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Magneto-optical Stern-Gerlach effect in an atomic ensemble

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We study the birefringence of quantized polarized light in a magneto-optically manipulated atomic ensemble as a generalized Stern-Gerlach effect of light. To explain this engineered birefringence microscopically, we derive an effective Shrödinger equation for the spatial motion of two orthogonally polarized components, which behave as a spin with an effective magnetic moment leading to a Stern-Gerlach split in a nonuniform magnetic field. We show that the electromagnetically-induced-transparency mechanism can enhance the magneto-optical Stern-Gerlach effect of light in the presence of a control field with a transverse spatial profile and a nonuniform magnetic field.

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I. INTRODUCTION

Particles with opposite spins and nonzero magnetic moments will go their separate ways in a nonuniform magnetic field, which is well known as the Stern-Gerlach effect [1]. Most recently, this kind of effect was theoretically predicted in a generalized version for some effective nonuniform fields [2], e.g., a chirality-dependent induced gauge potential for chiral molecules resulting from three nonuniform light fields even in the absence of the nonuniform magnetic field [3].

On the other hand, a similar effect for nonpolarized slow light was experimentally observed as the electromagnetically-induced-transparency (EIT) [4,5] enhanced deflection by a small magnetic field gradient [6]. It was also experimentally demonstrated with a spatially distributed control field [7]. Such spatial motion of light in the EIT atomic medium has been well explained by a semiclassical theory based on the spatial dependence of the refraction index of the atomic medium [8] and a fully quantum approach [9] based on the excitation of the dark polaritons—the mixtures of a photon and an atomic collective excitation [10–12]. The latter takes advantage of revealing the wave-particle duality of dark polaritons, and its crucial point of explanation is to derive the effective Schrödinger equation for the propagation of slow light in the EIT medium.

However, the above EIT-enhanced deflection of nonpolarized light cannot be simply explained as an analog of the conventional Stern-Gerlach effect since only one component of the "spin" is available [6–9]. An analog between light ray and atomic beam only appears in the polarized material. Birefringence—the decomposition of a ray of light into two rays dependent on the polarization when it passes through certain types of material, is classically formalized by assigning two different refractive indices to the material for different polarizations. In this sense, it is the Stern-Gerlach effect of light.

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In this paper, we study a generalized Stern-Gerlach effect of quantized polarized light as a phenomenon of birefringence. The anisotropic material is artificial. It is an atomic ensemble controlled by a specially designed magneto-optical manipulation based on the EIT mechanism. Two EIT configurations are formed by an optical field with a transverse spatial profile, since a magnetic field removes the degeneracy of the ground state. To represent an analog between birefringence of quantized light and the Stern-Gerlach effect, an effective Schrödinger equation is established for the spatial motion of two polarized components of light. Such an effective equation of motion describes a quasispin with an effective magnetic moment in an effective nonuniform magnetic field. The spatial gradient results from the transverse spatial profile of the optical field, and the effective magnetic moment is proportional to the two-photon detuning with connection to the corresponding optical field and the atomic transition.

This paper is organized as follows. In Sec. II, we present the theoretical model for four-level atoms with a tripod configuration in the presence of nonuniform external fields, and we give an analytical solution of the Heisenberg equations of this atomic ensemble in the atomic linear response with respect to the probe field. In Sec. III, an effective Schrödinger equation is derived for the spatial motion of two orthogonally polarized components, which behave as a spin with an effective magnetic moment. In Sec. IV, the symmetric and asymmetric Stern-Gerlach effects are investigated in the presence of a nonuniform magnetic field with a small transverse gradient. Then we investigate the optical Stern-Gerlach effect in Sec. V, which is caused by a nonuniform light field with a Gaussian profile in the transverse direction. In Sec. VI, we give an explanation based on dark polaritons, which are introduced as dressed fields to describe the spatial motion of collective excitation. We conclude our paper in the final section.

II. MAGNETO-OPTICALLY CONTROLLED ATOMIC ENSEMBLE

The system we consider is in a gas-cell ABCD shown in Fig. 1(b). It is an ensemble of $2N$ identical and noninteract-

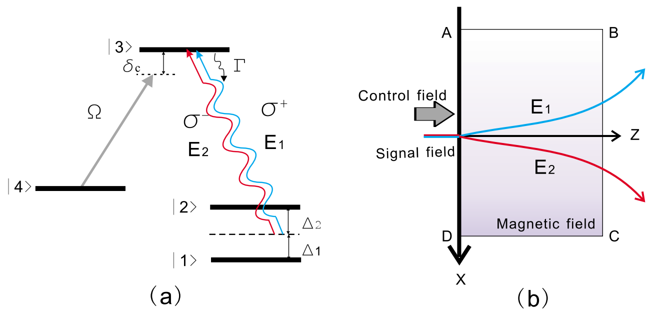

FIG. 1. (Color online) (a) Energy diagram of atoms interacting with a coupling field and a linear-polarized probe field in the presence of a magnetic field parallel to the field propagation direction. Ω is the Rabi frequency. (b) Schematic diagram of light deflection in the atomic medium.

ing atoms with a tripod configuration [13] of energy levels labeled as $|i\rangle$ $(i=1,2,3,4)$; see Fig. 1(a). Here, the submanifold is spanned by two Zeeman levels $|1\rangle$ and $|2\rangle$. The energy levels are shifted by the corresponding amount $\Delta_i=\mu_i B$, which is determined by the applied magnetic field along the z axis. Here, the magnetic moments $\mu_i=m_F^i g_F^i \mu_B$ are defined by the Bohr magneton μ_B, the gyromagnetic factor g_F^i, and the magnetic quantum number m_F^i of the corresponding state $|i\rangle$. The excited state $|3\rangle$ with $m_F^3=0$ is coupled to state $|1\rangle$ ($|2\rangle$) with $m_F^1=-1$ ($m_F^2=1$) via a σ^+ (σ^-) component E_1 (E_2) of the linear-polarized probe field with frequency ν and wave vector $\mathbf{k}=k\hat{e}_z$. A classical control field with frequency ν_c and wave vector $\mathbf{k}_c=k_c\hat{e}_z$ drives the atomic transition $|3\rangle$-$|4\rangle$ with spatially dependent Rabi frequency $\Omega(\mathbf{r})$.

The Hamiltonian of the total system $H=H^{(A)}+H^{(F)}+H^{(I)}$ is written in terms of the collective atomic operators $\tilde{\sigma}_{\mu\nu}(\mathbf{r},t)=(1/N_r)\Sigma_{r_j\in N_r}\tilde{\sigma}^j_{\mu\nu}(t)$, averaged over a small but macroscopic volume containing many atoms $N_r=(2N/V)dV \gg 1$ around position $\mathbf{r}$, where $2N$ is the total number of atoms and V is the volume of the medium [10,11], and $\tilde{\sigma}^j_{\mu\nu}(t) =|\mu\rangle_j\langle\nu|$. Here, $H^{(F)}$ is the free Hamiltonian of the radiation field. Neglecting the kinetic term of atoms, the Hamiltonian of the atomic part reads

$$H^{(A)}=\frac{2N}{V}\int d^3r[(\omega_0+\Delta_1)\tilde{\sigma}_{11}+(\omega_0+\Delta_2)\tilde{\sigma}_{22}+\omega_3\tilde{\sigma}_{33} + (\omega_4+\Delta_4)\tilde{\sigma}_{44}], \quad (1)$$

where $\omega_1=\omega_2=\omega_0$, and ω_μ $(\mu=1,2,3,4)$ are the atomic energy level spacing in the absence of the magnetic field. Under the electric-dipole approximation and the rotating-wave approximation, the interaction between the atomic ensemble and the electromagnetic fields reads [9–11]

$$H^{(I)}=-\frac{2N}{V}\int d^3r(\Omega\tilde{\sigma}_{34}e^{i(k_cz-\nu_ct)}+d_{31}\tilde{E}_1^+\tilde{\sigma}_{31}+d_{32}\tilde{E}_2^+\tilde{\sigma}_{32} + \text{H.c.}). \quad (2)$$

Here, $\tilde{E}_j^+$ are the positive frequency of the probe fields, $\Omega(\mathbf{r})$ is the Rabi frequency of the control field, which usually depends on the spatial coordinate through the spatial profile of the driving field, and d_{31} (d_{32}) is the dipole matrix element between the states $|3\rangle$ and $|1\rangle$ ($|2\rangle$).

As it is well known that EIT is a phenomenon specific to optically thick media in which both the optical fields and the material states are modified [5], we introduce the slow varying variables $E_j(\mathbf{r},t)$ for probe fields,

$$\tilde{E}_j^+(\mathbf{r},t)=\sqrt{\frac{\nu}{2\varepsilon_0V}}E_j(\mathbf{r},t)e^{i(kz-\nu t)} \quad (j=1,2) \quad (3)$$

and σ_{3j} $(j=1,2,4)$ for the atomic ensemble,

$$\tilde{\sigma}_{31}=\sigma_{31}\exp(-ikz), \quad (4a)$$

$$\tilde{\sigma}_{32}=\sigma_{32}\exp(-ikz), \quad (4b)$$

$$\tilde{\sigma}_{34}=\sigma_{34}\exp(-ik_cz). \quad (4c)$$

Then, the interaction Hamiltonian is rewritten as

$$H_I=\frac{2N}{V}\int d^3r[(\delta_1\sigma_{11}+\delta_2\sigma_{22}+\delta_c\sigma_{44}) -(\Omega\sigma_{34}+gE_1\sigma_{31}+gE_2\sigma_{32}+\text{H.c.})], \quad (5)$$

in a frame rotating with respect to the probe and driving fields, where $g=d_{31}\sqrt{v/(2\varepsilon_0V)}$, which is the same for both circular components $E_{1,2}$ due to the symmetry of the system ($|d_{31}|=|d_{32}|$), is the atom-field coupling constant and detunings are defined as

$$\delta_1=\omega_0-\omega_3+v+\Delta_1, \quad (6a)$$

$$\delta_2=\omega_0-\omega_3+v+\Delta_2, \quad (6b)$$

$$\delta_c=\omega_4-\omega_3+v_c+\Delta_4. \quad (6c)$$

Before going on, we remind the reader that tripod atoms are proven to be robust systems for "engineering" arbitrary coherent superpositions of atomic states [14] using an extension of the well known technique of stimulated Raman adiabatic passage.

The dynamics of this laser-driven atomic ensemble are described by the Heisenberg equations

$$\dot{\sigma}_{12}=[i(\delta_1-\delta_2)-\gamma]\sigma_{12}-igE_1\sigma_{32}+igE_2\sigma_{13}, \quad (7a)$$

$$\dot{\sigma}_{13}=(i\delta_1-\Gamma)\sigma_{13}+igE_1(\sigma_{11}-\sigma_{33})+igE_2\sigma_{12}+i\Omega\sigma_{24}, \quad (7b)$$

$$\dot{\sigma}_{14}=[i(\delta_1-\delta_c)-\gamma]\sigma_{14}-igE_1\sigma_{34}+i\Omega\sigma_{13}, \quad (7c)$$

$$\dot{\sigma}_{23}=(i\delta_2-\Gamma)\sigma_{23}+igE_2(\sigma_{22}-\sigma_{33})+igE_1\sigma_{21}+i\Omega\sigma_{24}, \quad (7d)$$

$$\dot{\sigma}_{24}=[i(\delta_2-\delta_c)-\gamma]\sigma_{24}-igE_2\sigma_{34}+i\Omega^*\sigma_{23}, \quad (7e)$$

$$\dot{\sigma}_{34}=-(i\delta_c+\Gamma)\sigma_{34}-igE_1^\dagger\sigma_{14}+igE_2^\dagger\sigma_{24}+i\Omega^*(\sigma_{33}-\sigma_{44}), \quad (7f)$$

where we have introduced the coherence relaxation rate of the ground state γ and the decay rate of the excited state Γ

phenomenologically. EIT is primarily concerned with the modification of the linear and nonlinear optical properties of the probe field perturbatively. We outline the solution of Eqs. (7) in the low-density approximation, where the intensity of the quantum probe field is much weaker than that of the coupling field, and the number of photons contained in the signal pulse is much less than the number of atoms in the sample. In the low-density approximation, the perturbation approach can be applied to the atomic part, which is introduced in terms of perturbation expansion,

$$\sigma_{\mu\nu} = \sigma_{\mu\nu}^{(0)} + \lambda\sigma_{\mu\nu}^{(1)} + \lambda^2\sigma_{\mu\nu}^{(2)} + \cdots, \tag{8}$$

where μ, $\nu=\{1,2,3,4\}$ and λ is a continuously varying parameter ranging from zero to unity. Here $\sigma_{\mu\nu}^{(0)}$ is of the zeroth order in gE_j, $\sigma_{\mu\nu}^{(1)}$ is of the first order in gE_j, and so on. By substituting Eq. (8) into Eqs. (7) and keeping the terms up to the first order in the probe field amplitude, the equations for the first-order atomic transition operators read

$$\dot{\sigma}_{13}^{(1)} = (i\delta_1 - \Gamma)\sigma_{13}^{(1)} + \frac{1}{2}igE_1 + i\Omega\sigma_{14}^{(1)}, \tag{9a}$$

$$\dot{\sigma}_{14}^{(1)} = [i(\delta_1 - \delta_c) - \gamma]\sigma_{14}^{(1)} + i\Omega^*\sigma_{13}^{(1)}, \tag{9b}$$

$$\dot{\sigma}_{23}^{(1)} = (i\delta_2 - \Gamma)\sigma_{23}^{(1)} + \frac{1}{2}igE_2 + i\Omega\sigma_{24}^{(1)}, \tag{9c}$$

$$\dot{\sigma}_{24}^{(1)} = [i(\delta_2 - \delta_c) - \gamma]\sigma_{24}^{(1)} + i\Omega^*\sigma_{23}^{(1)}, \tag{9d}$$

$$\dot{\sigma}_{34}^{(1)} = -(i\delta_c + \Gamma)\sigma_{34}^{(1)} + i\Omega^*(\sigma_{33}^{(1)} - \sigma_{44}^{(1)}), \tag{9e}$$

where we have assumed that the atoms are incoherently pumped in states $|1\rangle$ and $|2\rangle$ with equal population at the beginning. Under the adiabatic approximation that the evolution of the atomic system is much faster than the temporal change of the radiation field, the steady-state solutions are found,

$$\sigma_{j3}^{(1)} = \frac{gE_j(\delta_j - \delta_c)}{2|\Omega|^2} \quad (j=1,2), \tag{10}$$

where the condition $|\Omega|^2 \gg \Gamma\gamma$ for the observation of the important features of EIT is used, and we also set $\gamma=0$ for showing the basic principle of physics.

III. EFFECTIVE SCHRÖDINGER EQUATION DESCRIBING POLARIZATION AS SPIN PRECESSION

In this section, we derive an effective Schrödinger equation for the spatial motion of two orthogonally polarized components of light. In the linear optical response theory, the equations of motion for the optical fields are given by [9]

$$\left(i\partial_t + ic\partial_z + \frac{c}{2k}\nabla_T^2\right)E_j = -2g^*N\sigma_{j3}^{(1)}, \tag{11}$$

which are achieved straightforwardly from the Heisenberg equations. Here, c is the velocity of light in vacuum and the transverse Laplacian operator in the rectangular coordinates is defined as

$$\nabla_T^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{12}$$

Without loss of generality, we consider the propagating of a probe field confined in an x-z plane. From Eqs. (10) and (11), the equations of motion for the probe fields read

$$i\partial_t E_j = \left(-ic\partial_z - \frac{c}{2k}\frac{\partial^2}{\partial x^2} + \frac{|g|^2N(\delta_c - \delta_j)}{|\Omega|^2}\right)E_j. \tag{13}$$

In order to write Eqs. (13) in a more compact form and naturally show the superposition of the "quasispin" states of photons, we introduce the "spinor" Φ and the third Pauli matrix S_Z,

$$\Phi = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix}, \quad S_Z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{14}$$

Then Eqs. (13) become

$$i\partial_t\Phi = H_e\Phi, \tag{15}$$

which is a two-component Schrödinger equation with the effective Hamiltonian

$$H_e = T + V(x) - \mu_e B_e(x,z)S_Z. \tag{16}$$

Here the effective kinetic term is given in terms of the momentum operators $P_j \equiv -i\partial_j$ ($j \in \{x,z\}$),

$$T = cP_z + \frac{P_x^2}{2m}, \tag{17}$$

which represents an anisotropic dispersion relation in which the longitudinal motion is similar to an ultrarelativistic motion while the transverse motion is of nonrelativity with an effective mass $m=k/c$. The scalar potential is determined by the detunings and $\tan^2\theta$,

$$V(x) = \left(\delta_c - \frac{1}{2}(\delta_1 + \delta_2)\right)\tan^2\theta, \tag{18}$$

where we have defined $\tan^2\theta \equiv |g|^2N/|\Omega|^2$. The effective magnetic field $B_e(x,z)$ times magnetic moment μ_e gives the spin-dependent potential

$$\mu_e B_e(x,z) = (\delta_1 - \delta_2)\tan^2\theta. \tag{19}$$

Obviously, the above effective Hamiltonian totally determines the dynamics of the probe field with quasi-spin-orbit coupling $\mu_e B_e(x,z)S_Z$, which is also spatially dependent due to the inhomogeneity of the applied field.

IV. STERN-GERLACH EFFECTS IN NONUNIFORM MAGNETIC FIELD

In this section, we study the Stern-Gerlach effect of light when the magnetic field is inhomogeneous. Consider a linear magnetic field $B(\mathbf{r})=B_0+B_1x$, which is applied to the atomic ensemble driven by a uniform classical field. Due to the quasi-spin-orbit coupling, photons with orthogonal polarizations separate their ways by the small transverse gradient B_1. To go into this effect, we assume that both components of the

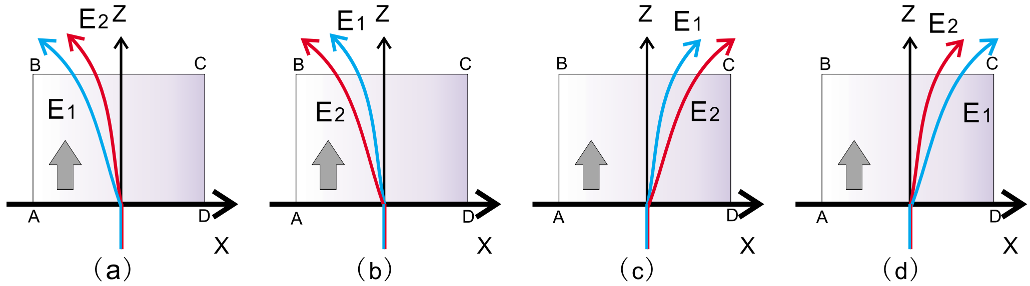

FIG. 2. (Color online) Schematic illustration of the asymmetric deflection of probe light by the nonuniform magnetic field in conditions (a)–(d).

linearly polarized probe beam are initially in a spatial Gaussian state,

$$\Phi(x,0)=\frac{1}{\sqrt{\pi b^2}}\begin{bmatrix}1\\1\end{bmatrix}\exp\left(-\frac{x^2}{2b^2}-\frac{z^2}{2b^2}\right), \tag{20}$$

which is centered at $(x,z)=(0,0)$ before it enters the gas cell. Here b is the width of the probe field profile. After a period of time, from Eq. (13) it is found that the initial Gaussian packet $\Phi(x,0)$ evolves into

$$\Phi(x,t)=\begin{pmatrix}E_1(t)\\E_2(t)\end{pmatrix}=\begin{pmatrix}e^{-iH_1t}E_1(0)\\e^{-iH_2t}E_2(0)\end{pmatrix}. \tag{21}$$

Here, the effective Hamiltonian is defined by

$$H_j=cP_z+\frac{1}{2m}P_x^2-\chi_j b_0-\chi_j\zeta x, \tag{22}$$

with the parameters

$$\zeta=B_1\tan^2\theta, \tag{23a}$$

$$b_0=B_0\tan^2\theta, \tag{23b}$$

$$\chi_j=\mu_j-\mu_4, \tag{23c}$$

where we have assumed that the optical fields are in resonance with the atomic transition in the absence of the external fields. By making use of the Wei-Norman algebraic method [15], the wave packet at time t can be explicitly obtained,

$$E_j(t)=\left(\frac{1/\pi}{b^2+i\dfrac{t}{m}}\right)^{1/2}e^{i\varphi_j-[(z-ct)^2/2b^2]}\times\exp\left(-\frac{\left(x-t^2\dfrac{\chi_j\zeta}{2m}\right)^2(b^2m^2-itm)}{2b^4m^2+2t^2}\right), \tag{24}$$

where we have introduced

$$\varphi_j=\chi_j t\left(\zeta x+b_0-\frac{t^2}{3}\frac{\chi_j\zeta^2}{2m}\right). \tag{25}$$

From Eq. (24), we can see that after passing through the atomic medium with length L, the initial center of the probe field moves to the position given by

$$z=L,\quad x_j=\frac{\chi_j B_1 N|g|^2L^2}{2\Omega^2kc}, \tag{26}$$

which indicate that, as long as the two Zeeman levels are not degenerate, the trajectories of σ^+- and σ^--polarized photons are bent in different directions. This means that the initial linearly polarized beam is split into two parts with opposite spins. Accompanying the split of light, the Stern-Gerlach effect along the transverse direction comes into being.

From Eq. (26), we can see that the Stern-Gerlach effect in the nonuniform magnetic field depends on the small magnetic field gradient B_1 and the parameter χ_j. Obviously, the Stern-Gerlach effect disappears when the nonuniform part of the magnetic field B_1 vanishes. Below we consider the split configurations of the probe light for different χ_j when $B_1>0$. When $x_1=-x_2$, e.g., $\mu_4=0$ and $\mu_1=-\mu_2$, a symmetric Stern-Gerlach effect is found—two components of probe beam propagate with a mirror symmetry around the $\hat{e}_z$ axis as shown in Fig. 1(b). Generally ($x_1\neq -x_2$, that is, $\chi_1\neq-\chi_2$), the asymmetric Stern-Gerlach effect occurs due to detuning mismatch, which is different from the split of atomic beam in the magnetic field. It is the difference of μ_i that exerts not identical forces on different polarized photons. The generalized Stern-Gerlach effect of light is caused by the asymmetric gradient force shown in Figs. 2–4. It can be found that when one of the following conditions is satisfied: (a) $\chi_1<\chi_2<0$; (b) $\chi_2<\chi_1<0$; (c) $\chi_2>\chi_1>0$; (d) $\chi_1>\chi_2>0$, two gradient forces are in the same direction with different magnitudes for photons, therefore two rays of light are bent in the same direction as shown in Fig. 2.

However, when one of the following conditions is satisfied: (e) $\chi_1>0>\chi_2$ and $|\chi_1|>|\chi_2|$; (f) $\chi_1>0>\chi_2$ and $|\chi_2|>|\chi_1|$; (g) $\chi_2>0>\chi_1$ and $|\chi_2|>|\chi_1|$; (h) $\chi_2>0>\chi_1$ and $|\chi_1|>|\chi_2|$, the polarized-dependent gradient forces are in the opposite directions with different magnitudes, therefore two rays of light are bent in the opposite direction with different angles as shown in Fig. 3.

In addition, when one of the following conditions is satisfied: (i) $\chi_1=0$ while $\chi_2>0$; (j) $\chi_1=0$ while $\chi_2<0$; (k) $\chi_2=0$ while $\chi_1>0$; (l) $\chi_2=0$ while $\chi_1<0$, only one of the two polarized photons is bent, as shown in Fig. 4.

V. STERN-GERLACH EFFECTS IN A NONUNIFORM LIGHT FIELD

The approach developed above can also be used to investigate the Stern-Gerlach effect of light caused by a nonuni-

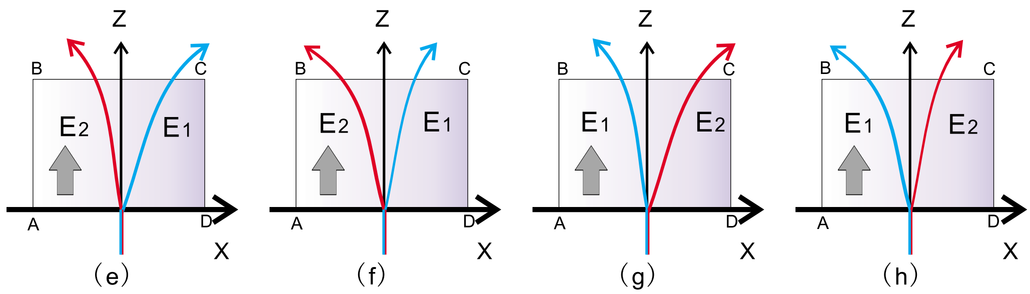

FIG. 3. (Color online) Schematic illustration of the asymmetric deflection of probe light by the nonuniform magnetic field in conditions (e)–(h).

form light field. In this section, we turn our attention to the Stern-Gerlach effect of slow light by the atomic ensemble driven by the optical field with a nonuniform profile while the magnetic field is uniform.

In most experiments, the control field is continuous and has a transverse spatial profile $\Omega(\mathbf{r})$ that changes little in the propagating direction. To study its transverse effects on the probe signal field, we choose the transverse spatial profile of the control field to be a Gaussian profile,

$$\Omega(\mathbf{r}) = \Omega_0 \exp\left(-\frac{x^2}{2\sigma^2}\right), \tag{27}$$

where σ is the width of the driving field profile. We also assume that both quasispins of photons are initially linearly polarized in a spatial Gaussian distribution,

$$\Phi(x,0) = \frac{1}{\sqrt{\pi b^2}}\begin{bmatrix}1\\1\end{bmatrix} e^{-[(x-a)^2/2b^2]-(z^2/2b^2)}, \tag{28}$$

where b is the width of the probe field profile and a is the initial wave-packet center of the probe field along the transverse direction. The sign of a indicates the incident position comparatively to the left- or right-hand side of the control beam's center $x_0=0$, and the magnitude $|a|$ denotes the distance from the control beam's center.

Since we are concerned with the situation in which the width of the probe field profile is much smaller than that of the driving field profile, we expand $|\Omega|^{-2}$ around a and retain the linear term proportional to $x-a$,

$$|\Omega|^{-2} \simeq \Omega_0^{-2}\left[\exp\left(\frac{a^2}{\sigma^2}\right) + \frac{2a}{\sigma^2}\exp\left(\frac{a^2}{\sigma^2}\right)(x-a)\right]. \tag{29}$$

Then the dynamics of the probe field is governed by the unitary operators $U_j(t)=\exp(-iH_{cj}t)$, which are generated by the effective Hamiltonians

$$H_{cj} = cP_z + \frac{1}{2m}P_x^2 + (\eta_{j0} + \eta_{j1}x), \tag{30}$$

where the parameters are defined as

$$\eta_{j0} = -\Omega_0^{-2}|g|^2 N\chi_j B \exp(a^2/\sigma^2), \tag{31a}$$

$$\eta_{j1} = -2a\Omega_0^{-2}|g|^2 N\chi_j B \exp(a^2/\sigma^2)/\sigma^2. \tag{31b}$$

By using the Wei-Norman algebraic method [15], we solve the time evolution problem of the probe field, and we find that at time t, two components of light become

$$E_j(t) = \left(\frac{1/\pi}{b^2 + i\frac{t}{m}}\right)^{1/2} e^{-i\varphi_j' - [(z-ct)^2/2b^2]} \times \exp\left(-\frac{\left(x-a-t^2\frac{\eta_{j1}}{2m}\right)^2 b^2 m^2}{2b^4m^2+2t^2}\right), \tag{32}$$

where we have introduced

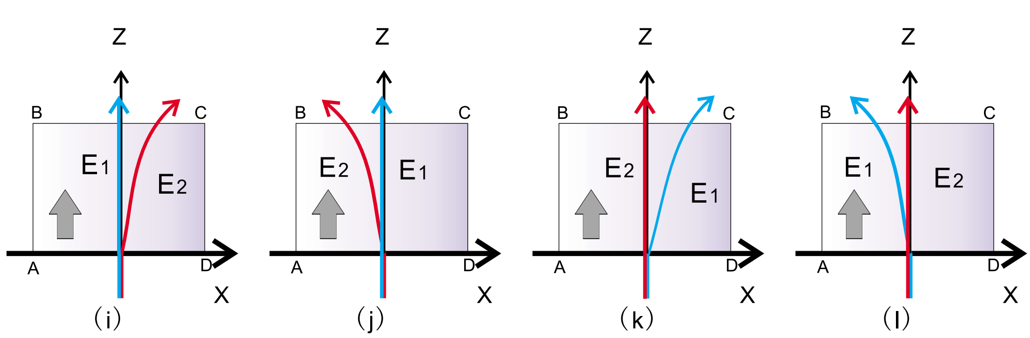

FIG. 4. (Color online) Schematic illustration of the asymmetric deflection of probe light by the nonuniform magnetic field in conditions (i)–(l).

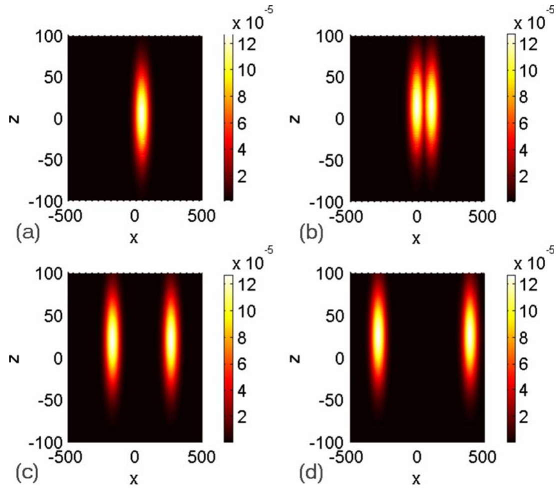

FIG. 5. (Color online) The density distributions (in arbitrary unit) of the two profiles of the probe light field for the symmetry Stern-Gerlach effect at different times: (a) $t=0$, (b) $t=1$, (c) $t=2$, and (d) $t=3$.

$$\varphi_j' = (\eta_{j0} - x\eta_{j1})t + \frac{mt}{2mb^4 + 2t^2} - \frac{\eta_{j1}^2 t^3}{6m}. \qquad (33)$$

In Fig. 5, we numerically demonstrate the wave-packet evolution of the two profiles of light fields.

From Eq. (32), it can be found that at the boundary of the medium, the center of the emergent wave function of the probe field is changed to

$$z = L, \quad x_j' = a + \frac{a\chi_j B \exp(a^2/\sigma^2)|g|^2 N L^2}{\Omega_0^2 \sigma^2 kc}, \qquad (34)$$

which indicates that the control field can split the linearly polarized probe beam into two as long as the magnetic field B and a are nonzero and $\chi_1 \neq \chi_2$. This is a generalized Stern-Gerlach effect in a nonuniform light field. The configurations of the splitting are completely determined by the detuning mismatch $\chi_j B$ and the incident position a of the probe light. Comparing Eq. (34) with Eq. (26), we find that an external controllable parameter a is offered in this case, which determines the splitting configurations of the probe light in a nonuniform light field. We denote $a>0$ ($a<0$) as the probe beam shifted to the right (left) with respect to the center of control light. Below we take $B>0$. When $a>0$, the probe beam E_j bends to the right of the axis at $x=a$ with $\chi_j>0$, while with $\chi_j<0$, the trajectory of E_j bends to the left. Actually, the splitting around the $x=a$ axis is the same as that shown in Figs. from 1(b) to 4(l) under the same conditions. When $a<0$, the split around the $x=a$ axis is similar to that in $a<0$, except exchanging the role of both components. Obviously, a spin current transverse to the energy flow in a nonmagnetic isotropic medium is generated by the spatial profile of a control field.

VI. DARK-STATE POLARITON EXPLANATION

In this section, we present a dark-state polariton (DSP) [5,10,11] explanation of the magneto-optical Stern-Gerlach effects for slow light [16] and show how the spatial motion of DSPs is associated with slow light propagation. Originally the quasiparticle picture is introduced to reveal the physical mechanism for the temporary transfer of excitations to and from the medium. To put it into a mathematical formalism, one defines the dark (Ψ_j) and bright (Φ_j) polariton fields [5,10,11],

$$\Psi_j = E_j \cos\theta - 2\sqrt{N}\sigma_{j4}^{(1)} \sin\theta, \qquad (35a)$$

$$\Phi_j = E_j \sin\theta + 2\sqrt{N}\sigma_{j4}^{(1)} \cos\theta, \qquad (35b)$$

which are relevant to different circular components here. They are atomic collective excitations dressed by the quantized probe light with its inverse relation,

$$E_j = \Psi_j \cos\theta + \Phi_j \sin\theta, \qquad (36a)$$

$$\sigma_{j4}^{(1)} = \frac{1}{2\sqrt{N}}(\Phi_j \cos\theta - \Psi_j \sin\theta). \qquad (36b)$$

It is demonstrated in the previous references [6,7,9] that the DSPs are matter particles with mass, momentum, magnetic moment, etc. However, in the system under our consideration, two dark polariton fields are excited. It is the two excited dark polariton fields that lead to the split of the emergent light. We will analytically show this split below. First we rewrite Eqs. (9) as

$$\sigma_{13}^{(1)} = -\frac{i}{\Omega^*}(\partial_t - d_1)\sigma_{14}^{(1)}, \qquad (37a)$$

$$\sigma_{23}^{(1)} = -\frac{i}{\Omega^*}(\partial_t - d_2)\sigma_{24}^{(1)}, \qquad (37b)$$

$$gE_1 = -2\left((\partial_t - d_{c1})\frac{1}{\Omega^*}(\partial_t - d_1) + \Omega\right)\sigma_{14}^{(1)}, \qquad (37c)$$

$$gE_2 = -2\left((\partial_t - d_{c2})\frac{1}{\Omega^*}(\partial_t - d_2) + \Omega\right)\sigma_{24}^{(1)}, \qquad (37d)$$

where we have defined

$$d_1 = i(\delta_1 - \delta_c) - \gamma, \qquad (38a)$$

$$d_2 = i(\delta_2 - \delta_c) - \gamma, \qquad (38b)$$

$$d_{c1} = i\delta_1 - \Gamma, \qquad (38c)$$

$$d_{c2} = i\delta_2 - \Gamma. \qquad (38d)$$

In terms of dark and bright polariton fields, Eqs. (11) and (37) can be rewritten as

$$\left(i\frac{\partial}{\partial t} + ic\frac{\partial}{\partial z} + \frac{c}{2k}\nabla_T^2\right)(\Psi_j \cos\theta + \Phi_j \sin\theta)$$
$$= i\frac{g^*\sqrt{N}}{\Omega^*}(\partial_t - d_j)(\Phi_j \cos\theta - \Psi_j \sin\theta) \qquad (39)$$

and

$$g\sqrt{N}(\Psi_j \cos\theta + \Phi_j \sin\theta) = -\left((\partial_t - d_{cj})\frac{1}{\Omega^*}(\partial_t - d_1) + \Omega\right) \times (\Phi_j \cos\theta - \Psi_j \sin\theta). \quad (40)$$

Under conditions of EIT, i.e., for negligible absorption, the excitation of bright polariton field Φ_j vanishes approximately. Then the dynamics of the dark polariton fields Ψ_j are governed by the Schrödinger-like equations [9],

$$i\partial_t \Psi_j = [T_B + V_{Bj}(\mathbf{r})]\Psi_j, \quad (41)$$

where the polarized-dependent effective potentials are given by

$$V_{Bj}(\mathbf{r}) = -\chi_j B(\mathbf{r})\sin^2\theta, \quad (42)$$

which are induced by the atomic response to the external spatial-dependent field. And the effective kinetic operator is defined by

$$T_B = v_g P_z + \frac{P_x^2}{2m_B}, \quad (43)$$

which also shows a similar anisotropic dispersion to that mentioned above. However, besides the effective mass $m_B = k/v_g$, the velocity $v_g = c\cos^2\theta$ along the z direction can be controlled by the amplitude of the control field. By adiabatically rotating the mixing angle θ from 0 to $\pi/2$, the polariton is decelerated to a full stop, thus all the information carried by different degrees of freedom of the probe field is stored in the atomic medium.

In the linear magnetic field $B(\mathbf{r}) = B_0 + B_1 x$, after the dark polaritons are excited in the atomic medium with Gaussian distribution,

$$\Psi_j(0) = \frac{1}{\sqrt{\pi b^2}} \exp\left(-\frac{x^2}{2b^2} - \frac{z^2}{2b^2}\right), \quad (44)$$

we find that these dark polaritons achieve different transverse velocities,

$$v_{jx} = \frac{\chi_j L}{m_B v_g}, \quad (45)$$

since the hybrid light-matter quasiparticles have different effective magnetic moments. Hence, a σ^+- and a σ^--polarized beam are emergent, which are separately centered at $(x_j, z_j) = (\chi_j B_1 L^2 \sin^2\theta/(2kv_g), L)$ on the boundary as long as a magnetic field with nonzero transverse magnetic gradient is applied. The deflection angle of the corresponding components of light is given by

$$\alpha_{Bj} = \frac{v_x}{v_g} = [(-1)^j g_F^j - m_F^4 g_F^4]\frac{\mu_B B_1 L \tan^2\theta}{kc}, \quad (46)$$

which implies that a mirror symmetry splitting around the z axis can be achieved if the magnetic quantum number $m_F^4 = 0$ can be selected. Figure 1(b) schematically illustrates the Stern-Gerlach effect at $m_F^4 = 0$.

VII. CONCLUSIONS

In summary, we have studied magneto-optical Stern-Gerlach effects in an EIT atomic ensemble. We have derived an effective Schrödinger equation for the spatial motion of two orthogonally polarized components of light. It has been shown that magneto-optical Stern-Gerlach effects can happen in the presence of both the nonuniform magnetic field and the nonuniform control light field. We have also presented a dark-state polariton explanation of magneto-optical Stern-Gerlach effects.

It should be pointed out that our present scheme is essentially different from the previous work on the light deflection in an atomic medium [8,9]. First, the present scheme relies solely on an intra-atomic process of the single-species atomic ensemble, which causes simultaneous EIT for both components of the probe field interacting with magnetically Zeeman split sublevels in the presence of a driving field. The split of the optical beam is based on the attainment of double-EIT for both components of the probe field. We note that the double-EIT effect is not simply the sum of two independent EIT effects, and two types of dark polaritons with different effective magnetic moments are needed to understand magneto-optical Stern-Gerlach effects in a double-EIT atomic ensemble. Secondly, our scheme concerns the exploitation of the two spin-state superposition of photons, and the predicted phenomena are more general than those in Refs. [8,9]. Thirdly, in our scheme the magnetic field is required for splitting the atomic sublevels, but it is not necessary in Refs. [8,9].

Finally, comparing magneto-optical Stern-Gerlach effects in an EIT atomic ensemble with the spin Hall effect [17–20], we can predict the existence of a polarization (or quasispin) current, which is transverse to the flow of photons and can be generated either by the transverse profile of electric fields or the transverse gradient of magnetic fields. The polarized current is carried by electrically neutral dark-state polaritons. Because the spectral position of dark resonances is very sensitive to magnetic fields, we hope that such an optical analog of such a "spin Hall effect" can be observed in the future.

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