Pair condensation of bosonic atoms induced by optical lattices

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We design a model of correlated hopping for bosonic atoms in optical lattices. Such a model exhibits three kinds of phases: a Mott insulator, a charge density wave, and a pair quasicondensate. One possible implementation of the model is based on two-state atoms embedded in an optical superlattice and having state-dependent interactions. Contrary to other models of pairing, correlated hopping is not a perturbative effect and should be observable in generalizations of current experiments.

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I. INTRODUCTION

Since the achievement of Bose-Einstein condensation in alkali atoms [1–3], we have witnessed two major breakthroughs in the many-body physics of cold atoms. One is the realization of Cooper pairing and the BCS to BEC transition with fermions [4–6]. The other one is the implementation of lattice Hamiltonians using neutral atoms in optical lattices [7,8]. Supported by this success, many theoretical papers suggest using cold atoms with two goals: the quantum simulation of well-known Hamiltonians such as Hubbard models [8] and spin lattices [9], and the quest for new physics such as bosonic quantum Hall effect [10,11] and lattice gauge theories [12,13]. In this work we aim at the latter, introducing a robust mechanism of pairing that works for bosonic atoms, opens theoretical challenges and is suited for the recent experiments in optical superlattices [14,15].

Pairing is a central concept in strongly correlated states. In particular, it is the essence of ordinary BCS superconductivity. In BCS theory, an attractive interaction mediated by a phonon bath is the basis by which electrons merge into bosonic pairs that conduct electricity without friction. Similar phenomena appear in the BCS pairing of fermionic atoms, where the natural attraction is enhanced by Feshbach resonances [4–6].

Another, less known mechanism for pairing is correlated hopping. It appears naturally in fermionic tight-binding models [16–22] and in quantum magnetism [23], consisting on the motion of particles being influenced by the environment. This is normally reflected by terms of the form $n_i a_j^\dagger a_k$ appearing in the Hamiltonian. Correlated hopping could lead to the formation of bound electron pairs [19,20] and it has been put forward as an explanation for high-T_c superconductivity [24,25].

We will introduce a mechanism for pairing, which is based on collisions that induce transport. As illustrated in Fig. 1(a), when atoms collide they can mutate their internal state. If the atoms are placed in a state-dependent optical lattice, whenever such a collision happens, the pair of atoms must tunnel to a different site associated to their new state [Fig. 1(b)]. For deep enough lattices, as in the Mott insulator experiments [7], this coordinated jump of pairs of particles will be the dominant process and the atoms will become a superfluid of pairs.

The structure of this paper is as follows. Section II introduces the model in two possible setups, one with cold atoms in a deep optical superlattice and the other with deep ordinary lattices. We show how collisions combined with state-dependent lattices lead to a strong correlated tunneling, as described before. The outcome is an effective Hamiltonian which can be related to the experimental parameters. The details of the derivation can be found in the Appendix. In Sec. III we then draw a realistic phase diagram using both Gutzwiller [26] and matrix product states (MPS) [27] variational methods. Both procedures essentially identify three kinds of phases: a charge-density wave (CDW), a Mott insu-

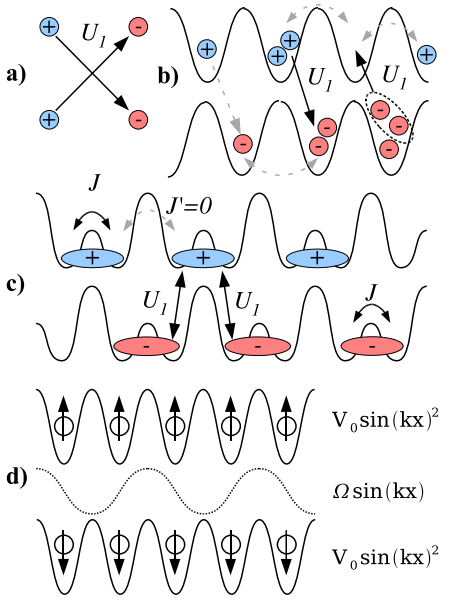

FIG. 1. (Color online) (a) Two atoms in state $|+\rangle$ collide and change into state $|-\rangle$. (b) When trapped in a state-dependent optical lattice, changing the state implies also jumping between lattice sites. We plot some forbidden (dashed) and allowed (solid) processes. Only pairs of atoms can hop between lattices. (c) In order to maximize the overlap between atoms in different states and thus the strength of correlated hopping, in this paper we consider two state-dependent superlattices. (d) The physical implementation of (c) is based on two-states atoms $|\pm\rangle \propto |\uparrow\rangle \pm |\downarrow\rangle$ confined in an optical lattice of strength V_0 and coupled by an oscillating Raman field, of strength $\Omega < V_0$.

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lator (MI), and a pair condensate or pair superfluid (PSF). In Sec. IV we consider first various experimental procedures to detect the different insulating and superfluid orders, and finish with a discussion on the atomic species and the currently accessible range of parameters.

II. THE MODEL

A. Superlattice model

In this section, we introduce a possible physical implementation of correlated hopping using the setup from Fig. 1(c). Bosonic atoms in two internal states are trapped in one-dimensional (1D) state-dependent superlattices that spatially overlap. Such superlattices can be created by combining a 2D lattice that tightly confines the atoms along two perpendicular directions Y and Z, with an adjustable 1D lattice and a suitable Raman coupling along X, as described in Fig. 1(d), in the Appendix and in Ref. [28].

We will assume that the potential barrier between superlattice sites is very large and prevents tunneling, but inside a superlattice cell the hopping amplitude J will be large compared to the on-site interactions. This implies that we can use a tight-binding approximation for the field operators [8]

$$\psi_+(x) = \sum_i a_{2i+} W(x-2il), \quad \psi_-(x) = \sum_i a_{2i-} W[x-(2i+1)l]. \tag{1}$$

Here, $a_{k\pm}$ denotes bosonic annihilation operators for particles in states (+) or (−) with a spatial wave function $W(x)$ given by the ground state of a particle in a superlattice cell. Note that the superlattice has a period $2l$ and that the underlying sublattice has period l and Wannier wave functions $w(x)$, such that $W(x) \simeq [w(x)+w(x-l)]/\sqrt{2}$.

We will now suggest one possible mechanism to engineer state-changing collisions. The idea begins by noticing that the trapped states are actually dressed states

$$\psi_\pm = (\psi_\uparrow \pm \psi_\downarrow)/\sqrt{2} \tag{2}$$

built from two collisionally stable internal states of the atom. One will normally find that the contact interaction between the atoms is diagonal in the undressed basis

$$H_{\text{int}} = \sum_{\alpha,\beta=\uparrow,\downarrow} \frac{g_{\alpha\beta}}{2} \int \psi_\alpha^\dagger(x)\psi_\beta^\dagger(x)\psi_\beta(x)\psi_\alpha(x)dx. \tag{3}$$

Such states also have in general both state-independent g_0 and state-dependent contact interactions $\{g_1, g_2\}$

$$g_{\uparrow\uparrow} = g_0 + g_2, \quad g_{\downarrow\downarrow} = g_0 - g_2, \quad g_{\uparrow\downarrow} = g_{\downarrow\uparrow} = g_0 + g_1. \tag{4}$$

Since the atoms are tightly confined along two directions, these constants are proportional to the effective one-dimensional scattering lengths.

To better understand the effect of our double hopping term, we will focus on $g_2=0$, a situation that can be found in rubidium [29]. When we replace the expansions (2) into Eq. (3), we obtain terms such as $g_1\psi_-^{\dagger 2}\psi_+^2$ inducing the processes from Fig. 1(a). As shown in the Appendix, once we collect all terms that involve Wannier wave functions separated by no more than a superlattice period, the outcome is a Hubbard model with correlated hopping

$$H = \sum_{i,j=i+1} [U{:}(n_i+n_j)^2{:} + Vn_in_j - tc_i^{\dagger 2}c_j^2 - tc_j^{\dagger 2}c_i^2]. \tag{5}$$

Here, we have renumbered the bosonic operators $c_{2k}=a_{k+}$, $c_{2k+1}=a_{k-}$, and $n_i=c_i^\dagger c_i$, with the natural zigzag order from Fig. 1(c). The colons $(:A:)$ denote normal order of creation and annihilation operators. Finally, we have introduced two nearest neighbor interactions U and V and a correlated hopping amplitude t

$$U = \frac{2U_0+U_1}{16}, \quad V = -\frac{U_1}{8}, \quad t = \frac{U_1}{16}, \tag{6}$$

which are functions of the interaction constants

$$U_i \simeq g_i \int dx|w(x)|^4 \quad \text{for} \quad i=0,1,2. \tag{7}$$

Out of the terms in Eq. (5), the correlated tunneling t favors the creation of delocalized states of pairs of atoms $|\psi_{\text{pair}}\rangle \sim \Sigma_i c_i^{\dagger 2}|0\rangle$. The nearest-neighbor interactions, on the other hand, either favor clustering $V<0$ or restrict the density preventing collapse $U\geq 0$. From these competing effects we expect a qualitative picture consisting on a quantum phase transition from a superfluid of paired particles in the noninteracting case to an insulator of incoherent bosons for large U,V. Finally, note that our Hamiltonian contains no single-particle hopping for the reasons stated at the beginning of this section.

B. Alternative setup

Correlated hopping also appears in a much weaker form in the setup from Fig. 1(b). There, two lattices trap atoms in different states and are shifted by half a period or $\lambda/4$ relative to each other. As before, the trapped states and the interaction are described in different basis. One can expand the interacting Hamiltonian in terms of Wannier functions together with operators defining the trapped states. This leads to Eq. (5) with

$$U = \frac{2U_0+U_1}{8}, \quad V = -\frac{2(1-\beta)U_0+U_1}{2}, \quad t = \beta\frac{U_1}{4}. \tag{8}$$

Here, U_0 and U_1 are the on-site interactions in the atomic basis and the factor

$$\beta = \int |w(x)|^2|w(x-\lambda/4)|^2dx \Big/ \int |w(x)|^4dx \ll 1, \tag{9}$$

computed using the Wannier functions $w(x)$, measures the relative strength of interaction between different lattices with respect to those on the same lattice site.

III. PHASE DIAGRAMS

In order to study the many-body physics of our model, we have used the Gutzwiller ansatz, which is a variational method based on the product state [26]

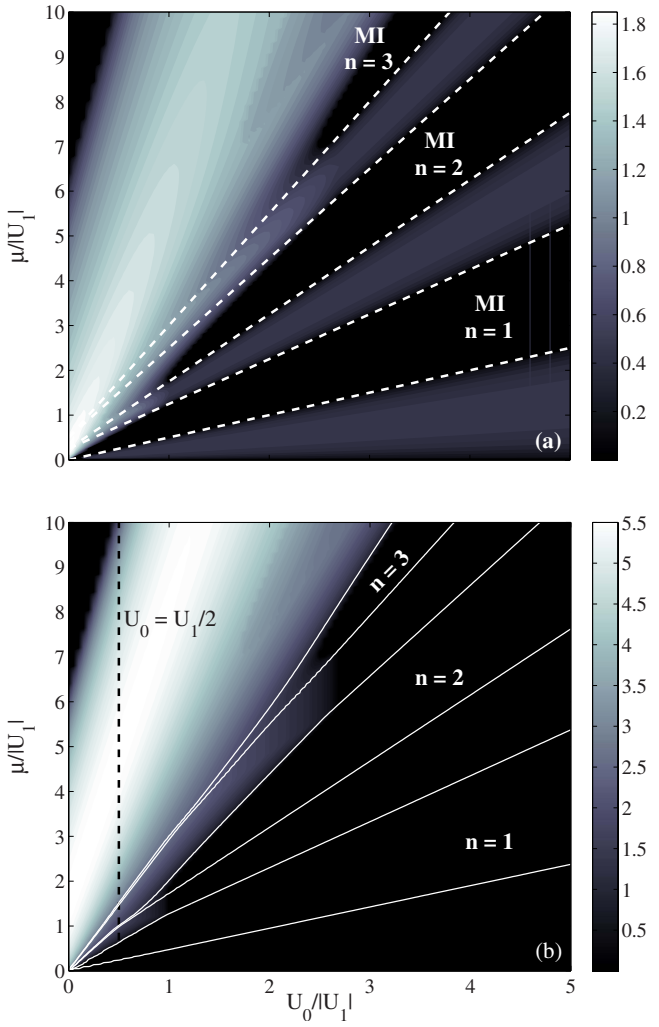

FIG. 2. (Color online) Ground-state properties estimated with the Gutzwiller wave function (10). Grayscale plots of (a) density fluctuations Δn and (b) pair condensate order parameter $\langle c^2 \rangle$. Dashed lines mark the analytical estimates, while solid lines delimit regions of integer filling. All plots cover the same region $(U_0, \mu) \in [0, 5U_1] \times [0, 10U_1]$.

$$|\psi_{\mathrm{GW}}\rangle = \prod_i \sum_n \frac{1}{\sqrt{n!}} f_n c_i^{\dagger n} |0\rangle. \tag{10}$$

Minimizing the expectation value of the free energy $F := H - \mu N$ with respect to the variables f_n under the constraint of a fixed norm $\Sigma |f_n|^2 = 1$, one obtains the phase diagram in the phase space of interaction and chemical potential (U_1, U_0, μ). For the sake of simplicity, we have used $U_1 := 1$ as unit of energy everywhere.

The results are shown in Fig. 2, where we plot the particle number variance $\Delta n := \sqrt{\langle n^2 - \bar{n}^2 \rangle}$ and what we identify as the order parameter of the paired superfluid $\langle c^2 \rangle$. From the zeros of the density fluctuations we can recognize the Mott regions with $\bar{n} = 1$, 2, and 3 particles per site. The boundary of the region with $\bar{n} = 1$ can be estimated analytically using a Gutzwiller state with nonzero components f_0, f_1, and f_2, which gives $\mu_p = \frac{U_0}{2}$ and $\mu_h = U_0 + \frac{1}{4}$. These are the energies to add a particle or to make a hole in this insulating lobe. If $\mu < \mu_h$, the Gutzwiller ansatz gives fractional densities but, as we will see below, this is an artifact of the uniform trial wave function. If $\mu > \mu_p$ and $U_0 < U_1/2$, we obtain a region of nonzero $\langle c^2 \rangle$. We take this as a sign of a long-range coherence in the two-body density matrix which cannot be attributed to an ordinary Bose-Einstein condensate, since the same simulation gives $\langle c \rangle = 0$.

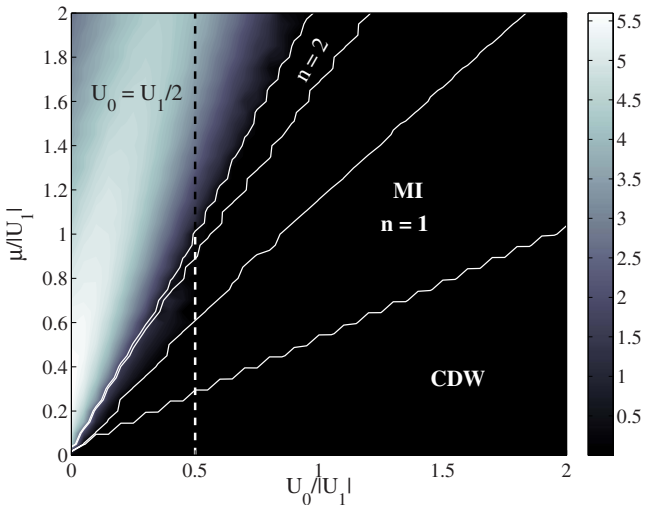

FIG. 3. (Color online) Ground-state properties estimated with the MPS method. We plot the averaged two-particle correlator $\Sigma_\Delta |C_\Delta^2|/L$, where $C_\Delta^2 := \langle c_i^{\dagger 2} c_{i+\Delta}^2 \rangle$. Solid lines delimit regions of integer filling.

The Gutzwiller ansatz does not reproduce accurately neither the location of phase transitions nor the behavior of correlators. For instance, $C_\Delta^1 := \langle c_i^\dagger c_{i+\Delta} \rangle$ and $C_\Delta^2 := \langle c_i^{\dagger 2} c_{i+\Delta}^2 \rangle$ computed with Eq. (10) are both uniform functions. In order to study these properties, we have used the MPS method [27] to estimate variationally the ground state of our model. The MPS is a more complex wave function that, for large enough computational resources and not too large systems, should reproduce the correlations in the superfluid regime. Calculations with up to 40 sites reveal that matrices of size $D = 40$ are enough to pinpoint the different phases.

As an example, in Fig. 3 we plot the averaged two-particle correlator. Density and number variance profiles are very similar to the Gutzwiller ones. We thus conclude that $\bar{n} = 1$ and $\bar{n} = 2$ insulating regions still exist, with a similar size and shape as in the Gutzwiller model. The biggest difference lays in the triangle $0 \leq \mu < \mu_h$. In this region of low density, the bosons arrange forming a charge density wave: a pattern alternating 0 and 1 atom per site. Above μ_p we find a paired superfluid, as conjectured. In this phase, quasi-long-range order is revealed by a slow decay of the off-diagonal elements in the two-particle density matrix. As shown in Fig. 4, the single particle correlator $C_\Delta^1 = \bar{\delta}_{\Delta,0}$ is only different from zero at $\Delta = 0$, where it becomes the density. This could have indicated the presence of a Mott phase, were it not for the nonzero value of Δn [Fig. 3(b)] and of the two-particle correlator C_Δ^2 that decays slowly at long distances. Due to the size of our simulations, we have not been able yet to determine the behavior of C_Δ^2, but numerical fits of curves such as

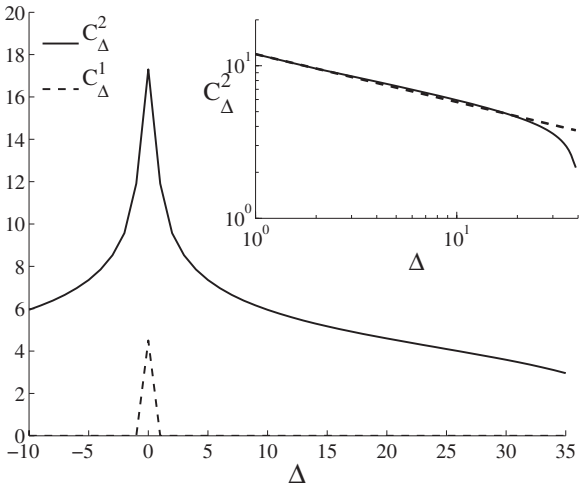

FIG. 4. Single particle $C^1_\Delta := \langle c_i^\dagger c_{i+\Delta} \rangle$ (dashed) and two particle $C^2_\Delta := \langle a_i^{\dagger 2} a^2_{i+\Delta} \rangle$ correlators (solid) for $\mu/U_1 = 0.9$, $U_0/U_1 = 0$, and a lattice with 40 sites. The inset repeats the plot for C^2_Δ in log-log scale, with a fit $C^2_\Delta \propto \Delta^{0.32}$ (dash).

the one in Fig. 4 suggest an algebraic decay $C^2_\Delta \sim \Delta^{-\alpha}$ with a nonuniversal exponent around $\alpha < 0.7$ that depends both on U_0 and the chemical potential μ.

IV. EXPERIMENTAL CONSIDERATIONS

A. State characterization

All the phases that appear in our setup can be recognized experimentally. First of all, the MI and the CDW have both a well defined number of particles per site and no coherence. While their time of flight pictures will show no interference fringes [7], the noise correlation should exhibit peaks at discrete momenta [30,31]. In addition, the CDW corresponds to a setup where either n_+ or n_- are uniformly zero. Finally, the insulator energy gap can be probed by static [7] or spectroscopic means [32].

Regarding the pair superfluid, it is a perfect "conductor" with a gapless excitation spectrum. It lacks single-particle order $C^1_\Delta \sim 0$ a.e. and will produce no interference fringes in time of flight images. In order to measure C^2_Δ and detect the pairing, we suggest to use photoassociation to build molecules out of pairs of atoms. Since the molecules will be built on-site, the nonzero correlator C^2_Δ will translate into long-range order for the resulting molecules. This order should reveal as an interference pattern in time-of-flight images, slightly blurred by the phase fluctuations which are inherent to 1D.

B. Microscopic details

Here we discuss the requirements for an atomic specie to produce correlated hopping in an optical lattice. First of all, the pair binding energy is proportional to our unit of energy U_1 and it should be large and comparable to the on-site interaction in a typical Mott insulator [7], which is about 1 kHz. While these values are normally large enough for the preparation of the ground state and the stability of the setup under dephasing and decoherence, there are two potential difficulties that one has to face.

The first problem is that the ratio of U_1/U_0 may be small. If this is the case, pairing will occur at large densities, making the experiment more difficult. This ratio can be increased using Feshbach resonances, as it has been done for the currently most promising atomic species ^{87}Rb. Having $g_2 = 0$, the value of U_1/U_0 has been enhanced up to 0.1–0.2 [29]. As numerical simulations confirm, this would already gives rise to pairing at densities between three and five particles per site.

While the use of Feshbach resonances is normally associated to strong losses, one should investigate the use of broader resonances not to dramatically increase g_1, but to moderately decrease the value of g_0, which would have the same effect. The second problem is that for species where g_1 is large, one typically finds a large value of g_2. Ongoing work suggests that this is not really an obstacle, as having both $g_{\uparrow\uparrow} \neq g_{\downarrow\downarrow}$ and $g_1 \neq 0$ leads to a richer family of Hamiltonians, with terms such as $n_k c^\dagger_{k+1} c_k$, known to produce pairing in fermionic models and which also cooperates with the correlated hopping in the bosonic case.

Finally, one does not need to restrict to only two degrees of freedom $a_\uparrow$ and $a_\downarrow$. Working with spinor condensates that have larger angular momenta, one already finds that state-changing collisions appear naturally, because their interactions can change the hyperfine state of the atoms while preserving total angular momentum [33]. The combination of these interactions with state-dependent lattices is expected to produce models which go beyond the simple correlated hopping introduced here.

V. CONCLUSIONS

Summing up, we have introduced a mechanism by which bosonic atoms with repulsive interactions can exhibit correlated hopping and pairing. The model (5) exhibits multiple phases, the most relevant being a superfluid of paired bosons. All phases are connected by second order quantum phase transitions and can be produced and identified using variations of current experiments [14,15].

The central idea of this paper is that state changing collisions can be turned into correlated hopping. We would like to emphasize that this is very general and thereby susceptible to be implemented in many different ways. We have provided enough evidence of this flexibility and, given the rapid increase in the number of trapped atomic species, we expect that in the near future better setups to test our proposals will appear.

ACKNOWLEDGMENTS

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APPENDIX: DETAILED DERIVATION OF THE MODEL IN SUPERLATTICES

As discussed before, the main idea behind atomic correlated hopping is to trap atoms whose interaction allows them to change their state. In this section we provide the technical details of one possible implementation of this idea using state-dependent superlattices that trap dressed states [Figs. 1(c) and 1(d)].

1. Dressed states trapping

Our starting point is the setup in Fig. 1(d), which was introduced in Ref. [28]. It consists on an optical lattice trapping atoms in states $|\uparrow\rangle$ and $|\downarrow\rangle$, together with a Raman coupling between these states. Mathematically, this configuration is described by the single-particle Hamiltonian

$$H_{\text{trap}} = V_0 \sin(kx)^2(|\uparrow\rangle\langle\uparrow| + |\downarrow\rangle\langle\downarrow|) + \Omega \sin(kx)(|\uparrow\rangle\langle\downarrow| + |\downarrow\rangle\langle\uparrow|), \tag{A1}$$

which contains a trapping term, equal for both species, together with a coupling between different species. In this model we have not included ac Stark shifts on the $|\uparrow\rangle$ and $|\downarrow\rangle$ states. If the left and right moving laser beams of the Raman configuration have different polarization, these shifts can only be produced by taking and putting photons on the same beam. Such Stark shifts must be uniform, proportional to the identity $|\uparrow\rangle\langle\uparrow|+|\downarrow\rangle\langle\downarrow|$ and can, for our purposes, be ignored.

By moving to the basis of dressed states $|\pm\rangle = \frac{1}{\sqrt{2}}(|\uparrow\rangle \pm |\downarrow\rangle)$, we find that the trapping is effectively equivalent to two superlattices with a relative displacement as in Fig. 1(c)

$$H_{\text{trap}} = [V_0 \sin(kx)^2 + \Omega \sin(kx)]|+\rangle\langle+| + [V_0 \sin(kx)^2 - \Omega \sin(kx)]|-\rangle\langle-|. \tag{A2}$$

Under appropriate circumstances, discussed in Ref. [28], we will find that each superlattice site has a unique ground state, energetically well differentiated from the next excited state, and which consists in a symmetric wave function spanning both lattice wells. If this is the case and if all energy scales, such as the interaction and the hopping, are small compared to the separation between Bloch bands, we can expand the field operators for the bosonic atoms using those localized wave functions, as in Eq. (A3). For convenience we will now adopt the final zigzag ordering

$$\psi_+(x) = \sum_i c_{2i}\ W(x-2il),$$
$$\psi_-(x) = \sum_i c_{2i+1}\ W[x-(2i+1)l], \tag{A3}$$

in which the bosonic operator c_j for even (odd) index j annihilates an atom in state $|+\rangle$ ($|-\rangle$) in the jth superlattice cell. We remind the reader that these bosonic modes are associated to localized wave functions $W(x)$ which are themselves a superposition of the Wannier functions $w(x')$ of the underlying lattice

$$W(x) \simeq \frac{1}{\sqrt{2}}[w(x) + w(x-l)]. \tag{A4}$$

2. State-changing collisions

We will now express the interaction (3) in the basis of dressed states. We proceed using the change of variables in Eq. (2) to find the expression of the densities

$$\rho_\uparrow(x) = \frac{1}{2}(\rho_+ + \rho_- + \psi_+^\dagger\psi_- + \psi_-^\dagger\psi_+),$$
$$\rho_\downarrow(x) = \frac{1}{2}(\rho_+ + \rho_- - \psi_+^\dagger\psi_- - \psi_-^\dagger\psi_+). \tag{A5}$$

The first obvious conclusion is that the total density is independent of the basis on which it is written

$$\rho(x) = \rho_\uparrow(x) + \rho_\downarrow(x) = \rho_+(x) + \rho_-(x). \tag{A6}$$

Hence, the terms proportional to g_0 are insensitive to the state of the atoms.

On the other hand, the spin-dependent terms do not look so simple. The g_1 interaction, depends on the product of two different densities

$$:\rho_\uparrow\rho_\downarrow: = \frac{1}{4}:(\rho_+ + \rho_-)^2: - \frac{1}{4}:(\psi_+^\dagger\psi_- + \psi_-^\dagger\psi_+)^2:$$
$$= \frac{1}{4}:(\rho_+ + \rho_-)^2: - \frac{1}{2}\rho_+\rho_- - \frac{1}{4}(\psi_+^{\dagger 2}\psi_-^2 + \text{H.c.})$$
$$= \frac{1}{4}:\rho_+^2 + \rho_-^2: - \frac{1}{4}(\psi_+^{\dagger 2}\psi_-^2 + \text{H.c.}) \tag{A7}$$

and gives rise processes that change the state of interacting atoms from $|-\rangle$ to $|+\rangle$ and vice versa [see Fig. 1(a)].

The term with g_2 has a rather different form

$$:\rho_\uparrow(x)^2 - \rho_\downarrow(x)^2: = :\rho(x)[\psi_+^\dagger(x)\psi_-(x) + \text{H . c.}]:, \tag{A8}$$

and describes processes in which one atom changes its state influenced by the surrounding environment. In the following subsections we will see what happens to the interaction terms (A6)–(A8), when the atoms are confined in a lattice.

3. Final model

It is now time to put everything together. We will take the tight-binding expansion of the field operators (A3) and use it together with Eqs. (A6)–(A8) to expand the interaction Hamiltonian (3).

Along the derivation, one obtains many integrals of ground state wave functions

$$C_{k,m} = \int |W(x-kl)|^2|W(x-ml)|^2dx. \tag{A9}$$

We will only keep those coefficients with a separation smaller than a superlattice period. Taking Eq. (A4), the

expression for the superlattice localized states, we can approximate such terms as follows:

$$C_{k,k} = \int |W(x)|^4 dx \simeq \frac{1}{2}\int |w(x)|^4 dx,$$

$$C_{k,k\pm 1} = \int |W(x)|^2 |W(x-l)|^2 dx \simeq \frac{1}{4}\int |w(x)|^4 dx, \tag{A10}$$

where $w(x)$ are the Wannier wave functions of the underlying sublattice. Using these tools, the spin-independent interaction term becomes

$$\frac{g_0}{2}\int d^3x :[\rho_\uparrow(x)+\rho_\downarrow(x)]^2: = \frac{g_0}{2}\sum_k^{N/2} :n_{2k}^2 C_{2k,2k} + n_{2k+1}^2 C_{2k+1,2k+1} + 2n_{2k}n_{2k+1}C_{2k,2k+1}:$$
$$\simeq \frac{g_0}{4}\int dx|w(x)|^4 \sum_k^N :n_k^2 + n_k n_{k+1}:. \tag{A11}$$

We will follow a similar procedure for the spin-dependent parts. The equivalent of Eq. (A7) has the expected two-body hopping terms

$$g_1\int d^3x :\rho_\uparrow(x)\rho_\downarrow(x): \simeq \frac{g_1}{8}\int dx|w(x)|^4 \sum_k^N \left(:n_k^2: - \frac{c_{k+1}^{\dagger 2}c_k^2 + c_k^{\dagger 2}c_{k+1}^2}{2}\right), \tag{A12}$$

while Eq. (A8), after a long calculation, simplifies to a single-particle hopping assisted by the environment

$$\frac{g_2}{2}\int d^3x :\rho_\uparrow(x)^2 - \rho_\downarrow(x)^2: = \frac{g_2}{8}\int dx|w(x)|^4 \sum_k^N :n_k \times (c_k^\dagger c_{k-1} + c_k^\dagger c_{k+1} + \text{H.c.}):. \tag{A13}$$

We have already introduced three constants (7) that represent the strength of the interactions in the underlying sublattice. Using these expressions, our final Hamiltonian looks as follows:

$$H = \frac{2U_0+U_1}{8}\sum_k :n_k^2: + \frac{U_0}{8}\sum_k :n_k n_{k+1}: - \frac{U_1}{16}\sum_k (c_{k+1}^{\dagger 2}c_k^2 + \text{H.c.}) - \frac{U_2}{8}\sum_k :(n_k+n_{k+1})(c_k^\dagger c_{k-1} + \text{H.c.}):. \tag{A14}$$

The final steps are to complete terms, replace the sum over k with a sum over nearest neighbors, and to set $U_2=0$. With this, one arrives to the desired model (5) with the parametrization already given in Eq. (6).

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