Effect of vacuum-induced coherences on coherent population trapping of moving atoms

Guojian Yang,* Min Xie, Ze Zhang, and Kaige Wang

Physics Department, Beijing Normal University, Beijing 100875, People's Republic of China and Applied Optics Beijing Area Major Laboratory, Beijing 100875, People's Republic of China

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We study an extended velocity-selective coherent population trapping (VSCPT) model where vacuum-induced coherence (VIC) is considered. This effect takes place when two nonorthogonal electric dipole moments of a Λ-type three-level atom couple with a common continuous vacuum. First, we introduce the main results of a conventional VSCPT model in the nondegenerate and unsymmetrically driven situation. Then, we work out the dynamic equation of the system in the presence of VIC expressed in both internal and external degrees of freedom. We report on the generation of atomic external coherences due to VIC, which counteract the feeding action of spontaneous emission to the trapping state and indicate its limited function negative to the formation of the trapping state if the atomic center-of-mass motion is considered.

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I. INTRODUCTION

Quantum interference between different transition pathways of a multilevel atomic system coupling with a common continuous vacuum results in a type of coherence, called vacuum-induced coherence (VIC). Since it was predicted by Agarwal [1], various schemes have been studied which release many interesting effects, such as, narrowing [2] or even cancellation [3–5] of spontaneous emission, amplification without population inversion [6], phase-dependent population inversion and phase control of spontaneous emission [7], and even exhibiting considerable squeezing [8], etc. To observe these effects, however, two electric-dipole moments of an atom with, for example, V- or Λ-type level structure, must be nonorthogonal and also if they share a common atomic state, the other two states should lie close. The problem of how to make the arrangement of nonorthogonal dipole moments has been addressed and alternative possibilities have been suggested [9].

Coherently driven Λ-type three-level atomic schemes have been frequently used in quantum optics and other related fields. This is due to their special property of the coherent population trapping (CPT) [10], the phenomenon that stems from nonabsorption as the atom is trapped in the linear combination of its two ground states, the so-called dark state. The applications of this effect include, e.g., electromagnetically induced transparency [11], storage of light [12], and light controlling of the atomic center-of-mass motion (CMM) [13,14], etc. In the presence of VIC, however, the systems may give rise to new behaviors. It was found that the CPT may disappear if the two transition dipole moments of the atom are parallel to each other and driven by the same laser fields [15]. Otherwise it can be preserved at the cost of lengthening the time scale for its formation [16]. A further study showed that in the presence of VIC, an inversionless gain is obtainable if an additional incoherent pumping field is added [17]. All these investigations and, to our knowledge, other existing works were made without taking either the atomic CMM or VIC into consideration. In this paper, we study the role of both VIC and CMM in the generation of the atomic trapping state via a typical coherently driven Λ-type atomic scheme, the velocity-selective coherent population trapping (VSCPT) [13]. We show how to describe VIC by using the master equation method and reveal the physics of the effect of VIC on the formation of this trapping state after the atomic CMM is taken into consideration. Optical coherence in quantum systems with cold multilevel atoms driven by light is perhaps the most important source available in modern optics, and quantum interference between different transition pathways usually plays an role of anticoherence or decoherence and thus cannot be neglected in some cases. The present work is our first attempt to look into this subject.

The paper is organized as follows. In Sec. II we introduce the VSCPT theory, then present the master equation description of VIC, and finally work out the equations of the density-matrix elements of the whole system. In Sec. III we make the comparative study on the dynamic evolutions of a VSCPT in the absence and presence of VIC via numerical simulations and present the relevant physical analysis. In Sec. IV, we make some concluding remarks.

II. DYNAMIC EQUATION OF THE SYSTEM

A. The VSCPT theory

We start by introducing the main part of original VSCPT theory [13]. The Λ-type atom-optical interaction scheme under consideration can be generalized as follows: a laser field of frequency ω_{L_1} (or wave number k_1) propagates toward $+Oz$, coupling with the atomic transition between the ground state $|g_1\rangle$ and the excited state $|e_0\rangle$. A counterpropagating laser field of frequency ω_{L_2} (or wave number k_2) couples with the another atomic transition between the second ground state $|g_2\rangle$ and $|e_0\rangle$. The pair of laser fields is assumed to be linearly polarized. In the dipole and rotating wave approximation, ignoring the interatomic action, we can express the Hamiltonian of the system as follows:

*yanggj@bnu.edu.cn

$$\hat{H}\prime = \frac{\hat{p}^2}{2m} + E_0|e_0\rangle\langle e_0| + E_1|g_1\rangle\langle g_1| + E_2|g_2\rangle\langle g_2| + \frac{\hbar}{2}\{\Omega_1|e_0\rangle \times\langle g_1|\exp[i(k_2 z - \omega_{L_1}t)] + \Omega_2|e_0\rangle \times\langle g_2|\exp[-i(k_2 z - \omega_{L_2}t)]\} + \text{H.c.}, \quad (1)$$

where m stands for the atomic mass and E_q $(q=0,1,2)$ the energy of the qth atomic state. The quantity $\Omega_q = d_q\mathcal{E}_q/\hbar$ is the related Rabi frequency, where $\mathcal{E}_q$ is the amplitude of the qth laser field and d_q the dipole matrix element defined by $d_q = \langle e_0|\hat{\boldsymbol{\epsilon}}_q\cdot\hat{d}|g_q\rangle$ and $\langle e_0|\hat{\boldsymbol{\epsilon}}_q\cdot\hat{d}|g_{q\prime}\rangle = 0$ for q,q' $(q\neq q\prime)=1,2$ with $\hat{\boldsymbol{\epsilon}}_q$ being the unit polarization vector of the qth laser field and $\hat{d}$ being the atomic dipole moment operator. In the later we'll show how to realize this atom-laser configuration in practice.

To simplify discussion, we denote $\Delta_q = \omega_{L_q} - \omega_q$ as the detuning between the qth laser field and atomic transition with frequency given by $\omega_q = (E_0 - E_q)/\hbar$, and make the transformation $\hat{H} \equiv \exp(\frac{i}{\hbar}t\hat{H}_0)\hat{H}' \exp(-\frac{i}{\hbar}t\hat{H}_0)$ with $\hat{H}_0 = (E_1 - \hbar\Delta_1)|g_1\rangle\langle g_1| + (E_2 - \hbar\Delta_2)|g_2\rangle\langle g_2| + E_0|e_0\rangle\langle e_0|$. Furthermore, what we are interested in takes place when atomic temperature is much lower, for example, even below the value corresponding to one-photon recoil energy, so that the atomic CMM must be treated quantum mechanically and the expansion $\exp(\pm ik_{1,2}z) = \Sigma_p|p\rangle\langle p\mp\hbar k_{1,2}|$ becomes suitable. For the above reasons, we can finally obtain the Hamiltonian of the system

$$\hat{H} = \frac{\hat{p}^2}{2m} + \hbar\Delta_1|g_1,p-\hbar k_1\rangle\langle g_1,p-\hbar k_1| + \hbar\Delta_2|g_2,p+\hbar k_2\rangle \times\langle g_2,p+\hbar k_2| + \frac{\hbar}{2}\sum_p[(\Omega_1|e_0,p\rangle\langle g_1,p-\hbar k_1| + \Omega_2|e_0,p\rangle \times\langle g_2,p+\hbar k_2|) + \text{H.c.}]. \quad (2)$$

Equation (2) indicates that the atomic states should now be described by $|e_0,p\rangle$, $|g_1,p-\hbar k_1\rangle$, $|g_2,p+\hbar k_2\rangle$, where each state is labeled by both internal and external quantum numbers. It can be easily seen that there exists a family $F(p) = \{|g_1,p-\hbar k_1\rangle, |g_2,p+\hbar k_2\rangle, |e_0,p\rangle\}$ [18], which is closed under the coherent action of $\hat{H}$.

The two states

$$|\Psi_-(p)\rangle = [\Omega_2|g_1,p-k_1\rangle - \Omega_1|g_2,p+k_2\rangle]/\Omega,$$

$$|\Psi_+(p)\rangle = [\Omega_1^*|g_1,p-k_1\rangle + \Omega_2^*|g_2,p+k_2\rangle]/\Omega \quad (3)$$

are of central importance in understanding the atomic trapping process, where $\Omega = [|\Omega_1|^2 + |\Omega_2|^2]^{1/2}$. Among them the state $|\Psi_-(p)\rangle$ is an optically nonabsorptive dark state. The atom oscillates between $|\Psi_-(p)\rangle$ and $|\Psi_+(p)\rangle$ at frequency $\Delta_{12}(p)$ under the coherent action of $\hat{H}$, where we have introduced new atom-field detunings with Doppler shift effects Δ_{12} for the two-photon Raman process and, for future use, Δ_{0i} $(i=1,2)$ for the one-photon process as follows:

$$\Delta_{12}(p) = \hbar k_1^2/2m - \hbar k_2^2/2m + p(k_1+k_2)/m + \Delta_1 - \Delta_2,$$

$$\Delta_{01}(p) = pk_1/m - \hbar k_1^2/2m - \Delta_1,$$

$$\Delta_{02}(p) = -pk_2/m - \hbar k_2^2/2m - \Delta_2. \quad (4)$$

If the atom falls into the state $|\Psi_-(p)\rangle$ with a momentum

$$p = p_0 = \frac{\hbar(k_1^2 - k_2^2) + 2m(\Delta_1 - \Delta_2)}{2(k_1+k_2)}, \quad (5)$$

however, the oscillation stops and the atom remains in $|\Psi_-(p_0)\rangle$ permanently.

The lifetimes of the states $|\Psi_\pm(p)\rangle$, which are described by their departure rates $\Gamma_\pm(p)$, can be estimated [19]. For example, as $\Omega_i \ll \Delta_q$, γ_q, and $\Omega_1 = \Omega_2 = \Omega$, $\gamma_1 = \gamma_2 = \gamma$, where γ_q is the decay rate of the atom from the state $|e_0\rangle$ to $|g_q\rangle$ $(q=1,2)$, and as the momentum p taking the value in the area very closed to p_0, the departure rate $\Gamma_-(p)$ from $|\Psi_-(p)\rangle$ takes a simple form

$$\Gamma_-(p) = 4\Delta_{12}^2(p,k_1,k_2,\Delta_1,\Delta_2)\gamma/\Omega^2. \quad (6)$$

Obviously, as $p=p_0$ we have $\Gamma_-|_{p=p_0}=0$, which means that the state $|\Psi_-(p_0)\rangle$ is indeed dark. While the departure rate $\Gamma_+(p)$ from $|\Psi_+(p)\rangle$ can be given approximately by its expression at the point p_0 and reads

$$\Gamma_+|_{p\simeq p_0} = \gamma(\Omega/2)^2/[\Delta(p_0,k_1,k_2,\Delta_1,\Delta_2)^2 + (\gamma/2)^2], \quad (7)$$

where $\Delta(p_0,k_1,k_2,\Delta_1,\Delta_2) = -\hbar k_1k_2/2m - (k_1\Delta_2 + k_2\Delta_1)/(k_1+k_2)$. We note that by taking $k_1=k_2=k$, $\Delta_1=\Delta_2=0$, and $\gamma \gg \omega_r$, where $\omega_r = \hbar k^2/2m$ is the one-photon recoil frequency, Eqs. (6) and (7) return to the results for a degenerated and resonant VSCPT system discussed in Ref. [13].

Spontaneous emission has a function redistributing the atomic population among the different momentum families by allowing the atoms to jump unilaterally from $|e_0,p_0+(-)^i\hbar k_i+u\rangle$ in the family $F[p_0+(-)^q\hbar k_q]$ into $|g_q,p_0+(-)^q\hbar k_q\rangle$ in the family $F(p_0)$ at the probability $W_{q+}(u)$ defined by [20]

$$W_{q\pm}(u) = \frac{3}{8}\frac{1}{\hbar k_q}\left(1 \pm \frac{u^2}{\hbar^2k_q^2}\right) \quad (q=1,2) \quad (8)$$

for $|u| \leq \hbar k_q$. In this way, all atoms, no matter what initial momenta they have, will finally accumulate in $|\Psi_-(p_0)\rangle$. The atomic temperature can be decided by the half-width of the momentum distribution peak, which is located at p_0 given by Eq. (5). If the interaction time is long enough, its half-width becomes smaller than the one-photon recoil momentum, corresponding to a temperature below the one-photon recoil energy.

B. Master equation description of VIC

The master equation method is an effective way to deal with problems related to the irreversible process experienced by an atomic system coupled with a continuous vacuum. In the interaction picture the Hamiltonian describing this coupling system in our case can be described by

$$H_I = \sum_{q=1,2} \sum_{\tilde{\mathbf{k}} \perp l} \left(\frac{\hbar c k}{2 \epsilon_0 V} \right)^{1/2} [\vec{d}_q \cdot \hat{\boldsymbol{\epsilon}}_l(\hat{\mathbf{k}})] \sigma_q^{+} a_{kl} \exp[i \tilde{\mathbf{k}} \cdot \tilde{\mathbf{r}} - i \Delta \omega_q t] + \text{H.c.}, \tag{9}$$

where we have adopted the electric dipole approximation and the rotating wave approximation. The quantity $\vec{d}_q$ is the vector of the qth atomic dipole moment and $\hat{\boldsymbol{\epsilon}}_l(\tilde{\mathbf{k}})$ $(l=1,2)$ the unit polarization vector of the vacuum mode characterized by the unit wave vector $\hat{\mathbf{k}}$ $(=\tilde{\mathbf{k}}/|\tilde{\mathbf{k}}|)$ and polarization number l. The operator $\hat{a}_{kl}$ $({}^{\dagger}\hat{a}_{kl})$ stands for the annihilation (creation) operator of this vacuum mode and fulfils the relation $[\hat{a}_{kl}, {}^{\dagger}\hat{a}_{k'l'}] = \delta(k-k')\delta(l-l')$, etc. σ_q^{+} (σ_q^{-}) is the atomic rising (lowering) operator corresponding to atomic transition $|g_q\rangle \leftrightarrow |e_0\rangle$. $\tilde{\mathbf{r}}$ represents the position the atom is located at and $\Delta\omega_q = \omega_k - \omega_q$ the detuning between the frequency ω_k $(=kc)$ of the k mode and the qth atomic transition frequency.

Since we are not interested in the details of the dynamics of the continuous vacuum modes, we derive a master equation for the reduced atomic operator by using the standard quantum optics theory [1,21,22], where two important assumptions are taken: (a) the coarse-grained factorization of the density operator of the combined atom-field system into the product of atom $[\rho(t)]$ and field density operators, (b) the Born-Markoff approximation. The Born approximation depends on the weak coupling between the vacuum and the atom. The Markoff approximation holds because the vacuum has fairly flat density of states. Using above approximations and tracing over the vacuum field states, we can finally derive the master equation for the reduced atomic operator. This equation reads

$$\begin{aligned} \mathcal{L}\rho(t) = -\sum_{q,q'=1,2} \frac{3}{4}\sqrt{\gamma_q \gamma_{q'}} \Bigg\{ & \left(\frac{\omega_{q'}}{\omega_q}\right)^{3/2} \int \frac{d\Omega(\hat{\mathbf{k}})}{4\pi} \mathcal{D}^{*}_{qq'}(\hat{\mathbf{k}},\hat{\mathbf{k}}) \sigma_q^{+}\sigma_{q'}^{-}\rho(t) \exp(-i\Delta\omega_{qq'}t) + \left(\frac{\omega_q}{\omega_{q'}}\right)^{3/2} \int \frac{d\Omega(\hat{\mathbf{k}})}{4\pi} \mathcal{D}^{*}_{qq'}(\hat{\mathbf{k}},\hat{\mathbf{k}}) \rho(t) \sigma_q^{+}\sigma_{q'}^{-} \\ & \times \exp(-i\Delta\omega_{qq'}t) + \left(\frac{\omega_{q'}}{\omega_q}\right)^{3/2} \int \frac{d\Omega(\hat{\mathbf{k}})}{4\pi} \mathcal{D}_{qq'}(\hat{\mathbf{k}},\hat{\mathbf{k}}) \sigma_q^{-} \exp\left(-i\frac{\omega_{q'}}{c}\hat{\mathbf{k}}\cdot\mathbf{r}\right) \rho(t) \exp\left(i\frac{\omega_{q'}}{c}\hat{\mathbf{k}}\cdot\mathbf{r}\right) \sigma_{q'}^{+} \exp(i\Delta\omega_{qq'}t) \\ & + \left(\frac{\omega_q}{\omega_{q'}}\right)^{3/2} \int \frac{d\Omega(\hat{\mathbf{k}})}{4\pi} \mathcal{D}_{qq'}(\hat{\mathbf{k}},\hat{\mathbf{k}}) \sigma_q^{-} \exp\left(-i\frac{\omega_q}{c}\hat{\mathbf{k}}\cdot\mathbf{r}\right) \rho(t) \exp\left(i\frac{\omega_q}{c}\hat{\mathbf{k}}\cdot\mathbf{r}\right) \sigma_{q'}^{+} \exp(i\Delta\omega_{qq'}t) \Bigg\}, \end{aligned} \tag{10}$$

where $\Delta\omega_{qq'} = \omega_q - \omega_{q'}$; $D_{qq'}(\hat{\mathbf{k}},\hat{\mathbf{k}})$ is the coupling pattern between the transition dipole moments and the vacuum mode propagating in the direction of the solid angle $\Omega(\hat{\mathbf{k}})$, and it is defined as

$$\mathcal{D}_{qq'}(\hat{\mathbf{k}},\hat{\mathbf{k}}) = \sum_{l \perp \hat{\mathbf{k}}} \left[\hat{\varepsilon}_q \cdot \hat{\boldsymbol{\epsilon}}_l(\hat{\mathbf{k}})\right]^{*} \left[\hat{\varepsilon}_{q'} \cdot \hat{\boldsymbol{\epsilon}}_l(\hat{\mathbf{k}})\right], \tag{11}$$

with $\hat{\boldsymbol{\varepsilon}}_q = \vec{d}_q / |\vec{d}_q|$ being the unit vector of the qth atomic dipole moment $\vec{d}_q$. The exponential functions $\exp(\pm i\frac{\omega_q}{c}\hat{\mathbf{k}}\cdot\mathbf{r})$ describe the recoil motion after the atom absorbs a photon from or emits a photon into this vacuum mode. To derive Eq. (10) we have ignored the Lamb shift dependent contributions.

In Eq. (10) the terms contain the factors $\exp(\pm i\Delta\omega_{qq'}t)$, which implies that any interesting phenomena including the VIC effect will be erased due to the optical coherent vibration at a frequency $\Delta\omega_{12} = \omega_1 - \omega_2$ if $\Delta\omega_{12}$ is large. In order to avoid this, we consider the nearly degenerated case of $\omega_1 \approx \omega_2 = \omega_0$ and obtain

$$\begin{aligned} \mathcal{L}\rho(t) = \mathcal{L}_q\rho(t) + \mathcal{L}_{qq'}\rho(t) = & -\sum_{q=1,2} \frac{\gamma_q}{2} [\sigma_q^{+}\sigma_q^{-}\rho(t) + \rho(t)\sigma_q^{+}\sigma_q^{-}] \\ & + \sum_{q=1,2} \frac{3}{2}\gamma_q \int \frac{d\Omega(\hat{\mathbf{k}})}{4\pi} \mathcal{D}_{qq}(\hat{\mathbf{k}},\hat{\mathbf{k}}) \sigma_q^{-} \\ & \times \exp\left(-i\frac{\omega_0}{c}\hat{\mathbf{k}}\cdot\mathbf{r}\right) \rho(t) \exp\left(i\frac{\omega_0}{c}\hat{\mathbf{k}}\cdot\mathbf{r}\right) \sigma_q^{+} \\ & + \sum_{q,q'(q\neq q')=1,2} \frac{3}{2}\sqrt{\gamma_q\gamma_{q'}} \int \frac{d\Omega(\hat{\mathbf{k}})}{4\pi} \mathcal{D}_{qq'}(\hat{\mathbf{k}},\hat{\mathbf{k}}) \sigma_q^{-} \\ & \times \exp\left(-i\frac{\omega_0}{c}\hat{\mathbf{k}}\cdot\mathbf{r}\right) \rho(t) \exp\left(i\frac{\omega_0}{c}\hat{\mathbf{k}}\cdot\mathbf{r}\right) \sigma_{q'}^{+}, \end{aligned} \tag{12}$$

where the notation $\mathcal{L}_q\rho(t)$ stands for the terms on the second and the third lines, describing the conventional spontaneous emission, and especially, the term on the third line describes the feeding action of the atom from the excited state to the ground states. The term $\mathcal{L}_{qq'}\rho(t)$, which is related to the last line in Eq. (12), describes the VIC effect.

To solve the master equation Eq. (12), we define the atomic density-matrix elements in the bases of $F(\mathbf{p})$ in the following way:

$$\rho_{00}(\mathbf{p},\mathbf{p}') = \langle e_0, \mathbf{p} | \rho | e_{0}, \mathbf{p}' \rangle,$$

$$\rho_{01}(\mathbf{p},\mathbf{p}') = \langle e_0,\mathbf{p}|\rho|g_{1,}\mathbf{p}' - \hbar k\hat{z}\rangle,$$

$$\rho_{02}(\mathbf{p},\mathbf{p}') = \langle e_0,\mathbf{p}|\rho|g_{2,}\mathbf{p}' + \hbar k\hat{z}\rangle,$$

$$\rho_{12}(\mathbf{p},\mathbf{p}') = \langle g_{1,}\mathbf{p} - \hbar k\hat{z}|\rho|g_{2,}\mathbf{p}' + \hbar k\hat{z}\rangle,$$

$$\rho_{11}(\mathbf{p},\mathbf{p}') = \langle g_{1,}\mathbf{p} - \hbar k\hat{z}|\rho|g_{1,}\mathbf{p}' - \hbar k\hat{z}\rangle,$$

$$\rho_{22}(\mathbf{p},\mathbf{p}') = \langle g_{2,}\mathbf{p} + \hbar k\hat{z}|\rho|g_{2,}\mathbf{p}' + \hbar k\hat{z}\rangle, \tag{13}$$

and the other matrix elements fulfill $\rho_{qq'}\mathbf{p},\mathbf{p}' = \rho^*_{q'q}(\mathbf{p}',\mathbf{p})$. Here we have taken $\hat{z}$ as the unit vector in the Oz direction and used the momentum vector $\mathbf{p}$ [$\mathbf{p}=(p_x,p_y,p_z=p)$], instead of a scalar p, because after considering spontaneous emission, the atomic CMM is direction dependent. At present, we look into the contribution of $\mathcal{L}_{qq'}\rho(t)$, which only leads to the equation for the element $\rho_{qq'}(\mathbf{p},\mathbf{p}')$ [$q,q'(q\neq q')=1,2$]

$$\frac{d}{dt}\rho_{qq'}(\mathbf{p},\mathbf{p}') = \frac{3}{2}\sqrt{\gamma_q\gamma_{q'}}\int \frac{d\Omega(\hat{\mathbf{k}})}{4\pi}\mathcal{D}_{qq'}(\hat{\mathbf{k}},\hat{\mathbf{k}})\rho_{00}[\mathbf{p} + (-1)^q\hbar k\hat{z} + \hbar k\hat{\mathbf{k}},\mathbf{p}' + (-1)^{q'}\hbar k\hat{z} + \hbar k\hat{\mathbf{k}}]. \tag{14}$$

Furthermore, by tracing $\frac{d}{dt}\rho_{qq'}(\mathbf{p},\mathbf{p}')$ over p_x and p_y, defining the unit vectors $\hat{\boldsymbol{\varepsilon}}_q$ ($q=1,2$) and $\hat{\mathbf{k}}$, respectively, as

$$\hat{\boldsymbol{\varepsilon}}_1 = (a_1,a_2,a_3),$$

$$\hat{\boldsymbol{\varepsilon}}_2 = (b_1,b_2,b_3),$$

$$\hat{\mathbf{k}} = (\sin\theta\cos\phi,\sin\theta\sin\phi,\cos\theta), \tag{15}$$

where a_i and b_i ($i=1,2,3$) can be decided via a specifically setting, and considering the equation

$$\mathcal{D}_{qq'}(\hat{\mathbf{k}},\hat{\mathbf{k}}) = \hat{\boldsymbol{\varepsilon}}_q\cdot\hat{\boldsymbol{\varepsilon}}_{q'} - (\hat{\boldsymbol{\varepsilon}}_q\cdot\hat{\mathbf{k}})^*(\hat{\boldsymbol{\varepsilon}}_{q'}\cdot\hat{\mathbf{k}}), \tag{16}$$

we finally obtain

$$\frac{d}{dt}\rho_{qq'}(p,p') = \sqrt{\gamma_q\gamma_{q'}}\int_{-\hbar k}^{\hbar k} du[(a_1^*b_1 + a_2^*b_2)W_+(u) + 2a_3^*b_3W_-(u)]\rho_{00}[p + (-1)^q\hbar k\hat{z} + u,p' + (-1)^{q'}\hbar k\hat{z} + u], \tag{17}$$

where we have carried out the integration of the variable ϕ, used the probability function $W_\pm(u)$ given by Eq. (8) by omitting the subscript i ($i=1,2$) because of the degenerate requirement $k_1=k_2=k$, and changed the integration variable $\cos\theta\to u/\hbar k$. Following a similar procedure, we can derive the equations for all elements of the whole system and will present them in the next section.

C. Dynamic equation of the VSCPT system

To derive the dynamic equation of the extended VSCPT system, we show first how to realize the Λ-type atom-light interaction described by Eq. (2) under the condition of two nonorthogonal dipole elements. Figure 1 suggests a possible arrangement where the polarization of one field acting on this transition dipole element is perpendicular to the another transition dipole element, so that one field drives only one transition. To this end we choose the following unit vectors of the atomic dipole elements:

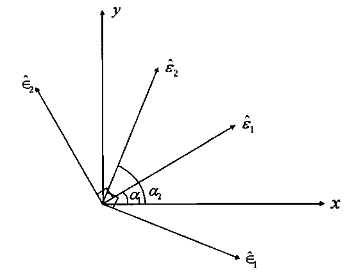

FIG. 1. Schematic diagram of atom-laser interaction. $\hat{\boldsymbol{\varepsilon}}_q$ ($q=1,2$) is the unit vector of the qth atomic dipole moment and $\hat{\boldsymbol{\epsilon}}_q$ the unit polarization vector of the qth laser field. $\hat{\boldsymbol{\epsilon}}_q$ is chosen so that one field drives only one transition.

$$\hat{\boldsymbol{\varepsilon}}_1 = (\sin\alpha_1,\cos\alpha_1,0),$$

$$\hat{\boldsymbol{\varepsilon}}_2 = (\sin\alpha_2,\cos\alpha_2,0), \tag{18}$$

and have $\hat{\boldsymbol{\varepsilon}}_1\cdot\hat{\boldsymbol{\varepsilon}}_2=\cos\alpha$ ($\alpha=\alpha_1-\alpha_2$). By taking both coherent and incoherent processes into consideration, we arrive at the full equations of the density-matrix elements for the whole system. They read

$$\frac{\partial\rho_{00}}{\partial t} = -\left[\frac{\gamma_1+\gamma_2}{2} + i\delta(p,p')\right]\rho_{00} - i[\Omega_1(\rho_{10}-\rho_{01}) + \Omega_2(\rho_{20}-\rho_{02})],$$

$$\frac{\partial\rho_{01}}{\partial t} = -\left(\frac{\gamma_1}{2} + i\Delta'_{01}(p,p')\right)\rho_{01} - i[\Omega_2\rho_{21} + \Omega_1(\rho_{11}-\rho_{00})],$$

$$\frac{\partial\rho_{02}}{\partial t} = -\left(\frac{\gamma_2}{2} + i\Delta'_{02}(p,p')\right)\rho_{02} - i[\Omega_1\rho_{12} + \Omega_2(\rho_{22}-\rho_{00})],$$

$$\frac{\partial\rho_{12}}{\partial t} = \frac{\sqrt{\gamma_1\gamma_2}}{2}\cos\alpha\int_{-\hbar k}^{\hbar k} duW_+(u)\rho_{00}(p-\hbar k+u,p'+\hbar k+u) - i\Delta'_{12}(p,p')\rho_{12} - i[\Omega_1\rho_{02} - \Omega_2\rho_{01}],$$

$$\frac{\partial\rho_{11}}{\partial t} = \frac{\gamma_1}{2}\int_{-\hbar k}^{\hbar k} duW_+(u)\rho_{00}(p-\hbar k+u,p'-\hbar k+u) - i\Omega_1(\rho_{01}-\rho_{10}),$$

$$\frac{\partial\rho_{22}}{\partial t} = \frac{\gamma_2}{2}\int_{-\hbar k}^{\hbar k} duW_+(u)\rho_{00}(p+\hbar k+u,p'+\hbar k+u) - i\Omega_2(\rho_{02}-\rho_{20}) \tag{19}$$

and their conjugations $\frac{\partial\rho_{10}}{\partial t}=(\frac{\partial\rho_{01}}{\partial t})^*$, $\frac{\partial\rho_{20}}{\partial t}=(\frac{\partial\rho_{02}}{\partial t})^*$, $\frac{\partial\rho_{21}}{\partial t}=(\frac{\partial\rho_{12}}{\partial t})^*$, where we have denoted $\rho_{ij}(p,p')$ as ρ_{ij} ($i,j=0,1,2$) for notation simplicity, and if no specification, will follow this rule. In Eq. (19) we have introduced a simple function $\delta(p,p')$ defined as $\delta(p,p')=p^2/2m\hbar - p'^2/2m\hbar$, and with it we redefined the following atom-field detunings with Doppler shift effect $\Delta'_{12}(p,p')$ for the two-photon Raman process and $\Delta'_{0i}(p,p')$ $(i=1,2)$ for the one-photon process:

$$\Delta'_{01}(p,p') = \delta(p,p'-\hbar k) - \Delta_1,$$

$$\Delta'_{02}(p,p') = \delta(p,p'+\hbar k) - \Delta_2,$$

$$\Delta'_{12}(p,p') = \delta(p-\hbar k,p'+\hbar k) + \Delta_1 - \Delta_2. \tag{20}$$

It is easy to see that if $p=p'$ Eq. (20) is the same as Eq. (4) for $k_1=k_2=k$.

From Eqs. (17) and (19) we find an interesting fact that the evolution of any density-matrix element couples not only to the single-family elements $\rho_{ij}(p,p)$ but also to the interfamily ones $\rho_{ij}(p,p')$ $(p\neq p')$. If ignoring $\rho_{ij}(p,p')$ $(p\neq p')$, from Eq. (19) we can recover the dynamic equation of the conventional VSCPT system [13]. It is known that the atom-laser interaction configuration described by Eq. (2) conserves angular momentum and thus prevents coherent redistribution of photons between the two counterpropagating waves from taking place, and furthermore, spontaneous emission in free space is of translational invariance. Therefore, in the evolution the system will keep the property of no interfamily density-matrix elements if it initially does. As to the elements $\rho_{ij}(p,p')$ with $p\neq p'$, they are known as atomic external coherences, e.g., $\rho_{00}(p,p')=\langle e_0,p|\rho|e_0 p'\rangle$ with $p\neq p'$, which means the coherence of the atomic population in the excited state in the external degree of freedom and comes into effects only due to the presence of VIC. In fact the feature of external coherence has been found in the dispersion of a two-level atom in a standing wave, where, due to coherent redistribution of photons between the two counterpropagating running waves by absorption in one wave and stimulated emission in the other wave, the atomic state $|e,p\rangle$ in the evolution couples to the states $|e,p\pm 2n\hbar k\rangle$ with n being an arbitrary natural number. Therefore, to solve the problems, an infinite number of elements $\langle e,p|\rho|e,p'\rangle$ with $|p-p'|=2n\hbar k$ must be considered. One of the differences between this two-level system and the one we are discussing is that, as a result of interacting with a continuous vacuum, the atomic momentum difference $|p-p'|$ for each external coherence $\langle e,p|\rho|e,p'\rangle$ in the presence of VIC can take value without any restriction.

III. NUMERICAL SIMULATION AND DISCUSSION

To look into the VSCPT effect under the influence of VIC in detail, in this section we solve numerically the Eq. (19) [23]. The following figures show the results under different parametric conditions. All parameters and quantities are dimensionless because we have made the following scale transformations $t\omega_r \Rightarrow t$, $p^2/2m\hbar\omega_r \Rightarrow p^2$, $\Omega_i/\omega_r \Rightarrow \Omega_i$, $\Delta_i/\omega_r \Rightarrow \Delta_i$, $\gamma_i/\omega_r \Rightarrow \gamma_i$, and $\hbar u^2/2m\omega_r \Rightarrow u^2$ $(i=1,2)$. In all of the situations considered in this paper, we have taken $\Delta_1=\Delta_2=0$, $\Omega_1=\Omega_2=10$, $\gamma_1=\gamma_2=10$, and assumed that initially the atom is in a symmetric superposition of its two ground states $|g_1\rangle$ and $|g_2\rangle$ and its momentum fulfills the Gaussian distribution with a standard half-width of $\Delta p=3\hbar k$. In the numerical simulations, we have set the range of the variable p from $-n\hbar k$ to $n\hbar k$ with the interval $\delta p=\hbar k/n$, where n is a natural number large enough that the interesting part of the solution of Eq. (19) (near p_0) is not affected by the truncation of p and that δp is small compared with the narrowest structure appearing in the p dependence of the solution.

The efficiency of the VSCPT system can be reflected directly through the momentum distribution of the atomic CMM along Oz, which is given by

$$\mathcal{P}(p^z_{\rm at}) = \rho_{00}(p^z_{\rm at},p^z_{\rm at}) + \rho_{11}(p^z_{\rm at}+\hbar k,p^z_{\rm at}+\hbar k) + \rho_2(p^z_{\rm at}-\hbar k,p^z_{\rm at}-\hbar k). \tag{21}$$

For the sake of comparison, first let us look into the situation in the absence of VIC. In order to get a complete knowledge of VSCPT, we consider a general atom-light coupling configuration, where the nondegeneracy is included. Figure 2 displays the results obtained at time $t=100$ for three values of β $(\beta=k_2/k_1)$: (a) $\beta=1.0$, (b) $\beta=0.75$, and (c) $\beta=0.50$. In each figure the atomic momentum distribution is with the features the same as or similar to those predicted by Aspect *et al.* [13]. That is, there are two peaks located, respectively, at $p^z_{\rm at}=(-)^i\hbar\frac{k_1+k_2}{2}$. The height of the peak indicates the probability of finding the atom trapped in the two ground state, and its width is proportional to the atomic temperature. In the nondegenerate situation of $k_2/k_1<1$ [Figs. 2(b) and 2(c)], in addition to these two peaks, there is a wide but lower bump in the right part of the figure, and it moves right further as k_2/k_1 decreases or as time develops as has been confirmed by the numerical simulation. This phenomenon is attributed mainly to the fact that after interacting with a pair of photons coming from the two counterpropagating laser fields of different wave numbers, the atom will get the recoil momentum proportional to $\hbar(k_1-k_2)$ in the same direction as that the photon with larger wave number propagates in. Obviously this phenomenon is harmful to the trapping effect we are concerned with. Compared to the degenerate configuration, more time is needed for a nondegenerate system to finish the trapping process.

In Fig. 3 we present a comparative study of VSCPT in the presence and absence of VIC, where we have chosen three different values of the angle α between the two transition dipole elements (a) $\alpha=\pi/2$, (b) $\alpha=\pi/3$, and (c) $\alpha=\pi/6$. It can be seen that the atomic momentum distribution in the presence of VIC keeps a feature similar to that in the absence of VIC. But in detail there are differences. The evident one is that the height of the peak is lowered and will be lowered more as the angle α decreases, thus the probability to find the atom in the trapping state is reduced. Compared to the feeding effect of spontaneous emission, however, the effect of VIC is weak and only lengthens the time for the trapping.

To get further knowledge of the effect of VIC on VSCPT, we study the situations without and with considering the atomic CMM. In the former case, the atomic population in the trapping state $|\Psi_-\rangle(|\Psi_-\rangle=|[\Omega_2|g_1\rangle-\Omega_1|g_2\rangle]/\Omega)$ fulfills

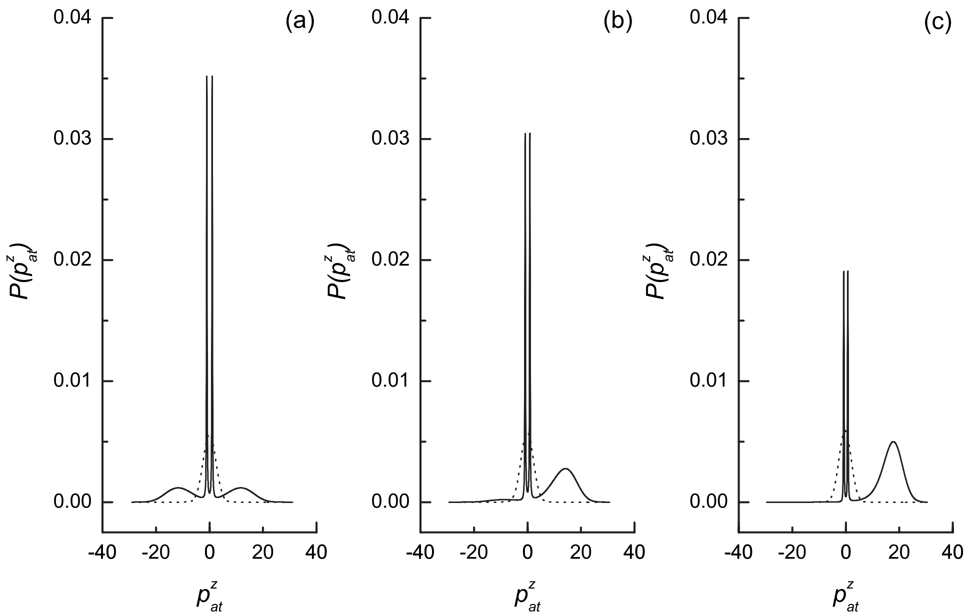

FIG. 2. Atomic momentum distribution $P(p^z_{\mathrm{at}})$ at time $t=100$ for different driven conditions: (a) $\beta=1.0$, (b) $\beta=0.75$, and (c) $\beta=0.50$. The other parameters are $\Delta_1=\Delta_2=0$, $\Omega_1=\Omega_2=10$, $\gamma_1=\gamma_2=10$, and $n=30$. The solid curves represent the final atomic momentum distributions and the dotted curves the initial Gaussian momentum distributions. All parameters and quantities in this and the following figures have been scaled and are dimensionless.

$$\frac{d}{dt}\langle\Psi_-|\rho|\Psi_-\rangle = \frac{1}{\Omega^2}[\gamma_1\Omega_1^2 + \gamma_2\Omega_2^2 - 2\sqrt{\gamma_1\gamma_2}\Omega_1\Omega_2\cos\alpha]\rho_{00}, \tag{22}$$

where $\Delta_1=\Delta_2$ has been chosen. The first two terms on the square bracket of Eq. (22) describe the feedings of spontaneous emission to the two ground states, and the last term shows the contribution of VIC. The effect of VIC on the CPT, which is found to be negative, cannot be underestimated because it can even cancel the feeding effect as long as a proper set of the parameters, e.g., $\gamma_1=\gamma_2=\gamma$, $\Omega_1=\Omega_2$ and $\alpha=0$, is taken [15,16] and the final population in the state $|\Psi_-\rangle$ will keep the same as that given by the initial condition. In the latter case, however, we have

$$\frac{d}{dt}\langle\Psi_-(p)|\rho|\Psi_-(p)\rangle = 2\gamma\int_{-\hbar k}^{\hbar k} du W_+(u)\{\rho_{00}(p-\hbar k-u,p-\hbar k-u) - \cos\alpha\,\mathrm{Re}[\rho_{00}(p-\hbar k+u,p+\hbar k+u)]\} \tag{23}$$

for $\gamma_1=\gamma_2=\gamma$, $\Omega_1=\Omega_2$, and $\Delta_1=\Delta_2$. The first and second terms on the right-hand side of Eq. (23) describe the contributions of spontaneous emission and VIC, respectively. To obtain this equation, we have assumed that the atom is initially in the excited state or the symmetric superposition of its two ground states, so that from the very beginning the two ground states are affected equally by spontaneous emission or by VIC. As the result of taking the atomic CMM and the

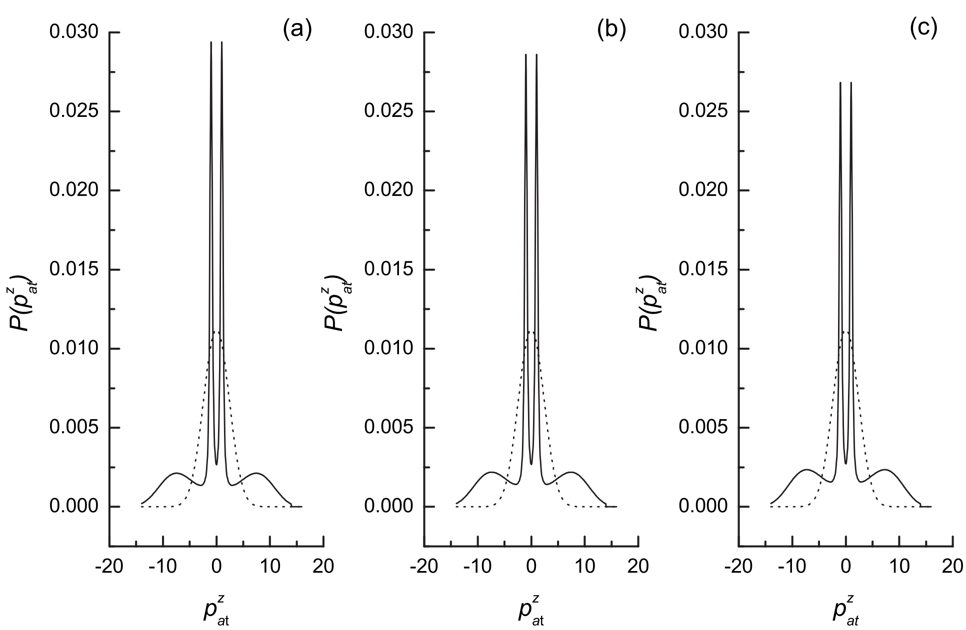

FIG. 3. Comparison between the trapping effects in the absence and the presence of VIC at time $t=20$ for (a) $\alpha=\pi/2$, (b) $\alpha=\pi/3$, (c) $\alpha=\pi/6$, and $n=15$. Other parameters and the definitions of the curves and the axis are the same as those in Fig. 2.

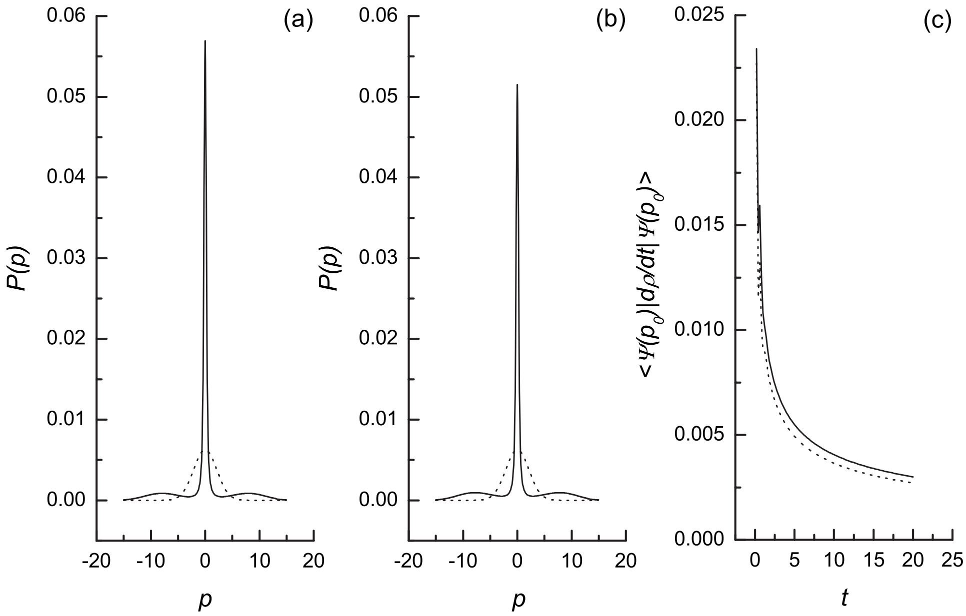

FIG. 4. Atomic population distribution in the state $|\Psi_{-}(p)\rangle$ at time $t=20$ for (a) $\alpha=\pi/2$ and (b) $\alpha=\pi/6$. The solid and dotted curves in (c) describe $\frac{d}{dt}\langle\Psi_{-}(p_0)|\rho|\Psi_{-}(p_0)\rangle$ in the corresponding cases (a) and (b), respectively. Other parameters are the same as those in Fig. 3.

propagating directions of the driven lasers into consideration, the atomic states are the p dependence and naturally, there is $\rho_{00}(p,p)\neq\rho_{00}(p,p')$ as $p\neq p'$ due to VIC. Thus, on the contrary to the situation described by Eq. (22), the effects of the feeding and VIC on CPT cannot cancel each other out completely, even if $\alpha=0$. Compared to the optical coherence, however, the external coherences are usually weak. If initially they are absent or there are no other special ways to strengthen them, the feeding effect of spontaneous emission plays a dominant role in the CPT process. This argument is illustrated further by the comparisons of the atomic population in the trapping state $|\Psi_{-}(p_0)\rangle$ [Figs. 4(a) and 4(b)] and of its the change rate $\langle\Psi_{-}(p_0)|\frac{d}{dt}\rho|\Psi_{-}(p_0)\rangle$ [Fig. 4(c)] between the case with the absence of VIC [see Fig. 4(a) and the solid curve in Fig. 4(c)] and the case with the presence of VIC [see Fig. 4(b) and the dotted curve in Fig. 4(c)]. In Fig. 4(c) the two curves are above the horizontal axis, which confirms that the CPT process exists in the both cases, but this process proceeds in the presence of VIC slowly than it does in the absence of VIC. Figures 4(a) and 4(b) indicates that the atomic population difference accumulated in $|\Psi_{-}(p_0)\rangle$ between the two cases is small, not more than 10% until $t=20$. Thus, compared to the feeding action, the VIC effect is indeed weak, at least under the present parametric condition. Both spontaneous emission and VIC are of the radiation-vacuum dependence and lead to a continuous variation of the atomic momentum. However, spontaneous emission transfers the atomic population from a family to another, which is known as the feeding effect, but cannot create any external coherence, while VIC behaves in the opposite way, and it counteracts the feeding action through external coherences.

IV. CONCLUSION

In conclusion, we have studied VSCPT of a coherently driven Λ-type three-level atomic system where the two electric-dipole moments are settled nonorthogonally. First we have shown the main results of a conventional VSCPT model with a nondegenerate and unsymmetrically driven configuration and indicated that it takes less time to trap the degenerated atoms with a pair of counterpropagating resonant laser fields. Then we worked out the dynamic equation of the atomic system in the presence of VIC and expressed it in both internal and external degrees of freedom. We have observed that the trapping effect for the moving atoms is weakened because the atomic external coherences induced by VIC counteract the feeding action of spontaneous emission. We have studied the behavior of the system operating in the situations without and with concerning atomic CMM, and found no evidence of the completely VIC-dependent cancellation of trapping effect in the latter case as a result of the cooperation of both internal and external coherences. To our knowledge there has been a number of papers discussing the VIC effect on the atomic trapping or other related properties of optical systems, but much less effort has been devoted to the subject of the atomic CMM controlled by lights. It may be worth extending the present study into other quantum optical systems where the decoherence action of the quantum interference between different transition pathways to desired optical coherences must be considered.

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