^{\text{bot}}$. The electric field at the top of the slit, E_x^{top} , is simply $E_x^{\text{top}} = (1 - 2iqI_0 k_0 l \mu) E_x^{\text{bot}}$. For $q \ll 1$, we can write Eq. (13) strictly in terms of elementary functions,

$$\tau \approx \frac{2\pi[\pi + k_0 l \mu q(3 - 2\gamma - 2 \ln q)]}{4 \ln q(2\gamma - 3 + \ln q) + \pi^2 + (2\gamma - 3)^2}, \quad (14)$$

demonstrating that for a given ratio of slit width to wavelength, τ is simply a linear function of the channel length l and permeability μ .

To illustrate how the transmission depends upon the geometry of the slit, we present in Fig. 2 a three-dimensional representation of τ [Eq. (13)] as a function of the slit dimensions, l and p , scaled by the wavelength. In this figure, the source field is normally incident with wavelength, $\lambda = 5 \mu\text{m}$, which is very close to the resonant plasma wavelength, so that the frequency dispersive ϵ nearly vanishes. It is evident from the plot that the transmission does not possess Fabry-

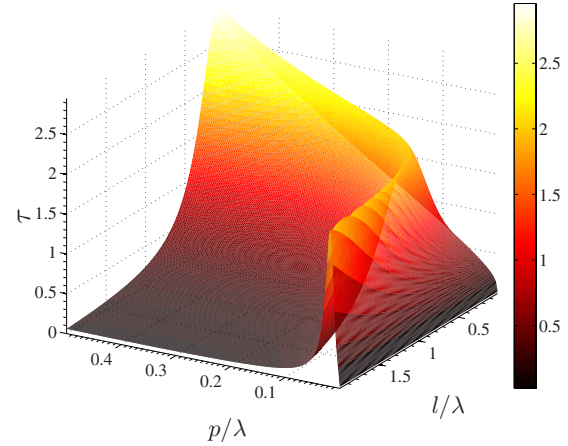


FIG. 2. (Color online) Transmission τ through a zero- ϵ slit as a function of the dimensionless geometrical parameters l/λ and p/λ .

Perot-like resonant oscillations as a function of l , as would be expected from a slit with conventional material. The figure shows that energy flow is generally restricted unless the geometrical parameters coincide with the brightly peaked regions that can decay quite rapidly.

The wave impedance, Z , defined as $Z = E_x/H_z$, is calculated from the field expressions above to yield the following impedance relation between the top and bottom of the openings, $Z^{\text{top}} = Z^{\text{bot}} - ik_0 l \mu$, with $Z^{\text{bot}} = 1/(2qI_0)$. A remarkable property of a narrow zero- ϵ slit is that despite the considerable wave impedance for $p \ll \lambda$, energy can still be transmitted if the slit length l is increased. This behavior is further illustrated in Fig. 3, where the transmission is shown to be robust for an extended range of channel lengths and widths. The maximum transmission, τ_{max} , seen contained within the continuous bright curve, is determined by examining the extrema of Eq. (13). By taking the appropriate derivative of the denominator, we find that τ_{max} is determined by the transcen-

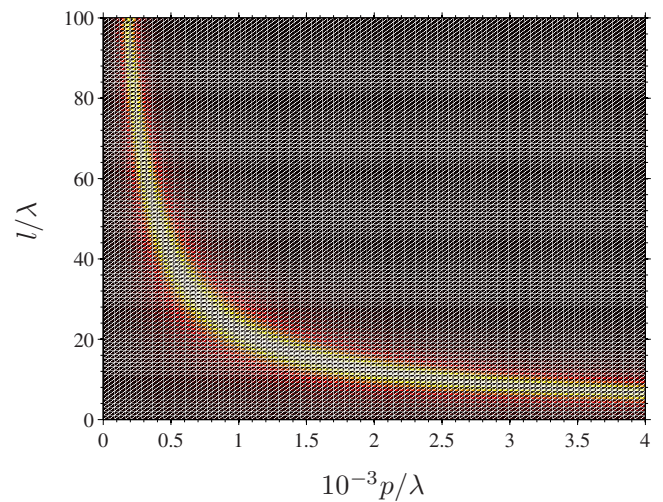


FIG. 3. (Color online) Transmission characteristics for a zero- ϵ metamaterial slit, demonstrating the l and p that give the optimal transmission at a given wavelength. Brighter regions indicate higher transmission. Relatively high transmission can persevere in deep slits ($l \gg \lambda$) and small openings ($p \ll \lambda$).

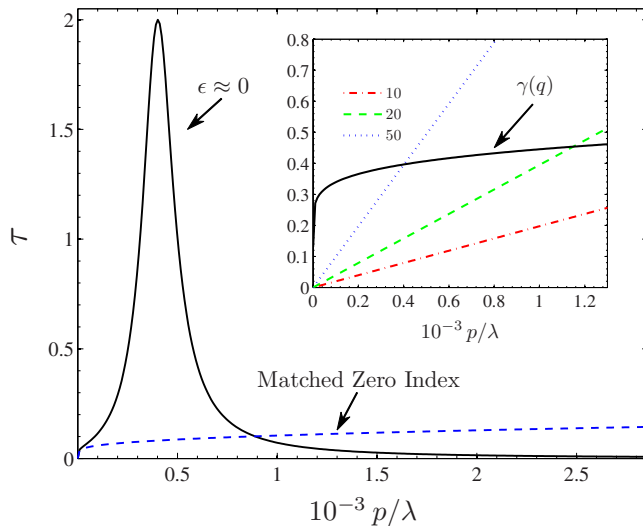


FIG. 4. (Color online) Transmission through deep metamaterial slits as a function of p/λ . A considerable resonance peak emerges for a small subwavelength width of zero- ϵ material. Also shown is a matched zero index medium where μ and ϵ are both nearly zero. The inset shows the intersections of the lines with the $\gamma(q)$ curve, corresponding to electrical widths that yield peak transmission. Differing ratios of l/λ (see legend) yield lines with different slopes. These results are consistent with the main plot, where the channel length corresponds to $l=50\lambda$.

dental equation, $\mu q k_0 l = \gamma(q)$, where $\gamma(q) = -\text{Im}[I_0]/|I_0|^2$ is a gradually increasing monotonic function of q . Thus as a function of q , and for each l , the intercept of $\gamma(q)$ with lines of slope $\mu k_0 l$ yields the permissible electrical slit widths that achieve peak light transmission. This technique is shown graphically in the inset of Fig. 4, where the inverse relationship between slit width and length are clearly seen.

It is also of interest to determine if the transmission can be enhanced for light incident upon a matched zero index material, whereby both ϵ and μ are vanishingly small. In the

main plot of Fig. 4, the transmission is therefore plotted as a function of slit width for a zero- ϵ slit (with $\mu=1$ as usual), and matched zero index slit. Surprisingly, for zero- ϵ media, the shown subwavelength widths support transmission resonances that are absent in matched zero index slits. Although the l dependence washes out for matched zero index channels, τ still depends strongly on the width dimension, and Eq. (13) approximately reduces to the expression for a slit filled with a conventional dielectric and length satisfying the Fabry-Perot geometric resonance condition. We can thus conclude that for zero- ϵ media and narrow enough openings, energy flow can be much greater than for slits loaded with metamaterials having no wave impedance mismatch between the ends of the slit. We also considered the effects of finite Γ and concluded that the presence of absorption loss simply reduces the overall magnitude of τ by a factor that becomes greater with increasing slit depth, but having no effect on the geometrical parameters leading to the particular resonance location. This bodes well for future development of miniaturized devices used for energy transport and narrow band frequency selective surfaces.

In conclusion, we have shown that significant transmission arises for a range of subwavelength widths in sufficiently deep slits containing zero- ϵ inclusions. We also demonstrated that these resonant channels can transmit greater energy compared to the corresponding matched zero index slits. Although there is currently limited metamaterial fabrication in the ir and optical frequencies, some recent progress has been made with negative-index metamaterials involving metal layers separated by a dielectric [17], alternating layers of InGaAs and AlInAs semiconductors [18], and photonic crystals in the near-infrared range [19].

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- [1] T. W. Ebbesen *et al.*, *Nature (London)* **391**, 667 (1998).
- [2] E. D. Palik, *Handbook of Optical Constants of Solids* (Academic, Washington, D.C., 1985).
- [3] M. Silveirinha and N. Engheta, *Phys. Rev. Lett.* **97**, 157403 (2006); *Phys. Rev. B* **76**, 245109 (2007).
- [4] S. Feng and K. Halterman, *Phys. Rev. Lett.* **100**, 063901 (2008).
- [5] R. Liu, Q. Cheng, T. Hand, and J. J. Mock, *Phys. Rev. Lett.* **100**, 023903 (2008).
- [6] B. Edwards, A. Alu, M. E. Young, M. Silveirinha, and N. Engheta, *Phys. Rev. Lett.* **100**, 033903 (2008).
- [7] M. Silveirinha and N. Engheta, *Phys. Rev. B* **75**, 075119 (2007).
- [8] S. Enoch, G. Tayeb, P. Sabouroux, N. Guerin, and P. Vincent, *Phys. Rev. Lett.* **89**, 213902 (2002).
- [9] R. W. Ziolkowski, *Phys. Rev. E* **70**, 046608 (2004).
- [10] Y. Takakura, *Phys. Rev. Lett.* **86**, 5601 (2001).
- [11] J. Bravo-Abad, L. Martin-Moreno, and F. J. Garcia-Vidal, *Phys. Rev. E* **69**, 026601 (2004).
- [12] F. Yang and J. R. Sambles, *Phys. Rev. Lett.* **89**, 063901 (2002).
- [13] M. Mechler, O. Samek, and S. V. Kulkhlevsky, *Phys. Rev. Lett.* **98**, 163901 (2007).
- [14] S. V. Kulkhlevsky, M. Mechler, L. Csapo, K. Janssens, and O. Samek, *Phys. Rev. B* **70**, 195428 (2004).
- [15] O. Mata-Mendez and J. Avendano, *J. Opt. Soc. Am. A* **24**, 1687 (2007).
- [16] R. Gordon, *Phys. Rev. B* **73**, 153405 (2006).
- [17] S. Zhang *et al.*, *Phys. Rev. Lett.* **95**, 137404 (2005).
- [18] A. J. Hoffman *et al.*, *Nat. Mater.* **6**, 946 (2007).
- [19] R. Chatterjee *et al.*, *Phys. Rev. Lett.* **100**, 187401 (2008).