PHYSICAL REVIEW A **78**, 011803(R) (2008)

Generation of a self-pulsed picosecond solitary wave train from a periodically amplifying Bragg structure

Jianhua Zeng,[1,2] Jianying Zhou,[1,*] Gershon Kurizki,[2,†] and Tomas Opatrny[3]
[1]*State Key Laboratory of Optoelectronic Materials and Technologies, Sun Yat-sen University, Guangzhou 510275, China*
[2]*Department of Chemical Physics, Weizmann Institute of Science, Rehovot 76100, Israel*
[3]*Department of Theoretical Physics, Palacký University, Olomouc 77200, Czech Republic*

(Received 27 December 2007; published 8 July 2008)

Self-pulsed lasing in the form of a coherent solitary wave train from a periodically amplifying Bragg structure is proposed. It is shown both analytically and numerically that the generated pulse train propagates in the periodically amplifying Bragg structure chip with superluminal group velocity. Its stability and coherence are in contrast with the inherent instability and chaoticity of self-pulsing from a uniform gain medium embedded in a Bragg reflector. Bragg-periodic semiconductor quantum-well heterostructures are candidates for realizing the required periodically amplifying Bragg structure.

DOI: 10.1103/PhysRevA.78.011803 PACS number(s): 42.55.Tv, 42.60.Mi, 42.65.Re

Techniques for the generation and control of high-repetition-rate ultrashort optical pulses play an increasingly important role in a variety of applications, ranging from high-speed optical communications, through optical computing to ultrafast signal processing. Among those, active [1] or passive [2] mode-locking techniques are considered to be very reliable for generating picosecond and subpicosecond (subps) optical pulses with ultrahigh repetition rate. The setback of mode-locking techniques is that they require a very long cavity to produce ultrashort optical pulses. In this Rapid Communication, we put forward a very compact and simple scheme for a laser chip that permits the generation of a train of stable ultrashort optical pulses. Although the scheme proposed here shares certain features with distributed feedback (DFB) lasers [3,4], its principles of operation and the resulting emission characteristics are essentially different.

Unlike cavity-mirror lasers, DFB lasers make use of optical feedback that is distributed throughout the structure. This feedback is via backward Bragg scattering from periodic modulations of the refractive index and the gain of the medium. Advantageous types of DFB lasers have been based on photonic crystals (PCs), i.e., structures with periodic variation of the refractive index where light is Bragg reflected in the linear low intensity regime within frequency bands corresponding to photonic band gaps (PBGs) [5]. In such structures, nonlinear pulse propagation near the photonic band edge gives rise to optical path length increase and small group velocity, leading to low-threshold laser action and DFB gain enhancement in both one-dimensional (1D) [6] and 2D [7] Bragg reflectors or PCs.

Here, we put forward a regime of laser action in a hitherto unexpected periodically amplifying Bragg structure (PABS). Analogously to the resonantly absorbing Bragg reflector (RABR) model introduced by Kurizki and co-workers [8], the PABS described here consists of a periodic array of thin layers of resonantly amplifying two-level systems (TLSs) separated by half-wavelength, nonabsorbing, passive dielectric layers, as shown in Fig. 1(a). As opposed to RABR, the TLSs are required to be inverted, i.e., predominantly pumped to the excited state. The feedback mechanism in PABS differs from that of existing DFB lasers [6,7], the essential innovation being that the active medium in PABS is arranged in thin layers, rather than uniformly distributed throughout the DFB structure. As shown below, it is this spatial gain pattern that is capable of suppressing the intrinsic instabilities of an ordinary gain medium, so as to stabilize the laser pulses generated in PABS.

Our analytical and numerical solutions demonstrate that a high-repetition-rate train of mutually coherent stable solitary waves with a bandwidth up to 90.1 GHz and pulse duration of few picoseconds (ps) can be generated in a PABS from a small seed pulse or from spontaneous emission noise. Remarkably, the solitary waves generated in the PABS exhibit superluminal group and energy velocities, previously demonstrated for weak-gain propagation [9,10].

Let us consider the coherent buildup of a light field starting from a weak signal or spontaneous emission noise in PABS. The relevant time scale considered here is longer than τ_c, the buildup time of a cooperative (superradiant) pulse from spontaneous emission [8,11]. $\tau_c=\bar{n}\mu_0^{-1}\sqrt{\varepsilon_0\hbar/2\pi\omega_0\rho_0}$, $\bar{n}$ being the mean refractive index of the structure and ω_0 the

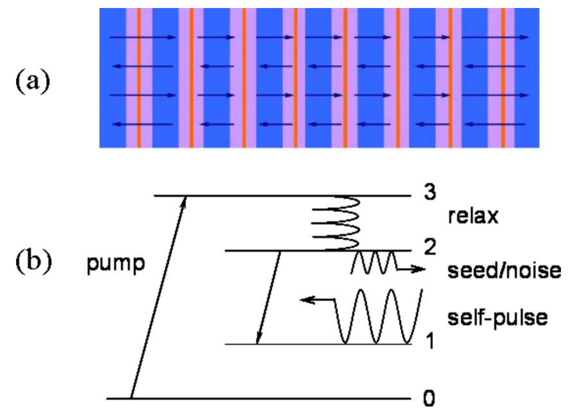

FIG. 1. (Color online) Schematic illustration of (a) propagation in PABS (thin layer embedded in a Bragg reflector); (b) pumping.

*stszjy@mail.sysu.edu.cn
†gershon.kurizki@weizmann.ac.il

TLS resonant frequency; μ_0 and ρ_0 are the transition dipole moment and density of the TLS, respectively. Under the slowly varying envelope and rotating-wave approximations, the semiclassical laser action in a PABS is then described by the coupled-mode two wave Maxwell-Bloch equations [8,12] in the dimensionless variables $x=x'/l_c$ and $t=t'/\tau_c$, $l_c=c\tau_c$:

$$\frac{\partial\Omega^+}{\partial t}+\frac{\partial\Omega^+}{\partial x}=i\eta\Omega^-+P+K(x,t),$$

$$\frac{\partial\Omega^-}{\partial t}-\frac{\partial\Omega^-}{\partial x}=i\eta\Omega^++P+K(x,t),$$

$$\frac{\partial P}{\partial t}=-\left(i\delta+\frac{1}{T_2}\right)P+(\Omega^++\Omega^-)w,$$

$$\frac{\partial w}{\partial t}=-\frac{1}{T_1}(w-1)-\mathrm{Re}[(\Omega^++\Omega^-)P^*]. \quad (1)$$

Here $\Omega^{\pm}=(2\tau_c\mu_0/\hbar)E^{\pm}$; $E^{\pm}$ are the smooth field envelopes of the forward and backward Bloch waves; $\eta=n_1\omega_0\tau_c/4$ is the Bragg reflectivity with the modulation depth of refractive index n_1; $K(x,t)$ is either an input signal (probe) or a random spontaneous-emission noise field that initiates the laser action; P and w are the spatially dependent polarization and density of population inversion, respectively; δ is the detuning from the exact Bragg condition; $T_{1,2}$ are the effective relaxation times of the $|0\rangle\leftrightarrow|3\rangle$ transition in Fig. 1(b), given by $T_{1,2}=\Gamma_{pump}+\Gamma_{1,2}$, where Γ_{pump} is the pumping rate and $\Gamma_{1,2}$ are the relevant longitudinal and transverse relaxation times. Population inversion on the lasing transition $|2\rangle\leftrightarrow|1\rangle$ can be realized using the standard four-level configuration. As depicted in Fig. 1(b), the lowest state $|0\rangle$ population is excited to the highest state $|3\rangle$ through optical pumping, and then rapidly relaxes to state $|2\rangle$, thereby achieving population inversion between states $|2\rangle$ and $|1\rangle$.

We assume that at $t=0$ there is no polarization in the PABS and the TLS population is inverted about the periodic positions of the active layers (labeled by index j), with spatial profile $f(x-x_j)$. The initial and boundary conditions are then given by

$$\Omega^+(x=0,t)=\Omega^-(x=L,t)=0,$$

$$\Omega^{\pm}(x,t=0)=0,\quad P(x,t=0)=0,$$

$$w(x,t=0)=w_0\sum_j f(x-x). \quad (2)$$

The analytical ansatz for the output field that solves Eqs. (1) and (2) upon taking $T_{1,2}\to\infty$, the active layers infinitely thin [8] $f(x-x_j)\to\delta(x-x_j)$, and the seed term as $K(x=0,t)=A_0\sec h(t)$, is then found to be a periodic, phase-coherent train of sech solitons,

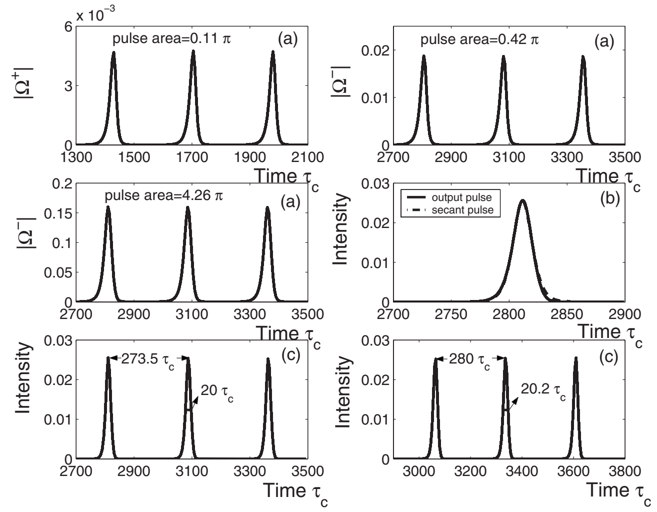

FIG. 2. (a) Solitary wave train amplitudes as a function of time (in units of τ_c) for forward $|\Omega^+|$ and backward $|\Omega^-|$ generated pulses with various pulse areas for $\tau_c=200$ fs, structure length $L=20l_c$, and the longitudinal and transverse relaxation times $T_1=10^4\tau_c$ and $T_2=5\tau_c$, respectively. (b) Zoom-in view on a single solitary wave pulse within the train: pulse form (solid) in comparison with sech pulse (dashed). (c) Left: repetition rate (18.3 GHz) and full width at half maximum (4 ps) of the output pulse train in a PABS with infinitely thin active layers. Right: same, in a PABS with active layers of 20-nm atomic width.

$$\Sigma_+(x=L,t)=\tau_c(\Omega^++\Omega^-)$$
$$=A\sum_{n=1}^{N}e^{i\chi_0(t-nt_0)}\sec h[v(t-nt_0)/t_p]. \quad (3)$$

Here, all the solitary wave-pulse amplitudes A are the same, χ_0 is the phase shift of each pulse, v its group velocity, and t_p its pulse width. The corresponding population inversion yields

$$w=-1-\frac{\chi_0}{2(\eta-\delta)}A^2\sum_{n=1}^{N}\frac{1}{\cosh^2[v(t-nt_0)/t_p]}. \quad (4)$$

Using these explicit expressions, we can relate their parameters as follows:

$$\chi_0=\frac{-\delta\pm\sqrt{\delta^2+3\eta^2+6+3v^2/\tau_p^2}}{3},$$

$$|A|=\sqrt{\frac{6v^2(\eta-\delta)/\tau_p^2}{-\delta+\sqrt{\delta^2+3\eta^2+6+3v^2/\tau_p^2}}}. \quad (5)$$

To compare this solitary wave train solution to numerical simulations, we have used an implicit fourth-order Runge-Kutta numerical method [13] to solve the Maxwell-Bloch equations (1) with the initial and boundary condition (2) and noise as input. Very good agreement has been obtained for ultrashort laser-pulse generation between Eq. (3) and our numerical simulation (Fig. 2) which illustrates the output pulse train. It is seen from the picture that the shape of the pulses generated in the PABS differs only slightly from that of the

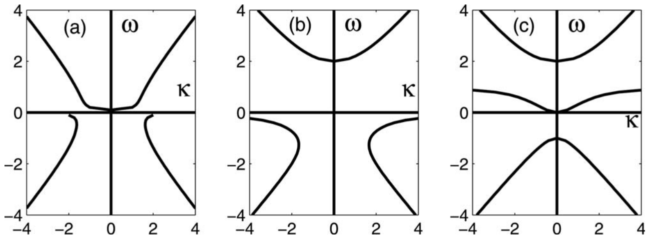

FIG. 3. The dispersion relation in Eq. (4) for (a) $\eta=0.1$, $\delta=0$, (b) $\eta=2$, $\delta=0$, and (c) $\eta=2$, $\delta=1$.

sech soliton of the exact sine-Gordon equation [14]. Remarkably, the output sech pulse area can be much smaller than 2π, as opposed to self-induced transparency (SIT) solitons in bulk media [9,14]. The multiple reflections in the structure compensate for this pulse weakness.

Whereas analytically we have assumed that the TLS active layers are infinitely thin, we have also numerically modeled a more realistic PABS by taking into account the finite width of active layers [8]. Numerical checks prove that the ultrashort pulses can be generated in a PABS even when the layer width is up to 20 nm, as shown in Fig. 2(c).

The single-pass time spent by the solitary wave pulse corresponds to a group velocity of the solitary waves that is 2.5 times larger than the vacuum speed of light. This superluminal (fast light) effect is characteristic of propagation in a gain medium [9,15]. This conforms to the linearized dispersion relation in PABS (fixing $w=1$), which has the form

$$(\delta-\omega)[k^2+\eta^2-(\omega^2+2)]-2(\eta-\delta)=0\cdot \qquad (6)$$

As seen from Fig. 3, this dispersion relation has branches corresponding to superluminal propagation whenever $d\omega/dk>1$.

To investigate the mechanism of a self-pulse generation in the PABS, we have plotted the spatiotemporal evolution process in Fig. 4. The internal DFB effect causes the spontaneous emission noise to grow fast into a certain laser mode. As described in Fig. 4(c), this process can be repeated many times whenever the threshold of relaxation oscillation is crossed, allowing us to observe the output of the solitary wave train pulses. This formation process is seen [Fig. 4(a)] to be analogous to superradiant buildup of the output [11], but the spatially periodic inversion imposed here makes it unique. As can be seen in Fig. 4(c), the threshold inversion is very small because of the very weak loss in a PABS. The unique feature of the PABS is that the field intensities in different layers of the gain medium is correlated due to interlayer coupling, giving rise to a well determined cavity lifetime [16]. This results in an increased repetition rate as well as improved stability of the laser output by relaxation oscillation [17]. The inversion periodicity acts as a spatial mode filter that regularizes the output pulses. Consequently, the system is robust, i.e., insensitive to the initial noise intensity, as shown in Fig. 4(d).

For comparison, we investigated the laser behavior in a conventional PC (Bragg reflector) structure—uniformly doped by active gain material [12]. Equations (1) are then modified as follows:

$$w,P\rightarrow w_{1,2},P_{1,2},$$

where P_1 and P_2 are slowly varying envelope components of the TLS polarization driving Ω^+ and Ω^-, respectively; w_1 is the density of the inverted population, and w_2 is its modulation depth of population. The numerical simulation of the output intensity of laser pulse generation in a uniformly doped PC is presented in Fig. 4(e), which shows that self-pulsing instability can occur. This comes about since in the uniformly doped structure, the internal gain DFB stimulates

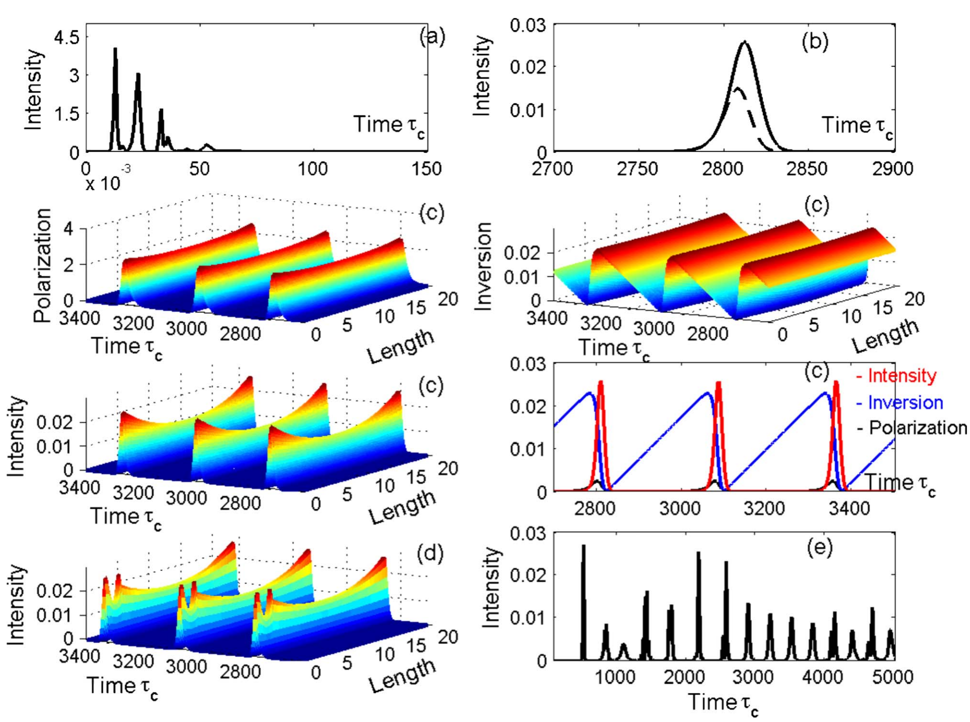

FIG. 4. (Color online) (a) The coherent superradiant emission pattern prior to the formation of solitary wave train pulses. (b) Emission intensity of solitary wave pulses at the middle (dashed) and the end (solid) of the PABS length. (c) The spatiotemporal evolution process of the solitary wave pulse in a PABS. (d) Effects of additive noise on the output pulses. (e) Emission intensity of pulse generation in a uniformly doped nonlinear PBG material.

the self-pulsing and leads to the chaotic behavior of the pulse. As we have described above, it is the spatially periodic gain pattern in PABS that is capable of suppressing these intrinsic instabilities. Thus, the PABS lasing structure described here is a highly advantageous source of stable, mutually coherent, ultrashort laser pulses.

We have considered the realization of such multiple laser pulses using longitudinal and transverse relaxation times for doped semiconductor heterostructures [18]. For many kinds of dopants, we have numerically checked that ultrashort multiple-pulse generation can be obtained. Remarkably, repetition rates up to 90.1 GHz and full width at half maximum (FWHM) of 1.84 picoseconds can be directly obtained from our numerical simulation of a PABS based on a quantum-well semiconductor structure. The corresponding parameter values are as follows: longitudinal and transverse relaxation times $T_1=10^3\tau_c$ and $T_2=5\tau_c$, respectively; mean refractive index $\bar{n}=3.6$, angular frequency $\omega_0=2.26\times10^{15}$ s^{-1}, transition dipole moment $\mu_0=4.5\times10^{-29}$ C m, and two-level system (exciton) density $\rho_0=1\times10^{18}$ cm^{-3}, respectively. Then the corresponding characteristic absorption time is $\tau_c=200$ fs. With the development of crystal growth and microfabrication techniques, this PABS device can be fabricated as a Bragg-periodic semiconductor quantum-well structure [19].

In conclusion, we have demonstrated analytically and numerically a lasing regime: the generation of highly stable high-repetition-rate picosecond solitary wave pulse train in a PABS chip. In contrast, the generation of ultrashort laser pulses in uniformly doped gain media tends to be chaotic because of the existence of instabilities. Bragg-reflecting periodic semiconductor quantum-well heterostructures are shown to be viable candidates for realizing these effects.

Support for this project is by the National Key Basic Research Special Foundation (Grant No. G2004CB719805) and the Chinese National Natural Science Foundation (Grants No. 60677051 and No. 10774193). G.K. acknowledges support from EC (SCALA), GIF, and DIP.

[1] T. F. Carruthers and I. N. Duling III, Opt. Lett. **21**, 1927 (1996); E. Yoshida, N. Shimizu, and M. Nakazawa, IEEE Photon. Technol. Lett. **11**, 1587 (1999); J. M. Melkonian *et al.*, Opt. Lett. **32**, 1701 (2007).
[2] Y. K. Chen *et al.*, Appl. Phys. Lett. **58**, 1253 (1991); J. Schröder *et al.*, Opt. Lett. **31**, 3489 (2006).
[3] H. Kogelnik and C. V. Shank, Appl. Phys. Lett. **18**, 152 (1971); C. V. Shank, J. E. Bjorkholm, and H. Kogelnik, *ibid.* **18**, 395 (1971); H. Kogelnik and C. V. Shank, J. Appl. Phys. **43**, 2327 (1972).
[4] H. G. Winful, J. H. Marburger, and E. Garmire, Appl. Phys. Lett. **35**, 379 (1979); J. E. Ehrlich, G. Assanto, and G. I. Stegeman, *ibid.* **56**, 602 (1990); Y. H. Liao and H. G. Winful, Opt. Lett. **21**, 471 (1996); H. J. Wunsche *et al.*, Phys. Rev. Lett. **94**, 163901 (2005).
[5] E. Yablonovitch, Phys. Rev. Lett. **58**, 2059 (1987); S. John, *ibid.* **58**, 2486 (1987); C. M. Soukoulis, *Photonic Band Gap Materials* (Kluwer, Dordrecht, 1996); K. Busch *et al.*, Phys. Rep. **444**, 101 (2007).
[6] J. P. Dowling *et al.*, J. Appl. Phys. **75**, 1896 (1994); V. I. Kopp *et al.*, Opt. Lett. **23**, 1707 (1998); G. Strangi *et al.*, Phys. Rev. Lett. **94**, 063903 (2005); R. D. Xia *et al.*, Adv. Mater. (Weinheim, Ger.) **19**, 4054 (2007).
[7] M. Meier *et al.*, Appl. Phys. Lett. **74**, 7 (1999); K. Sakoda, Opt. Express **4**, 167 (1999); K. Sakoda *et al.*, *ibid.* **4**, 481 (1999); S. Riechel *et al.*, Appl. Phys. Lett. **77**, 2310 (2000); M. Notomi *et al.*, *ibid.* **78**, 1325 (2001); K. Forberich *et al.*, J. Appl. Phys. **100**, 023110 (2006); I. V. Konoplev *et al.*, Phys. Rev. Lett. **96**, 035002 (2006); B. Ellis *et al.*, Appl. Phys. Lett. **90**, 151102 (2007).
[8] A. Kozhekin and G. Kurizki, Phys. Rev. Lett. **74**, 5020 (1995); A. E. Kozhekin, G. Kurizki, and B. Malomed, *ibid.* **81**, 3647 (1998); T. Opatrny, B. A. Malomed, and G. Kurizki, Phys. Rev. E **60**, 6137 (1999); G. Kurizki *et al.*, Prog. Opt. **42**, 93 (2001).
[9] R. Y. Chiao, A. E. Kozhekin, and G. Kurizki, Phys. Rev. Lett. **77**, 1254 (1996).
[10] M. Blaauboer, A. G. Kofman, A. E. Kozhekin, G. Kurizki, D. Lenstra, and A. Lodder, Phys. Rev. A **57**, 4905 (1998).
[11] R. H. Dicke, Phys. Rev. **93**, 99 (1954); C. Sibilia *et al.*, J. Opt. A, Pure Appl. Opt. **7**, S198 (2005).
[12] N. Akozbek and S. John, Phys. Rev. E **58**, 3876 (1998); L. Florescu, K. Busch, and S. John, J. Opt. Soc. Am. B **19**, 2215 (2002).
[13] C. M de Sterke, K. R. Jackson, and B. D. Robert, J. Opt. Soc. Am. B **8**, 403 (1991).
[14] B. I. Mantsyzov and R. N. Kuz'min, Sov. Phys. JETP **64**, 37 (1986).
[15] B. D. Clader and J. H. Eberly, J. Opt. Soc. Am. B **24**, 916 (2007).
[16] M. Hübner *et al.*, Phys. Rev. Lett. **76**, 4199 (1996).
[17] A. E. Siegman, *Lasers* (University Science Books, Mill Valley, CA, 1986).
[18] J.-C. Diels and W. Rudolph, *Ultrashort Laser Pulse Phenomena: Fundamentals, Techniques, and Applications on a Picosecond Timescale* (Academic, San Diego, 1996).
[19] G. Khitrova *et al.*, Rev. Mod. Phys. **71**, 1591 (1999); J. P. Prineas *et al.*, Appl. Phys. Lett. **81**, 4332 (2002); D. Goulding *et al.*, Phys. Rev. Lett. **98**, 153903 (2007); Johnny C. Ho *et al.*, Nat. Mater. **7**, 62 (2008).