Greenberger-Horne-Zeilinger versus *W* states: Quantum teleportation through noisy channels

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Which state loses less quantum information between Greenberger-Horne-Zeilinger (GHZ) and *W* states when they are prepared for two-party quantum teleportation through a noisy channel? We address this issue by solving analytically a master equation in the Lindblad form with introducing the noisy channels that cause the quantum channels to be mixed states. It is found that the answer to this question is dependent on the type of noisy channel. If, for example, the noisy channel is $(L_{2,x}, L_{3,x}, L_{4,x})$ type, where the L's denote the Lindblad operators, the GHZ state is always more robust than the *W* state, i.e., the GHZ state preserves more quantum information. In, however, the $(L_{2,y}, L_{3,y}, L_{4,y})$-type channel the situation becomes completely reversed. In the $(L_{2,z}, L_{3,z}, L_{4,z})$-type channel, the *W* state is more robust than the GHZ state when the noisy parameter (κ) is comparatively small while the GHZ state becomes more robust when κ is large. In isotropic noisy channel we found that both states preserve an equal amount of quantum information. A relation between the average fidelity and entanglement for the mixed state quantum channels are discussed.

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I. INTRODUCTION

Quantum teleportation is an important process in quantum-information theories [1]. This process enables us to transmit an unknown quantum state from a sender, called Alice, to a remote recipient, called Bob, via dual classical channels. Bennett *et al.* have shown this process first in Ref. [2]. In that paper the authors used an Einstein-Podolsky-Rosen (EPR) state

$$|\mathrm{EPR}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{1.1}$$

as a quantum channel between Alice and Bob. In fact, $|\mathrm{EPR}\rangle$ is not the only two-qubit state which allows a perfect quantum teleportation. Any states that are local-unitary (LU) equivalent with $|\mathrm{EPR}\rangle$ also can be used as quantum channels for the perfect teleportation. This set of states forms a set of maximally entangled states.

Subsequently, quantum teleportation using three-qubit quantum channels are discussed. In the three-qubit system it is well known that there are two LU-inequivalent types of the maximally entangled states, called the Greenberger-Horne-Zeilinger (GHZ) [3] and the *W* [4] states whose general expressions are

$$|\mathrm{GHZ}\rangle = a|000\rangle + b|111\rangle, \quad (|a|^2 + |b|^2 = 1),$$

$$|W\rangle = a|001\rangle + b|010\rangle + c|100\rangle, \quad (|a|^2 + |b|^2 + |c|^2 = 1). \tag{1.2}$$

The perfect two-party quantum teleportation with exact GHZ state[1] was discussed in Ref. [5]. Furthermore, the authors of Ref. [5] discussed three-party teleportation (Alice, Bob, Cliff) with the GHZ state. This can be used as an imperfect quantum cloning machine [6].

Recently, it was shown [7] that not only GHZ state

$$|\psi_{\mathrm{GHZ}}\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{1.3}$$

but also *W* state

$$|\psi_W\rangle = \frac{1}{2}(|100\rangle + |010\rangle + \sqrt{2}|001\rangle) \tag{1.4}$$

can be used as quantum channels for the perfect two-party teleportation. As shown in Ref. [8] both $|\psi_{\mathrm{GHZ}}\rangle$ and $|\psi_W\rangle$ have $G(\psi)=1/\sqrt{2}$, where $G(\psi)$ is a Groverian entanglement measure [9]. Motivated from the fact that $|\psi_{\mathrm{GHZ}}\rangle$ and $|\psi_W\rangle$ have the same Groverian entanglement measure, the authors of Ref. [10] have shown that the state

$$|\tilde{\psi}\rangle = \frac{1}{\sqrt{2}}(|00q_1\rangle + |11q_2\rangle), \tag{1.5}$$

where $|q_1\rangle$ and $|q_2\rangle$ are arbitrary normalized one-qubit states, has also $G(\psi)=1/\sqrt{2}$ and it can be used as a perfect two-party teleportation.

[1]Exact GHZ state is $|\mathrm{GHZ}\rangle$ in Eq.(1.2) with $a=b=1/\sqrt{2}$.

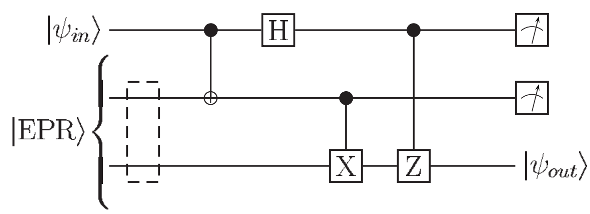

FIG. 1. A quantum circuit for quantum teleportation through noisy channels with the EPR state. The top two lines belong to Alice while the bottom line belongs to Bob. The dotted box represents noisy channels, which causes the quantum channel to be a mixed state.

The fact that both $|\psi_{\mathrm{GHZ}}\rangle$ and $|\psi_W\rangle$ allow the perfect two-party teleportation naturally arises the following question: Which state is better if noisy channels are introduced in the process of teleportation? The purpose of this paper is to address this issue by solving analytically a master equation in the Lindblad form [11]

$$\frac{\partial\rho}{\partial t} = -i[H_S,\rho] + \sum_{i,\alpha}\left(L_{i,\alpha}\rho L_{i,\alpha}^{\dagger} - \frac{1}{2}\{L_{i,\alpha}^{\dagger}L_{i,\alpha},\rho\}\right), \quad (1.6)$$

where the Lindblad operator $L_{i,\alpha} \equiv \sqrt{\kappa_{i,\alpha}}\sigma_{\alpha}^{(i)}$ acts on the ith qubit and describes decoherence, where $\sigma_{\alpha}^{(i)}$ denotes the Pauli matrix of the ith qubit with $\alpha = x,y,z$. The constant $\kappa_{i,\alpha}$ is approximately equal to the inverse of decoherence time. The master equation approach is shown to be equivalent to the usual quantum operation approach for the description of open quantum systems [1].

To reduce the effect of the noisy channels in the teleportation process the special purification protocols have been developed in Refs. [12,13]. Via this purification process for the noisy quantum channel one can increase the fidelity of teleportation. One can also directly compute the fidelity between initial unknown state and final state. This was discussed in Ref. [14] when the two-qubit EPR quantum channel interacts with various noisy channels. The quantum circuit for teleportation with $|\mathrm{EPR}\rangle$ through a noisy channel is illustrated in Fig. 1. The two top lines belong to Alice, while the bottom one belongs to Bob. The dotted box denotes noisy channel. Although different noisy channels were discussed in Ref. [14], we will concentrate on the noisy channel which makes the quantum channel to be mixed because our main purpose is comparison of $|\psi_{\mathrm{GHZ}}\rangle$ with $|\psi_W\rangle$ in the teleportation process.

How much quantum information is lost due to noisy channel can be measured by fidelity between $|\psi_{\mathrm{in}}\rangle$ and $|\psi_{\mathrm{out}}\rangle$. In order to quantify this quantity it is more convenient to use the density matrix. Let $\rho_{\mathrm{in}} = |\psi_{\mathrm{in}}\rangle\langle\psi_{\mathrm{in}}|$ and $\rho_{\mathrm{EPR}} = |\mathrm{EPR}\rangle\langle\mathrm{EPR}|$. Then the density matrix for the output state reduces to

$$\rho_{\mathrm{out}} = \mathrm{Tr}_{1,2}[U_{\mathrm{EPR}}\rho_{\mathrm{in}} \otimes \varepsilon(\rho_{\mathrm{EPR}})U_{\mathrm{EPR}}^{\dagger}], \quad (1.7)$$

where $\mathrm{Tr}_{1,2}$ is partial trace over Alice's qubits and U_{EPR} is a unitary operator implemented by quantum circuit in Fig. 1. In Eq. (1.7) ε denotes a quantum operation which maps from ρ_{EPR} to $\varepsilon(\rho_{\mathrm{EPR}})$ due to noisy channel. The explicit expressions for $\varepsilon(\rho_{\mathrm{EPR}})$ can be derived by solving the master equation. Then the quantity which measures how much information is preserved or lost can be written as

$$F = \langle\psi_{\mathrm{in}}|\rho_{\mathrm{out}}|\psi_{\mathrm{in}}\rangle \quad (1.8)$$

which is the square of the usual fidelity $F(\rho,\sigma) = \mathrm{Tr}\sqrt{\rho^{1/2}\sigma\rho^{1/2}}$. Thus $F=1$ implies the perfect teleportation. If $1-F$ becomes larger and larger, this indicates that we lost quantum information more and more.

The paper is organized as follows. In Sec. II we consider the two-party quantum teleportation with $|\psi_{\mathrm{GHZ}}\rangle$ as quantum channel when noisy channel makes $|\psi_{\mathrm{GHZ}}\rangle$ to be a mixed state. Solving the master equation (1.6) analytically, we compute F in Eq. (1.8) explicitly when Lindblad operators are $(L_{2,x},L_{3,x},L_{4,x})$, $(L_{2,y},L_{3,y},L_{4,y})$, $(L_{2,z},L_{3,z},L_{4,z})$, and isotropy, respectively. In Sec. III the calculation in the preceding section is repeated with changing the quantum channel from $|\psi_{\mathrm{GHZ}}\rangle$ to $|\psi_W\rangle$. In Sec. IV the results of Sec. II and Sec. III are compared with each other. It is shown that the answer of the question "which state is more robust[2] in the noisy channel?" is completely dependent on the type of the noisy channel. In the $(L_{2,x},L_{3,x},L_{4,x})$, for example, $|\psi_{\mathrm{GHZ}}\rangle$ preserves more information than $|\psi_W\rangle$, while the reverse situation occurs in the $(L_{2,y},L_{3,y},L_{4,y})$ channel. The situation in the $(L_{2,z},L_{3,z},L_{4,z})$ channel is more delicate. When the multiplication of a noisy parameter with time parameter, i.e., $\kappa_{i,z}t$, is small, $|\psi_W\rangle$ is slightly more robust than $|\psi_{\mathrm{GHZ}}\rangle$. If, however, $\kappa_{i,z}t$ becomes larger, $|\psi_{\mathrm{GHZ}}\rangle$ preserves more information than $|\psi_W\rangle$. In isotropy noisy with $\kappa_{i,x}=\kappa_{i,y}=\kappa_{i,z}=\kappa$ the average of F over all input states $|\psi_{\mathrm{in}}\rangle$ becomes identical for $|\psi_{\mathrm{GHZ}}\rangle$ and $|\psi_W\rangle$. In Sec. IV we give a brief conclusion. Also we discuss in this section a relation between average fidelity and entanglement for the mixed state quantum channels.

II. GHZ STATE WITH NOISY CHANNELS

In this section we would like to explore the effect of the noisy channels when the teleportation is performed with $|\psi_{\mathrm{GHZ}}\rangle$. It is convenient to write the unknown state $|\psi_{\mathrm{in}}\rangle$ to be teleported as a Bloch vector on a Bloch sphere in a form

$$|\psi_{\mathrm{in}}\rangle = \cos\left(\frac{\theta}{2}\right)e^{i\phi/2}|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{-i\phi/2}|1\rangle, \quad (2.1)$$

where θ and ϕ are the polar and azimuthal angles.

The quantum circuit for teleportation with $|\psi_{\mathrm{GHZ}}\rangle$ is shown in Fig. 2. The three top lines belong to Alice, while the bottom one belongs to Bob. The dotted box denotes noisy channel. Comparing Fig. 2 to Fig. 1 there appears one more controlled-NOT (CNOT) gate between the unknown state and GHZ state.

The density for the output state can be computed by

$$\rho_{\mathrm{out}} = \mathrm{Tr}_{1,2,3}[U_{\mathrm{GHZ}}\rho_{in} \otimes \varepsilon(\rho_{\mathrm{GHZ}})U_{\mathrm{GHZ}}^{\dagger}], \quad (2.2)$$

where $\mathrm{Tr}_{1,2,3}$ is partial trace over Alice's qubits and U_{GHZ} is a unitary operator, which can be read from Fig. 2. In Eq. (2.2) $\rho_{\mathrm{in}} = |\psi_{\mathrm{in}}\rangle\langle\psi_{\mathrm{in}}|$ and $\rho_{\mathrm{GHZ}} = |\psi_{\mathrm{GHZ}}\rangle\langle\psi_{\mathrm{GHZ}}|$.

[2] Throughout this paper "a given state is more robust" means that the state does lose less quantum information in the quantum teleportation through noise channels.

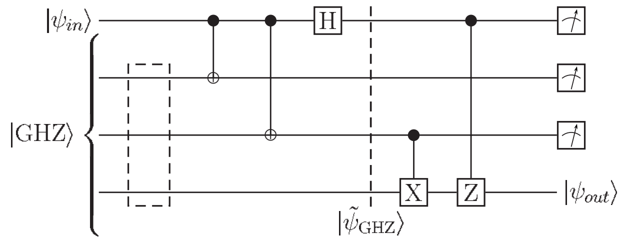

FIG. 2. A quantum circuit for quantum teleportation through noisy channels with the GHZ state. The top three lines belong to Alice while the bottom line belongs to Bob. The dotted box represents noisy channels, which causes the quantum channel to be a mixed state.

Now, we consider $(L_{2,z},L_{3,z},L_{4,z})$ noise channel because it is most simple to solve the master equation (1.6). Setting $\sigma_{ij}=\varepsilon_{ij}(\rho_{\mathrm{GHZ}})$ with $i,j=0,\ldots,7$ and assuming $H_S=0$ and $\kappa_{2,z}=\kappa_{3,z}=\kappa_{4,z}=\kappa$, the master equation reduces to eight diagonal and 28 off-diagonal first-order differential equations. Most of them simply give trivial solution and the only nonvanishing components are $\sigma_{00}=\sigma_{77}=1/2$ and $\sigma_{07}=\sigma_{70}=e^{-6\kappa t}/2$. Thus, in this noisy channel $\varepsilon(\rho_{\mathrm{GHZ}})$ becomes

$$\varepsilon(\rho_{\mathrm{GHZ}})=\frac{1}{2}(|000\rangle\langle 000|+|111\rangle\langle 111|)+\frac{1}{2}e^{-6\kappa t}(|000\rangle\langle 111|+|111\rangle\langle 000|). \tag{2.3}$$

Inserting Eq. (2.3) into Eq. (2.2) it is straightforward to derive ρ_{out}. Then the quantity F defined in Eq. (1.8) is dependent on input angle θ as follows:

$$F(\theta,\phi)=1-\frac{1}{2}(1-e^{-6\kappa t})\sin^2\theta. \tag{2.4}$$

The average $F(\theta,\phi)$ over all possible input unknown states defined as

$$\bar{F}\equiv\frac{1}{4\pi}\int_0^{2\pi}d\phi\int_0^{\pi}d\theta\sin\theta F(\theta,\phi) \tag{2.5}$$

reduces to

$$\bar{F}=\frac{2}{3}+\frac{1}{3}e^{-6\kappa t}. \tag{2.6}$$

It is easy to check that $F(\theta,\phi)=\bar{F}=1$ when there is no noisy channel, i.e., $\kappa=0$, which implies the perfect teleportation.

Next, we consider $(L_{2,x},L_{3,x},L_{4,x})$ noisy channel. Setting again $\sigma_{ij}=\varepsilon_{ij}(\rho_{\mathrm{GHZ}})$ and assuming again $H_S=0$ and $\kappa_{2,x}=\kappa_{3,x}=\kappa_{4,x}=\kappa$, one can show that the master equation (1.6) reduces to eight diagonal coupled linear differential equations and 28 off-diagonal coupled linear differential equations. The eight diagonal equations imply $\Sigma_{i=0}^{3}\sigma_{ii}=\Sigma_{i=4}^{7}\sigma_{ii}=1/2$. Thus we can write $\sigma_{00}=1/2+\delta q_0$, $\sigma_{11}=\delta q_1$, $\sigma_{22}=\delta q_2$, $\sigma_{33}=-\delta q_0-\delta q_1-\delta q_2$, $\sigma_{44}=\delta q_4$, $\sigma_{55}=\delta q_5$, $\sigma_{66}=\delta q_6$, and $\sigma_{77}=1/2-(\delta q_4+\delta q_5+\delta q_6)$ with $\delta q_i(t=0)=0$ for all i. Inserting these expressions into the original coupled equations, one can easily derive the diagonal components of σ, which are

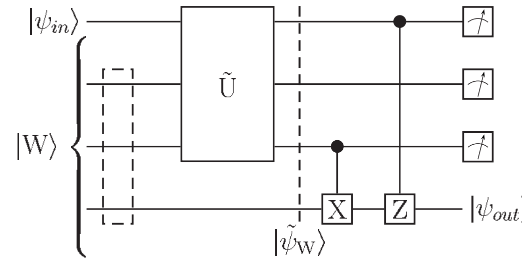

FIG. 3. A quantum circuit for quantum teleportation through noisy channels with the W state. The top three lines belong to Alice while the bottom line belongs to Bob. The dotted box represents noisy channels, which causes the quantum channel to be a mixed state. The unitary operator $\tilde{U}$ makes $|\tilde{\psi}_W\rangle$ coincide with $|\tilde{\psi}_{\mathrm{GHZ}}\rangle$ expressed in Fig. 2.

$$\sigma_{00}=\sigma_{77}=\frac{1}{8}(1+3e^{-4\kappa t}),$$

$$\sigma_{11}=\cdots=\sigma_{66}=\frac{1}{8}(1-e^{-4\kappa t}). \tag{2.7}$$

The off-diagonal 28 coupled equations consist of seven groups, each of which are four coupled differential equations in the closed form. Thus, we can solve all of them by similar way. Most of them give the trivial solutions and the nonvanishing components are

$$\sigma_{07}=\frac{1}{8}(1+3e^{-4\kappa t}),$$

$$\sigma_{16}=\sigma_{25}=\sigma_{34}=\frac{1}{8}(1-e^{-4\kappa t}) \tag{2.8}$$

with $\sigma_{ij}=\sigma_{ji}$. Defining

$$\alpha_+\equiv 1+3e^{-4\kappa t},$$

$$\alpha_-\equiv 1-e^{-4\kappa t}, \tag{2.9}$$

we can express $\varepsilon(\rho_{\mathrm{GHZ}})$ analytically in a form

$$\varepsilon(\rho_{\mathrm{GHZ}})=\frac{1}{8}\begin{pmatrix}\alpha_+ & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_+\\ 0 & \alpha_- & 0 & 0 & 0 & 0 & \alpha_- & 0\\ 0 & 0 & \alpha_- & 0 & 0 & \alpha_- & 0 & 0\\ 0 & 0 & 0 & \alpha_- & \alpha_- & 0 & 0 & 0\\ 0 & 0 & 0 & \alpha_- & \alpha_- & 0 & 0 & 0\\ 0 & 0 & \alpha_- & 0 & 0 & \alpha_- & 0 & 0\\ 0 & \alpha_- & 0 & 0 & 0 & 0 & \alpha_- & 0\\ \alpha_+ & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_+\end{pmatrix}. \tag{2.10}$$

Then using Eq. (2.2) one can compute $F(\theta,\phi)$ and $\bar{F}$, whose expressions are

$$F(\theta,\phi)=\frac{1}{2}[(1+\sin^2\theta\cos^2\phi)+e^{-4\kappa t}(\cos^2\theta+\sin^2\theta\sin^2\phi)],$$

$$\bar{F}=\frac{2}{3}+\frac{1}{3}e^{-4\kappa t}. \quad (2.11)$$

Similar calculation shows that $\varepsilon(\rho_{\rm GHZ})$ for $(L_{2,y},L_{3,y},L_{4,y})$ noisy channel becomes

$$\varepsilon(\rho_{\rm GHZ})=\frac{1}{8}\begin{pmatrix}\alpha_+ & 0 & 0 & 0 & 0 & 0 & 0 & \beta_1\\ 0 & \alpha_- & 0 & 0 & 0 & 0 & -\beta_2 & 0\\ 0 & 0 & \alpha_- & 0 & 0 & -\beta_2 & 0 & 0\\ 0 & 0 & 0 & \alpha_- & -\beta_2 & 0 & 0 & 0\\ 0 & 0 & 0 & -\beta_2 & \alpha_- & 0 & 0 & 0\\ 0 & 0 & -\beta_2 & 0 & 0 & \alpha_- & 0 & 0\\ 0 & -\beta_2 & 0 & 0 & 0 & 0 & \alpha_- & 0\\ \beta_1 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_+\end{pmatrix}, \quad (2.12)$$

where $\alpha_\pm$ are given in Eq. (2.9) and, β_1 and β_2 are defined as

$$\beta_1=3e^{-2\kappa t}+e^{-6\kappa t},$$

$$\beta_2=e^{-2\kappa t}-e^{-6\kappa t}. \quad (2.13)$$

One may wonder why the off-diagonal components of Eq. (2.12) is much different from those of Eq. (2.10) because of the following consideration: If σ_{ij}^x and σ_{ij}^y are density matrices for (L_{2x},L_{3x},L_{4x}) and (L_{2y},L_{3y},L_{4y}) noises, respectively, then $(u\otimes u\otimes u)\sigma_{ij}^x(u\otimes u\otimes u)^\dagger$ satisfies the master equation for the (L_{2y},L_{3y},L_{4y}) provided that u is a unitary operator satisfying $u\sigma_x u^\dagger=\sigma_y$. Although this is completely correct, this does not guarantee $\sigma_{ij}^y=(u\otimes u\otimes u)\sigma_{ij}^x(u\otimes u\otimes u)^\dagger$ because σ_{ij}^x and σ_{ij}^y should satisfy the boundary condition, i.e., $\sigma_{ij}^x=\sigma_{ij}^y=\rho_{\rm GHZ}$ when $\kappa t=0$. The detailed computation for the off-diagonal components of σ_{ij}^x and σ_{ij}^y is briefly summarized in the Appendix.

One can show that Eq. (2.2) generates

$$F(\theta,\phi)=\frac{1}{2}[1+\sin^2\theta\sin^2\phi e^{-2\kappa t}+\cos^2\theta e^{-4\kappa t}+\sin^2\theta\cos^2\phi e^{-6\kappa t}],$$

$$\bar{F}=\frac{1}{6}(3+e^{-2\kappa t}+e^{-4\kappa t}+e^{-6\kappa t}). \quad (2.14)$$

For isotropic noise, which is described by nine Lindblad operators, $L_{2,\alpha}$, $L_{3,\alpha}$, and $L_{4,\alpha}$ with $\alpha=x,y,z$, $\varepsilon(\rho_{\rm GHZ})$ becomes

$$\varepsilon(\rho_{\rm GHZ})=\frac{1}{8}\begin{pmatrix}\tilde{\alpha}_+ & 0 & 0 & 0 & 0 & 0 & 0 & \gamma\\ 0 & \tilde{\alpha}_- & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & \tilde{\alpha}_- & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \tilde{\alpha}_- & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & \tilde{\alpha}_- & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \tilde{\alpha}_- & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\alpha}_- & 0\\ \gamma & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\alpha}_+\end{pmatrix}, \quad (2.15)$$

where

$$\tilde{\alpha}_+=1+3e^{-8\kappa t},$$

$$\tilde{\alpha}_-=1-e^{-8\kappa t},$$

$$\gamma=4e^{-12\kappa t}. \quad (2.16)$$

In this case $F(\theta,\phi)$ and $\bar{F}$ becomes

$$F(\theta,\phi)=\frac{1}{2}[1+e^{-8\kappa t}\cos^2\theta+e^{-12\kappa t}\sin^2\theta],$$

$$\bar{F}=\frac{1}{6}(3+e^{-8\kappa t}+2e^{-12\kappa t}). \quad (2.17)$$

It is interesting to note that $F(\theta,\phi)$ for the isotropic noisy channel is dependent on angle parameter θ, while the same quantity is independent of θ in Ref. [14], where the two-qubit EPR state was used. The final results of $F(\theta,\phi)$ and $\bar{F}$ are summarized in Table I and will be compared to those derived from $|\psi_W\rangle$. In the next section we will discuss the effect of noisy channels when we prepare $|\psi_W\rangle$ for the quantum teleportation.

III. W STATE WITH NOISY CHANNELS

In this section we would like to repeat calculation of the preceding section when $|\psi_{\mathrm{GHZ}}\rangle$ is replaced by $|\psi_W\rangle$. In order to compute $F(\theta,\phi)$ we need a quantum circuit, which should be, of course, different from Fig. 2. The quantum circuit for the quantum teleportation with $|\psi_W\rangle$ described in Fig. 3 is not simple like the GHZ state. It cannot be represented by the usual controlled-NOT (CNOT) and Hardmard gates. In fact, we do not know how to express the $\tilde{U}$ gate described in Fig. 3 as a combination of the usual well-known gates such as CNOT, Hardmard, Pauli X,Y,Z, and Toffoli gates. The $\tilde{U}$ gate is made to make $|\tilde{\psi}_W\rangle$ in Fig. 3 equal to $|\tilde{\psi}_{\mathrm{GHZ}}\rangle$ in Fig 2. The explicit expression for the $\tilde{U}$ gate is

$$\tilde{U}=\frac{1}{2}\begin{pmatrix} 0 & 1 & 1 & 0 & \sqrt{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ \sqrt{2} & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 & -\sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & -\sqrt{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sqrt{2} & -\sqrt{2} & 0 \\ \sqrt{2} & 0 & 0 & 0 & 0 & -1 & -1 & 0 \end{pmatrix}. \tag{3.1}$$

In spite of, therefore, lack of knowledge on $\tilde{U}$-gate ρ_{out}, the density matrix for the output state, can be derived by

$$\rho_{\mathrm{out}}=\mathrm{Tr}_{1,2,3}[U_W\rho_{\mathrm{in}}\otimes\varepsilon(\rho_W)U_W^{\dagger}], \tag{3.2}$$

where the unitary operator U_W can be read easily from Fig. 3 and $\varepsilon(\rho_W)$ is a density matrix constructed by $\rho_W\equiv|\psi_W\rangle\langle\psi_W|$ and noisy channels described by the dotted box in Fig. 3.

Now we first consider the $(L_{2,z},L_{3,z},L_{4,z})$ channel. In this case the master equation (1.6) with assuming, for simplicity, $\kappa_{2,z}=\kappa_{3,z}=\kappa_{4,z}=\kappa$ reduces to the simple first-order differential equations, which gives

$$\varepsilon(\rho_W)=\frac{1}{4}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & \sqrt{2}e^{-4\kappa t} & 0 & \sqrt{2}e^{-4\kappa t} & 0 & 0 & 0 \\ 0 & \sqrt{2}e^{-4\kappa t} & 1 & 0 & e^{-4\kappa t} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{2}e^{-4\kappa t} & e^{-4\kappa t} & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{3.3}$$

Then Eq. (3.2) allows us to compute ρ_{out} directly and Eq. (1.8) gives

$$F(\theta,\phi)=1-\frac{1}{4}(1-e^{-4\kappa t})(1+\sin^2\theta),$$

$$\bar{F}=\frac{1}{12}(7+5e^{-4\kappa t}). \tag{3.4}$$

Next we consider the $(L_{2,x},L_{3,x},L_{4,x})$ noisy channel with $\kappa_{2,x}=\kappa_{3,x}=\kappa_{4,x}=\kappa$. In this case the master equation reduces to eight diagonal coupled equations and 28 off-diagonal coupled equations. The diagonal equations imply $\sum_{i=0}^{3}\sigma_{ii}=1/2+e^{-2\kappa t}/4$ and $\sum_{i=4}^{7}\sigma_{ii}=1/2-e^{-2\kappa t}/4$, where $\sigma_{ij}=\varepsilon_{ij}(\rho_W)$ with $i,j=0,\ldots,7$. Using these two constraints one can compute all diagonal components, which are

$$\sigma_{00}=\frac{1}{8}(1+e^{-2\kappa t}-e^{-4\kappa t}-e^{-6\kappa t}),$$

$$\sigma_{11}=\frac{1}{8}(1+e^{-2\kappa t}+e^{-4\kappa t}+e^{-6\kappa t}),$$

$$\sigma_{22}=\sigma_{44}=\frac{1}{8}(1+e^{-6\kappa t}),$$

$$\sigma_{33}=\sigma_{55}=\frac{1}{8}(1-e^{-6\kappa t}),$$

$$\sigma_{66}=\frac{1}{8}(1-e^{-2\kappa t}+e^{-4\kappa t}-e^{-6\kappa t}),$$

$$\sigma_{77}=\frac{1}{8}(1-e^{-2\kappa t}-e^{-4\kappa t}+e^{-6\kappa t}). \tag{3.5}$$

The equations for the off-diagonal components are more complicated. However, these equations consist of seven groups, each of which are four closed coupled equations. This fact allows us to compute all components analytically, whose explicit expressions are

$$\sigma_{03}=\sigma_{05}=\sqrt{2}\sigma_{06}=\frac{\sqrt{2}}{16}(1+e^{-2\kappa t}-e^{-4\kappa t}-e^{-6\kappa t}),$$

$$\sigma_{12}=\sigma_{14}=\sqrt{2}\sigma_{24}=\frac{\sqrt{2}}{16}(1+e^{-2\kappa t}+e^{-4\kappa t}+e^{-6\kappa t}),$$

$$\sigma_{27}=\sigma_{47}=\sqrt{2}\sigma_{17}=\frac{\sqrt{2}}{16}(1-e^{-2\kappa t}-e^{-4\kappa t}+e^{-6\kappa t}),$$

$$\sigma_{36}=\sigma_{56}=\sqrt{2}\sigma_{35}=\frac{\sqrt{2}}{16}(1-e^{-2\kappa t}+e^{-4\kappa t}-e^{-6\kappa t}) \tag{3.6}$$

with $\sigma_{ij}=\sigma_{ji}$. Defining

$$\alpha_1=1+e^{-2\kappa t}+e^{-4\kappa t}+e^{-6\kappa t},$$

$$\alpha_2=1+e^{-2\kappa t}-e^{-4\kappa t}-e^{-6\kappa t},$$

$$\alpha_3 = 1 - e^{-2\kappa t} - e^{-4\kappa t} + e^{-6\kappa t},$$

$$\alpha_4 = 1 - e^{-2\kappa t} + e^{-4\kappa t} - e^{-6\kappa t},$$

$$\beta_\pm = 1 \pm e^{-6\kappa t}, \tag{3.7}$$

one can express $\varepsilon(\rho_W)$ as follows:

$$\varepsilon(\rho_W) = \frac{1}{16}\begin{pmatrix} 2\alpha_2 & 0 & 0 & \sqrt{2}\alpha_2 & 0 & \sqrt{2}\alpha_2 & \alpha_2 & 0 \\ 0 & 2\alpha_1 & \sqrt{2}\alpha_1 & 0 & \sqrt{2}\alpha_1 & 0 & 0 & \alpha_3 \\ 0 & \sqrt{2}\alpha_1 & 2\beta_+ & 0 & \alpha_1 & 0 & 0 & \sqrt{2}\alpha_3 \\ \sqrt{2}\alpha_2 & 0 & 0 & 2\beta_- & 0 & \alpha_4 & \sqrt{2}\alpha_4 & 0 \\ 0 & \sqrt{2}\alpha_1 & \alpha_1 & 0 & 2\beta_+ & 0 & 0 & \sqrt{2}\alpha_3 \\ \sqrt{2}\alpha_2 & 0 & 0 & \alpha_4 & 0 & 2\beta_- & \sqrt{2}\alpha_4 & 0 \\ \alpha_2 & 0 & 0 & \sqrt{2}\alpha_4 & 0 & \sqrt{2}\alpha_4 & 2\alpha_4 & 0 \\ 0 & \alpha_3 & \sqrt{2}\alpha_3 & 0 & \sqrt{2}\alpha_3 & 0 & 0 & 2\alpha_3 \end{pmatrix}. \tag{3.8}$$

Inserting Eq. (3.8) into (3.2), one can compute ρ_{out} directly. Thus using ρ_{out} and Eq. (1.8), one can compute $F(\theta,\phi)$ and $\bar{F}$ whose expressions are

$$F(\theta,\phi) = \frac{1}{8}[(4 + 2\sin^2\theta\cos^2\phi) + e^{-2\kappa t}(\cos^2\theta + 2\sin^2\theta\cos^2\phi) + e^{-4\kappa t}(2\sin^2\theta\sin^2\phi) + e^{-6\kappa t}(3\cos^2\theta + 2\sin^2\theta\sin^2\phi)],$$

$$\bar{F} = \frac{1}{24}(14 + 3e^{-2\kappa t} + 2e^{-4\kappa t} + 5e^{-6\kappa t}). \tag{3.9}$$

For the $(L_{2,y}, L_{3,y}, L_{4,y})$ noisy channel similar calculation shows that $\varepsilon(\rho_W)$ reduces to

$$\varepsilon(\rho_W) = \frac{1}{16}\begin{pmatrix} 2\alpha_2 & 0 & 0 & -\sqrt{2}\alpha_2 & 0 & -\sqrt{2}\alpha_2 & -\alpha_2 & 0 \\ 0 & 2\alpha_1 & \sqrt{2}\alpha_1 & 0 & \sqrt{2}\alpha_1 & 0 & 0 & -\alpha_3 \\ 0 & \sqrt{2}\alpha_1 & 2\beta_+ & 0 & \alpha_1 & 0 & 0 & -\sqrt{2}\alpha_3 \\ -\sqrt{2}\alpha_2 & 0 & 0 & 2\beta_- & 0 & \alpha_4 & \sqrt{2}\alpha_4 & 0 \\ 0 & \sqrt{2}\alpha_1 & \alpha_1 & 0 & 2\beta_+ & 0 & 0 & -\sqrt{2}\alpha_3 \\ -\sqrt{2}\alpha_2 & 0 & 0 & \alpha_4 & 0 & 2\beta_- & \sqrt{2}\alpha_4 & 0 \\ -\alpha_2 & 0 & 0 & \sqrt{2}\alpha_4 & 0 & \sqrt{2}\alpha_4 & 2\alpha_4 & 0 \\ 0 & -\alpha_3 & -\sqrt{2}\alpha_3 & 0 & -\sqrt{2}\alpha_3 & 0 & 0 & 2\alpha_3 \end{pmatrix} \tag{3.10}$$

and, as a result, $F(\theta,\phi)$ and $\bar{F}$ reduce to

$$F(\theta,\phi) = \frac{1}{8}[(4 + 2\sin^2\theta\sin^2\phi) + e^{-2\kappa t}(\cos^2\theta + 2\sin^2\theta\sin^2\phi) + e^{-4\kappa t}(2\sin^2\theta\cos^2\phi) + e^{-6\kappa t}(3\cos^2\theta + 2\sin^2\theta\cos^2\phi)],$$

$$\bar{F} = \frac{1}{24}(14 + 3e^{-2\kappa t} + 2e^{-4\kappa t} + 5e^{-6\kappa t}). \tag{3.11}$$

It is interesting to note that $\bar{F}$ for the $(L_{2,x}, L_{3,x}, L_{4,x})$ noisy channel is the same with $\bar{F}$ for the $(L_{2,y}, L_{3,y}, L_{4,y})$ noisy channel.

Finally for isotropic noisy channel $\varepsilon(\rho_W)$ becomes

$$\varepsilon(\rho_W) = \frac{1}{8}\begin{pmatrix} \tilde{\alpha}_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{\alpha}_1 & \sqrt{2}\tilde{\gamma}_+ & 0 & \sqrt{2}\tilde{\gamma}_+ & 0 & 0 & 0 \\ 0 & \sqrt{2}\tilde{\gamma}_+ & \tilde{\beta}_+ & 0 & \tilde{\gamma}_+ & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{\beta}_- & 0 & \tilde{\gamma}_- & \sqrt{2}\tilde{\gamma}_- & 0 \\ 0 & \sqrt{2}\tilde{\gamma}_+ & \tilde{\gamma}_+ & 0 & \tilde{\beta}_+ & 0 & 0 & 0 \\ 0 & 0 & 0 & \tilde{\gamma}_- & 0 & \tilde{\beta}_- & \sqrt{2}\tilde{\gamma}_- & 0 \\ 0 & 0 & 0 & \sqrt{2}\tilde{\gamma}_- & 0 & \sqrt{2}\tilde{\gamma}_- & \tilde{\alpha}_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \tilde{\alpha}_3 \end{pmatrix}, \tag{3.12}$$

where

$$\tilde{\alpha}_1 = 1 + e^{-4\kappa t} + e^{-8\kappa t} + e^{-12\kappa t},$$

$$\tilde{\alpha}_2 = 1 + e^{-4\kappa t} - e^{-8\kappa t} - e^{-12\kappa t},$$

$$\tilde{\alpha}_3 = 1 - e^{-4\kappa t} - e^{-8\kappa t} + e^{-12\kappa t},$$

$$\tilde{\alpha}_4 = 1 - e^{-4\kappa t} + e^{-8\kappa t} - e^{-12\kappa t},$$

$$\tilde{\beta}_\pm = 1 \pm e^{-12\kappa t},$$

$$\tilde{\gamma}_\pm = e^{-8\kappa t} \pm e^{-12\kappa t}. \tag{3.13}$$

Thus, one can compute $F(\theta,\phi)$ and $\bar{F}$ for this noisy channel, which are

$$F(\theta,\phi) = \frac{1}{4}[2 + e^{-8\kappa t}\sin^2\theta + e^{-12\kappa t}(1+\cos^2\theta)],$$

$$\bar{F} = \frac{1}{6}(3 + e^{-8\kappa t} + 2e^{-12\kappa t}). \tag{3.14}$$

The measures $F(\theta,\phi)$ and $\bar{F}$ for the various noisy channels are summarized in Table I with those for the GHZ state. In the next section we will compare $F(\theta,\phi)$ and $\bar{F}$ for the GHZ state with those for the W state.

IV. GHZ VERSUS W

The quantities $F(\theta,\phi)$ and $\bar{F}$ for various noisy channels are summarized in Table I when GHZ and W states are prepared for the quantum teleportation. The most interesting feature in Table I is the fact that $\bar{F}$ for GHZ is exactly the same with that for W in the isotropic channel. Since the isotropic noisy channel can be regarded roughly as a sum of $(L_{2,x},L_{3,x},L_{4,x})$, $(L_{2,y},L_{3,y},L_{4,y})$, and $(L_{2,z},L_{3,z},L_{4,z})$ noisy channels, this fact indicates that the robustness of the quantum channel is noise dependent.

In order to show this fact explicitly we plot the κt dependence of $\bar{F}$ for $(L_{2,x},L_{3,x},L_{4,x})$ [Fig. 4(a)], $(L_{2,y},L_{3,y},L_{4,y})$ [Fig. 4(b)], and $(L_{2,z},L_{3,z},L_{4,z})$ [Fig. 4(c)] noisy channels. Figure 4 shows that $\bar{F}$ for $|\psi_{\rm GHZ}\rangle$ is always larger than that for $|\psi_W\rangle$ in the $(L_{2,x},L_{3,x},L_{4,x})$ noisy channel. This means that $|\psi_{\rm GHZ}\rangle$ does lose less quantum information compared to $|\psi_W\rangle$ in this noisy channel. However, the situation is changed in the $(L_{2,y},L_{3,y},L_{4,y})$ noisy channel. In this case $\bar{F}$ for $|\psi_W\rangle$ is always larger than that for $|\psi_{\rm GHZ}\rangle$. This means that $|\psi_W\rangle$ is more robust than $|\psi_{\rm GHZ}\rangle$ in this noisy channel. In the $(L_{2,z},L_{3,z},L_{4,z})$ noisy channel the situation is more delicate.

TABLE I. Summary of $F(\theta,\phi)$ and $\bar{F}$ in various noisy channels.

	Noise	GHZ	W
$F(\theta,\phi)$	(L_{2x},L_{3x},L_{4x})	$\frac{1}{2}[(1+\sin^2\theta\cos^2\phi)+e^{-4\kappa t}(1-\sin^2\theta\cos^2\phi)]$	$\frac{1}{8}[(4+2\sin^2\theta\cos^2\phi)+e^{-2\kappa t}(\cos^2\theta+2\sin^2\theta\cos^2\phi)$ $+e^{-4\kappa t}(2\sin^2\theta\sin^2\phi)+e^{-6\kappa t}(3\cos^2\theta+2\sin^2\theta\sin^2\phi)]$
	(L_{2y},L_{3y},L_{4y})	$\frac{1}{2}(1+\sin^2\theta\sin^2\phi e^{-2\kappa t}+\cos^2\theta e^{-4\kappa t}+\sin^2\theta\cos^2\phi e^{-6\kappa t})$	$\frac{1}{8}[(4+2\sin^2\theta\sin^2\phi)+e^{-2\kappa t}(\cos^2\theta+2\sin^2\theta\sin^2\phi)$ $+e^{-4\kappa t}(2\sin^2\theta\cos^2\phi)+e^{-6\kappa t}(3\cos^2\theta+2\sin^2\theta\cos^2\phi)]$
	(L_{2z},L_{3z},L_{4z})	$1-\frac{1}{2}(1-e^{-6\kappa t})\sin^2\theta$	$1-\frac{1}{4}(1-e^{-4\kappa t})(1+\sin^2\theta)$
	Isotropy	$\frac{1}{2}(1+\cos^2\theta e^{-8\kappa t}+\sin^2\theta e^{-12\kappa t})$	$\frac{1}{4}[2+\sin^2\theta e^{-8\kappa t}+(1+\cos^2\theta)e^{-12\kappa t}]$
$\bar{F}$	(L_{2x},L_{3x},L_{4x})	$\frac{2}{3}+\frac{1}{3}e^{-4\kappa t}$	$\frac{1}{24}(14+3e^{-2\kappa t}+2e^{-4\kappa t}+5e^{-6\kappa t})$
	(L_{2y},L_{3y},L_{4y})	$\frac{1}{6}(3+e^{-2\kappa t}+e^{-4\kappa t}+e^{-6\kappa t})$	$\frac{1}{24}(14+3e^{-2\kappa t}+2e^{-4\kappa t}+5e^{-6\kappa t})$
	(L_{2z},L_{3z},L_{4z})	$\frac{2}{3}+\frac{1}{3}e^{-6\kappa t}$	$\frac{1}{12}(7+5e^{-4\kappa t})$
	Isotropy	$\frac{1}{6}(3+e^{-8\kappa t}+2e^{-12\kappa t})$	$\frac{1}{6}(3+e^{-8\kappa t}+2e^{-12\kappa t})$

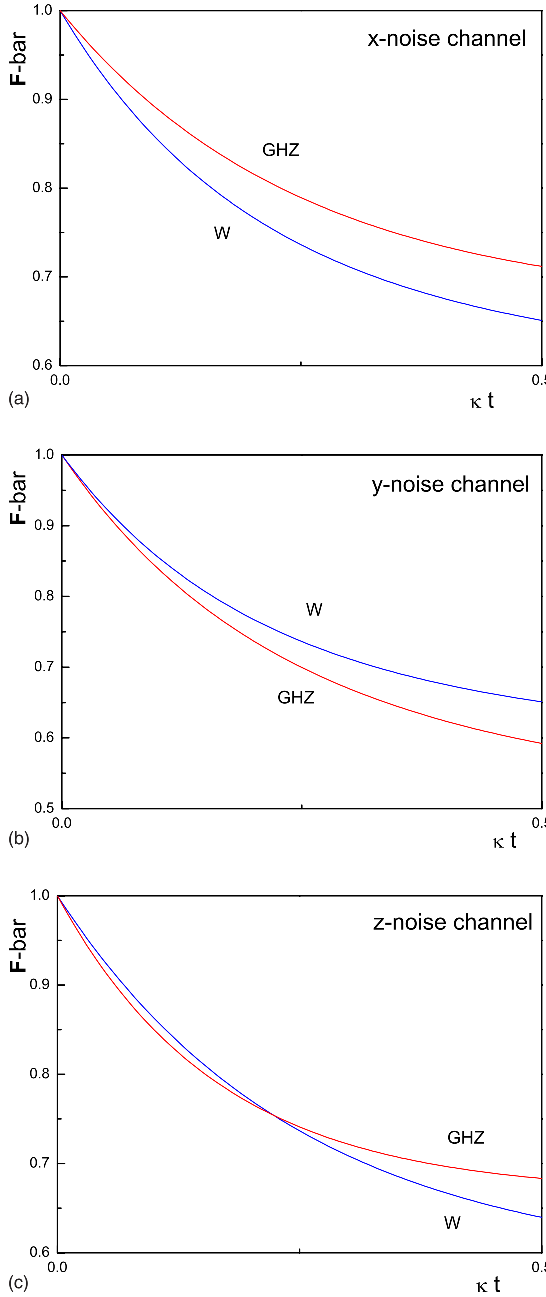

FIG. 4. (Color online) The plot of κt dependence of $\bar{F}$ for $(L_{2,x},L_{3,x},L_{4,x})$ (a), $(L_{2,y},L_{3,y},L_{4,y})$ (b), and $(L_{2,z},L_{3,z},L_{4,z})$ (c) noisy channels. (a) shows that $\bar{F}$ for the GHZ state is always larger than that for the W state, which implies that the GHZ state does lose less quantum information than the W state in the $(L_{2,x},L_{3,x},L_{4,x})$ noisy channel. (b) shows, however, that the situation is completely reversed in the $(L_{2,y},L_{3,y},L_{4,y})$ noisy channel. In the $(L_{2,z},L_{3,z},L_{4,z})$ noisy channel (c) indicates that the W state is more robust when $\kappa t < 0.223$ while the GHZ state becomes more robust when $\kappa t > 0.223$.

In this channel $\bar{F}$ for $|\psi_W\rangle$ is larger than that for $|\psi_{\mathrm{GHZ}}\rangle$ when $\kappa t \leq 0.223$. If, however, $\kappa t \geq 0.223$, $\bar{F}$ for $|\psi_{\mathrm{GHZ}}\rangle$ becomes larger than that for $|\psi_W\rangle$. Summing over all those phenomena seems to make the same $\bar{F}$ for $|\psi_{\mathrm{GHZ}}\rangle$ and $|\psi_W\rangle$ in the isotropic channel.

However, we should note that the result of Fig. 4 is dependent on the choice of the basis. To show this explicitly let us consider a unitary operator $U=(\sigma_x+\sigma_y)/\sqrt{2}$, which yields $U\sigma_x U^\dagger=\sigma_y$ and $U\sigma_y U^\dagger=\sigma_x$. Now, let us consider the noisy teleportation when quantum channels are $|\psi'_{\mathrm{GHZ}}\rangle=U\otimes U \otimes U|\psi_{\mathrm{GHZ}}\rangle$ and $|\psi'_W\rangle=U\otimes U\otimes U|\psi_W\rangle$, respectively. Then it is obvious that Figs. 4(a) and 4(b) would be interchanged with each other if one computes the average fidelity. This indicates that Fig. 4 is dependent on the choice of the basis states.

Another interesting point in Table I is the fact that $\bar{F}$ for the GHZ state decays to 2/3 in $(L_{2,x},L_{3,x},L_{4,x})$ and $(L_{2,z},L_{3,z},L_{4,z})$ noisy channels. The number $\bar{F}=2/3$ corresponds to the average fidelity obtained only by the classical communication [15]. However, in the $(L_{2,y},L_{3,y},L_{4,y})$ noisy channel, $\bar{F}$ for the GHZ state decays to 1/2, which corresponds to no-communication between Alice and Bob. When quantum channel is subject to noise in one direction, the average fidelity for the W state always decays to 7/12, which is slightly smaller than 2/3. In the isotropic noisy channel, $\bar{F}$ for both GHZ and W states decays to 1/2 when $\kappa t\to\infty$ as with the two-qubit EPR quantum channel [14].

Figure 5 is a plot of θ and ϕ dependence of $F(\theta,\phi)$ for $(L_{2,x},L_{3,x},L_{4,x})$ [Fig. 5(a)], $(L_{2,y},L_{3,y},L_{4,y})$ [Fig. 5(b)], $(L_{2,z},L_{3,z},L_{4,z})$ [(Fig. 5(c)], and isotropic [Fig. 5(d)] noisy channels when κt is fixed to 0.5. The transparent and opaque surfaces correspond to GHZ and W states, respectively. Figure 5(a) indicates that in the $(L_{2,x},L_{3,x},L_{4,x})$ noisy channel, $F(\theta,\phi)$ for the GHZ state is larger than that for the W state in the entire range of θ and ϕ. Figure 5(b) shows that in the $(L_{2,y},L_{3,y},L_{4,y})$ noisy channel, $F(\theta,\phi)$ for the W state is larger in almost every range of θ and ϕ except for the small boundary region. This is consistent with the fact that $\bar{F}$ for the W state is larger than that for the GHZ state in this noisy channel. Figures 5(c) and 5(d) show that in $(L_{2,z},L_{3,z},L_{4,z})$ and isotropic noisy channels, $F(\theta,\phi)$ for the GHZ state is generally larger than that for the W state in small θ region (approximately $0\leq\theta<1$) and large θ region (approximately $2<\theta\leq\pi$) while in the middle θ region (approximately $1<\theta<2$) $F(\theta,\phi)$ for the W state is larger.

V. CONCLUSION

In this paper we consider the quantum teleportation with GHZ and W states, respectively, when the noisy channels cause the quantum channels to be mixed states. The issue of robustness between GHZ and W, i.e., which state does lose less quantum information, in the noisy channels is completely dependent on the type of noisy channel. If, for example, the noisy channel is $(L_{2,x},L_{3,x},L_{4,x})$ type, the GHZ state is always robust compared to the W state while the reverse situation occurs in the $(L_{2,y},L_{3,y},L_{4,y})$ noisy channel. In the $(L_{2,z},L_{3,z},L_{4,z})$ noisy channel, the W state does lose less information than the GHZ state when κt is comparatively small. If, however, $\kappa t\geq 0.223$, the GHZ state becomes more robust in this noisy channel.

Since the decoherence mechanism in each qubit is obviously independent, one can explore the different noisy chan-

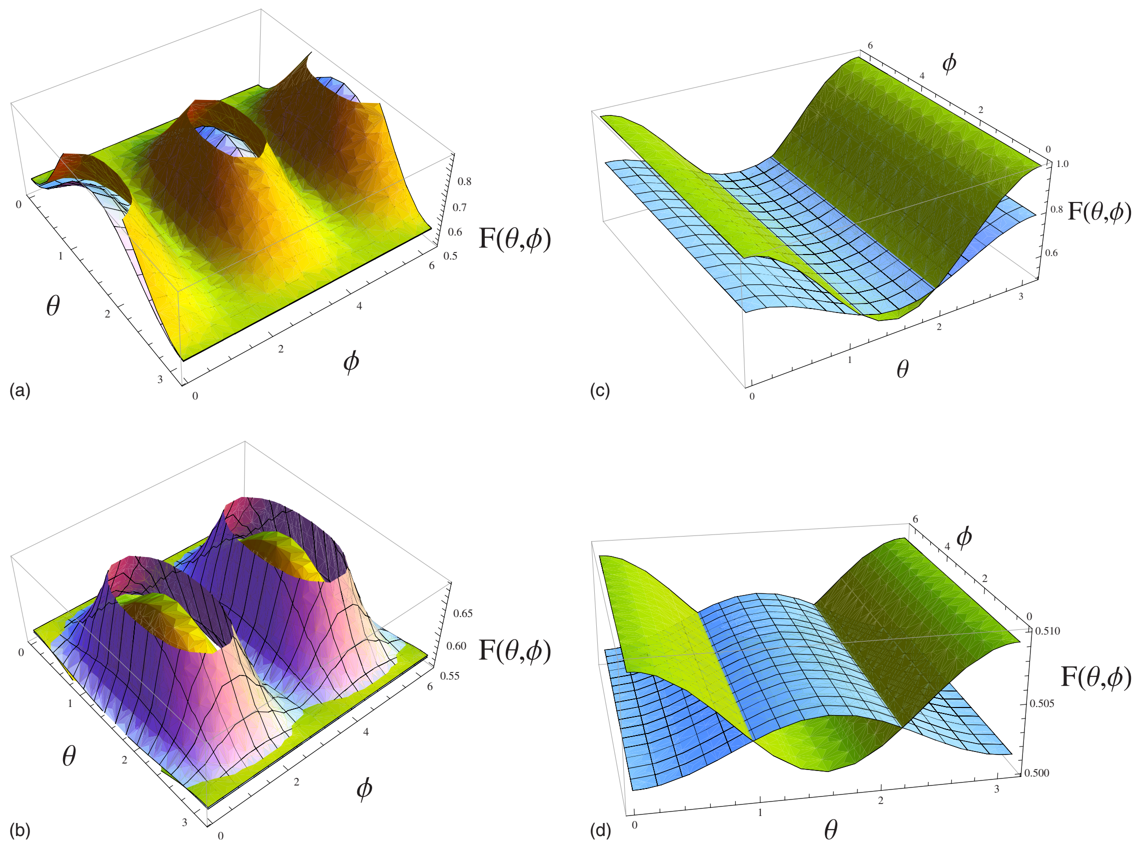

FIG. 5. (Color online) The plot of $(\theta,\ \phi)$ dependence of $F(\theta,\phi)$ for $(L_{2,x},L_{3,x},L_{4,x})$ (a), $(L_{2,y},L_{3,y},L_{4,y})$ (b), $(L_{2,z},L_{3,z},L_{4,z})$ (c), and isotropic (d) noisy channels. The transparent and opaque surfaces correspond to GHZ and W states, respectively. All figures are consistent with $\bar{F}$ given in Table I.

nels for each qubit in the given quantum channel such as the $(L_{2,x},L_{3,y},L_{4,z})$ noisy channel. In this sense the noisy channels discussed in this paper can be said to be oversimplified. The reason that we consider only the noisy channels with the same axis in this paper can be summarized as follows. First, the main purpose of this paper is to show explicitly that the robustness between GHZ and W states in the noisy teleportation is dependent on the noisy types. Thus, as shown in Fig. 4 it is sufficient to introduce the same-axis noisy channels. Another reason is that we would like to explore the cases of high fidelity because the quantum channels become useless if $\bar{F}$ is comparatively small. We conjecture that $\bar{F}$ with same-axis noisy channels are in general larger than $\bar{F}$ with different-axis noisy channels. For example, let us consider the teleportation with the EPR state depicted in Fig. 1. When the quantum channel is subject to $(L_{2,x},L_{3,x})$ or $(L_{2,z},L_{3,z})$ noisy channels, the average fidelity $\bar{F}$ is always

$$\bar{F}_1 = \frac{2}{3} + \frac{1}{3}e^{-4\kappa t}. \tag{5.1}$$

If, however, the quantum channel is subject to $(L_{2,x},L_{3,z})$ or $(L_{2,z},L_{3,x})$ noisy channels, direct calculation shows that the average fidelity reduces to

$$\bar{F}_D = \frac{1}{6}(3 + 2e^{-2\kappa t} + e^{-4\kappa t}), \tag{5.2}$$

which is smaller than $\bar{F}_1$ in full range of κt. This supports our conjecture although detailed calculation is needed for the complete proof.

Probably one may be able to increase $F(\theta,\phi)$ and $\bar{F}$ summarized in Table I via the purification of noisy channels discussed in Refs. [12,13]. To explore this issue, of course, we need another detailed calculation, which is beyond the scope of the present paper.

It is of interest to extend our paper to examine the fidelity measures $F(\theta,\phi)$ and $\bar{F}$ when other types of noisy channels such as amplitude damping or depolarizing channels are introduced. It is also equally interesting to examine the same noisy channels in other places such as noisy channels during Bell's measurement or the unitary operation.

The most important point we would like to explore in the future is to understand the physical reason why and how the robustness of GHZ and W states is dependent on the noisy types. In our opinion the most nice approach to understand the physical reason is to investigate the entanglement of the mixed states $\varepsilon(\rho_{\mathrm{GHZ}})$ and $\varepsilon(\rho_W)$. For example, let us consider the quantum teleportation through the noisy channels with the EPR state for brevity, which is fully discussed in Ref. [14]. In this case when the quantum channel is subject to

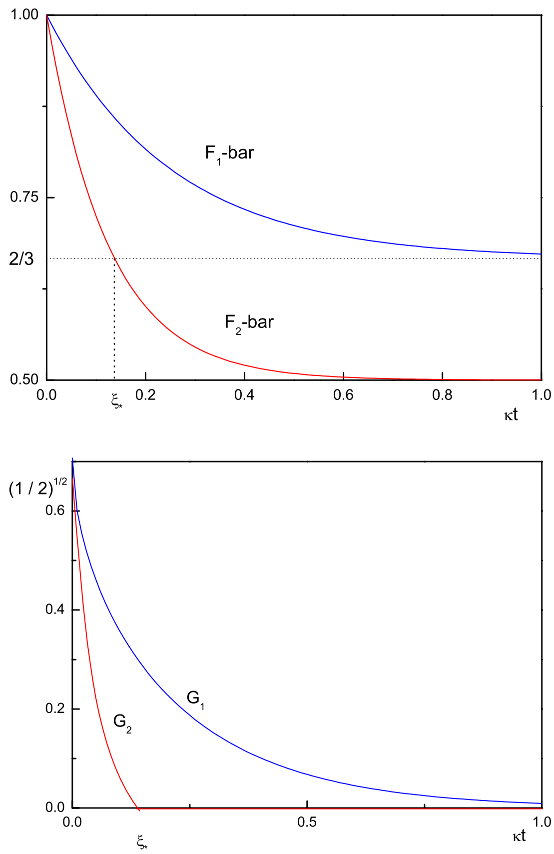

FIG. 6. (Color online) Conjecture of relation between $\bar{F}$ and mixed states Groverian measure. Since $\bar{F}_2$ becomes smaller than 2/3 when $\kappa t \geq \xi_* = -\ln(\sqrt{2}-1)/2$, the corresponding Groverian measure G_2 is expected to vanish in the same region.

$(L_{2,x}, L_{3,x})$, $(L_{2,y}, L_{3,y})$ or $(L_{2,z}, L_{3,z})$ noisy channels, the average fidelity $\bar{F}$ is always the same with Eq. (5.1), while the isotropic noisy channel gives

$$\bar{F}_2 = \frac{1}{2} + \frac{1}{2}e^{-8\kappa t}. \tag{5.3}$$

Then we think that an appropriate entanglement measure should have the following properties. The measure for the mixed state $\varepsilon_1(\rho)$ generated by $(L_{2,x}, L_{3,x})$, $(L_{2,y}, L_{3,y})$, and $(L_{2,z}, L_{3,z})$ noisy channels should decay to zero at $\kappa t \to \infty$ because $\bar{F} = 2/3$ implies that the mixed states do not play any role as quantum channels. By the same reason the measure for the mixed state $\varepsilon_2(\rho)$ generated by the isotropic noisy channel should vanish at $\kappa t \geq (1/8)\ln 3$.

If we take a Groverian entanglement measure $G(\rho)$ [9,16] as an entanglement measure, there is another constraint $G(\rho) = 1/\sqrt{2}$ at $\kappa t = 0$ because the Groverian measure for the pure EPR state is $1/\sqrt{2}$. As a result, we can conjecture that the Groverian measure G_1 and G_2 for $\varepsilon_1(\rho)$ and $\varepsilon_2(\rho)$ may exhibit as Fig. 6. We would like to show whether or not our conjecture is correct. In addition we would like to extend our conjecture to the quantum teleportation through noisy channels with GHZ and W states discussed in this paper.

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APPENDIX

Let, for simplicity, σ_{ij}^x and σ_{ij}^y be the density matrices for (L_{2x}, L_{3x}, L_{4x}) and (L_{2y}, L_{3y}, L_{4y}) noises, respectively. Then the master equation (1.6) makes the off-diagonal components of σ_{ij}^x and σ_{ij}^y satisfy the following coupled equations:

$$\frac{d\sigma_{07}^x}{dt} = -\kappa(3\sigma_{07}^x - \sigma_{16}^x - \sigma_{25}^x - \sigma_{43}^x),$$

$$\frac{d\sigma_{16}^x}{dt} = -\kappa(3\sigma_{16}^x - \sigma_{07}^x - \sigma_{34}^x - \sigma_{52}^x),$$

$$\frac{d\sigma_{25}^x}{dt} = -\kappa(3\sigma_{25}^x - \sigma_{07}^x - \sigma_{34}^x - \sigma_{61}^x),$$

$$\frac{d\sigma_{34}^x}{dt} = -\kappa(3\sigma_{34}^x - \sigma_{16}^x - \sigma_{25}^x - \sigma_{70}^x) \tag{A1}$$

and

$$\frac{d\sigma_{07}^y}{dt} = -\kappa(3\sigma_{07}^y + \sigma_{16}^y + \sigma_{25}^y + \sigma_{43}^y),$$

$$\frac{d\sigma_{16}^y}{dt} = -\kappa(3\sigma_{16}^y + \sigma_{07}^y + \sigma_{34}^y + \sigma_{52}^y),$$

$$\frac{d\sigma_{25}^y}{dt} = -\kappa(3\sigma_{25}^y + \sigma_{07}^y + \sigma_{34}^y + \sigma_{61}^y),$$

$$\frac{d\sigma_{34}^y}{dt} = -\kappa(3\sigma_{34}^y + \sigma_{16}^y + \sigma_{25}^y + \sigma_{70}^y), \tag{A2}$$

and their complex conjugates. Then it is easy to show that $\sigma_{07}^x = \sigma_{70}^x = \alpha_+/8$, $\sigma_{16}^x = \sigma_{61}^x = \sigma_{25}^x = \sigma_{52}^x = \sigma_{34}^x = \sigma_{43}^x = \alpha_-/8$, $\sigma_{07}^y = \sigma_{70}^y = \beta_1/8$, and $\sigma_{16}^y = \sigma_{61}^y = \sigma_{25}^y = \sigma_{52}^y = \sigma_{34}^y = \sigma_{43}^y = -\beta_2/8$ satisfy Eq. (A1) and Eq. (A2). Also these solutions satisfy the boundary condition $\sigma_{ij}^x = \sigma_{ij}^y = \rho_{\rm GHZ}$ at $\kappa t = 0$.

If we ignore the boundary condition, many different solutions for σ_{ij}^y can be obtained from σ_{ij}^x. For example, $\sigma_{07}^y = -\sigma_{70}^y = i\alpha_+$ and $\sigma_{16}^y = -\sigma_{61}^y = \sigma_{25}^y = -\sigma_{52}^y = \sigma_{43}^y = -\sigma_{34}^y = -i\alpha_-$ are also solutions of Eq. (A2). These are the solutions derived from $(u \otimes u \otimes u)\sigma_{ij}^x (u \otimes u \otimes u)^\dagger$ when

$$u = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1-i \\ 1+i & 0 \end{pmatrix}. \tag{A3}$$

Even if these are solutions of Eq. (A2), they do not satisfy the proper boundary condition.

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