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Dispersive coupling between the superconducting transmission line resonator and the double quantum dots

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Realization of controllable interaction between distant qubits is one of the major problems in scalable solid-state quantum computing. We study a superconducting transmission line resonator (TLR) as a tunable dispersive coupler for the double-dot molecules. A general interaction Hamiltonian of n two-electron spin-based qubits and the TLR is presented, where the double-dot qubits are biased at the large detuning region and the TLR is always empty and virtually excited. Our analysis of the main decoherence sources indicates that various major quantum operations can be reliably implemented with current technology.

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Semiconductor quantum dot is one of the most promising candidates for quantum computation [1,2]. With the help of external electromagnetic bias, one can coherently manipulate the quantum state of the trapped electron [3–5]. Recently, the singlet and triplet states of the two electrons in a double-dot molecule have been proposed as logical qubits [6]. Compared with the single-electron qubit, this two-electron spin-based qubit has proved to be more robust against noise from the quasistatic nuclear spins [4] as well as low frequency fluctuation of the electrostatic gates [7].

Stimulated by these advances, we focus on further steps towards realizing scalable quantum computing in this solid-state system. One of the critical next steps is to implement two-qubit quantum gates. Capacitive coupling, i.e., directly coupling via Coulomb interaction of electrons in quantum dots, is the most straightforward way [8–11]. However, it fails for distant qubits because the Coulomb interaction decays rapidly as the distance between the qubits increases. Moreover, even if one could place many qubits in a small area to obtain enough coupling strength, problems such as the residual coupling arise, hence the design of quantum gates would become more and more complicated. All these issues make alternative coupling methods desirable for scalable quantum computing in the future.

In the very latest papers, the idea of combining quantum dots with the circuit QED system [12–14] is attracting more and more attention. References [7,15] have suggested that the charge degree of freedom of the double dot and the quantized voltage of a transmission line resonator (TLR) can be coupled via a capacitor. These works suggested that in the resonant strong coupling limit, the energy and the quantum information can be transferred between the double dot and the TLR. However, only one double-dot qubit is considered to interact with the TLR in these previous works (Refs. [7,15]). Therefore, it is nontrivial to find a general interaction formalism involving n double-dot qubits and the TLR, which can be exploited to realize various important quantum-information processes (QIP) such as the control-NOT gate between any double-dot qubits and multiqubit entanglement generation.

In this Rapid Communication, instead of the resonant coupling of one double-dot qubit with TLR in previous papers, we consider the dispersive operations [16] of multi double-dot molecules and a circuit QED combined system. A general interaction Hamiltonian of n double-dot qubits and the TLR is presented. These two-electron spin-based qubits are biased at the large detuning region and the qubit-TLR interaction can be tuned on and off by the control of the qubit level splitting. Since the TLR is always empty and only virtually excited, the quantum coherence of this system is immune to the energy loss of the cavity. This scalable architecture is flexible enough to allow for various major quantum operations such as nontrivial multiqubit quantum gates and entanglement generation. Since a long lifetime for the double-dot qubit has already been achieved, the proposed quantum gates could be realized with very high fidelity.

Let us illustrate our idea intuitively. The system we study is a TLR of length L, with inductance F and capacitance C per unit length, coupled to n double-dot qubits by coupling capacitors C_{ci} and to external input or output by wiring capacitor C_0 from left and right, as shown in Fig. 1. In the following, we treat the TLR and the individual qubits as independent systems with the small coupling as perturbation [17].

TLR. A single TLR can be well described by a series of inductors with each node capacitively connected to the ground, as shown in Fig. 2. To transmit input and output signals, the TLR should be coupled to the external coil by a wiring capacitor C_0. Denoting $\epsilon_0=C_0/LC$ and focusing only on the full wave mode, we can get the Hamiltonian of the TLR,

$$H_{\text{cavity}}=\hbar\omega a^{\dagger}a, \tag{1}$$

where a is the annihilation operator of the full-wave mode. The frequency of this mode is slightly renormalized by the wiring capacitor as $\omega\approx\omega_0[1-2\epsilon_0]$, where $\omega_0=2\pi/(L\sqrt{FC})$. The voltage distribution inside the TLR has the form

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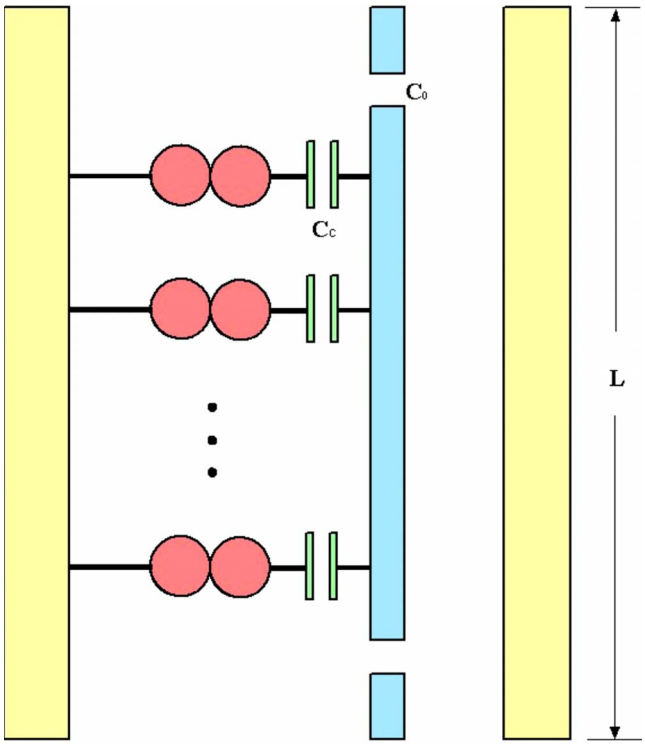

FIG. 1. (Color online) Coupled system of a TLR and n double-dot molecules. The length of the TLR is L (about ten millimeters). C_0 is the wiring capacitor between TLR and external input or output. The double-dot qubits are connected to the TLR via the coupling capacitors C_{ci}. The TLR acts as a data bus between the double-dot qubits.

$$V_{TLR}(x) = \sqrt{\hbar\omega/LC}(a^\dagger + a)\cos[kx + \delta], \tag{2}$$

where the small phase shift δ satisfies the condition $\tan\delta = 2\pi\epsilon_0$.

Double-dot qubits. As shown in Fig. 3 a double-dot molecule is formed by a layer of two-dimensional electron gas restrained by several electrostatic gates used to control the potentials of individual dots and the interdot tunneling. Its low-energy spectrum is plotted in Fig. 4. Due to the Coulomb interaction and Pauli principle, near the "saddle" point (gray area in Fig. 4) the molecule can be reduced to an artificial three-level system with the Hamiltonian

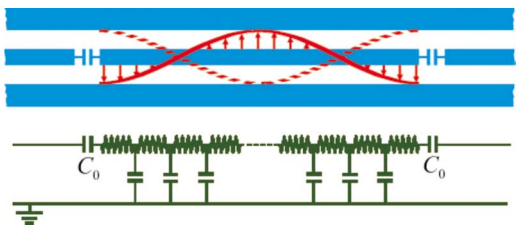

FIG. 2. (Color online) Circuit representation of a TLR. The TLR consists of a full-wave section of superconducting coplanar waveguide. The red line represents the full-wave mode. The coupling capacitors connected to the input or output wiring slightly modify the frequency and phase of the TLR's modes.

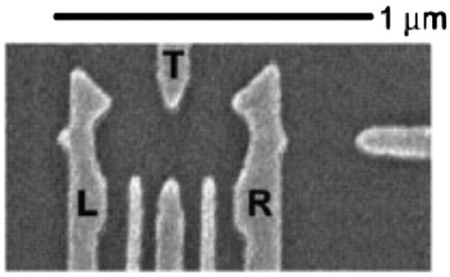

FIG. 3. Micrograph of a double-dot sample. This figure is reproduced from Fig. 1 of Ref. [4]. The L and R gates represent the left and right gates, respectively, in the text, and the T gate can be used to adjust the tunneling coupling strength between two dots.

$$H_{DD} = E_T|(1,1)T_0\rangle\langle(1,1)T_0| + E_S|(1,1)S\rangle\langle(1,1)S| - \varepsilon|(0,2)S\rangle \times\langle(0,2)S| + T_C(|(1,1)S\rangle\langle(0,2)S| + |(0,2)S\rangle\langle(1,1)S|), \tag{3}$$

where the notation (n_l, n_r) indicates n_l electrons on the "left" dot and n_r electrons on the "right" dot, S and T represent spin singlet and triplet states, respectively, and ε and T_C denote the external voltage bias and tunneling amplitude between two dots. It is a good approximation to set $E_T \approx E_S = 0$ [5], therefore Eq. (3) has eigenstates

$$|\tilde{S}\rangle = \cos\theta|(1,1)S\rangle + \sin\theta|(0,2)S\rangle,$$

$$|\tilde{G}\rangle = -\sin\theta|(1,1)S\rangle + \cos\theta|(0,2)S\rangle. \tag{4}$$

Here the parameter θ rests on the external bias ε and T_C [5]. The voltage of the left dot is

$$V_{dot} = \frac{2e}{C_{tot}}|(0,2)S\rangle\langle(0,2)S| + \frac{e}{C_{tot}}|(1,1)S\rangle\langle(1,1)S|, \tag{5}$$

where C_{tot} is the total capacitance of the double-dot molecule. In realistic experiments, the choice $\theta = \pi/4$ is favorable for quantum operation because at this point the energy difference between $|\tilde{S}\rangle$ and $|\tilde{G}\rangle$ is insensitive to the first-order fluctuation of ε [19] and the nuclear hyperfine field [7].

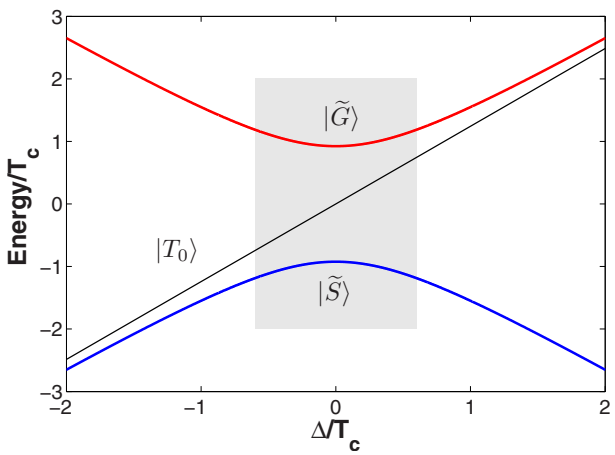

FIG. 4. (Color online) Lowest-energy states of the double-dot molecule. The gray region is the "saddle" point in the text.

Therefore, we denote $|0(1)\rangle=(|\downarrow\uparrow\rangle\mp|(0,2)S\rangle)/\sqrt{2}$ and rewrite Eq. (5) as

$$V_{dot}=e(I+\sigma_x)/2C_{tot} \tag{6}$$

in the basis $\{|0\rangle, |1\rangle\}$.

Combined system. As shown in Fig. 1, when a double dot is coupled to a TLR with a capacitor C_c, the charging energy of C_c provides a qubit-resonator interaction

$$H_{int}=C_cV_{TLR}(x)V_{dot}. \tag{7}$$

With Eqs. (2) and (6), we can get the standard Jaynes-Cumming (JC) model

$$H_{int}=g(x)(a\sigma^+ + a^\dagger\sigma^-), \tag{8}$$

where $\sigma^+=|1\rangle\langle 0|$, $\sigma^-=|0\rangle\langle 1|$, and the coupling g factor

$$g(x)=\frac{eC_c}{C_{tot}}\sqrt{\frac{\hbar\omega}{LC}}\cos[kx+\delta]. \tag{9}$$

It is interesting to note the single-dot qubit which exploits the spin degree of freedom of the trapped electron cannot be coupled to the TLR because of its charge degeneracy.

Previous papers have suggested putting the double-dot qubits on the two leads of the cavity and to operate the circuit QED system resonantly. Alternatively, we consider putting n double-dot molecules inside the cavity and operating them in the region of large detuning. As shown in Fig. 1, there are n double-dot molecules coupled to a TLR. Since the TLR length is in the range of 10 mm, we can place the qubits far enough from each other, thus the Coulomb interaction between distant qubits can be neglected. In the interaction picture, we can get an effective Hamiltonian of n double-dot molecules and the TLR as

$$H_{int}=\sum_{j=1}^{n} g_j(e^{-i\tau_j t}a^\dagger\sigma_j^- + e^{i\tau_j t}a\sigma_j^+), \quad j=1,2,\ldots,n, \tag{10}$$

where τ_j is the detuning between the transition frequency of qubit j and the full mode frequency of the TLR. When the condition of dispersive detuning, i.e., $\tau_j \gg g_j$ for $j=1,2,\ldots,n$, is fulfilled, the Hamiltonian Eq. (10) can be transformed to [18]

$$H_{eff}=\lambda\left[\sum_{i,j=1}^{n}(\sigma_j^+\sigma_i^- aa^\dagger - \sigma_j^-\sigma_i^+ a^\dagger a)\right], \tag{11}$$

where $\lambda=g^2/\tau$ (for simplicity we assume identical coupling strength g and detuning τ).

Various important quantum-information processes could be realized with the above Hamiltonian Eq. (11). As an example, we show how to perform a controlled phase (CPHASE) gate between any two double-dot qubits. Initially we prepare the cavity field in the vacuum state and the two target qubits in the state $|1\rangle_1|0\rangle_2$ [16]. As the effective coupling strength λ depends on the detuning τ, we can selectively tune the two target qubits into the dispersive region by choosing a relatively small (but still dispersive) detuning while retaining other qubits decoupled. In this circumstance, the reduced form of H_{eff} in the basis $\{|00\rangle, |10\rangle, |01\rangle, |11\rangle\}$ is

$$H_{20}=\lambda\left[\sum_{j=1,2}^{n}|1\rangle_{jj}\langle 1| + (\sigma_1^+\sigma_2^- + \sigma_1^-\sigma_2^+)\right], \tag{12}$$

and the evolution operator is

$$U(t)=\exp(-itH_{20})=\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{e^{-2i\lambda t}+1}{2} & \frac{e^{-2i\lambda t}-1}{2} & 0 \\ 0 & \frac{e^{-2i\lambda t}-1}{2} & \frac{e^{-2i\lambda t}+1}{2} & 0 \\ 0 & 0 & 0 & \frac{e^{-2i\lambda t}}{2} \end{pmatrix}. \tag{13}$$

After a period $t_0=\pi/4\lambda$, the initial state $|1\rangle_1|0\rangle_2$ evolves into the maximally entangled Einstein-Podolsky-Rosen (EPR) state

$$|\Psi_{EPR}\rangle=\frac{1}{\sqrt{2}}(|1\rangle_1|0\rangle_2 - i|0\rangle_1|1\rangle_2), \tag{14}$$

where we have omitted the common phase factor $\pi/4$.

Now we discuss the decoherence problem resulting from the environmental and systematical error, which is the main obstacle of implementing quantum computing in a solid-state system. The dissipation of the single TLR occurs mainly through coupling to the external leads. In general the magnitude of this process can be described by the decay factor $\kappa=\omega/Q$, where Q is the quality factor of the TLRs [17]. In the reported high-finesse TLR cavity with eigenfrequency $\omega_0/2\pi=10$ GHz and $Q=1\times10^5$, the κ factor is of the order 0.1 MHz [13]. Moreover, in the dispersive coupling condition, the TLR is always empty and only virtually excited. The influence of the cavity loss is negligible even when the quality factor of TLR is lower, which greatly simplifies the experimental implementation.

For the double-dot qubit, things become more complicated. Phonons in the substrate as well as the finite impedance of the voltage bias can cause fluctuations in the voltage bias and induce relaxation to the qubits. This relaxation process can be suppressed by electrical circuit engineering [20,21] and very long spin-relaxation time T_1 has already been observed experimentally [22]. Another main noise source is the nuclear spins. The fluctuations of the nuclear spins in the substrate can lead to phase randomization of the electron spin via hyperfine interaction. Nevertheless, the speed of nuclear field variation is much slower than that of the electron spin evolution. Due to the special singlet and triplet states encoding strategy, this kind of decoherence can be also greatly reduced [5]. By further exploring spin echo technique, dephasing time $T_2\approx 1$ μs has been shown experimentally.

We should also mention the low frequency fluctuations of the electrostatic bias. This noise, typically with a $1/f$ spectrum in its low frequency part, is believed to be mainly produced by the charge defect of the gate electrodes. The gate bias of the qubit drifts randomly when an electron tunnels in or out of the metallic electrode. Due to the low frequency

property, the effect of the $1/f$ noise on the qubit is dephasing rather than relaxation. In recent experiments, the optimal point control is widely used to fight against the $1/f$ noise [19]. By operating the qubit at the saddle point where the linear longitudinal qubit-noise coupling vanishes (i.e., $\theta = \pi/4$ of our proposal), one can prolong the dephasing time by several orders [23].

Following the standard quantum theory of damping, we investigate the combined influence of all the above decoherence processes on the coupled system. After tracing out the reservoir degrees of freedom, we obtain the master equation for the reduced density matrix ρ of the three-party system

$$\frac{d\rho}{dt} = -i[H_{int}^{eff}, \rho] + \frac{\gamma_{\varphi 1}}{2}[\sigma_{z1}\rho\sigma_{z1} - \rho] + \left(\frac{\gamma_1}{4}\right) \times \left[\sigma_1^- \rho \sigma_1^+ - \frac{1}{2}\sigma_1^+\sigma_1^-\rho - \frac{1}{2}\rho\sigma_1^+\sigma_1^-\right] + \frac{\gamma_{\varphi 2}}{2}[\sigma_{z2}\rho\sigma_{z2} - \rho] + \left(\frac{\gamma_2}{4}\right)\left[\sigma_2^- \rho \sigma_2^+ - \frac{1}{2}\sigma_2^+\sigma_2^-\rho - \frac{1}{2}\rho\sigma_2^+\sigma_2^-\right],$$

where $\gamma_{\varphi i}$ and γ_i are the pure dephasing rate and relaxation rate, respectively, of individual qubits. Choosing $\gamma_\varphi = \gamma_{\varphi 1} = \gamma_{\varphi 2}$ and $\gamma = \gamma_1 = \gamma_2$, we calculate the error probability D of the EPR generation versus γ_φ and γ, as shown in Fig. 5. In this calculation we choose $g/2\pi = 100$ MHz and the large detuning $\tau = 10g$ [7]. This simulation shows that the error probability D can be lower than 1% when we use the previously reported decoherence rates $\gamma/2\pi = 0.2$ MHz and $\gamma_\varphi/2\pi = 0.5$ MHz.

In conclusion, we propose a dispersive coupling scheme and find a general interaction Hamiltonian for n double-dot qubits and the TLR. These double-dot qubits are biased at the large detuning region and the TLR is always empty and virtually excited. The energy loss of the cavity has little influence on the quantum coherence of this system. After analyzing the main decoherence sources, we show that various quantum operations of scalable solid-state quantum computing could be reliably implemented with current technology.

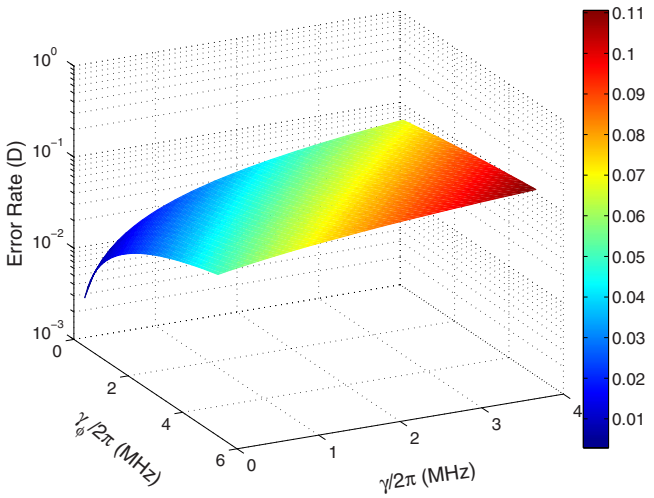

FIG. 5. (Color online) Dependence of the error probability D in entanglement generation on dissipation factors γ_φ and γ.

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