Influence of non-Markovian relaxation processes on self-induced transparency: Memory function theory for optical solitons

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The influence of transverse relaxation on the resonance soliton (2π hyperbolic-secant pulse) in solids is considered. It is shown that memory effects can be considered in terms of generalized non-Markovian optical Bloch equations. The equations of self-induced transparency with a memory function are presented. Explicit forms for the first-order effects of this relaxation on the frequency shift and the pulse height of a resonant 2π soliton in solids are given and discussed. It is shown that memory effects do lead to a qualitative change in the dynamics of the influence of transverse relaxations on a 2π pulse in comparison to the McCall-Hahn theory and others. The dynamics of the changes of the inverse width of the pulse are obtained in both the Markovian and the non-Markovian cases with realistic parameters that can be achieved in current experiments.

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I. INTRODUCTION

Resonant optical solitons can be created by means of the McCall-Hahn mechanism, i.e., from the nonlinear coherent interaction of an electromagnetic pulse with resonant atoms in a medium (such as gaseous atoms or impurity atoms in solids), when the conditions $\omega T \gg 1$ and $T \ll T_{1,2}$ are satisfied [1]. Here, ω and T are the pulse frequency and duration, respectively; T_1 and T_2 are the longitudinal and transverse relaxation times of the resonant atoms. Exploring the evolution of a coherent light pulse in a resonant absorbing medium, McCall and Hahn observed the amazing phenomenon of anomalously low energy loss when the pulse power exceeds some critical value. This effect was called self-induced transparency (SIT) [1]. The physical explanation of the SIT effect is based on the representation of the absorber by an ensemble of two-level atoms whose evolution is caused by induced processes due to the interaction with the coherent light pulse. In general the theory of the interaction of electromagnetic radiation with an ensemble of two-level atoms is based on the Bloch equations for atoms and the Maxwell equations for the classical electromagnetic field. From the Maxwell-Bloch equations, many of the principal theoretical results of SIT were originally obtained by McCall and Hahn, using quite simple methods, in particular, the fact that certain solutions of this system of equations, known as 2π pulses, did indeed have solitonlike properties [1].

In real physical systems, relaxation effects invariably exert an important influence on nonlinear wave processes. These relaxation effects modify the evolution of the soliton parameters, which is one of the principal problems in the physics of nonlinear waves. In the case of ideal SIT, i.e. without relaxation processes considered, one has an advantage in that one can make use of the inverse scattering transform (IST) [2–4], by which it is possible to obtain the complete solution of any integrable nonlinear system. There are many situations when relaxation processes can be ignored. Mostly due to the works of Lamb [5,6], the soliton part of the solution was developed and studied, which allowed one to understand the asymptotic form of the solution of SIT in the attenuator case. Following upon the work of Lamb [6], Ablowitz, Kaup, and Newell [7] obtained the complete solution for the Maxwell-Bloch equations, including the nonsoliton part (called "radiation") as well as the soliton part. The simplified version of the theory of SIT, based on a sine-Gordon equation, was analyzed by means of its IST in Ref. [8]. Since that time, additional results have shown an even closer connection between the McCall-Hahn theory and the IST solution than was at first suspected [9–15].

Although there have been many physical problems, among them SIT, solved by these ISTs, at the same time there continue to be many related problems which cannot be solved exactly by these techniques. As an example, the

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addition of relaxation processes generally will cause the method of the IST to fail. On the other hand, there are many situations where these relaxation effects can be taken to be small and, when that is the case, one can expect a perturbation expansion to allow one to gain some insight into the influence of these effects on the fully nonlinear state. Of the fully nonlinear states, the most important are the soliton solutions. In the absence of any perturbations, single solitons have a fixed wave form (envelope) and velocity. When the system is perturbed by relaxation, we can expect the shape of the wave form, as well as its velocity and other parameters, to change slowly [16–18]. A perturbation expansion based on the IST was used in Ref. [17] to consider the first-order effects of various relaxation processes on the propagation of a 2π pulse. Explicit analytical but approximate expressions for the phase modulation, the time delay, the instantaneous frequency (the frequency shifts), and the decay rate of the pulse were determined. The results from Ref. [17] generally agree with those of McCall and Hahn's theory but provide corrections for the effects of relaxation. Effects due to relaxation result in a time delay and a changed rate at which an off-resonance 2π pulse would drift away from the resonance due to the homogeneous broadening.

In Ref. [19], the IST perturbation expansion was used to investigate the influence of a Markovian transverse relaxation process on the 2π pulse of SIT due to the inhomogeneous broadening of spectral lines. In this work, the influence of the relaxation on the continuous spectrum of the scattering data of the Zakharov-Shabat (ZS) eigenvalue problem was investigated. Explicit analytical expressions for the continuous part of the scattering coefficients were determined. It was shown that the transverse relaxation induces the excitation of the continuous spectrum of the scattering data of the ZS spectral problem. It was found that the transverse relaxation effect leads to changes in the wave form and the energy of the pulse of SIT, and explicit first-order, analytical expressions for these quantities were given. Asymptotic expressions for the amplitude and energy of the pulse were also calculated in Refs. [19,20].

The above-mentioned work [1,17,19,20] gives the complete physical picture of the influence of relaxation processes on optical SIT in the framework of the Bloch-Maxwell equations if Markovian relaxation processes are present. In these semiphenomenological Bloch equations, the effect of transverse relaxation was taken into account by means of the Markovian relaxation terms P_x/T_2 and P_y/T_2, where P_x and P_y are the in-phase (with respect to the electrical field of the pulse) and quadrature (shifted in phase by $\pi/2$) components of the electrical dipole moment of optical atoms [12,14,21]. Consequently, the components of the transverse polarization, P_x and P_y, and the longitudinal component, P_z, evolve according to

$$\frac{\partial P_x}{\partial t} = \gamma_E[\vec{P} \times \vec{\mathcal{E}}]_x - \frac{P_x}{T_2},$$

$$\frac{\partial P_y}{\partial t} = \gamma_E[\vec{P} \times \vec{\mathcal{E}}]_y - \frac{P_y}{T_2},$$

$$\frac{\partial P_z}{\partial t} = \gamma_E[\vec{P} \times \vec{\mathcal{E}}]_z - \frac{P_z - P_0}{T_1}, \tag{1}$$

where γ_E is the gyroelectric ratio, and $\vec{P}$ and $\vec{\mathcal{E}}$ are the vectors of the polarization and the electric field, where P_0 is the equilibrium value toward which the function P_z relaxes when the electric field equals zero [1,21]. Equations (1) are called the "T_1-T_2 model" of the optical Bloch equations [21,22] and follow from the so-called Markov approximation wherein one takes the time evolution of the optical polarization $P_{x,y}$ to depend only on the values of $P_{x,y}$ at that same time.

Numerical investigations of optical processes are typically based on these optical Bloch Eqs. (1) [21,22]. It is to be noted that this T_1-T_2 model of the Bloch equations is a good approximation for gases and other systems. But this is not always the case, particularly for solid state systems and certain dielectrics, where relaxation processes occur due to electron-phonon interactions which need time to be built up (retarded interaction). Even in some cases where the transverse components do exhibit exponential decay, the dynamics of the polarization does not follow Eqs. (1) [22]. In solids, the Markovian model is only a rough approximation at best and in many cases is actually inapplicable. Because of this, it becomes appropriate to consider certain non-Markovian models which would be more appropriate for light-matter interactions in solids, for which there exist numerous experimental results [22,23].

To do this, it becomes necessary to include contributions of memory effects into the Bloch equations, i.e., Eq. (1) would have to be modified so that the evolution would contain time integrations of $P_{x,y}$ over all previous times $t' \leqslant t$. In this case, the appropriate terms in Eq. (1) would be replaced according to

$$\frac{P_{x,y}(t)}{T_2} \to \int_{-\infty}^{t} K(t-t')P_{x,y}(t')dt',$$

where $K(t-t')$ is a memory function [24–27]. Such a generalization of the Bloch equations would significantly improve their suitability for the description of optical relaxation in solids. That is the purpose of this paper, wherein we shall incorporate into the SIT theory memory effects in transverse relaxations so as to better approximate the evolution of SIT solitons in solids. Examples for the microscopic derivation of such memory effects are given in Refs. [22,28–32].

II. T_1-T_2 MODEL OF THE OPTICAL BLOCH EQUATIONS

To introduce the formalism let us first review the T_1-T_2 model of the optical Bloch equations. Then we will consider the influence of relaxation processes on resonance SIT solitons in solids, which contain optical active impurity atoms.

We shall take the optical pulse to be linearly polarized with width $T \ll T_{1,2}$, frequency $\omega \gg T^{-1}$, and wave vector $\vec{k}$. The electric field $\vec{E}(z,t) = \vec{e}E(z,t)$ is taken to be propagating along the positive z axis, where $\vec{e}$ is the polarization vector, directed along the x axis.

For the description of impurity atoms we use the two-level model which can be described by the states $|1\rangle$ and $|2\rangle$

with energies $E_1=0$ and $E_2=\hbar\omega_0$, respectively, where $|1\rangle$ is the ground state. The Hamiltonian and wave function of this system are

$$H = H_0 + \hat{V},$$

$$|\Psi\rangle = \sum_{n=1,2} c_n(t)\exp\left(-\frac{i}{\hbar}E_n t\right)|n\rangle,$$

where $H_0=\hbar\omega_0|2\rangle\langle 2|$ is the Hamiltonian of the two-level atom with frequency of excitation ω_0,

$$\hat{V} = -\mu_{12}(|2\rangle\langle 1| + |1\rangle\langle 2|)E$$

is the interaction Hamiltonian, $\hbar$ is Planck's constant, and $\mu_{12}=\vec{\mu}_{12}\cdot\vec{e}$ where $\vec{\mu}_{12}$ is the electric dipole moment for the corresponding transition, assumed to be real. The quantities E_1 and E_2 are eigenvalues of the Hamiltonian H_0. From the Schrödinger equation, the probability amplitudes c_1 and c_2 are determined by [33]

$$i\hbar\frac{\partial c_1(t)}{\partial t} = -\mu_{12}Ec_2(t)e^{-i\omega_0 t},$$

$$i\hbar\frac{\partial c_2(t)}{\partial t} = -\mu_{12}Ec_1(t)e^{i\omega_0 t}. \qquad (2)$$

Using the method of slowly changing envelopes

$$E = \sum_{l=\pm 1}\hat{E}_l Z_l, \qquad (3)$$

where $\hat{E}_l$ is the slowly varying complex envelope of the electric field and $Z_l=e^{il(kz-\omega t)}$ contains the rapidly varying phase of the carrier wave, we may apply

$$\left|\frac{\partial \hat{E}_l}{\partial t}\right| \ll \omega|\hat{E}_l|, \qquad \left|\frac{\partial \hat{E}_l}{\partial z}\right| \ll k|\hat{E}_l|. \qquad (4)$$

We also take E to be real in which case $\hat{E}_l=\hat{E}^*_{-l}$.

Substituting Eq. (3) into Eq. (2) and taking the probability amplitudes to be of the form

$$c_1 = iv_1^* e^{-i(\Delta/2)t}, \qquad c_2 = v_2^* e^{i[kz+(\Delta/2)t]}, \qquad (5)$$

where $\Delta=\omega_0-\omega$, the Schrödinger equations (2) will be transformed into the Zakharov-Shabat spectral problem [4]:

$$\frac{\partial v_1}{\partial t} + i\zeta v_1 = qv_2, \qquad \frac{\partial v_2}{\partial t} - i\zeta v_2 = rv_1, \qquad (6)$$

where $r=-q^*$ and

$$\zeta = \frac{\Delta}{2}, \qquad r = -\frac{\mu_{12}}{\hbar}\hat{E}_1, \qquad q = \frac{\mu_{12}}{\hbar}\hat{E}_1^*. \qquad (7)$$

The average values of the Pauli operators $\hat{\sigma}_i$ which describe the induced dipole and inversion probability for the state $|\Psi\rangle$, $s_i=\mathrm{Tr}\langle\Psi|\hat{\sigma}_i|\Psi\rangle$ (where $i=1,2,3$) are [21]

$$s_x = c_1^*(t)c_2(t)e^{-i\omega_0 t} + c_1(t)c_2^*(t)e^{i\omega_0 t},$$

$$s_y = ic_1^*(t)c_2(t)e^{-i\omega_0 t} - ic_1(t)c_2^*(t)e^{i\omega_0 t},$$

$$s_z = c_2^*(t)c_2(t) - c_1^*(t)c_1(t). \qquad (8)$$

Defining $s^{\pm}=\frac{1}{2}(s_x \pm is_y)$, then from Eqs. (5) and (8), we obtain

$$s^+ = iv_1^* v_2 e^{-i(kz-\omega t)},$$

$$s^- = -iv_1 v_2^* e^{i(kz-\omega t)},$$

$$s_z = |v_2(t)|^2 - |v_1(t)|^2. \qquad (9)$$

Substituting Eq. (9) into Eq. (6) and defining the quantities

$$\rho^+ = v_1^* v_2, \qquad \rho^- = v_1 v_2^*,$$

we obtain the undamped Bloch equations [21]

$$\frac{\partial\rho^+}{\partial t} = i\Delta\rho^+ - rs_z,$$

$$\frac{\partial\rho^-}{\partial t} = -i\Delta\rho^- + qs_z,$$

$$\frac{\partial s_z}{\partial t} = 2(r\rho^- - q\rho^+). \qquad (10)$$

It also becomes convenient to represent the functions $\rho^{\pm}$ as

$$\rho^{\pm} = \frac{1}{2}(\mp iu + v)e^{\pm i\varphi},$$

where φ is a phase function, and u and $-v$ are the components, in units of the transition moment μ_{12}, of the atomic dipole moment in-phase and in-quadrature components with respect to the field E. In other words, v is the absorptive component of the atomic dipole moment, while u is the dispersive component. In the absence of phase modulation, where $\varphi=0$, the functions $\hat{E}_1=\hat{E}_1^*=\hat{E}$ are real, and Eq. (10) can be transformed to the usual form of the optical Bloch equations in the rotating frame [21]:

$$\frac{\partial u}{\partial t} = -\Delta v, \qquad \frac{\partial v}{\partial t} = \Delta u + \frac{2\mu_{12}}{\hbar}\hat{E}w, \qquad \frac{\partial w}{\partial t} = -\frac{2\mu_{12}}{\hbar}\hat{E}v,$$

where $\frac{1}{2}\hbar\omega_0 w$ is the expectation of the atom's unperturbed energy.

Next we define the quantities

$$\Lambda^{\pm} = \pm i\mu_{12}n_0\rho^{\pm}, \qquad W = \mu_{12}n_0 s_z,$$

where n_0 is the concentration of optically active atoms, and introduce phenomenological Markovian decay constants into Eq. (10), which transforms the Bloch equations into the form

$$\frac{\partial\Lambda^+}{\partial t} = i\Delta\Lambda^+ - irW - \frac{\Lambda^+}{T_2},$$

$$\frac{\partial\Lambda^-}{\partial t} = -i\Delta\Lambda^- - iqW - \frac{\Lambda^-}{T_2},$$

$$\frac{\partial W}{\partial t} = 2i(r\Lambda^- + q\Lambda^+) - \frac{W - W_0}{T_1}, \qquad (11)$$

where W_0 is the equilibrium value toward which the inversion W relaxes when $r=q=0$. Equation (11) forms the optical semiphenomenological Bloch equations for the T_1-T_2 model in the rotating frame [12,14,21]. To study solitons as self-consistent solutions of the Maxwell-Bloch equations, we need, in addition to Eq. (11) for the material, a description of the pulse propagation in the medium. The wave equation for the electric field $E(z,t)$ of the optical pulse in medium is given by

$$\frac{\partial^2 E}{\partial z^2} - \frac{\varepsilon}{c^2}\frac{\partial^2 E}{\partial t^2} = \frac{4\pi}{c^2}\frac{\partial^2 P}{\partial t^2}, \qquad (12)$$

where c is the light velocity in vacuum, ε is the permittivity constant, and P is the polarization of the two-level system. Upon taking into account inhomogeneous broadening of the spectral line, the polarization of the two-level system is equal to

$$P = \int g(\Delta)(\Lambda^+ Z_{-1} + \Lambda^- Z_1) d\Delta, \qquad (13)$$

where $g(\Delta)$ is the inhomogeneous broadening function. In order for the polarization P to be real, it follows that $\rho^{+*} = \rho^-$. Assuming that the envelopes $\rho^{\pm}$ vary sufficiently slowly in space and time as compared with the carrier wave parts, it follows that we may take

$$\left|\frac{\partial \rho^{\pm}}{\partial t}\right| \ll \omega|\rho^{\pm}|, \qquad \left|\frac{\partial \rho^{\pm}}{\partial z}\right| \ll k|\rho^{\pm}|.$$

The above together with the Eq. (4) is known as the slowly varying envelope approximation [1,9,12,14,21].

Substituting the polarization (13) into Eq. (12), using Eqs. (3) and then taking into account Eq. (7), we obtain the following nonlinear wave equations for the envelopes:

$$\frac{\partial r}{\partial z} + \frac{\eta}{c}\frac{\partial r}{\partial t} = -\kappa_0 \int g(\Delta)\rho^+ d\Delta,$$

$$\frac{\partial q}{\partial z} + \frac{\eta}{c}\frac{\partial q}{\partial t} = \kappa_0 \int g(\Delta)\rho^- d\Delta,$$

where

$$\kappa_0 = \frac{2\pi n_0 \mu_{12}^2 \omega}{c\eta\hbar},$$

and η is the refractive index ($\eta^2 = \varepsilon$). Introducing the retarded time τ and the spatial distance as new independent variables

$$\chi = z, \qquad \tau = t - \frac{\eta}{c}z, \qquad (14)$$

the material and wave equations obtain the form

$$\dot{\Lambda}^+ = i\Delta\Lambda^+ - irW - \frac{\Lambda^+}{T_2},$$

$$\dot{\Lambda}^- = -i\Delta\Lambda^- - iqW - \frac{\Lambda^-}{T_2},$$

$$\dot{W} = 2i(r\Lambda^- + q\Lambda^+) - \frac{W - W_0}{T_1}, \qquad (15)$$

$$r_\chi = \frac{\partial r}{\partial \chi} = -\langle \rho^+ \rangle, \quad q_\chi = \frac{\partial q}{\partial \chi} = \langle \rho^- \rangle, \qquad (16)$$

where the overdot denotes differentiation by the variable τ and $\langle \cdots \rangle = \kappa_0 \int_{-\infty}^{\infty} g(\Delta) \cdots d\Delta$. Equations (15) and (16) are the system of equations of the SIT with damping which have been investigated in Refs. [1,5,17,19,20,34–36]. This model is the T_1-T_2 model of the Maxwell-Bloch equations. As already mentioned, this system of equations is not generally valid in solids. We next will take into account memory effects and include them into Eqs. (15) to study a model more suitable for relaxation processes in solids.

III. THE MEMORY FUNCTION

Microscopic, non-Markovian relaxation processes in the Bloch equations can be described in a time-local [37] or time-nonlocal description [28,37] via a self-consistent calculation using a Hamiltonian containing the electronic system, the optical field, and the bath as well as respective interactions. Both approaches agree in their results for the weak coupling case [38]. A critical discussion of the *ad hoc* introduction of memory terms into Bloch equations is given in Ref. [39]. Therefore it is appropriate to use a non-Markovian, time-nonlocal description similar to those derived microscopically in Refs. [28–32]. Here we shall simply focus on using a well-behaved model of a memory function in order to obtain analytical results. Our goal in this paper is not to redo the microscopic derivation of the relaxation contributions, but rather only to solve the Maxwell-Bloch system for soliton propagation when memory effects are present.

Here we shall suppose that the longitudinal relaxations are longer than the transverse relaxations, i.e., we take $T_1 \gg T_2$, and will ignore the effects of the longitudinal relaxations by taking $T_1 \to \infty$. This limit is called the pure dephasing model [32]. We note that the influence of T_1 relaxations on SIT have already been previously investigated in Refs. [1,17,34,35]. In these works the T_1-T_2 model of the optical Bloch equations (15) has been used, which is valid in gaseous media, and this model has also been used as a rough approximation for solids. For dielectrics that contain optically active impurity atoms or for semiconductor nanostructures, the Markovian T_1-T_2 model of the optical Bloch equations loses its validity [22,28–32]. It then becomes appropriate to determine the shape of the spectral line and the memory dynamics of the transverse relaxations by the use of the fluctuation-dissipation theorem [40] and to express this shape by means of the Fourier transform of the correlation function,

$$G(t) = \frac{\mathrm{Tr}[\exp(-iH_{\mathrm{int}}t)S_x \exp(iH_{\mathrm{int}}t)S_x]}{\mathrm{Tr}(S_x^2)}.$$

The function $G(t)$ is called the function of free induction decay, where S_x is the x component of the pseudospin [22],

H_{int} is the interaction Hamiltonian which determines the broadening of the spectral line. A typical example for such an application is the electron-phonon coupling in solids. In most cases it is impossible to obtain an exact expression for the function $G(t)$ and therefore approximations are usually used. For dielectrics containing optically active impurity atoms, the function $G(t)$ is governed by the memory function [25]

$$\frac{dG(t)}{dt} = -\int_0^t K(t-t')G(t')dt'. \tag{17}$$

Because $G(t) \sim P_{x,y}(t)$, the dependence of $G(t)$ on the memory is the same as for the x and y components of the polarization. As a consequence of the time retardation in t', the free induction decay does not have the exponential decay character [22,28–32] that is often characteristic of the T_1-T_2 model. This modification is well known from SIT experiments on different solid materials, and particularly for those dielectrics that contain optically active impurity atoms. Thus, to better describe SIT in solids, it becomes important to include memory effects into any such model.

Since the x and y components of the polarization, as well as the functions $G(t)$ and $K(t)$, change rapidly with respect to the characteristic dephasing time [25], one should note that as a consequence of the presence of memory in Eq. (17), the pseudospin will not commute with the interaction Hamiltonian H_{int}. Therefore the pseudospin will not be a quasi-integral of the motion.

Since one cannot generally obtain exact expressions for $G(t)$, it becomes appropriate to develop approximate analytical expressions for this function in various limits [25]. For example, for small memory depth and times, we may take

$$G(t) = 1 - \frac{M_2}{2!}t^2 + \frac{M_4}{4!}t^4 - \cdots,$$

where the moments M_{2n} are

$$M_{2n} = (-1)^n\left(\frac{d^{2n}G}{dt^{2n}}\right)_{t=0}, \quad M_{2n+1} = 0, \quad n = 0,1,2,\ldots.$$

The memory function $K(t)$ may have a rather complex form, and consequently for material-specific calculations it is necessary to use different, adopted correlation functions. Often, Gaussian approximations (Refs. [25,41] and references therein) or exponentials $\exp(-\tau/\tau_c)$ [42] can be used. In this paper, as a first attack on the very complex problem of simultaneous pulse propagation and system-bath interaction, we focus on the Gaussian memory structure. It contains well-defined parameters (memory depth and strength) as well as being sufficiently localized in the frequency domain to provide a well-defined model system. Investigation of the more specific mechanism of the relaxation is not the goal here, but is left to future investigations on specific materials.

Similarly to $G(t)$, for the memory function $K(t)$,

$$K(t) = M_2\left(1 - \frac{N_2}{2!}t^2 + \frac{N_4}{4!}t^4 - \cdots\right),$$

we may take as a simple approximation for $K(t)$ the form

$$K(t) = M_2 e^{-(N_2/2)t^2}, \tag{18}$$

where the quantity N_2 is the second moment of the memory function. Note that for small times we still have the correct form for $K(t)$. For large t, due to the Gaussian shape, there is a rapid decay in this function, whose rate is governed by N_2. We emphasize that the use of the Gaussian approximation for the function of memory does not mean that the shape of the spectral line should be Gaussian too. Thus there is some reason to expect that the Gaussian approximation could give reasonable results for optical SIT in solids.

Continuing, it is convenient to introduce the functions $P^{\pm}$ defined by

$$P^{\pm}(t) = P_x(t) \pm iP_y(t),$$

in which case, in the absence of the optical fields, since $G(t) \sim P_{x,y}$, from Eq. (17), we would have [25]

$$\frac{dP^{\pm}(t)}{dt} = -\int_{-\infty}^t K(t-t')P^{\pm}(t')dt'. \tag{19}$$

The lower limit of the integral has been taken to be $-\infty$ since we seek a pulse shape bounded at $\pm\infty$.

We need to point out that the driving pulse must be sufficiently weak as not to probe the reservoir during the interaction, i.e., we assume that the bath approximation is well suited for the system under investigation. Otherwise the dephasing rate will depend on the driving field intensity [43].

We define the unitary operator

$$U = e^{i\omega_0 S_z t},$$

where S_z is the z component of the pseudospin. Upon carrying out the corresponding unitary transformation, we obtain

$$P^{\pm}(t) = \hat{P}^{\pm}(t)e^{\mp i\omega_0 t}, \tag{20}$$

in which case Eq. (19) becomes

$$\frac{d\hat{P}^{\pm}(t)}{dt} = \pm i\omega_0\hat{P}^{\pm}(t) - \int_{-\infty}^t K(t-t')\hat{P}^{\pm}(t')e^{\pm i\omega_0(t-t')}dt'. \tag{21}$$

Inserting the above expression (18) for K into Eq. (21), using the expression $\hat{P}^{\pm}(t) = \Lambda^{\pm}e^{\pm i\omega t}$, where the spatial and temporal variables are given in Eq. (14), and now including the optical field we obtain the equations

$$\dot{\Lambda}^+ = i\Delta\Lambda^+ - irW - \rho_T^+,$$

$$\dot{\Lambda}^- = -i\Delta\Lambda^- - iqW - \rho_T^-,$$

$$\dot{W} = 2i(r\Lambda^- + q\Lambda^+), \tag{22}$$

where

$$\rho_T^{\pm} = M_2\int_{-\infty}^{\tau}\Lambda^{\pm}e^{\pm i\Delta(\tau-t')}e^{-(N_2/2)(\tau-t')^2}dt'.$$

As mentioned before, we consider here only the case where $T_1 \to \infty$.

The system of equations (16) and (22) are the equations for SIT with pure dephasing only, described by a non-Markovian memory of the dipole and are the main object of our investigations. For the solution of these equations we will use the IST and its perturbation theory to study the evolution of a soliton's parameters [17,18].

IV. ZAKHAROV-SHABAT EQUATIONS

We shall now show that by solving the Zakharov-Shabat equations (ZSEs)

$$\dot{v}_1 = -i\zeta v_1 + q v_2, \quad \dot{v}_2 = i\zeta v_2 + r v_1, \tag{23}$$

where $r=-q^*$, we can also solve the system of equations (16) and (22) when the damping terms $\rho_T^{\pm}$ are included or absent, respectively. First we delineate certain properties of these equations that we shall need.

There are two pairs of linearly independent solutions (Jost functions) of the ZSE: the first pair is denoted by Φ and $\bar{\Phi}$,

$$\Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}, \quad \bar{\Phi} = \begin{pmatrix} \bar{\phi}_1 \\ \bar{\phi}_2 \end{pmatrix},$$

and the second pair is Ψ and $\bar{\Psi}$

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \quad \bar{\Psi} = \begin{pmatrix} \bar{\psi}_1 \\ \bar{\psi}_2 \end{pmatrix}.$$

The first pair Φ and $\bar{\Phi}$ is defined by the asymptotic limit as $\tau \to -\infty$ to be

$$\Phi \to \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\zeta\tau}, \quad \bar{\Phi} \to \begin{pmatrix} 0 \\ -1 \end{pmatrix} e^{i\zeta\tau},$$

and the second pair Ψ and $\bar{\Psi}$ is defined by the asymptotic limit as $\tau \to +\infty$ to be

$$\Psi \to \begin{pmatrix} 0 \\ 1 \end{pmatrix} e^{i\zeta\tau}, \quad \bar{\Psi} \to \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-i\zeta\tau}.$$

For real ζ, the scattering coefficients a, b, $\bar{a}$, and $\bar{b}$ are defined by the asymptotic limits at $\tau \to +\infty$, where

$$\Phi \to \begin{pmatrix} a e^{-i\zeta\tau} \\ b e^{i\zeta\tau} \end{pmatrix}, \quad \bar{\Phi} \to \begin{pmatrix} \bar{b} e^{-i\zeta\tau} \\ -\bar{a} e^{-i\zeta\tau} \end{pmatrix}.$$

On the real axis, one finds that $a\bar{a}+b\bar{b}=1$. From the above definitions, one observes that in general the two pairs of solutions can be related by

$$\Phi = a\bar{\Psi} + b\Psi, \quad \bar{\Phi} = -\bar{a}\Psi + \bar{b}\bar{\Psi}.$$

From the relation $r=-q^*$, it follows that $\bar{\Phi}$ and $\bar{\Psi}$ can be given in terms of Φ and $\bar{\Psi}$:

$$\bar{\Phi} = \begin{pmatrix} \phi_2^* \\ -\phi_1^* \end{pmatrix}, \quad \bar{\Psi} = \begin{pmatrix} \psi_2^* \\ -\psi_1^* \end{pmatrix},$$

and

$$\bar{a}(\zeta) = a^*(\zeta^*), \quad \bar{b}(\zeta) = b^*(\zeta^*)$$

where for real ζ

$$\bar{a}a + \bar{b}b = 1.$$

In addition to the continuous spectra, the ZSE [Eq. (23)] can also possess bound states. These occur whenever $a(\zeta)$ has a zero in the upper half complex ζ plane. Here we shall consider the situation where a has only one zero. If we designate the zero of a by $\zeta_1=\xi_1+i\eta_1$, with both ξ_1 and η_1 real, then since $a(\zeta_1)=0$, $\Phi(\zeta_1)=b_1\Psi(\zeta_1)$, where $b_1=b(\zeta_1)=e^{i\beta_1}e^{2\eta_1 x_0}$, which defines β_1 and x_0.

Next we note that the undamped form of Eq. (22), i.e., $\rho_T^{\pm}=0$, can be "factored" into parts which are Eqs. (6), upon taking $\zeta=\Delta/2$. In fact, for $\rho_T^{\pm}=0$ this becomes obvious if we define

$$\rho^+ = \phi_1^*(\zeta)\phi_2(\zeta)|_{\zeta=\Delta/2},$$

$$\rho^- = \phi_1(\zeta)\phi_2^*(\zeta)|_{\zeta=\Delta/2},$$

$$s_z = [|\phi_2(\zeta)|^2 - |\phi_1(\zeta)|^2]|_{\zeta=\Delta/2}, \tag{24}$$

where ϕ is the above mentioned eigensolution of the ZSE [17].

The IST allows one to find exact solutions of a certain class of nonlinear equations [15]. A well-known application is obtaining the SIT solutions of the Maxwell-Bloch equations without damping [Eqs. (10) and (16)]. For this system of equations, the ZSE, Eqs. (6), and Eqs. (8) for the average values of the Pauli operators $\hat{\sigma}_i$ are used. On the other hand, for the study of transverse non-Markovian relaxation effects we must consider the more general Eqs. (22), forming a generalization of Eqs. (10). The terms $\rho_T^{\pm}$ in Eqs. (22) are responsible for the influence of the non-Markovian relaxation effects on the SIT soliton. Following from the general theory of the IST, a small perturbation of the exactly solvable models (e.g., Bloch equations without damping) does not lead to a large change in the scattering data of the Zakharov-Shabat spectral problem [16–18]. Therefore, a small perturbation does not destroy the soliton solution and first-order perturbation theory leads only to small perturbations of the soliton parameters [17]. These parameters, including the relaxation effects, are calculated below. Also, it is important to note that the perturbation approach is valid for both Markovian [Eqs. (15) and (16); [17]] and also non-Markovian [Eqs. (22) and (16)] cases, considered in the present work, as long as the perturbed solution can be expressed as a small perturbation of the exactly solvable Eqs. (10). Consequently, in the following, the terms $\rho_T^{\pm}$ in the Bloch equations (22) are assumed to be small and their influence is considered in perturbation theory on the basis of the exactly solvable Eqs. (10) and (16) as in the unperturbed solution. Thus we now have a means for constructing damped SIT solutions. We shall be mainly interested in the soliton solutions.

Now, let us consider the solution of these equations when we have a single SIT soliton. In this case, the scattering coefficient $b=0$ on the real axis and $a(\zeta)=(\zeta-\zeta_1)/(\zeta-\zeta_1^*)$ has a single zero in the upper half complex ζ plane at $\zeta=\zeta_1$.

The eigenfunctions of the ZSE Ψ for such a single soliton ($\zeta=\zeta_1$) have the following form:

$$\psi_1(\zeta_1)=\frac{b_1^* e^{-(i\xi_1+3\eta_1)t}}{z_0}, \quad \psi_2(\zeta_1)=\frac{e^{(i\xi_1-\eta_1)t}}{z_0}, \qquad (25)$$

where $z_0=1+|b_1|^2e^{-4\eta_1 t}$.

Substituting the wave functions ϕ_1 and ϕ_2 in Eqs. (23), we obtain

$$\rho^+=A\frac{e^{2(i\xi_1-\eta_1)t}}{z_0^2}+B\frac{e^{2(i\xi_1-3\eta_1)t}}{z_0^2},$$

$$\rho^-=\rho^{+*}, \quad s_z=-1+\frac{2\alpha|b_1|^2e^{-4\eta_1 t}}{z_0^2}, \qquad (26)$$

where

$$A=\frac{4i\eta b_1}{\Delta-2\zeta_1}, \quad B=\frac{4i\eta_1 b_1|b_1|^2}{\Delta-2\zeta_1^*},$$

$$\alpha=\frac{4\eta_1{}^2}{(\Delta/2-\zeta_1^*)(\Delta/2-\zeta_1)}.$$

Substituting these expressions for $\rho^\pm$ into Eq. (13), we obtain the general form of the polarization P which is valid for soliton propagation in all three cases: undamped and damped for Markovian and for non-Markovian damping. These different cases will differ in the dependence of the scattering data on the distance χ. This dependence for the non-Markovian case, not known up to now, will be considered in the next section.

V. EVOLUTION OF THE PARAMETERS OF THE SOLITON

Let us now apply the perturbation expansion for the ZSE developed in Ref. [17]. We can then determine the perturbed evolution of the soliton parameters ξ_1 and η_1 due to the influence of the memory relaxations on the pulse width (pulse height) $1/2\eta_1$, and the instantaneous frequency $\omega_0-2\xi_1$ by means of the equation [3,17]

$$\zeta_{1_\chi}=\frac{b_1}{a_1'}I(\psi,\psi), \qquad (27)$$

where

$$I(\psi,\psi)=f(\zeta,\tau)|_{\tau\to-\infty}^{\tau\to\infty}+\int_{-\infty}^{\infty}h(\zeta,\tau)d\tau,$$

$$f(\zeta,\tau)=-i\left\langle\frac{\rho^+}{2\zeta-\Delta}\right\rangle\psi_1^2+i\left\langle\frac{\rho^-}{2\zeta-\Delta}\right\rangle\psi_2^2 -i\left\langle\frac{N}{2\zeta-\Delta}\right\rangle(\psi_1\psi_2+\psi_2\psi_1),$$

$$h(\zeta,\tau)=-i\left\langle\frac{\rho_T^+}{2\zeta-\Delta}\right\rangle\psi_1^2+i\left\langle\frac{\rho_T^-}{2\zeta-\Delta}\right\rangle\psi_2^2,$$

$$a_1'=\left(\frac{\partial a}{\partial\zeta}\right)\bigg|_{\zeta=\zeta_1}.$$

Substituting Eqs. (24)–(26) into Eq. (27), after long analytical calculations with the aid of the computer software MATHEMATICA, we obtain the evolution equations of these parameters:

$$\zeta_{1_\chi}=-i\frac{M_2\pi^2}{8\eta_1^2}\left(A_1-A_2+iB_2 -(2.5A_1-1.5A_2+1.5iB_2)\frac{N_2}{4\eta_1^2}\right), \qquad (28)$$

where

$$A_n=\kappa_0\int_{-\infty}^{\infty}\frac{g(\Delta)}{\left[\left(\frac{\Delta-2\xi_1}{2\eta_1}\right)^2+1\right]^n}d\Delta,$$

$$B_n=\kappa_0\int_{-\infty}^{\infty}\frac{g(\Delta)\frac{\Delta-2\xi_1}{2\eta_1}}{\left[\left(\frac{\Delta-2\xi_1}{2\eta_1}\right)^2+1\right]^n}d\Delta, \quad n=1,2.$$

From Eq. (28), after separating the real and imaginary parts, we obtain the pair of equations

$$\xi_{1_\chi}=D(1-1.5L)B_2, \qquad (29)$$

$$\eta_{1_\chi}=-D[A_1(1-2.5L)-A_2(1-1.5L)], \qquad (30)$$

where

$$D=\frac{M_2\pi^2}{16\eta_1^2}, \quad L=\frac{N_2}{4\eta_1^2}.$$

These equations give the evolution of the soliton's width and frequency shift due to the transverse relaxations due to memory, and are the main results of this work. Thus we now have a model of the evolution of the parameters of an SIT soliton in a solid when the relaxations have a memory. Next, we will consider some of the implications of these results.

VI. CONCLUSION

In Eqs. (29) and (30), we have first-order equations of motion for the 2π-soliton action parameters $\xi_1(\chi)$ and $\eta_1(\chi)$ as a function of distance. These parameters occur in the one-soliton solution in the following manner [12,14,17]:

$$\hat{E}_1=\frac{2\eta_1\hbar}{\mu_{12}}e^{i\beta_1}e^{-2i\xi_1\tau}\operatorname{sech}[2\eta_1(\tau-\tau_0)]. \qquad (31)$$

From Eqs. (14) and (31), we see that $\frac{2\eta_1\hbar}{\mu_{12}}$ is the pulse height, $\omega_0-2\xi_1$ is the instantaneous frequency, $\beta_1+2\eta_1\chi/c$ is the phase at fixed χ, and τ_0 is the central position of the pulse [17].

For the study of the influence of transverse relaxation on the resonance solitons of SIT in solids, in the present work we investigate the scattering data η_1 and ξ_1, i.e., on the pulse height and the instantaneous frequency.

Let us take the limit of a large inhomogeneous broadening and let T_2^* be a measure of the width of this inhomogeneous broadening (thus T_2^* is small.) Then using the limit $\eta_1 T_2^* \ll 1$, the above quantities A_n are found to be [17]:

$$A_n = 2\eta_1 \alpha_0(2\xi_1)\frac{\pi(2n-2)!}{4^{n-1}[(n-1)!]^2}, \quad n=1,2,\ldots,$$

where

$$\alpha_0(2\xi_1) = \kappa_0 g(2\xi_1).$$

With this, we then obtain, in lowest order,

$$A_1 = 2A_2 = 2\pi\eta_1\alpha_0(2\xi_1), \quad B_2 = -\xi_1\eta_1 T_2^{*2} A_1. \tag{32}$$

Combining Eqs. (29), (30), and (32) we have

$$\xi_{1_\chi} = -\xi_1\eta_1 T_2^{*2} D(1-1.5L)A_1, \tag{33}$$

$$\eta_{1_\chi} = -D[1-3.5L]A_2. \tag{34}$$

Equations (33) and (34) give us the evolution of the scattering data (parameters) of the SIT soliton (2π hyperbolic-secant pulse), Eq. (31). These equations take into account memory effects and are valid for non-Lorentzian spectral lines.

In the special case of the Markov approximation, which is the T_1-T_2 model, the line shape is Lorentzian. For this case, the evolution of the scattering data have already been investigated [17,19,20]. To obtain the Markovian limit from the memory function $K(t)=M_2\exp[-(N_2/2)t^2]$, we consider the case when N_2 is very large and $M_2\sqrt{N_2}$ is held constant. Under this condition the function $K(t)$ essentially becomes a δ function of zero argument. Then it follows that we may replace all the memory terms involved with the function $G(t)$, the transverse components of the polarization $P_{x,y}$, and Eq. (17) with the constant value of $1/T_2$, in which case, the non-Markovian optical Bloch equations (22) will reduce to the usual form of the optical Bloch equations given by Eqs. (15) (for $T_1\to\infty$). In this case the evolution equations for the scattering data become [17,33]

$$\xi_{0_\chi} = -\frac{4\pi\alpha_0(2\xi_0)}{3T_2}\xi_0\eta_0 T_2^{*2}, \tag{35}$$

$$\eta_{0_\chi} = -\frac{4\pi\alpha_0(2\xi_0)}{3T_2}, \tag{36}$$

where the quantities η_0 and ξ_0 are the corresponding values of η and ξ in the special situation when the line shape is Lorentzian as considered earlier in Refs. [17,34,35].

Let us now return to the non-Markovian case in the limit when $\eta_1 T_2^* \ll 1$. Equations (33) and (34) now become

$$\xi_{1_\chi} = -\xi_1 T_2^{*2}\frac{M_2\pi^3}{8}(1-1.5L)\alpha_0(2\xi_1), \tag{37}$$

$$\eta_{1_\chi} = -\frac{M_2\pi^3}{16\eta_1}(1-3.5L)\alpha_0(2\xi_1). \tag{38}$$

Now we have the case that when N_2 is sufficiently small such that $L<2/3$, Eq. (37) shows that if a 2π pulse (soliton), Eq. (31), is off resonance, it exhibits a stable evolution and ξ_1 will move back toward the resonance. On the other hand, when N_2 is larger than this limit, ξ_1 will be unstable and will move further off resonance. The dependencies seen here for solids containing memory effects [Eq. (37)] are therefore different from what one would obtain for the Markovian approximation [Eq. (35)]. From Eq. (38) we can obtain an implicit equation for η_1 or $\mu(\chi)=\eta_1^2(\chi)+b\ln|\eta_1^2(\chi)-b|$:

$$\mu(\chi) = \mu(0) - \frac{\pi^3 M_2\alpha_0(2\xi_1)}{4}\chi \tag{39}$$

where $b=\frac{7}{8}N_2$.

Let us now compare this result, where memory effects are present, with the Markovian result, Eq. (36), where the spectral line is Lorentzian [17] and memory effects are absent. In the latter case,

$$\eta_0(\chi) = \eta_0(0)\left(1 - \frac{4\pi\alpha_0(2\xi_0)}{3\eta_0(0)T_2}\chi\right). \tag{40}$$

Comparing these two expressions, one can see that memory effects do lead to different dynamics in the quantity η. In particular, if one takes into account that in the non-Markovian case we have assumed that $N_2/\eta_1^2 \ll 1$ and if we suppose that the initial inverse pulse widths in both cases were the same, i.e., $\eta_1(0)=\eta_0(0)$, then Eq. (39) would simplify and the inverse of the pulse duration, $\eta_1(\chi)$, would be determined by a square root law, in contrast to the linear dependence found for the Markovian case [Eq. (40)]:

$$\eta_1(\chi) = \eta_0(0)\sqrt{1 - \frac{\pi^3 M_2\alpha_0(2\xi_1)}{4\eta_0^2(0)}\chi}. \tag{41}$$

Using typical parameters for the pulse and the materials [44], we can construct a plot of the inverse width of a Markovian pulse $\eta_0(\chi)/\eta_0(0)$ as a function of the distance χ in comparison to that of a non-Markovian pulse $\eta_1(\chi)/\eta_0(0)$ for the same value of the coefficient of χ. The result is shown in Fig. 1.

From the figure it is clear that in both cases, the propagating pulse width $(2\eta_{1,0})^{-1}$ will be growing but there is a different functional for the Markovian and non-Markovian cases. The different slopes of the curves could be experimentally addressed. We do note that each of these theories is valid only for distances where the condition $T\ll T_2$ is satisfied. We have to note that because the quantity N_2 characterizes the memory function shape, after neglecting in Eq. (39) the terms proportional to b, we lost very important information about the shape of the memory function and therefore for a more detailed consideration, it will be preferable to use Eq. (39) instead of Eq. (41).

Summarizing the above results, we see that the influence of memory effects on the process of nonlinear pulse propagation under the condition of SIT leads to qualitatively new results. In particular, the dynamics of the bound state eigen-

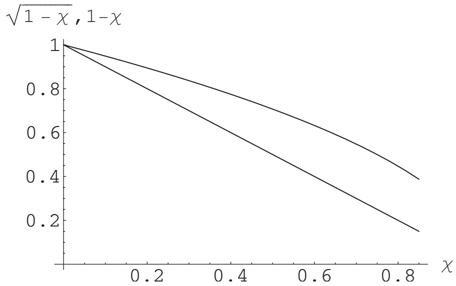

FIG. 1. Dependence of the inverse width of the pulses $\eta_0(\chi)/\eta_0(0)$ and $\eta_1(\chi)/\eta_0(0)$ on the distance χ for the Markovian (bottom curve) and non-Markovian case. The initial values of the widths of the pulses $[2\eta_0(0)]^{-1}$ are the same. The χ axis is in units of 10^{-4} cm. All quantities are measured in the CGS system.

value becomes different—whence the evolution of the frequency shift and the pulse height of SIT solitons in solids with memory effects will differ from those known from studies in gases.

The influence of transverse relaxations on the evolution of a SIT soliton in the general case is given by Eqs. (16) and (22). The solution of these equations for different shapes of the spectral line is connected with different mathematical problems. Therefore the solution of these two different physically interesting situations has to be considered separately. The first situation is when the spectral line has a Lorentzian shape and the second is when it has a non-Lorentzian one. In the first case the memory function $K(t)$ is damping exponentially very fast compared to the transverse components of the polarization $P_{x,y}$. In this case, the appropriate equations are (15) and (16) and the evolution of the scattering data has the form (35), (36), and (40). This is the case investigated in Refs. [1,17,19,20,34–36]. In the second case when the spectral line has a non-Lorentzian shape, we must use the SIT equations with memory, (16) and (22), and the evolution of the scattering data follows the form given by Eqs. (37)–(39) and (41).

It is clear that Eqs. (22) are more general and contain Eqs. (15) in the limiting case where $N_2/\eta_1^2 \gg 1$. However, it must also be noted that we have to be very careful when we consider the limiting case. This is because in solving the system of equations (16) and (22), we have used the assumption that $N_2/\eta_1^2 \ll 1$ and therefore Eqs. (37) and (38) do not contain in the limit the case of Eqs. (35) and (36). In other words, we consider two cases: the first case is where $N_2/\eta_1^2 \gg 1$ and the second case is where $N_2/\eta_1^2 \ll 1$. The corresponding values of the scattering data ξ_0, η_0 and ξ_1, η_1 do not transform into each other and are two different limiting values. But at the same time we can see that in both limiting cases the functions η_{0_χ} and η_{1_χ} are negative and therefore in both cases the pulses must always decay.

In the present work we have neglected not only the effects connected with longitudinal relaxations but also with all the other additional effects considered in Ref. [17], since memory effects would not influence them. The approach that we have used is of a rather general character and can be used for investigations of the influence of memory not only on other aspects of SIT solitons, but also for other types of nonlinear waves (breathers, double breathers) [14,45–48] and for interactions between solitons. It could also be used to study the properties of the resonant solitons in acoustic waves [49–51] as well as nonlinear electromagnetic waves involved in electron microwave spectra [52,53].

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