Time-reversal operation of tunneling dynamics of Bose-Einstein condensates in optical lattices

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We propose schemes that can perform perfect time-reversal operation of the tunneling dynamics of Bose-Einstein condensates in optical lattices. One of the time-reversal schemes is to reverse the sign of the tunneling coefficient by a time-periodic potential, maintaining its magnitude at a constant value. The other scheme utilizes a staggered 0-π phase imprinting on the optical lattices. If the reversal of the tunneling coefficient or the phase imprinting is performed at $\tau=\tau_p$, the initial distribution of the condensates is regained at $\tau=2\tau_p$. Although the nonlinear interatomic interaction deteriorates the fidelity of the time-reversal operation, if the sign of the nonlinearity is reversed simultaneously with the phase imprinting at $\tau=\tau_p$, the time-reversal operation at $\tau=2\tau_p$ becomes nearly perfect.

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I. INTRODUCTION

Quantum dynamics in a periodic lattice is one of the oldest problems of quantum mechanics, whose basis was settled by Bloch [1] and Zener [2], around 1930. Aimed at a description of electron motion in crystalline lattices, this problem has been considered for about half a century as an academic one, because dissipation effects forbid the observation of most quantum effects in the motion of a crystalline electron. Laser cooling of atoms has brought a revival of interest in such problems, as it produces atoms whose de Broglie wavelength is comparable to the wavelength of the light interacting with the atoms. Light potentials generated by standing waves have been used in many experimental studies of quantum dynamics. For example, Bloch oscillations have been observed both with single atoms [3] and with a Bose-Einstein condensate [4] in an accelerated standing wave. Wannier-Stark ladders [5] and collective tunneling effects [6] have also been studied with such systems.

The aim of this paper is to investigate spatiotemporal dynamics for a system of M condensates with tunneling allowed only between neighboring condensates. We propose a scheme that can perform a complete time-reversal operation of the distribution of Bose-Einstein condensates (BECs) in optical lattices. One of the time-reversal schemes is to reverse the sign of the tunneling coefficient by a time-periodic field, maintaining its magnitude a constant value. The other scheme utilizes staggered 0-π phase imprinting on the optical lattices. If the reversal of the tunneling coefficient or the phase imprinting occurs at $\tau=\tau_p$, the initial distribution of the condensates is regained at $\tau=2\tau_p$. The nonlinear atom-atom interaction degrades the fidelity of the time-reversal operation. However, if the sign of the nonlinearity is reversed simultaneously with the phase imprinting at $\tau=\tau_p$, the time-reversal operation at $\tau=2\tau_p$ becomes nearly perfect.

We note that there is certainly a connection between the question of equilibration of an isolated quantum system, which is presently much debated in the literature [7], and the time-reversal operation described in this paper. It has been shown that a generic isolated quantum many-body system approaches a distinctly nonequilibrium steady state that bears strong memory of the initial conditions [7], which is required to achieve the time-reversal operation.

Although, our study is done in the context of BEC physics, it is applicable to ubiquitous discrete problems such as nonlinear lattices of condensed matter [8], arrays of Josephson junctions [9], localized modes in electrical lattices [10], and optical waveguide arrays [11].

II. BASIC MODEL

We consider M identical small condensates with tunneling allowed only between neighboring condensates. All numerical results presented in this paper are performed for $M=20$. When the heights of the interwell barriers of the periodic potential are much higher than the condensate chemical potential, the evolution of the quantum particles can be well described by the discrete nonlinear Schrödinger equation (DNLSE) which is derived from the Gross-Pitaevskii equation [12–14] with $\hbar=1$:

$$i\frac{dC_j}{dt}=(\varepsilon_j-\Lambda|C_j|^2)C_j+\kappa(C_{j+1}+C_{j-1}), \tag{1}$$

where $C_j(t)$ is the probability amplitude of finding the BEC atoms on site j $(j=1,2,\ldots,M)$ at time t, ε_j represents the site energies, κ is the nearest-neighbor hopping (or tunneling) term, and $\Lambda=4\pi a/m$ is the nonlinear coefficient, characterized by the s-wave scattering length a and the mass m. The normalized particle number on site j is given by $n_j=|C_j(t)|^2N$ with the total number of particles N and the total probability amplitude is maintained as unity, i.e., $\sum_{j=1}^{M}|C_j(t)|^2=1$. Note that Eq. (1) is invariant with respect to the transformations $\Lambda/\kappa\to-\Lambda/\kappa$, $\varepsilon_j/\kappa\to-\varepsilon_j/\kappa$, and $C_j\to C_j^*e^{i\pi j}$. If we introduce a dimensionless time $\tau=\kappa t$, $\xi_j=\varepsilon_j/\kappa$ and $\lambda=\Lambda/\kappa$ become the only two variable parameters in Eq. (1).

III. TIME-REVERSAL BY κ-REVERSAL OPERATION

We notice that, if all of the site energies are equal ($\varepsilon_j=\varepsilon$) and the interatomic interaction is neglected ($\Lambda=0$), Eq. (1) is invariant with respect to the transformation $\kappa\to$

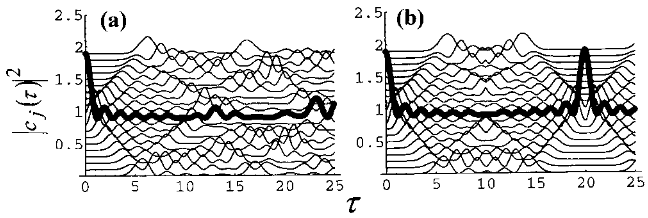

FIG. 1. Time development of atomic populations $n_j(\tau) = |C_j(\tau)|^2$ in a linear lattice obtained for lattice number $M=20$ and for the initial condition $C_{10}(0)=1.0$, without (a) and with (b) κ reversal at $\tau=10$. The perfect recovery of the initial state is realized at $\tau=20$, where all atoms are again occupied in site $j=10$ (shown by the solid line).

$-\kappa$ and $t \to -t$. This transformation implies that the time-reversal process is realized by reversing the sign of the tunneling coefficient κ; hereafter we call this operation "κ reversal."

First we consider a case where the BECs are initially loaded into a single site $j=10$ and the sign of the coupling coefficient is reversed from positive to negative at $\tau=10$, while $|\kappa|=1$ remains constatnt. The numerical result is shown in Fig. 1(b), where each curve corresponds to the particle density $n_j(\tau) = |C_j(\tau)|^2$. The lowest (highest) curve corresponds to $n_1(\tau)$ $[n_{20}(\tau)]$. They are successively shifted to the upper side by the amount of 0.1. The BEC initially at a single site gradually spreads to the neighboring sites due to quantum tunneling. After $\tau \approx 5$, the BEC has spread over all sites and then coherently reflects at both boundaries, resulting in a complicated pattern due to the interference of the matter waves. After κ reversal at $\tau=10$, the spreading waves show time-reversal behavior and then perfect recovery of the initial state is realized at $\tau=20$, where all atoms again occupy site $j=10$. In order to compare spatiotemporal behaviors with and without κ reversal, we show numerical result without κ reversal [see Fig. 1(a)]. In this case, the matter waves show strong dispersive nature as the development time is increased.

Next, we consider how to realize the κ-reversal process in experiments. It is well known that the tunneling coefficient is modified by a driving ac field. The effective coupling coefficient under the influence of an ac field, whose amplitude and frequency are described by the parameters V and ω, is given by $\kappa_{\mathrm{eff}} = J_0(Vx/\omega)\kappa$, where J_0 is the zeroth Bessel function of the first kind and x is the intersite separation of the optical lattice [15–17]. If we choose two modulation amplitudes V_1 and V_2 satisfying $J_0(V_1 x/\omega)\kappa = -J_0(V_2 x/\omega)\kappa$, the perfect time-reversal process may be obtained as shown in Fig. 1(b). κ reversal is also realized by periodic δ kicks. Recently, we have shown that the effective coupling coefficient is modified by periodic kicks as $\kappa_{\mathrm{eff}} = \kappa \cos(Ax)$, where A is the pulse area of the kick field [18]. The κ-reversal process is therefore realized for $A_1 x = 2n\pi$ and $A_2 x = (2n+1)\pi$, where n is integer. We confirmed by numerical simulations that the ac modulation of the coupling coefficient really gives rise to a time-reversal process. In order to obtain high fidelity of the time-reversal process, the ac frequency ω should be much higher than the tunneling frequency $\omega_t (=2\pi/\kappa)$ such as $\omega/\omega_t \geq 30$.

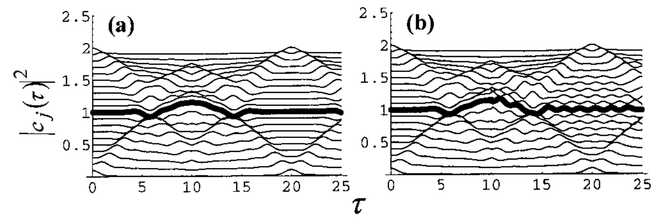

FIG. 2. Time development of atomic populations obtained for the initial condition of equally distributed BECs on all sites, i.e., $C_j(t) = 1/\sqrt{20}$. The κ-reversal operation at $\tau=10$ is shown in (a). The phase-imprinting operation at $\tau=\tau_p=10$ is shown in (b), with $\varepsilon_0 = 100$, and $\tau_0 = 0.01$.

IV. TIME REVERSAL BY STAGGERED PHASE IMPRINTING PROCESS

Considering again the invariant transformations $\Lambda/\kappa \to -\Lambda/\kappa$ $(\lambda \to -\lambda)$, $\varepsilon_j/\kappa \to -\varepsilon_j/\kappa$ $(\xi_j \to -\xi_j)$, and $C_j \to C_j^* e^{i\pi j}$, we can expect that staggered 0-π phase imprinting on the lattice sites also gives rise to a time-reversal process. For staggered 0-π phase imprinting with the potential $\varepsilon_p(\tau) = \varepsilon_0 \operatorname{sech}[(\tau-\tau_p)/\tau_0]$ for odd sites with $\varepsilon_0 = 50, \tau_p = 10, \tau_0 = 0.02$, i.e., $\int_{-\infty}^{+\infty} \varepsilon_p(\tau) d\tau = \pi$, we obtain time-reversal dynamics that is almost identical with that shown in Fig. 1(b). Thus, we know that the staggered phase imprinting process is equivalent to the κ-reversal process.

In Fig. 2, we show another numerical result which is obtained for a different initial condition in which the BECs are equally distributed in all lattice sites, i.e., $C_j(t) = 1/\sqrt{M}$ with $M=20$. The time-reversal processes obtained for the κ-reversal process and the phase-imprinting process are shown in Figs. 3(a) and 3(b), respectively. The parameters of the phase-imprinting potential are $\tau_p = 10$, $\varepsilon_0 = 100$, and $\tau_0 = 0.01$ which satisfy $\int_{-\infty}^{+\infty} \varepsilon_p(\tau) d\tau = \pi$. Regardless of the different initial conditions, the time-reversal behavior is clearly seen at $\tau = 2\tau_p = 20$. For the phase-imprinting scheme [see Fig. 2(b)], residual ripples are seen in the time-reversed waves, which can be sufficiently reduced by applying much narrower phase-imprinting pulses.

Here, we consider how to realize staggered 0-π phase imprinting. The optical lattice in which BECs are loaded is created by two laser beams intersecting at angle θ. The resulting periodic potential $V(x) = V_0 \sin^2(\pi x/d_L)$ has lattice spacing $d_L = \lambda/[2\sin(\theta/2)]$. Another time-dependent optical lattice with lattice spacing $2d_L$ is superimposed on this optical lattice to give staggered phase imprinting; one of the lattice sites j has a potential minimum (π phase imprinting)

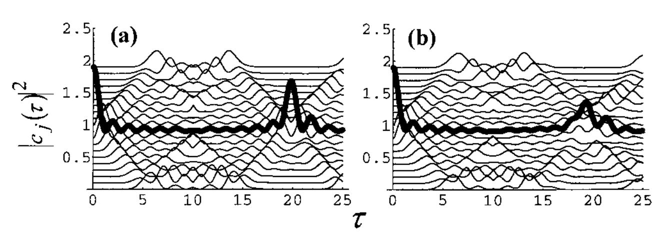

FIG. 3. Time-reversal process in nonlinear systems obtained by staggered phase imprinting for different values of the nonlinear coefficient λ= (a) 1.0 and (b) 2.0.

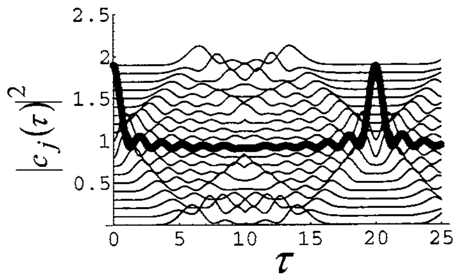

FIG. 4. Time-reversal dynamics obtained for simultaneous utilization of the phase-imprinting and nonlinearity-reversal processes for $|\lambda|=2.0$ with $\tau_p=10$.

and its two nearest neighbor sites $j\pm1$ have potential maxima (0 phase imprinting). Phase imprinting by a phase mask and potentials created by lithographically patterned electrodes are other candidates to realize staggered phase imprinting in experiments.

V. TIME-REVERSAL OPERATION IN NONLINEAR SYSTEMS

So far, we have investigated a linear system in which the interatomic interactions can be neglected. Here we study a nonlinear system in which the interatomic interactions have an important role. In Fig. 3, we show the time-reversal process in a nonlinear system obtained by staggered phase imprinting with different values of the nonlinear coefficient λ. The parameters used for the calculations are $\varepsilon_0=50$, $\tau_p=10$, $\tau_0=0.02$, which satisfy $\int_{-\infty}^{+\infty}\varepsilon_p(\tau)d\tau=\pi$, but $\lambda=1.0$ (a) and 2.0 (b). It is clear from comparing Figs. 3(a) and 3(b) with Fig. 1(b) that the time-reversal property deteriorates as the nonlinearity is increased. The degradation of the time reversal can fortunately be reversed by simultaneous reversal of the sign of the nonlinear coefficient at the phase-imprinting time $\tau=\tau_p$. In Fig. 4, we show the time-reversal dynamics obtained for simultaneous utilization of the phase-imprinting and nonlinearity reversal processes for $|\lambda|=2.0$ with $\tau_p=10$. Comparing Fig. 3(b) with Fig. 4, we can see that the simultaneous introduction of the nonlinearity reversal with the staggered phase imprinting brings perfect recovery of the initial state, i.e., perfect time reversal.

It is possible to control the interatomic interaction by applying an external magnetic field B, which controls the scattering length. Specifically, the behavior of the scattering length near a Feshbach resonant magnetic field B_0 is typically of the form [19]

$$a(B)=\tilde{a}\left(1-\frac{\Delta}{B-B_0}\right), \tag{2}$$

where $\tilde{a}$ is the value of the scattering length far from resonance and Δ represents the width of the resonance. Keeping in mind the relation $\Lambda=4\pi a(B)/m$, we know that the sign and the magnitude of the nonlinearity can be changed by an external magnetic field. Magnetic fields B ($<B_0$) give a positive nonlinearity and magnetic fields B ($>B_0$) a negative one.

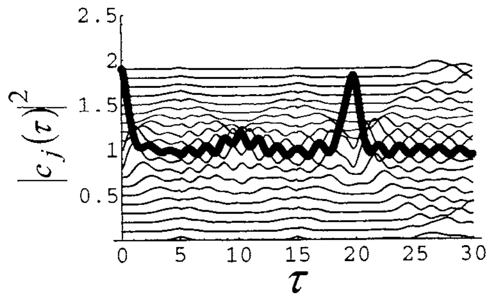

FIG. 5. Time development of matter waves when two phase-imprinting pulses are successively applied. The nonlinear coefficient used for the calculation is $\lambda=3.6$ and the first and second phase imprintings are applied at $\tau=5$ and 15, respectively.

Finally, we point out a peculiar time-reversal phenomenon in the nonlinear system. This is observed in two successive phase-printing processes. Figure 5 shows the time development of the matter waves when two phase-imprinting pulses are successively applied. The nonlinear coefficient used for the calculation is $\lambda=3.6$ and the first and second phase imprinting are applied at $\tau=5$ and 15, respectively. Here we note that the first time-reversal behavior expected at $\tau=10$ shows poor recovery but the recovery by the second time reversal expected at $\tau=20$ is quite good. This behavior can be clearly seen for relatively high nonlinearities, typically $\lambda\approx2.4-4.0$, and disappears above the self-localization threshold for $\lambda\geq4$. At present, we cannot understand the physical mechanism of this peculiar behavior, but it seems as if the nonlinear coefficient has effectively changed its sign at the first time-reversal time at $\tau=5$, and therefore the accumulated phase of the positive nonlinearity for $\tau=0-10$ can be compensated in the successive development of the second time reversal for $\tau=10-20$. From a practical point of view, this scheme is very useful even if the time-reversal process is not perfect, because it is a very simple process which does not require a nonlinearity-reversal process.

Finally, we point out that the time-reversal mechanism in nonlinear systems may have a connection to the question of the equilibration of an isolated quantum system [7]: for large values of the interatomic interaction the system approaches a distinctly nonequilibrium steady state that bears a strong memory of the initial conditions.

VI. CONCLUSION

We have proposed the two schemes that can perform a complete time-reversal operation of the distribution of BECs in optical lattices. One is the κ-reversal scheme in which the sign of the tunneling coefficient is reversed by the time-periodic field. The other scheme is to utilize staggered 0-π phase imprinting on the optical lattices. Furthermore, it was shown that, if the sign of the nonlinearity is reversed simultaneously with the phase-imprinting process, the time-reversal operation could be perfect even in a nonlinear system.

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