Trichromatic phase dependence of squeezing in resonance fluorescence

Xiang-ming Hu and Xue-hua Zhang

Department of Physics, Huazhong Normal University, Wuhan 430079, People's Republic of China

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We investigate the quantum noise in phase quadratures of resonance fluorescence from a two-level atom driven by a trichromatic exciting field. It is shown that the noise spectra are crucially dependent on the sum of relative phases of the sideband components compared to the central component although the sideband exciting components are much weaker than the central component. When we tune the exciting sidebands on two-photon resonance with the Mollow sideband transitions and set the phase sum to π , sideband squeezing in the in-phase quadrature remains almost the same as in the monochromatic case. Once the phase sum is set to zero, the squeezing is destroyed. The responsible mechanism is explained in terms of two-photon emission processes induced by the trichromatic components and the interference between these processes.

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Optically coherent control of light-matter interactions lies at the heart of quantum optics and laser physics. One such example is resonance fluorescence, i.e., the scattering of light from free atoms that are irradiated with a near-resonant field. It was predicted $\lceil 1 \rceil$ $\lceil 1 \rceil$ $\lceil 1 \rceil$ and demonstrated $\lceil 2 \rceil$ $\lceil 2 \rceil$ $\lceil 2 \rceil$ that the spectrum of the resonance fluorescence from a strongly driven twolevel atom has a three-peaked structure. A central peak is located at the driving field frequency, and two sidebands are distant from the central peak by an amount equal to the Rabi frequency. Theoretical and experimental studies have shown that the resonance fluorescence has the potential for producing nonclassical light. In particular, photon antibunching was presented by Carmichael and Walls $\left[3\right]$ $\left[3\right]$ $\left[3\right]$ and Kimble and Mandel $[4]$ $[4]$ $[4]$ and demonstrated in many laboratories $[5-8]$ $[5-8]$ $[5-8]$; sub-Poissonian photon statistics was predicted by Mandel [[9](#page-4-6)] and verified by several groups $[10,11]$ $[10,11]$ $[10,11]$ $[10,11]$; squeezing in the phase quadratures was reported by Walls and Zoller $\lceil 12 \rceil$ $\lceil 12 \rceil$ $\lceil 12 \rceil$ and Loudon [[13](#page-4-10)], and observed by Lu et al. [[14](#page-4-11)].

On the other hand, polychromatic excitation leads to many novel and important effects in light-matter interactions. Examples include dressed-state lasers $[15]$ $[15]$ $[15]$, multiphoton processes $[16]$ $[16]$ $[16]$, intrinsically irreversible laser gain mechanisms [[17](#page-4-14)], modifications of the Autler-Townes spectrum [[18](#page-4-15)], polychromatic electromagnetically induced transparency [[19](#page-4-16)], large self-phase-modulation [[20](#page-4-17)], population inversion of a single driven atom in a cavity $\lceil 21 \rceil$ $\lceil 21 \rceil$ $\lceil 21 \rceil$, bichromatic laser cooling $\lceil 22 \rceil$ $\lceil 22 \rceil$ $\lceil 22 \rceil$, and subhalfwavelength atom localization $\lceil 23 \rceil$ $\lceil 23 \rceil$ $\lceil 23 \rceil$. The spectrum of fluorescence excited by polychromatic excitation exhibits a comblike structure $[24-30]$ $[24-30]$ $[24-30]$, in which the peak interval is determined by the modulation frequency, not by the Rabi frequency as in the monochromatic case. It has been predicted that bichromatic excitation leads to a strong photon correlation $[31]$ $[31]$ $[31]$ and an enhancement of noise squeez-ing in the resonance fluorescence of two-level atoms [[32](#page-5-2)]. Wu *et al.* [[33](#page-5-3)] demonstrated the phase-sensitive dynamics of bichromatically driven two-level atoms. However, the phase dependence no longer exists for the steady state $[25]$ $[25]$ $[25]$. A phase-dependent spectrum occurs when there are more than two exciting frequency components. Ficek *et al.* [[30](#page-5-0)] have investigated the phase-dependent effects when the symmetrical sideband components have equal relative phases compared to the central component. In particular, in the quintuchromatic excitation case, there is a flip of the spectral peaks between two different frequencies when two pairs of sideband driving components simultaneously change their respective phases from 0 to π . When a very large number of frequency components have the same relative phase, the fluorescence spectrum can disappear or return to the Mollow structure, depending on whether or not the sideband Rabi frequencies are equal to that of the central component. Most recently we have presented the effects of the sum of the relative phases of the sideband components compared to the central component when the symmetrical sidebands have different relative phases $\lceil 34 \rceil$ $\lceil 34 \rceil$ $\lceil 34 \rceil$. In the trichromatic case, the fluorescence spectrum is determined by the phase sum, not simply by the respective phases. The spectral lines are selectively eliminated simply by varying the phase sum.

Some questions remain: What is the phase dependence of the noise spectra? How do the exciting sidebands change the sideband squeezing in the phase quadratures? In this paper we address these two questions. We show that the exciting sidebands have remarkable effects on the noise spectra even when the sidebands are much weaker than the central component. When the exciting sidebands are tuned on twophoton resonance with the Mollow sideband transitions and the phase sum is set to π , the sideband squeezing that appears in the monochromatic case is kept almost unchanged. However, when we switch the phase sum to zero, the squeezing is almost washed out. Such phase dependence is attributed to the sideband-induced two-photon transitions in the dressed-state picture created by the central component of the trichromatic field.

Consider a two-level atom, which has ground and excited states $|1\rangle$ and $|2\rangle$. The atom is driven by a trichromatic field $\frac{1}{2}(E_0 + E_1 e^{i\delta t} + E_2 e^{-i\delta t})e^{-i\omega_0 t} + \text{c.c.,}$ where c.c. is the complex conjugate, E_j ($j=0,1,2$) are the amplitudes of the various components, ω_0 is the central frequency, and δ is the frequency difference. The master equation for the reduced density ρ is derived in an appropriate rotating frame and in the dipole approximation as

$$
\frac{d\rho}{dt} = -\frac{i}{\hbar} [H,\rho] + \mathcal{L}\rho, \qquad (1)
$$

where *H* is the Hamiltonian of the system,

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$$
H = -\hbar \Delta \sigma_{22} - \frac{\hbar}{2} (\Omega_0 + \Omega_1 e^{i\delta t} + \Omega_2 e^{-i\delta t}) \sigma_{21} + \text{H.c.}, \quad (2)
$$

with H.c. being the Hermitian conjugate. $\mathcal{L}\rho$ is the damping term

$$
\mathcal{L}\rho = \frac{\Gamma}{2}(2\sigma_{12}\rho\sigma_{21} - \sigma_{22}\rho - \rho\sigma_{22}),\tag{3}
$$

which describes the atomic spontaneous emission from $|2\rangle$ to [1](#page-0-0)) at the rate Γ . In Eqs. (1)–([3](#page-1-0)), $\sigma_{jk} = |j\rangle\langle k|$ represent the atomic projection operators for $j = k$ and the flip operators for $j \neq k$ (*j*, $k=1,2$). $\Omega_j = dE_j / \hbar$ (*j*=0,1,2) are complex Rabi frequencies associated with the respective components, and *d* is the atomic transition electric dipole moment. $\Delta = \omega_0 - \omega_{21}$ is the detuning between the central field frequency ω_0 and the atomic frequency ω_{21} . By arranging the expectation values into a column vector $X(t) = (\langle \sigma_{12} \rangle, \langle \sigma_{21} \rangle, \langle \sigma_{22} \rangle)^T$, we write the equation of motion in the compact form

$$
\frac{d}{dt}X(t) = Q(t)X(t) + R(t),
$$
\n(4)

where $Q(t)$ is the coupling matrix

$$
Q(t) = \begin{pmatrix} -\gamma & 0 & -2b \\ 0 & -\gamma^* & -2b^* \\ b^* & b & -\Gamma \end{pmatrix},
$$
 (5)

and *R*(*t*) is the inhomogeneous term *R*(*t*)=(*b*,*b*^{*},0)^{*T*}, $\gamma = \frac{\Gamma}{2}$ $+ i\Delta$, and $b(t) = \frac{i}{2}(\Omega_0 + \Omega_1 e^{i\delta t} + \Omega_2 e^{-i\delta t})$. $\langle \sigma_{11} \rangle$ is obtained by the closure relation $\langle \sigma_{11} \rangle + \langle \sigma_{22} \rangle = 1$.

Note that the coupling matrix *Q* and the inhomogeneous term *R* are both time dependent via the parameter *b*. Following the same techniques as in Ref. $[35]$ $[35]$ $[35]$, we solve Eq. (4) (4) (4) in three steps. (i) Make the harmonic expansion on the components $X_k(t) = \sum_{l=-\infty}^{\infty} X_k^{(l)}(t) e^{il\delta t}$, where $X_k^{(l)}$ $(k=1,2,3)$ represent the slowly varying amplitudes. (ii) Substitute the expanded terms into Eq. (4) (4) (4) , equate the coefficients of the harmonic of δ , and obtain an infinite series of equations for the slowly varying amplitudes. (iii) Arrange the slowly varying amplitudes into a column vector \bar{X} as

$$
\overline{X} = (\dots; X_1^{(-N)}, X_2^{(-N)}, X_3^{(-N)}; \dots; X_1^{(0)}, X_2^{(0)}, X_3^{(0)}; \dots; X_1^{(N)}, X_2^{(N)}, X_3^{(N)}; \dots)^T,
$$
\n
$$
(6)
$$

write the infinite set of equations in a compact form, and then obtain the steady state solution $(t \rightarrow \infty)$ as

$$
\overline{X} = -\overline{Q}^{-1}\overline{R},\tag{7}
$$

where the matrix \overline{Q} and the column vector \overline{R} , which are not written here, are obtained from Eq. (4) (4) (4) and the above three steps. The numerical calculation is performed by truncating the set of equations at a large value *N* that is necessary to achieve accuracy.

The incoherent spectrum of the scattered field $E_{\text{in}}(t)$ is usually defined as $[36]$ $[36]$ $[36]$

$$
S_{\rm in}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\tau \lim_{t \to \infty} \langle E_{\rm in}^{(-)}(r, t + \tau) E_{\rm in}^{(+)}(r, t) \rangle e^{-i\omega \tau}.
$$
 (8)

Relating the field operators to the atomic operators, we write the spectrum $\lceil 36 \rceil$ $\lceil 36 \rceil$ $\lceil 36 \rceil$

$$
S_{\rm in}(\omega) = \Gamma u(\mathbf{r}) \text{Re} \int_0^\infty d\tau \lim_{t \to \infty} \langle \Delta \sigma_{21}(t + \tau) \Delta \sigma_{12}(t) \rangle e^{-i\omega \tau}, \tag{9}
$$

where $\Delta A = A - \langle A \rangle$, and $u(\mathbf{r}) = (3/8\pi) \sin^2 \theta$, with θ the angle between the observation direction **r** and the atomic transition dipole moment **d**. The two-time correlation function in Eq. ([9](#page-1-2)) can be calculated by introducing the correlation matrix $Y(\tau)$ with its three components

$$
Y(\tau) = \left[\langle \Delta \sigma_{12}(t + \tau) \Delta \sigma_{12}(t) \rangle, \langle \Delta \sigma_{21}(t + \tau) \times \Delta \sigma_{12}(t) \rangle, \langle \Delta \sigma_{22}(t + \tau) \Delta \sigma_{12}(t) \rangle \right]^T.
$$
 (10)

According to the quantum regression theorem [[37](#page-5-7)], for τ $>$ 0, the two-time average $Y(\tau)$ satisfies the same equation of motion as the one-time average $X(t)$ with vanishing inhomogeneous term,

$$
\frac{d}{d\tau}Y = QY.\tag{11}
$$

Using the same techniques for solving $X(t)$ and the Laplace transform $Y(z) = \frac{1}{2\pi} \int_0^\infty Y(\tau) e^{-zt}$ we obtain the incoherent spectrum

$$
S_{\text{in}}(\omega) = \frac{\Gamma}{2} u(\mathbf{r}) \text{Re}[Y_2^{(0)}(i\omega) + Y_2^{(0)}(-i\omega)],\tag{12}
$$

where the indices 2 and 0 in $Y_2^{(0)}$ have the same meanings as those in $X_2^{(0)}$.

The normally ordered noise spectrum of the fluorescence field is usually defined as $[36]$ $[36]$ $[36]$

$$
S_{x(y)} = \int_{-\infty}^{\infty} d\tau \lim_{t \to \infty} \langle : \Delta E_{x(y)}(r, t + \tau) \Delta E_{x(y)}(r, t) : \rangle e^{i\omega \tau}, \tag{13}
$$

where $E_{x(y)}(r, t)$ is the slowly varying in-phase (out-of-phase) quadrature operator of the fluorescent radiation field. Using the relation between the fluorescent field and the atomic flip operators in the far radiation zone, we write the spectrum in the form

$$
S_{x(y)}(\omega) = \Gamma \operatorname{Re} \int_0^{\infty} d\tau \lim_{t \to \infty} \cos(\omega \tau)
$$

$$
\times [\langle \Delta \sigma_{21}(t + \tau) \Delta \sigma_{12}(t) \rangle
$$

$$
\pm \langle \Delta \sigma_{12}(t + \tau) \Delta \sigma_{12}(t) \rangle].
$$
 (14)

When $S_{x(y)}(\omega)$ < 0 the fluorescent light shows quadrature squeezing. The value $S_{x(y)}(\omega) = -\frac{1}{4}$ corresponds to the maximal degree 100% of squeezing. The noise spectrum for $x(y)$ quadrature is obtained as

$$
S_{x(y)}(\omega) = \frac{\Gamma}{2} \text{Re}[Y_2^{(0)}(i\omega) + Y_2^{(0)}(-i\omega) \pm Y_1^{(0)}(i\omega) \n\pm Y_1^{(0)}(-i\omega)].
$$
\n(15)

 $S_{x(y)}(\omega)$ is a measure of the deviation from the shot noise limit. A negative spectrum is a signature of a nonclassical state of the field.

In our calculations, we scale the Rabi frequencies $(|\Omega_j|, j=0,1,2)$ and the frequency differences (Δ, δ, ω) in units of Γ . Without loss of generality, we assume the Rabi frequency Ω_0 to be real, and choose $\Omega_j = |\Omega_j|e^{-i\phi_j}$ (*j*=1,2), where ϕ_i is the phase of the sideband component E_i relative to the central component E_0 . For convenience we define the phase sum $\Phi = \phi_1 + \phi_2$. First, we show the phase dependence of the fluorescence spectrum. In Fig. [1](#page-2-0) we plot $S_{\text{in}}(\omega)$ for Φ =0 (solid lines) and π (dotted lines). The other parameters are chosen as (a) $\Omega_0 = 16$, $|\Omega_1| = |\Omega_2| = 3$, $\Delta = 12$, $\delta = 10$, (b) Ω_0 =40, $|\Omega_1|$ = $|\Omega_2|$ =5, Δ =30, δ =25. The frequencies of the exciting sidebands are on two-photon resonance with the Mollow sideband transitions, $\delta = \frac{1}{2}\overline{\Omega}$, where $\overline{\Omega} = \sqrt{\Delta^2 + \Omega_0^2}$ is the separation of the dressed states in the same doublet, as will be shown below. For comparison, we have plotted the spectra for monochromatic excitation $(\Omega_1 = \Omega_2 = 0$, thin dashdot-dotted lines). It has become clear that for the monochromatic case sideband squeezing is present only when $\Delta \neq 0$ [[38](#page-5-8)]. So we focus on the case of $\Delta \neq 0$. In the presence of the exciting sidebands, the spectrum has an asymmetrical structure when $\Delta \neq 0$. This is in sharp contrast to the monochromatic case, in which the fluorescence spectrum has symmetrical structure even when the exciting field is detuned from the atomic transition $[1]$ $[1]$ $[1]$. In the figure we have chosen different ratios of the sideband Rabi frequencies to that of the central component, $\frac{|\Omega_{1,2}|}{\bar{\Omega}}$ $\frac{|\Omega_{1,2}|}{\bar{\Omega}}$ $\frac{|\Omega_{1,2}|}{\bar{\Omega}}$ = 0.15 [Fig. 1(a)] and 0.1 [Fig. 1(b)]. The condition $|\Omega_{1,2}| \ll \overline{\Omega}$ is better satisfied for the latter than for the former. When the conditions $\Gamma \ll |\Omega_{1,2}| \ll |\Omega_0, |\Delta|$ and $\delta \sim \frac{\Omega}{2}$ are satisfied, the fluorescence spectrum for the trichromatic excitation displays the following features. (i) The frequencies at which spectral peaks occur for the monochromatic case are hardly modified. (ii) There appear additional minor sidebands at $\omega \approx (m + \frac{1}{2})\overline{\Omega}$, $m = 0, 1, 2, ...$ These originate from the harmonics of the modulating field.

FIG. 1. (Color online) Fluorescence spectrum $S_{\text{in}}(\omega)$ for $\Phi = 0$ (solid lines) and π (dotted lines). The other parameters are chosen as (a) $\Omega_0 = 16$, $|\Omega_1| = |\Omega_2| = 3$, $\Delta = 12$, $\delta = 10$, (b) $\Omega_0 = 40$, $|\Omega_1| = |\Omega_2|$ $= 5$, $\Delta = 30$, $\delta = 25$. The spectrum (thin dash-dot-dotted lines) for $\Omega_1 = \Omega_2 = 0$ is plotted for comparison.

(iii) The heights of the spectral peaks at $\omega = 0, \pm \overline{\Omega}$ are greatly increased or decreased for $\Phi = 0$, but are kept almost unchanged for $\Phi = \pi$. In addition, when we take $\Delta < 0$, the spectrum switches symmetrically from the left to the right half regime.

Next, we show the phase dependence of the noise spec-trum. In Fig. [2](#page-3-0) we plot the noise spectrum $S_x(\omega)$ for the same parameters as in Fig. [1.](#page-2-0) Similar features are found. In Fig. $2(a)$ $2(a)$, the spectrum for the monochromatic case has its minimum value $[S_x]_{\text{min}}$ =−0.0638 at a pair of sidebands $\omega \approx \pm 20$. This corresponds to 25% squeezing at the Mollow sidebands $\omega = \pm \Omega = \pm 20$. For the trichromatic case, the spectrum for $\Phi = \pi$ takes its minimum value $[S_x]_{\text{min}}$ =−0.050 at the sidebands $\omega \approx \pm 20.6$. This corresponds to sideband squeezing of 20%. Compared with the monochromatic case, the squeezing for the trichromatic case is only slightly decreased. In sharp contrast, for $\Phi = 0$ (solid lines), the spectra are close to or above the shot noise limit, which indicates that squeezing is almost canceled. In Fig. $2(b)$ $2(b)$, $[S_x]_{\text{min}} = 0.0641$ at $\omega \approx \pm 50.0$, squeezing of 25.6% occurs for the monochromatic case. For the trichromatic case, the spectrum for $\Phi = \pi$ takes its minimum $[S_x]_{\text{min}} = -0.057$ at the sidebands $\omega \approx \pm 50.6$. We have 23% squeezing. This shows that the squeezing is kept almost the same as in the monochromatic case. For $\Phi = 0$, squeezing no longer exists. So far, we have considered the effects of the weak sidebands

FIG. 2. (Color online) Noise spectrum $S_x(\omega)$ for the same parameters as in Fig [1.](#page-2-0)

 $[\Gamma \ll |\Omega_{1,2}| \ll (\Omega_0, |\Delta|)]$ on the noise spectrum. When we increase the Rabi frequencies of the exciting sidebands such that the above relations are not satisfied, the above noise squeezing is spoiled. Strong sidebands will wash out any squeezing. For bichromatic excitation as the extreme case of trichromatic excitation, no squeezing exists; this is not shown in our figures.

The mechanism responsible for the squeezing can be traced to the two-photon emission processes, as shown in Fig. $3(a)$ $3(a)$. There are three channels for the two-photon emission: (i) the absorption of photons of the strong central driving component (ω_0) and the emission of photons of sidebands $[\omega_0 \pm \overline{\Omega}$, also see Fig. [3](#page-3-1)(b)]; (ii) the successive

FIG. 3. (Color online) (a) Trichromatic-component-induced $(\omega_0, \omega_0 \pm \delta)$ channels for two-photon emission at sidebands $(\omega_0 \pm \overline{\Omega})$. (b) The dressed states associated with the central exciting component and the transitions induced by the sideband components. *n* is the number of quanta for the central exciting component.

absorption of photons of the weak high- and low-frequency driving components $(\omega_0 \pm \delta)$ and the emission of photons of sidebands $(\omega_0 \pm \bar{\Omega})$; and (iii) the same as in (ii) except for the exchange of the high- and low-frequency driving components. The two-photon emission occurs at sidebands $(\omega_0 \pm \overline{\Omega})$ due to the Stark splitting by the strong central driving component $(\Omega_0 \ge |\Omega_{1,2}|)$. At the same time, the weak modulation components cause the two-photon emission of the same sidebands as above. That is why the squeezing occurs at sidebands but not at the central frequency as in the bichromatic case considered by Jakob and Kryuchkyan [[32](#page-5-2)]. In their case, there is no central component, and the bichromatic components are symmetrically detuned from the atomic resonance frequency. The bichromatic driving field induces degenerate two-photon emission $(\omega_0 = \omega_{21})$ [[31](#page-5-1)]. This determines squeezing at the central frequency [[32](#page-5-2)]. For the trichromatic case, however, the central driving component is much stronger than the driving sidebands and determines the two-photon emission at sidebands. Usually one expects that the driving sidebands spoil the squeezing effects. However, for this particular phase sum, the destructive effects of the driving sidebands can be suppressed to a negligible degree. This can be roughly understood by noting that the above three channels for two-photon emission interfere with each other. We rewrite the time-dependent Rabi frequency $\Omega(t) = \Omega_0 + 2|\Omega_1|\cos(\delta t + \frac{\phi_2 - \phi_1}{2})\exp(i\frac{\Phi}{2}),$ where we have used $\Phi = \phi_1 + \phi_2$, $|\Omega_1| = |\Omega_2|$. As time passes, the effects of the respective phases (ϕ_1, ϕ_2) in the term cos($\delta t + \frac{\phi_2 - \phi_1}{2}$) are averaged out. This indicates that the dynamics and squeezing properties of the system are independent of the phases (ϕ_1, ϕ_2) . However, the phase sum Φ always plays its role. For $\Phi = 0$, the Rabi frequency reduces to $\Omega(t) = \Omega_0$ +2 Ω_1 cos($\delta t + \frac{\phi_2 - \phi_1}{2}$), the time-dependent part of which is real and ranges from $-2\Omega_1$ to $2\Omega_1$. In this case the interference decreases the squeezing to such a degree that the squeezing tends to vanish. For $\Phi = \pi$, the Rabi frequency becomes $\Omega(t) = \Omega_0 + i2\Omega_1 \cos(\delta t + \frac{\phi_2 - \phi_1}{2})$, the time-dependent part of which is purely imaginary and changes from $-i2\Omega_1$ to $i2\Omega_1$. In this case, the destructive interference is at the lowest level and thus the squeezing is kept almost the same as in the monochromatic case.

Further, the sideband squeezing and the phase dependence can be explained in terms of dressed states [[39](#page-5-9)] that are associated with the strong central component of the trichromatic field. By diagnosing the Hamiltonian $H = -\hbar \Delta \sigma_{22}$ $-\frac{\hbar}{2}\Omega_0(\sigma_{12}+\sigma_{21}),$ we obtain the dressed states $|+\rangle = s|1\rangle$ + \bar{c} (2), $|-\rangle = c|1\rangle - s|2\rangle$, and the corresponding eigenvalues $\lambda_{\pm} = \frac{\hbar}{2}(-\Delta \pm \overline{\Omega})$, where $s = \sqrt{\frac{1}{2} \pm \frac{\Delta}{2\Omega}}$. This means that the atomic level is split into two levels, which are shifted from the original position by λ_{\pm} and separated from each other by λ_{+} – λ_{-} = $\hbar \Omega$. Using the dressed states we rewrite the Hamiltonian for the interaction of the atom with the exciting sidebands,

$$
H_s = -\frac{\hbar}{2} (\Omega_1 e^{i\delta t} + \Omega_2 e^{-i\delta t}) (cs\sigma_{++} - cs\sigma_{--} + c^2 \sigma_{+-} e^{i\overline{\Omega}t} - s^2 \sigma_{+-} e^{-i\overline{\Omega}t}) + \text{H.c.} \tag{16}
$$

We focus on the case of $|\Omega_{1,2}| \ll (\Omega_0, \Delta)$, i.e., $|\Omega_{1,2}| \ll \overline{\Omega}$. When we choose $\delta = \frac{1}{2}\overline{\Omega}$, all the one-photon transitions $|\pm\rangle \rightarrow |\pm\rangle$ induced by the sidebands have the detunings $\delta = \overline{\Omega}/2$ $\pm \delta$, as shown in Fig. [3](#page-3-1)(b). Since $1 \le |\Omega_{1,2}| \le \delta$, the onephoton transitions are far off resonance with the dressed transitions. The transitions $|+\rangle \rightarrow | \pm \rangle \rightarrow | + \rangle$ and $\delta = \overline{\Omega}/2$ $\delta = \overline{\Omega}/2$ $\ket{-} \rightarrow \ket{\pm} \rightarrow \ket{-}$ are on the two-photon resonances. The $\overline{\delta} = \overline{\Omega}/2$ $\delta = \overline{\Omega}/2$ effective Rabi frequency is small, $\Omega_{\text{eff}} = \frac{\Omega_1 \Omega_2}{\delta} \ll \overline{\Omega}$. The exciting sidebands have no significant effect on the level shift of the dressed states $|\pm\rangle$. Therefore, the spectral lines remain at $\omega = \pm \overline{\Omega}$, as in the monochromatic case. Minor peaks in the spectrum at $\omega = (m + \frac{1}{2})\overline{\Omega}$ are the signature of such $(2m+1)$ -photon transitions, $m=0,1,2,...$ It should be noted that the two-photon transitions have the same phase Φ .

The system dynamics and quantum noise are determined by the phase sum Φ but not by the respective phases $\phi_{1,2}$. As the phase sum is varied from $\Phi = 0$ to π , both the fluorescence and noise spectra are significantly changed, as shown by the numerical results in Figs. [1](#page-2-0) and [2.](#page-3-0)

Finally, we should note that homodyne detection also causes phase dependence $[14]$ $[14]$ $[14]$, which was employed to enhance the squeezing, as shown by Jakob and Kryuchkyan [[32](#page-5-2)]. In that case, a strong local oscillator field is used, which has the same frequency as the fluorescent field. The atomic radiation field is mixed with a strong local oscillator field $E_L = |E_L|e^{i\theta}$ (θ is the phase of the local oscillator field relative to the driving field) at a diode detector. The atomic radiation

field contains three parts $E(t) = \langle E_a \rangle + \Delta E_x + i \Delta E_y$, the mean field $\langle E_a \rangle$, and the fluctuating components in phase, ΔE_x , and 90° out of phase with the driving field, ΔE_v . In the measurements, the detector measures the total power of the combined field, proportional to $|E_L + E|^2$. The fluctuations in the detected power are determined by the interference terms $\Delta P(t) \propto E_L^* E(t) + E_L E^*(t) = |E_L| [e^{-i\theta} E(t) + e^{i\theta} E^*(t)].$ For a weak driving field, the cross correlation between in-phase and outof-phase fluctuations is maximized at the central frequency for $\theta = \pm \frac{\pi}{4}$. For the previous bichromatic case [[32](#page-5-2)], the maximum degree of squeezing (\approx 50%) is reached, which is approximately twice as strong as it is in the monochromatic case. In the present work, the two-photon emission processes are modulated by the driving sidebands. Since no local oscillator field is used, the squeezing is kept at almost the same level as in the monochromatic excitation case $\lceil 38 \rceil$ $\lceil 38 \rceil$ $\lceil 38 \rceil$.

In conclusion, we have shown the phase dependence of the quantum noise in phase quadratures of resonance fluorescence from a two-level atom driven by a trichromatic exciting field. When the conditions (i) $\Gamma \ll |\Omega_{1,2}| \ll (\Omega_0, |\Delta|)$, (ii) $\delta \sim \frac{\overline{\Omega}}{2}$, and (iii) $\Phi = \pi$ are satisfied, one has almost the same amount of sideband squeezing as for the monochromatic case. Switching the phase sum to zero or increasing the sidebands to a value comparable to that of the central component leads to disappearance of the sideband squeezing.

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