# Slow light by coherent hole burnings

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We show that the simultaneous application of a copropagating saturating pump and a counterpropagating coherent beam can be used to burn a narrow spectral hole within the absorption line of the optical transition in a Doppler-broadened medium. The large index of refraction of this hole slows down a light pulse by a factor of about $10^4$. In addition, we propose a method to create two-color slow light pulses with simultaneous gain by employing a bichromatic field to saturate the medium.

## I. INTRODUCTION

In dispersive media, the reduction of the optical group velocity by several orders of magnitude has attracted a great deal of interest [1–6]. There are many applications that can benefit from light with slow group velocities, for example, controllable optical delay lines [7], low-light-level nonlinear optics [8], optical communication, and quantum-information storage. Several methods have been used to achieve slow light: electromagnetically induced transparency (EIT) [9–12], population oscillations [13], nonlinear magneto-optics [3], gain resonances [14,15], and photonic lattices [16]. In particular, Agarwal and Dey [17] have proposed a Doppler-broadened two-level system to produce slow light using optical hole burning (OHB). Shakhmuratov *et al.* [18] extended this work to a persistent spectral hole-burning (PSHB) technique that can be realized in many solid materials.

Considering both EIT and OHB, Dong and Gao [19] proposed a coherent hole-burning (CHB) technique that allows the simultaneous observation of an EIT window and four hole burnings when two laser beams (one saturating and one coupling) drive a Doppler-broadened atomic medium in the ladder configuration. Wu *et al.* [20,21] extended the CHB theory into a Λ-type Doppler-broadened atomic system and observed better CHBs because of matched transition wavelengths.

In this paper, we propose an approach to achieve slow-down of a light pulse with CHB. We burn a narrow spectral hole in the Doppler-broadened absorption line by the simultaneous application of a copropagating saturating pump and a counterpropagating coherent beam. Using this method, we show that the group velocity in the atomic vapor is reduced by several orders of magnitude, a larger slowdown than that induced by general hole burning under the same conditions. We can also obtain a steeper slope of the refractive index with larger transmission when the probe pulse is slightly detuned from resonance. An important advantage of using CHB is that two narrow propagating pulses accompanied by am-

plification can be retarded when a bichromatic laser field is used as the saturating pump.

## II. THE MODEL AND EQUATIONS

The Λ-type system under consideration is shown in Fig. 1 for two different excitation schemes. In the first scheme, shown in Fig. 1(a), levels $|2\rangle$ and $|3\rangle$ are coupled by an intense laser with frequency $\omega_c$ and Rabi frequency $\Omega_c$. A weak probe field with frequency $\omega_p$ and Rabi frequency $\Omega_p$ detects the absorption on transition $|1\rangle \leftrightarrow |3\rangle$. Another laser with frequency $\omega_s$ and Rabi frequency $\Omega_s$ also acts on transition $|1\rangle \leftrightarrow |3\rangle$, and is used to saturate the medium. $\Delta_c = \omega_{32} - \omega_c$, $\Delta_s = \omega_{31} - \omega_s$, and $\Delta_p = \omega_{31} - \omega_p$ are detunings of the three fields. $\Gamma_{31}$ and $\Gamma_{32}$ denote the spontaneous decay rates from level $|3\rangle$ to levels $|1\rangle$ and $|2\rangle$. $\Gamma_{21}$ designates the population damping rate of levels $|1\rangle$ and $|2\rangle$.

Based on the semiclassical theory, using the standard density-matrix formalism with the dipole approximation and the rotating wave approximation, the interaction Hamiltonian $H_I$ for this system is

$$H_I = (\Delta_s - \Delta_c)|2\rangle\langle2| + \Delta_s|3\rangle\langle3| - (\Omega_s|1\rangle\langle3| + \Omega_c|2\rangle\langle3| + \text{c.c.}).$$

$$(1)$$

For this scheme, the detailed calculation of the density-matrix equations in the interaction picture and the generation or degeneracy of CHBs under the dressed-state representa-
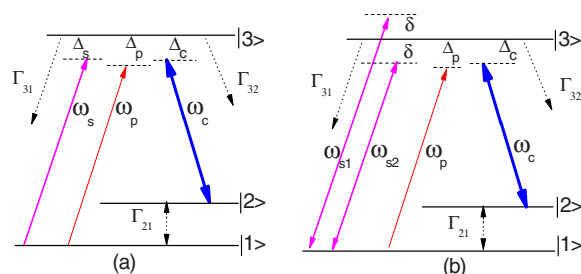


FIG. 1. (Color online) Energy level diagram of a Doppler-broadened Λ-type system driven by probe $\Omega_p$, coupling $\Omega_c$, and (a) saturating $\Omega_s$ or (b) bichromatic saturating $\Omega_{s1,s2}$ beams.

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tion has been explained in our previous work [20,21].

Here, we emphasize the method of calculation for the second excitation scheme, shown in Fig. 1(b), where we extend the CHB technique by employing a bichromatic linearly polarized laser field to excite the atoms to saturation,

$$E(t) = \frac{1}{2}(E_{s1}e^{i\delta t} + E_{s2}e^{-i\delta t})e^{-i\omega_{31}t} + \text{c.c.} \tag{2}$$

$E_{s1}$ and $E_{s2}$ are the amplitudes of the two frequency components $\omega_{s1}$ and $\omega_{s2}$ of the bichromatic field. $\Delta_{s1(s2)} = \omega_{s1(s2)} - \omega_{31} = +(-)\delta$ is the corresponding detuning. $2\delta$ is the frequency difference between the two laser field components. Therefore, the master equation for the density-matrix operators should be

$$\dot{\rho}_{11} = i(\Omega_{s1}^* e^{-i\delta t} + \Omega_{s2}^* e^{i\delta t})\rho_{31} - i(\Omega_{s1}e^{i\delta t} + \Omega_{s2}e^{-i\delta t})\rho_{13}$$
$$+ \Gamma_{31}\rho_{33} + \Gamma_s(\rho_{22} - \rho_{11}),$$

$$\dot{\rho}_{22} = i\Omega_c^*\rho_{32} - \Omega_c\rho_{23} + \Gamma_{32}\rho_{33} + \Gamma_s(\rho_{11} - \rho_{22}),$$

$$\dot{\rho}_{12} = [i(\Delta_s - \Delta_c) - \gamma_{12}]\rho_{12} + i\Omega_s\rho_{32} - i\Omega_c^*\rho_{13},$$

$$\dot{\rho}_{13} = (i\Delta_s - \gamma_{13})\rho_{13} + i(\Omega_{s1}e^{i\delta t} + \Omega_{s2}e^{-i\delta t})(\rho_{33} - \rho_{11}) - i\Omega_c\rho_{12},$$

$$\dot{\rho}_{23} = (i\Delta_c - \gamma_{23})\rho_{23} + i\Omega_c(\rho_{33} - \rho_{22}) - i(\Omega_{s1}e^{i\delta t} + \Omega_{s2}e^{-i\delta t})\rho_{21},$$

$$\rho_{11} + \rho_{22} + \rho_{33} = 1,$$

$$\rho_{ij} = \rho_{ji}^*. \tag{3}$$

Here, $\Omega_{s1} = E_{s1}\mu_{13}/2\hbar$, $\Omega_{s2} = E_{s2}\mu_{13}/2\hbar$, and $\Omega_c = E_c\mu_{23}/2\hbar$ are the Rabi frequencies, where $\mu_{ij}$ are the electric dipole moments. $\gamma_{ij}$ refers to the rate of coherence decay between levels $|i\rangle$ and $|j\rangle$: $\gamma_{12} = \gamma_{21} = \Gamma_s$, $\gamma_{13} = \gamma_{31} = \gamma_{23} = \gamma_{32} = \frac{1}{2}(\Gamma_{31} + \Gamma_{32} + \Gamma_s)$.

In order to solve these equations, we expand the density-matrix elements $\rho_{jk}$ in a Fourier series as

$$\rho_{jk} = \sum_{n=-\infty}^{+\infty} X_k^{(n)} e^{in\delta t} \quad (j,k = 1,2,3), \tag{4}$$

where $X_k^{(n)}$ represent the slowly varying amplitudes and are needed to determine the absorption spectra, and $X_{k(k=1-8)}^n = (\rho_{31}, \rho_{13}, \rho_{21}, \rho_{12}, \rho_{32}, \rho_{23}, \rho_{11}, \rho_{22})^T$ [22].

According to the linear response theory, the steady-state absorption and refraction spectrum of the weak probe laser can be written as

$$A(\Delta_p) = \text{Re}\left(\int_0^\infty \lim_{t\to\infty}\langle[P^-(t+\tau), P^+(t)]\rangle e^{i\omega_p\tau}\right), \tag{5}$$

$$n(\Delta_p) = \text{Im}\left(\int_0^\infty \lim_{t\to\infty}\langle[P^-(t+\tau), P^+(t)]\rangle e^{i\omega_p\tau}\right), \tag{6}$$

where $P^- = \mu_{31}|3\rangle\langle1|$ and $P^+ = \mu_{13}|1\rangle\langle3|$ are atomic polarization operators with dipole moments $\mu_{31}$ and $\mu_{13}$ on transition $|1\rangle \leftrightarrow |3\rangle$.

Following the method of calculations for the bichromatic manipulation of absorption spectrum [23], we can get the

refractive index spectra of the probe laser. To consider the Doppler frequency shift, we have to substitute $\Delta_p$, $\Delta_s$, and $\Delta_c$ with $\Delta_p + \omega_{31}v/c$, $\Delta_s + \omega_{31}v/c$, and $\Delta_c - \omega_{32}v/c$, where $+$ $(-)$ denotes that the saturating (coupling) laser is copropagating (counterpropagating) with the probe laser. If the number of atoms per unit volume with velocity $v$ is $N(v)dv$, then the absorption and refraction spectra of the probe laser read

$$A(\Delta_p) = \int_{-\infty}^\infty A(\Delta_p, v)N(v)dv = \int_{-\infty}^\infty A(\Delta_p, v)\frac{N_0}{v_p\sqrt{\pi}}e^{-v^2/v_p^2}dv, \tag{7}$$

$$n(\Delta_p) = \int_{-\infty}^\infty n(\Delta_p, v)N(v)dv = \int_{-\infty}^\infty n(\Delta_p, v)\frac{N_0}{v_p\sqrt{\pi}}e^{-v^2/v_p^2}dv, \tag{8}$$

where $N_0$ is the total number of atoms, and $v_p = \sqrt{\frac{2kT}{m}} = \sqrt{\frac{2RT}{M}}$ represents the most probable atomic velocity. The Doppler width of the probe absorption spectrum is given by $\delta\omega_D = \frac{\omega_{31}}{c}\sqrt{\frac{8kT\ln 2}{m}} = \frac{2\omega_{31}}{c}\sqrt{\frac{2RT\ln 2}{M}} = 2\sqrt{\ln 2}\frac{\omega_{31}v_p}{c}$.

Of particular interest to us is the group velocity of pulse light, which is determined by the relation $v_g = c/n_g$, where $n_g$ is the group index: $n_g = n(\Delta_p) + \omega\frac{\partial n(\Delta_p)}{\partial \Delta_p}$.

## III. NUMERICAL RESULTS AND DISCUSSION

### A. The first scheme [Fig. 1(a)]

The Doppler-broadened probe absorption, refractive index, and group index as functions of the frequency detuning of probe field are given in Fig. 2. At resonance, there exists a rather narrow and deep hole. The other two side CHBs are asymmetric both in position and in depth. This is consistent with our previous theoretical analysis [20,21,23], which predicts that CHBs should be located at $\Delta_p = \Delta_s$ and $\Delta_p = [\Delta_c + 5\Delta_s \pm 3\sqrt{(\Delta_c + \Delta_s)^2 + 8\Omega_c^2}]$. The steep dispersion of the central hole creating the large group index at the same frequency will cause slow light with large slowdown factor. We have also shown the behavior in the region of resonance with different intensity of $\Omega_s$ (4, 3, 2.5, and 2 MHz) in Figs. 2(a')–2(c'). There is a coherent contribution to this hole from the response for the coupling and probe fields and an incoherent contribution from the hole burning.

If we take out the pump $\Omega_s$ and change the propagation direction of the coupling field $\Omega_c$ in order to eliminate the Doppler broadening, this scheme becomes the typical EIT. It is well known that EIT allows us to obtain a steeper slope of the refractive index and thereby a large group index; however, it is usually necessary to cool the medium to very low temperatures. We need a technique, such as OHB, in which it is possible to provide a high slowdown factor of the group velocity at room temperature. Here, we propose another option, using CHB technology, to create slow light at room temperature. It is well known that the Doppler-broadening effect usually degrades the slow light effect, while in our approach, it can instead be used to spectrally tune the slow light propagation.

For the detuning value of $\Delta_s = 20$ MHz and $\Delta_c = 0, 50, 100, 150$ MHz [Figs. 2(a'')–2(c'')], we can find
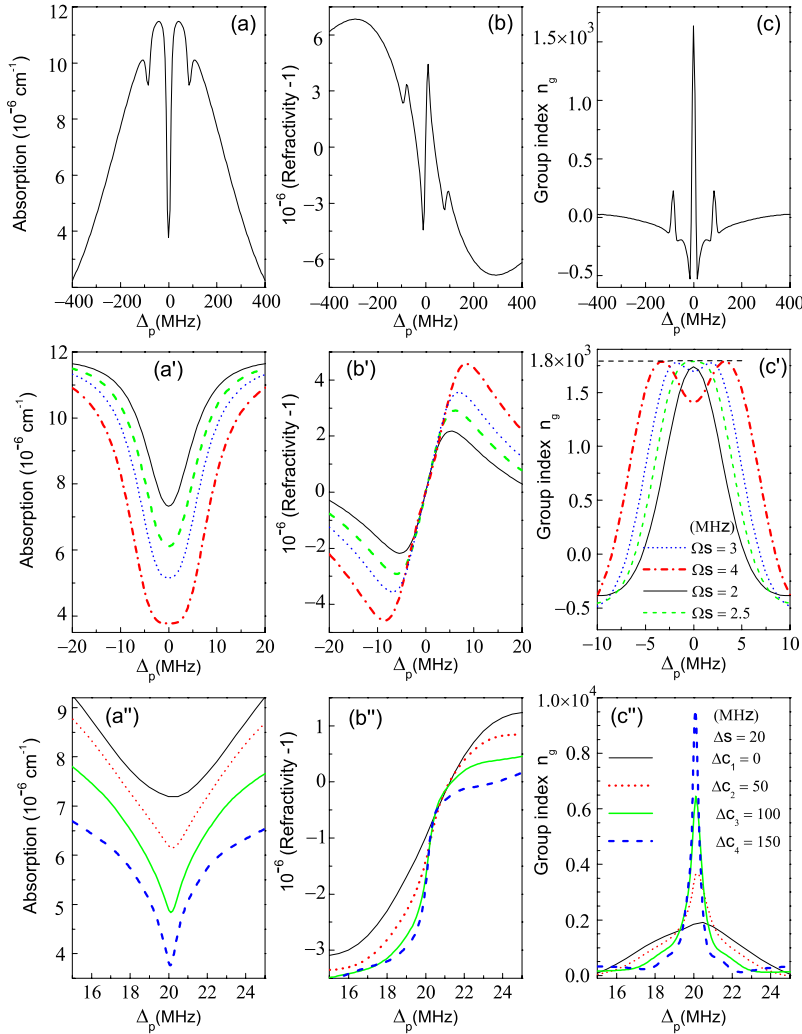
FIG. 2. (Color online) (a) Absorption, (b) refraction, and (c) group index variation with detuning of the probe field in the presence of a co-propagating saturating field $\Omega_s=4$ MHz and a counterpropagating coupling field $\Omega_c=40$ MHz. $(a')-(c')$ show blowups of (a)–(c) with different $\Omega_s$. $(a'')-(c'')$ show the same as the black curve in $(a')-(c')$ with different detunings.

that the CHB at resonance moves to $\Delta_p=20$ MHz. At the same time, a larger $\Delta_c$ can be chosen to obtain a steeper slope of the refractive index and thereby a larger group index with higher transmission. This phenomenon can be understood as follows. On the one hand, the Maxwell speed distribution shows that the atoms with the most probable velocity occupy the large scale within the whole group. In our case, we assume that the most probable atomic velocity is $v_p=250$ m/s. On the other hand, two groups of atoms with velocity $v_{1,2}=[(\Delta_c-3\Delta_s)\pm\sqrt{(\Delta_c+\Delta_s)^2+8\Omega_c^2}]\frac{c}{4\omega_s}$ at level $|1\rangle$ can be excited to saturation due to the presence of the coupling field $(\omega_p\approx\omega_s\approx\omega_c,\ \eta_s=-\eta_c=1)$ [20,21]. So, for the central CHB, we can adjust the appropriate parameters to make $v_{1,2}$ both close to $v_p$ so as to excite more atoms to saturation and thereby obtain a steeper slope of the refractive index.

Fixing the parameters $\Delta_s=20$ MHz, $\Omega_c=40$ MHz, and $\lambda_s=794.969$ nm for the $^{87}$Rb $D1$ line, we can give detailed values of $v_{1,2}=11,-35;\ 24,-28;\ 41,-25;$ and $58,-22$ (m/s) corresponding to different $\Delta_c=0,\ 50,\ 100,$ and $150$ MHz. It is clear that $v_{1,2}=58,-22$ resulting from $\Delta_c=150$ MHz are closest of these to $v_p$. If $\Delta_c$ is increased even more, for example, $\Delta_c=200$ MHz, we can get $v_{12}=76,-21$, which are closer to $v_p$ so that the group velocity continues to increase.

But we should note that it is better not to increase $\Delta_c$ too much because the spectral detuning of the coupling field decreases the width of the spectrum of the transparent probe pulse, which is shown by the blue curve in Fig. 2(c'').

It is clear that the peak value of the group index induced by CHB is 25 times larger than that obtained by general OHB under the same conditions, as shown in Fig. 3. Here, we give the typical inhomogeneous hole burning, which becomes deeper and wider with increasing intensity of $\Omega_s$.

### B. The second scheme [Fig. 1(b)]

For this excitation scheme, more interesting phenomena can be observed: By reducing $\delta$, we can find a deep and narrow hole at resonance [Fig. 4(a), dashed curve] in the absorption profile that is similar to the EIT window (solid curve). It is clearly shown in the blowup that the two borders of the hole at $\Delta_s'=\Delta_s\pm\delta\ (\Delta_s=0,\ \delta=1$ MHz) go deep below zero, that is, gain appears, because of the strong saturation excited by the two field components of the bichromatic laser. The slopes of the refractive index at the edges of the hole are especially steep [Fig. 4(b)], which can produce two narrow slow light pulses simultaneously [Fig. 4(c)]. When the detunings are changed to $\Delta_s=20$ MHz and $\Delta_c=150$ MHz, the
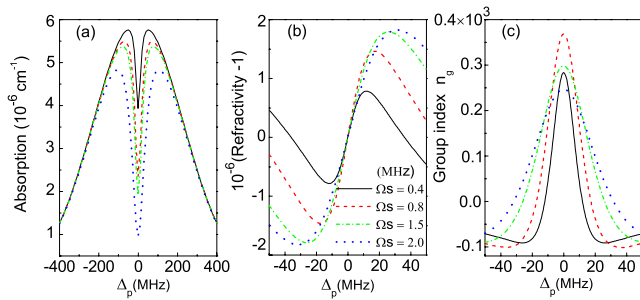
FIG. 3. (Color online) (a) Absorption, (b) refraction, and (c) group index variation with detuning of the probe field in the presence of a copropagating pump field $\Omega_s$. The common parameters for $^{87}$Rb vapor are chosen as Doppler width parameter $D=500$ MHz, density $N=2\times10^{12}/cm^3$, $\Gamma_{31}=\Gamma_{32}=6$ MHz, $\Gamma_{21}=10$ KHz, and $\Delta_s=0$.

hole at resonance moves to $\Delta_p=\Delta_s=20$ MHz [Figs. 4(a')–4(c')]. The transparency window becomes wider and the edges of the hole at $\Delta'_s=\Delta_s\pm\delta$ ($\Delta_s=20$ MHz, $\delta=1$ MHz) become deeper; gain increases. The two slopes of the refractive index at the edges of the hole are steeper and thereby produce larger group index value. This method offers the possibility of observing the reduced propagation speed of a two-color field accompanied by amplification. Also, we can conveniently control the frequency and slowdown factor of the probe field by modulating the detunings of both coupling and saturating beams.

This figure shows clearly that the slow light induced by bichromatic coherent hole burning (dashed curves) has great advantages: higher slowdown factor with gain and a two-color slow light field. On the one hand, the two retarded propagation pulses with gain have many advantages in high-resolution spectroscopy and frequency stabilization. On the other hand, several holes can be burnt in different parts of the inhomogeneous absorption line and thereby several probe pulses with frequencies coinciding with the frequency positions of these holes can be slowed down simultaneously. This can be used for application in the construction of multichannel optical storage devices.

## IV. CONCLUSION

In summary, the coherent hole burning in a Doppler-broadened $\Lambda$-type system enables us to achieve slow light. When the system is excited by a bichromatic laser field, we can slow down two narrow pulses with large gain simultaneously. This can be demonstrated experimentally because the detailed analyses in our paper are carried out for the relevant hyperfine levels of the $^{87}$Rb $D1$ line, $\lambda_{31}$
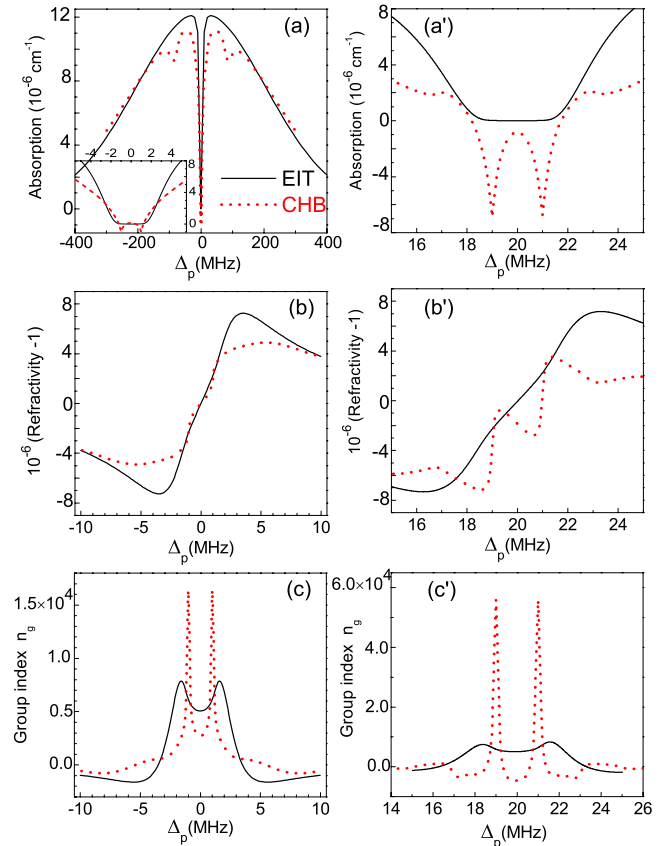


FIG. 4. (Color online) (a) Absorption, (b) refractivion, and (c) group index variation with detuning of the probe field. Comparison of EIT (black, solid) with $\Omega_c=30$ MHz and coherent hole burning (red, dashed) induced by a copropagating bichromatic saturating field $\Omega_{s1}=\Omega_{s2}=4$ MHz, $\delta=1$ MHz, and a counterpropagating coupling field $\Omega_c=40$ MHz. The inset shows the blowup. (a')–(c') show the same as (a)–(c) but with different detunings $\Delta_s=20$ MHz and $\Delta_c=150$ MHz.

$=794.969$ nm and $\lambda_{32}=794.983$ nm. If we can extend this CHB technique from an atomic vapor with slow decay time and transient spectral hole-burning properties into solid materials exhibiting fast decay time and PSHB properties, we believe that the present approach has excellent potential for applications in controlling the width of the spectral holes and the optical group velocity of the propagation pulse.

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