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Nonlocal density correlations as a signature of Hawking radiation from acoustic black holes

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We have used the analogy between gravitational systems and nonhomogeneous fluid flows to calculate the density-density correlation function of an atomic Bose-Einstein condensate in the presence of an acoustic black hole. The emission of correlated pairs of phonons by Hawking-like process results into a peculiar long-range density correlation. Quantitative estimations of the effect are provided for realistic experimental configurations.

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Hawking's prediction of black holes evaporation is generally regarded as a milestone of modern theoretical physics. Combining Einstein's general relativity and quantum mechanics, Hawking was able to show that black holes are not "black," but emit thermal radiation at a temperature inversely proportional to their mass [1]. This quantum mechanical process is triggered by the formation of a horizon and proceeds via the conversion of vacuum fluctuations into on-shell particles. Unfortunately so far there is no experimental support for this amazing theoretical prediction. The emission temperature (Hawking temperature) for a solar mass black hole is expected to be of the order of 10^{-8} K, far below the 3 K cosmic microwave background. No evidence has been found so far of an x-ray background from a hypothetical primordial population of low mass black holes ($\sim 10^{10}$ kg) in the final stages of their evaporation [2]. Expectations to directly observe Hawking radiation from miniblack holes formed in colliders like Large Hadron Collider (LHC) or next generation ones, are based on models where the quantum gravity scale (Planck scale, 10^{19} GeV) is lowered down to the TeV scale by the presence of extra dimensions [3]. It is perhaps fair to say that the prospects to have a direct experimental detection of Hawking radiation from black holes in the near future are not very optimistic.

In a remarkable work Unruh [4] showed that Hawking radiation is not peculiar to gravity, but is rather a purely kinematic effect of quantum field theory which only depends on field propagation on a black hole-type curved space-time background. This opens the concrete possibility of studying the Hawking radiation process in completely different physical systems. As an example, the propagation of sound waves in Eulerian fluids can be described in terms of the same equation describing a massless scalar field on a curved space time characterized by an acoustic metric $G_{\mu\nu}$ which is a function of the background flow: The curvature of the acoustic geometry is induced by the inhomogeneity of the fluid flow, while flat Minkowskian space time is recovered in the case of a homogeneous system. In particular, an acoustic black hole (or dumb hole) configuration is obtained whenever a subsonic flow turns supersonic: Sound waves in the supersonic region are in fact dragged away by the flow and cannot propagate back towards the acoustic horizon separating the supersonic and subsonic regions. Upon quantization, Hawking radiation is expected to appear as a flux of thermal phonons emitted from the horizon at a temperature proportional to its surface gravity. Even though a substantial effort has been spent on a variety of analog models [5], e.g., superfluid Helium [6], phonons in atomic Bose-Einstein condensates [7], degenerate Fermi gases [8], slow light in moving media [9], or travelling refractive index interfaces in nonlinear media [10], the weakness of Hawking radiation has so far prevented experimental verification of these predictions.

In the present paper we propose a route to detect the emission by an acoustic black hole in a flowing Bose-Einstein condensate. Differently from the case of astrophysical black holes, both the external and the internal region of the acoustic black hole are in fact accessible to experiment: The quantum-optical correlations between the particles emitted inside and outside the acoustic black hole are responsible for a significant long-range correlation between the density fluctuations at points, respectively, inside and outside the acoustic black hole. This unique signature can be exploited to isolate Hawking radiation from the background of competing processes and experimental noise.

We start by briefly recalling those features of Hawking radiation that we shall use in what follows. Consider the quantum field associated to Hawking radiation to be in its vacuum state at early times before the horizon formation (let us call it $|\mathrm{in}\rangle$). In the Heisenberg picture, the state of the field at late times will still be $|\mathrm{in}\rangle$, but because of the horizon formation the $|\mathrm{in}\rangle$ state does not correspond to the late time vacuum state $|\mathrm{out}\rangle$ of the field. A basis for the out modes (see Fig. 1) is composed by ingoing, i.e., propagating downstream in the free-falling frame, modes and outgoing, i.e., modes propagating upstream, modes. Let $|0_{\mathrm{ing}}\rangle$ and $|0_{\mathrm{outg}}\rangle$ be the vacuum states of, respectively, the ingoing and outgoing modes. Among the outgoing modes, we must distinguish between the ones which propagate outside the horizon with a sound velocity faster than the flow velocity and are therefore able to reach the asymptotic region far from the horizon, and the ones that are trapped inside the horizon and are dragged inwards by the flow which is here faster than the speed of sound.

Hawking radiation corresponds to particles creation in the outgoing sector only and, limiting to this latter, the late-time decomposition for the $|\mathrm{in}\rangle$ vacuum in terms of these escaping and trapped modes has the form of a two-mode squeezed vacuum state [11]:

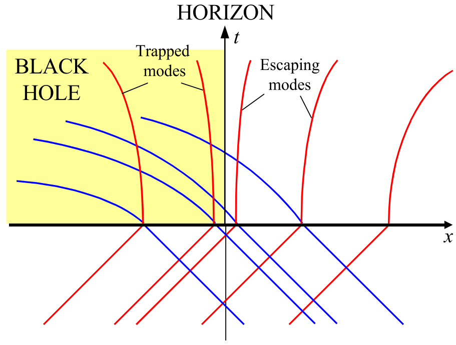

FIG. 1. (Color online) Space-time diagram describing the ingoing (blue lines) and outgoing (red) modes. Before the black hole formation ($t<0$) modes propagate along straight lines. When the horizon (in $x=0$) is formed propagation gets significantly distorted and the outgoing modes inside the black hole remain trapped.

$$|\text{in}\rangle \propto \exp\left(\sum_{\omega} e^{-\hbar\omega/2\kappa_B T_H} a_{\omega}^{(\text{esc})\dagger} a_{\omega}^{(\text{tr})\dagger}\right)|0_{\text{outg}}\rangle, \tag{1}$$

where $a_{\omega}^{(\text{esc,tr})\dagger}$ are creation operators for, respectively, the outgoing escaping and trapped modes, T_H is the Hawking temperature, and κ_B is the Boltzmann constant. Thus, the Hawking process consists of the creation of correlated pairs of outgoing quanta triggered by the horizon formation [12,13]: One member is emitted into an escaping mode, and corresponds to what is usually called Hawking particle; the other member, the so-called partner, is emitted into a trapped mode inside the horizon. Of course, this latter particle is definitively lost and can not be detected in the case of astrophysical black holes.

Let us now move to an analog model of black hole based on a flowing atomic Bose-Einstein condensate (BEC). As we are considering small fluctuations around a stationary and fully condensed state, we write the Bose field operator as $\hat{\Psi}=e^{i\hat{\theta}}\sqrt{\hat{n}}$ in terms of the number density $\hat{n}$ and the phase $\hat{\theta}$ [14] and we expand the density $\hat{n}=n+\hat{n}_1$ and phase $\hat{\theta}=\theta+\hat{\theta}_1$ operators around the classical background values n and θ fixed by the mean-field Gross-Pitaevskii equation.

We limit our attention to the so-called hydrodynamic limit, where perturbations are considered with wavelengths much longer than the healing length $\xi=\hbar/\sqrt{mgn}$ (m is the atomic mass and g the atom-atom nonlinear interaction constant in the Gross-Pitaevskii equation [14]). The linearized equation of motion for the phase $\hat{\theta}_1$ field is then formally equivalent to a curved space-time field equation for a massless scalar field [7]

$$\Box\hat{\theta}_1 = \frac{1}{\sqrt{-G}}\partial_{\mu}(\sqrt{-G}G^{\mu\nu}\partial_{\nu})\hat{\theta}_1 = 0, \tag{2}$$

where $\Box$ is the curved d'Alembertian for the acoustic metric $G_{\mu\nu}$ of line element

$$ds^2 = G_{\mu\nu}dx^{\mu}dx^{\nu} = \frac{n}{mc}[-c^2dt^2 + (d\vec{x}-\vec{v}dt)(d\vec{x}-\vec{v}dt)], \tag{3}$$

and G is the metric determinant, $\vec{v}=\hbar\vec{\nabla}\theta/m$ is the local flow velocity, and $c=\sqrt{gn/m}$ is the local sound speed. As the density operator $\hat{n}_1$ is algebraically related to $\hat{\theta}_1$ by $\hat{n}_1=-\frac{\hbar}{g}(\partial_t\hat{\theta}_1+\frac{\hbar}{m}\vec{\nabla}\theta\cdot\vec{\nabla}\hat{\theta}_1)$, the one-time density-density correlation function

$$G_2(x,x') = \langle\hat{n}(x)\hat{n}(x')\rangle - \langle\hat{n}(x)\rangle\langle\hat{n}(x')\rangle \tag{4}$$

can be simply expressed in term of the two-points function for the field $\hat{\theta}_1$,

$$G_2(x,x') = \frac{\hbar^2}{g(x,t)g(x',t)}\lim_{t'\to t}\mathcal{D}\langle\hat{\theta}_1(x,t)\hat{\theta}_1(x',t')\rangle, \tag{5}$$

the operator $\mathcal{D}$ being defined as $\mathcal{D}=[\partial_t\partial_{t'}+v(\vec{x})\vec{\nabla}_{\vec{x}}\partial_{t'}+v(\vec{x}')\partial_t\vec{\nabla}_{\vec{x}'}+v(\vec{x})v(\vec{x}')\vec{\nabla}_{\vec{x}}\cdot\vec{\nabla}_{\vec{x}'}]$.

To obtain workable analytical expressions we restrict our attention to the simplest case of a one-dimensional condensate, whose transverse size $\ell_{\perp}$ is assumed to be much smaller than the healing length ξ. Configurations of this kind can be realized in the laboratory by outcoupling atoms from a mother condensate into an atomic fiber to form a so-called atom laser beam [15–17]. Performing a dimensional reduction along the transverse direction [18], the two-point function for the field $\hat{\theta}_1$ can be approximated as

$$\langle\hat{\theta}_1(x,t)\hat{\theta}_1(x',t')\rangle \simeq -\frac{1}{4\pi\sqrt{C(x,t)C(x',t')}}\ln(\Delta x^-\Delta x^+), \tag{6}$$

where $C=n_{1\text{D}}\xi$ is the conformal factor for the metric (3), with $n_{1D}=n\ell_{\perp}^2$, and $x^{\pm}=t\pm\int\frac{dx}{(c\mp v)}$ are light (sound) cone coordinates. The expectation value entering the two-point function (6) is to be taken in the vacuum state of the positive frequency modes with respect to the $x^{\pm}$ coordinates [19,11]. After dimensional reduction, an effective potential term appears in the $(1+1)$-dimensional wave equation (2), which is responsible for backscattering of the modes. In what follows we neglect such a term, to obtain a conformally invariant $(1+1)$-dimensional theory that can be handled in a fully analytical way [20]. This is expected to produce only slight quantitative overestimations of the final result [21].

The consistency of this approach can be validated on the simplest case of a spatially homogeneous one-dimensional BEC of density $n_{1\text{D}}$ moving at a constant and spatially uniform speed v. In this case the light cone coordinates associated to the vacuum are simply $x^{\pm}=t\pm\frac{x}{c\mp v}$. Using (5), this leads to the result $G_2^{1\text{D}}(x,x')=-n_{1\text{D}}\xi/[2\pi(x-x')^2]$ which fully agrees with standard BEC theory [22] in the long-distance $|x-x'|\gg\xi$ limit where the hydrodynamic approximation is valid.

We now turn to the most interesting case of an acoustic black hole. Consider a one-dimensional BEC of uniform density $n_{1\text{D}}$ which is flowing with constant speed v along the $-\hat{x}$ direction. For $t<0$ the sound velocity $c(x,t)$ is constant in space and equal to $c(x,t)=c_r>v$. At $t=0$, the sound velocity in the $x<0$ region is rapidly switched to a lower $c_l<v$ value in order to create a horizon that separates a region of subsonic $v<c_r$ flow for $x>0$ from a supersonic $v>c_l$ one for $x<0$. In the crossover region $x\in[-x_0,x_0]$, the sound velocity is assumed to vary linearly in x. In order for the hydrodynamic approximation to be valid x_0 must be larger than ξ.

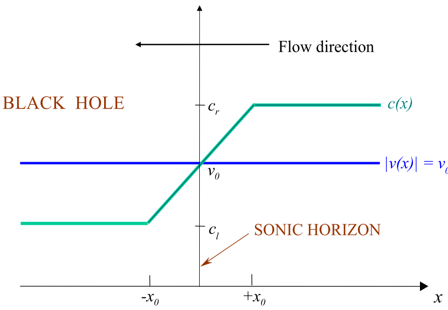

FIG. 2. (Color online) Sketch of the proposed setup. The density n and the flow velocity v are kept constant, while the sound velocity c is modified around $t=0$ according to the figure. The sonic horizon is at $x=0$.

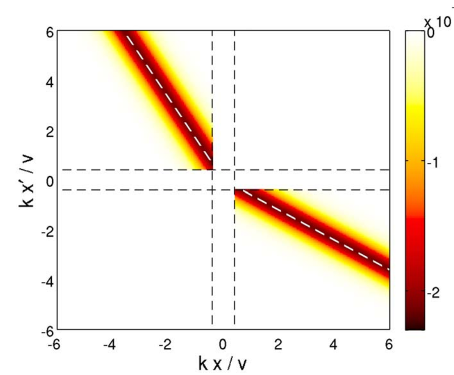

FIG. 3. (Color online) Density-density correlation pattern $G_2^{1D}(x,x')/n_{1D}^2$. The valley-shaped feature indicated as a white dashed line is the signature of Hawking radiation. The black dashed lines indicate the $[-x_0,x_0]$ horizon region.

The sonic horizon (i.e., the locus where $c=v$) lies at $x=0$ (see Fig. 2).

In practice, such a modulation can be obtained by spatially modulating the transverse confinement of the waveguide [15,23] and/or the nonlinear interaction constant $g(x)$ by means of a spatially varying magnetic field in the vicinity of a Feshbach resonance [14]. In order to minimize competing processes such as back-scattering of condensate atoms, Landau-Čerenkov phonon emission [24], and soliton shedding [15] from the horizon region, the change in $g(x)$ must be compensated by a corresponding change in the external potential $V(x)$ so to keep the local potential $ng(x)+V(x)$ constant.

While the ingoing modes remain positive frequency modes with respect to the x^+ light cone coordinate even after the formation of the black hole, the outgoing modes are no longer positive frequency with respect to the x^- light cone coordinate. Indeed they emerge as positive frequency with respect to the generalized coordinate $\tilde{x}^- = \pm \frac{e^{-kx^-}}{k}$ [19,18], where the surface gravity at the horizon (located at $x^-=\infty$ or $\tilde{x}^-=0$) is defined as

$$k = \frac{1}{2v}\frac{d}{dx}(c^2 - v^2)|_H = \left(\frac{dc}{dx}\right)_H \tag{7}$$

and fully determines the Hawking temperature $T_H = \hbar k/(2\pi\kappa_B)$ [19]. The $\pm$ signs in the definition of $\tilde{x}^-$ refer to outgoing modes which are, respectively, trapped inside the horizon ($x<0$) or able to escape towards the asymptotic subsonic region ($x>0$).

The correlation function $G_2^{1D}(x,x')$ can then be derived from the general formulas (5) and (6) using $\ln(\Delta\tilde{x}^-\Delta x^+)$ as the master function for the $|\text{in}\rangle$ state. A compact formula can then be obtained for the most interesting case when the x,x' points are located on opposite sides with respect to the horizon and lie well outside the modulation region, i.e., $x>x_0$ and $x'<-x_0$

$$\frac{G_2^{1D}(x,x')}{n_{1D}^2} \simeq -\frac{k^2\xi_l\xi_r}{16\pi c_l c_r}\frac{1}{\sqrt{(n_{1D}\xi_r)(n_{1D}\xi_l)}} \times \frac{c_r c_l}{(c_r-v)(v-c_l)}\frac{1}{\cosh^2\left[\frac{k}{2}\left(\frac{x}{c_r-v}+\frac{x'}{v-c_l}\right)\right]}. \tag{8}$$

The x,x' dependence contained in the cosh term is the central result of the present paper: As shown in Fig. 3, $G_2^{1D}(x,x')$ has a quite narrow, valley-shaped feature centered on the $x'/(v-c_l)=-x/(c_r-v)$ straight line that describes correlations between pairs of points located, respectively, inside and outside the black hole. The bottom value of the valley goes as the squared surface gravity and it is inversely proportional to the diluteness parameter $n_{1D}\xi$ of the gas. The neglected subleading term $O(|x-x'|^{-2})$ contains the vacuum contribution of the ingoing modes and it is a consequence of the short-range repulsion of bosons [22]. On the other hand, no specific signal appears for pairs of points on the same side of the horizon.

The position of the valley has a transparent physical interpretation: At all times, pairs of Hawking phonons almost simultaneously emerge from the horizon region and propagate into the subsonic and supersonic regions at speeds, respectively, c_r-v and $v-c_l$. After a propagation time Δt, the initial quantum correlation then reflects into a density correlation between $x=(c_r-v)\Delta t$ and $x'=-(v-c_l)\Delta t$. The valley-shaped feature is immediately recovered once we integrate over all possible values of Δt. The width of the valley is proportional to the inverse of the surface gravity k^{-1} and it provides a simple way of estimating the Hawking temperature of the emission. Note that under the hydrodynamic assumption the width is much larger than the healing length ξ.

The contribution of mode backscattering, here neglected, quickly vanishes in the hydrodynamic limit: The corresponding terms are in fact of higher order in k. Although all our predictions have been obtained for the simplest configuration, the behavior of the correlation function at late (t

$\gg k^{-1}$) times can be shown to be generic and independent of the details of the horizon formation process [19].

We conclude by providing quantitative estimations of the Hawking signal for parameters inspired to an existing guided Rb-atom laser experiment [16]. For such systems, we can consider flow speeds on the order of $v=4$ mm/s and sound speed modulations such that $c_l/v=0.7$ and $c_r/v=1.5$, which implies that the healing length ξ is on the order of 0.2 μm. A value $2x_0=6\xi=1.2$ μm for the thickness of the modulation region is expected to be enough to fulfill the hydrodynamic assumption. Inserting these values into the expression for T_H, one obtains a quite low [25] Hawking temperature in the $T_H \approx 4$ nK range. For a reasonable value of the diluteness parameter $n_{1D}\xi \simeq 10$, the maximum of the density correlation signal (8) turns out to be $G_2^{1D}/n_{1D}^2 \sim -2.5\times 10^{-4}$. However, as our predictions follow from quantum hydrodynamics, their validity goes beyond weakly interacting BECs and in particular in one dimension they extend to a generic Luttinger liquid [26]: Stronger correlation signals can therefore be observed in stronger interacting systems where the diluteness parameter is smaller.

Furthermore, its peculiar valley shape should allow to isolate its contribution on top of the background of thermal fluctuations [27] and experimental noise. To this purpose several detection schemes have been developed during recent years to experimentally characterize local density fluctuations in ultracold atomic gases [28–30], some of which are able to resolve even individual atoms [31]. To overcome the atomic shot noise, one may take advantage of the large number of atoms that are available in the quasi-cw atom laser beam [16,17].

In summary, we have shown that the gravitational analogy predicts a characteristic peak in the density-density correlation function in a flowing atomic Bose-Einstein condensate in the presence of a horizon: The quantum correlation within the pairs of Hawking phonons reflects into long-ranged density correlations that extend far from the horizon. This constitutes a promising signature to detect and isolate the Hawking emission of phonons from acoustic black holes.

Note added. Recently, numerical evidence supporting the predictions in this paper has been obtained starting from a microscopic theory of the atomic condensate [32].

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