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Spin squeezing via ladder operations on an atomic coherent state

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We study the generation of spin squeezing by the repeated action of the angular momentum (Dicke) lowering operator on an atomic coherent state prepared for a collection of N two-level atoms or ions. The atoms or ions of the atomic coherent state are not entangled, but the action of the lowering operator (or similarly of the raising operator) generates entanglement among the atoms, and spin squeezing occurs for some ranges of the relevant parameter. Spin squeezing in a collection of two-level atoms or ions is of importance for precision spectroscopy. We discuss methods by which our spin-squeezed states could be generated in the contexts of cavity QED and trapped ions.

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Recently, there has been considerable interest in the generation of so-called spin-squeezed states [1] in a collection of N two-level atoms or ions [2]. This interest stems from the desire to reduce projection noise in high-precession population spectroscopy, important in atomic frequency standards [3] and in the improvement of the performance of atomic clocks [4]. Spin squeezing is also of interest in connection with quantum-information processing as it turns out that a collection of squeezed spins is entangled. As pointed out by Kitagawa and Ueda [1], spin squeezing is the result of correlations (entanglement) that can occur in a multiparticle system consisting of a collection of two-level system atoms or spin-1/2 particles. Sørensen *et al.* [5] have shown that a collection of N two-level systems is entangled if

$$\xi_1^2 = \frac{2J\langle(\Delta \hat{J}_{\mathbf{n}_1})^2\rangle}{\langle \hat{J}_{\mathbf{n}_2}\rangle^2 + \langle \hat{J}_{\mathbf{n}_3}\rangle^2} < 1, \tag{1}$$

where $J=N/2$, $\hat{J}_{\mathbf{n}_i}=\mathbf{n}_i\cdot\hat{\mathbf{J}}$ (the $\mathbf{n}$'s being mutually orthogonal unit vectors) are the usual collective angular momentum operators of Dicke [6] given by $\hat{J}=\Sigma_{i=1}^N \hat{s}^{(i)}$ and where the $\hat{s}^{(i)}$ are the spin-1/2 operators acting in the two-level space of the ith atom. The condition in Eq. (1) may be taken as the condition for spin squeezing and is, in fact, closely related to the definition discussed by Wineland *et al.* [3], which we denote as

$$\xi_2^2 = \frac{2J\langle(\Delta \hat{J}_{\mathbf{n}})^2\rangle}{|\langle \hat{J}_{\mathbf{n}_\perp}\rangle|^2} < 1, \tag{2}$$

where $\mathbf{n}$ and $\mathbf{n}_\perp$ are orthogonal unit vectors. This latter definition is of importance in connection with noise reduction in spectroscopy [2(a),3]. We point out that the condition $\xi_1^2 < 1$ is not inclusive of all entangled states: If the condition holds, the atoms are entangled as was proved in [5], but if it does not hold, the atoms may, or may not, be entangled.

Numerous methods have been proposed for generating spin-squeezed states (see Refs. [2,3] for examples). In this paper we take the following approach: We assume that a collection of two-level atoms initially all in their ground states are excited into an atomic coherent state (ACS) [7], also known a spin-coherent state [8]. The ACS involves no entanglement between the atoms, each atom merely being in a superposition of its ground and excited states. We consider multiple actions of the collective ladder operators $\hat{J}_\pm$ on the ACS and show that entanglement and spin squeezing are generated. We concentrate on the lowering operator $\hat{J}_-$ as the actions of the raising operator $\hat{J}_+$ are essentially identical with those of the actions of the former owing to the fact that the Hilbert space is of finite dimension $2J+1$. Furthermore, it turns out that the lowering operator approach may be the most important in regard to experimental feasibility.

Our approach is inspired by work some years ago of Agarwal and Tara [9] on excitations on photonic coherent states of a single-mode field obtained by multiple actions of the field creation operator and more recent work that examines the effect of the actions of photonic raising *and* lowering operators on other kinds of states, such as the squeezed vacuum state which can be transformed into a weak Schrödinger cat state, a "kitten" state, for the purposes of quantum-information processing [10]. We note that the laboratory generation of a one-photon added state has been reported [11]. The photonic states generated by such methods are highly nonclassical, as is indicated by negativities in their corresponding Wigner functions over some regions of phase space.

The ACSs are defined according to

$$\begin{aligned}|\zeta,J\rangle &= \exp\left(\frac{\theta}{2}e^{i\varphi}\hat{J}_+ - \frac{\theta}{2}e^{-i\varphi}\hat{J}_-\right)|J,-J\rangle \\ &= (1+|\zeta|^2)^{-J}\sum_{M=-J}^{J}\binom{2J}{J+M}^{1/2}\zeta^{J+M}|J,M\rangle,\end{aligned} \tag{3}$$

where $\zeta=e^{i\varphi}\tan(\theta/2)$ and where the angles θ and φ, $0\le\theta<\pi, 0\le\varphi<2\pi$, parametrize the Bloch sphere. The angular momentum states $|J,M\rangle$ are the Dicke states [6] where the "ground" state $|J,-J\rangle$ is a product of the ground states of each of the atoms—i.e., $|J,-J\rangle = |g\rangle_1 \otimes |g\rangle_2 \otimes \cdots \otimes |g\rangle_N$. Because the raising and lowering operators are sums of the corresponding operators acting in the spaces of each of the atoms, the ACS of Eq. (3) contains no entanglement or spin squeezing as can be checked by evaluating $\xi_{1,2}^2$, which both come out to be unity. The probability that the spin ensemble is in the Dicke state $|J,M\rangle$ is given by

$$P_M^{(J)} = (1+|\zeta|^2)^{-2J}\binom{2J}{J+M}|\zeta|^{2(J+M)}, \tag{4}$$

which is, of course, a binomial distribution.

We now act n times with the lowering operator $\hat{J}_-$ on the ACS to obtain the states

$$|\zeta,J;n-\rangle \sim (\hat{J}_-)^n|\zeta,J\rangle \tag{5}$$

or, in normalized form,

$$|\zeta,J;n-\rangle = \mathcal{N}_n^{(-)}\sum_{M=-J+n}^{J}\left[\binom{2J}{J+M}\frac{(J+M)!}{(J-M)!}\frac{(J-M+n)!}{(J+M-n)!}\right]^{1/2} \times \zeta^{J+M}|J,M-n\rangle, \tag{6}$$

where

$$\mathcal{N}_n^{(-)} = \left\{\sum_{M=-J+n}^{J}\left[\binom{2J}{J+M}\frac{(J+M)!}{(J-M)!}\frac{(J-M+n)!}{(J+M-n)!}\right] \times |\zeta|^{2(J+M)}\right\}^{-1/2}. \tag{7}$$

Similarly, for n actions of the raising operator on the ACS we have

$$|\zeta,J;n+\rangle \sim (\hat{J}_+)^n|\zeta,J\rangle, \tag{8}$$

which in normalized form is

$$|\zeta,J;n+\rangle = \mathcal{N}_n^{(+)}\sum_{M=-J}^{J-n}\left[\binom{2J}{J+M}\frac{(J-M)!}{(J+M)!}\frac{(J+M+n)!}{(J-M-n)!}\right]^{1/2} \times \zeta^{J+M}|J,M+n\rangle, \tag{9}$$

where

$$\mathcal{N}_n^{(+)} = \left\{\sum_{M=-J}^{J-n}\left[\binom{2J}{J+M}\frac{(J-M)!}{(J+M)!}\frac{(J+M+n)!}{(J-M-n)!}\right] \times |\zeta|^{2(J+M)}\right\}^{-1/2}. \tag{10}$$

The corresponding probabilities for finding the system in the Dicke state $|J,M\rangle$ are

$$P_M^{(J,n-)} = \mathcal{N}_n^{(-)2}\binom{2J}{J+M}\frac{(J+M)!}{(J-M)!}\frac{(J-M+n)!}{(J+M-n)!}|\zeta|^{2(J+M)}, \tag{11}$$

$$P_M^{(J,n+)} = \mathcal{N}_n^{(+)2}\binom{2J}{J+M}\frac{(J-M)!}{(J+M)!}\frac{(J+M+n)!}{(J-M-n)!}|\zeta|^{2(J+M)}. \tag{12}$$

Clearly we must restrict n to the range $0<n<2J$. Henceforth we shall restrict our attention to the cases of actions by the lowering operator on the ACS. Results obtained through the actions of the raising operator are identical to those obtained by the raising operator, as we have checked numerically.

To see how entanglement comes about in our approach, consider the following: The operator $\hat{J}_-$ acts on the Dicke states $|J,M\rangle$, but the Dicke states themselves, apart from the extremal states $|J,\pm J\rangle$, are entangled states of the collection of two-level atoms [6,12]. The *specific* superpositions of Dicke states that form the ACS contain no atomic entanglement. However, entanglement arises upon the sequential actions of the lowering operator which remove the states $|J,J\rangle, |J,J-1\rangle$, etc., such that total remaining state in Eq. (6), in terms of the states of the individual atoms, can no longer be factored. As an illustration, consider the case for $N=2$ and $J=1$. The ACS is

$$|\zeta,1\rangle = (1+|\zeta|^2)^{-1}[|1,-1\rangle + \sqrt{2}\zeta|1,0\rangle + \zeta^2|1,1\rangle]. \tag{13}$$

In terms of the states of the two two-level atoms, the Dicke states are given by $|1,-1\rangle = |g\rangle_1|g\rangle_2$, $|1,0\rangle = (|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2)/\sqrt{2}$, and $|1,1\rangle = |e\rangle_1|e\rangle_2$. So, in terms of the individual atom states, the ACS reads

$$\begin{aligned}|\zeta,1\rangle &= (1+|\zeta|^2)^{-1}[|g\rangle_1|g\rangle_2 + \zeta(|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2) + \zeta^2|e\rangle_1|e\rangle_2] \\ &= (1+|\zeta|^2)^{-1}[(|g\rangle_1 + \zeta|e\rangle_1)(|g\rangle_2 + \zeta|e\rangle_2)],\end{aligned} \tag{14}$$

where we clearly see that the ACS is factorized and thus contains no entanglement. But the action of the operator $\hat{J}_-$ yields

$$\begin{aligned}\hat{J}_-|\zeta,1\rangle &\sim [\sqrt{2}\zeta|1,-1\rangle + \zeta^2|1,0\rangle] \\ &\sim \left[\sqrt{2}\zeta|g\rangle_1|g\rangle_2 + \frac{\zeta^2}{\sqrt{2}}(|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2)\right].\end{aligned} \tag{15}$$

The atomic states can no longer be factorized. In fact, the second term consists of one of the so-called Bell states—i.e., $(|e\rangle_1|g\rangle_2 + |g\rangle_1|e\rangle_2)/\sqrt{2}$—known to be entangled as it leads to violations of Bell's inequality [13]. Note also that this becomes the dominant component for large $|\zeta|$. Of course, if we make one more application of $\hat{J}_-$, we end up with the nonentangled state $|1,-1\rangle = |g\rangle_1|g\rangle_2$. Evidently, for a larger numbers of atoms, entanglement will be enhanced by repeated applications of the lowering operator, at least up to a point, beyond which the degree of entanglement will start to decrease as the reduced atomic state approaches the extremal state $|J,-J\rangle = \Pi_{k=1}^{N}\otimes|g\rangle_k$.

To characterize the entanglement and spin squeezing obtained, we examined the three squeezing parameters

$$\xi_{1x}^2 = \frac{2J\langle(\Delta\hat{J}_x)^2\rangle}{\langle\hat{J}_y\rangle^2 + \langle\hat{J}_z\rangle^2}, \quad \xi_{1y}^2 = \frac{2J\langle(\Delta\hat{J}_y)^2\rangle}{\langle\hat{J}_x\rangle^2 + \langle\hat{J}_z\rangle^2}, \quad \xi_{1z}^2 = \frac{2J\langle(\Delta\hat{J}_z)^2\rangle}{\langle\hat{J}_x\rangle^2 + \langle\hat{J}_y\rangle^2}. \tag{16}$$

In addition, we examined the parameters

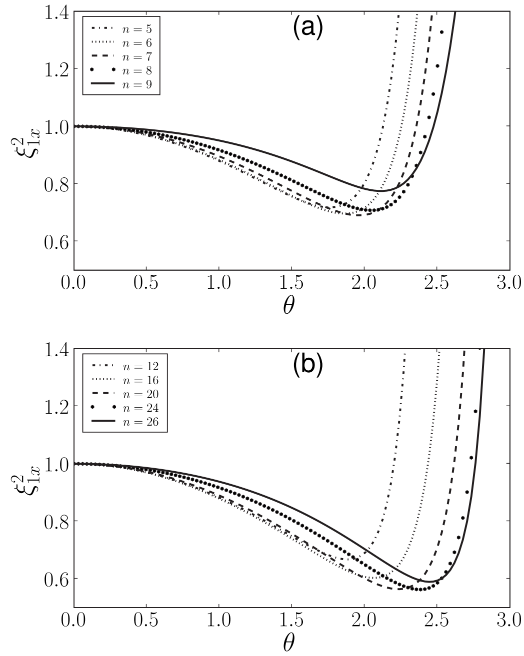

FIG. 1. Plot of ξ^2_{1x} versus θ for (a) $J=5$ (10 atoms) and (b) $J=15$ (30 atoms) for $\varphi=0$ and for different numbers of lowering operations n as indicated.

$$\xi^2_{2x} = \frac{2J\langle(\Delta\hat{J}_x)^2\rangle}{|\langle\hat{J}_z\rangle|^2}, \quad \xi^2_{2y} = \frac{2J\langle(\Delta\hat{J}_y)^2\rangle}{|\langle\hat{J}_z\rangle|^2}, \tag{17}$$

which turn out to be virtually identical to the ξ^2_{1x} and ξ^2_{1y} parameters for certain choices of φ. Numerically we have found that the greatest degree of squeezing occurs in ξ^2_{1x} for $\varphi=0$ and $\pi/2$ and in ξ^2_{1y} for $\varphi=\pi$ and $3\pi/2$. Also, we have found that $\xi^2_{2x}\approx\xi^2_{1x}$ for $\varphi=0$ and $\pi/2$ and that $\xi^2_{2y}\approx\xi^2_{1y}$ for $\varphi=\pi$ and $3\pi/2$. This occurs because in the former case $\langle\hat{J}_y\rangle\approx 0$ and in the latter $\langle\hat{J}_x\rangle\approx 0$. Henceforth we shall not consider ξ^2_{2x} and ξ^2_{2y}. We also found that ξ^2_{1z} always shows squeezing, but is independent of φ.

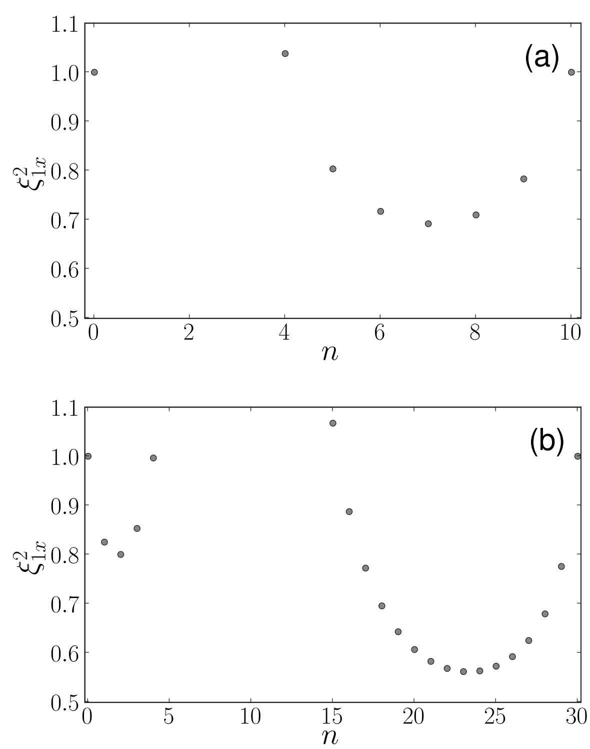

FIG. 2. Plot of ξ^2_{1x} versus n for (a) $J=5$, $\theta=2.2$ and (b) $J=15$, $\theta=2.4$ for $\varphi=0$.

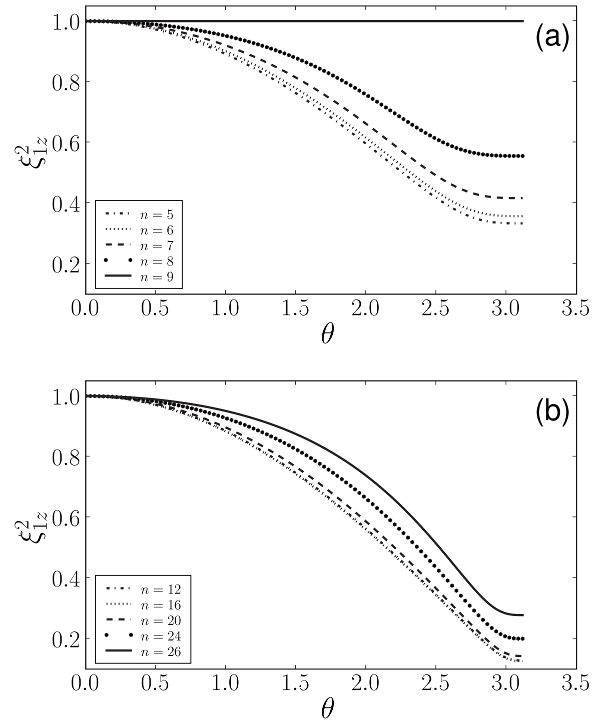

FIG. 3. Plot of ξ^2_{1z} versus θ for (a) $J=5$ and (b) $J=15$ for φ arbitrary and for different numbers of lowering operations n as indicated.

In Fig. 1 we plot, for $\varphi=0$, ξ^2_{1x} versus θ for different numbers of laddering operations n for (a) $J=5$ (10 atoms) and (b) $J=15$ (30 atoms) We notice that the maximum degree of squeezing increases with the number of atoms, N, as the θ value of maximal squeezing also increases with N. We also notice that there is an optimal number of lowerings, $n_{\max}$, for achieving the maximum amount of squeezing for a given number of atoms. We find for (a) $n_{\max}=7$ and for (b) $n_{\max}=24$. The number $n_{\max}$ is always greater than $(2J+1)/2$, the $2J+1$ being the dimension of the space of the Dicke states for a given number of atoms ($J=N/2$). But from the graphs, it is apparent that substantial spin squeezing may be obtained with $n<n_{\max}$. For the case $J=15$, a good amount of squeezing, nearly maximal, is obtained for $n=16$, one-third the number of lowerings required to attain the maximum squeezing. An alternative viewpoint is given in Fig. 2, where we plot ξ^2_{1x} versus n, again for (a) $J=5$ and (b) $J=15$, for $\varphi=0$, but with θ chosen for maximum squeezing based on the results displayed in Fig. 1.

In Fig. 3 we plot ξ^2_{1z} against θ for different n for (a) $J=5$ and (b) $J=15$. ξ^2_{1z} is independent of φ. In Fig. 4 we plot ξ^2_{1z} against n for (a) $J=5$ for $\theta=2.0$ and (b) $J=15$ for $\theta=2.8$. We find that there is always spin squeezing in this parameter. This is because, as we have checked numerically,

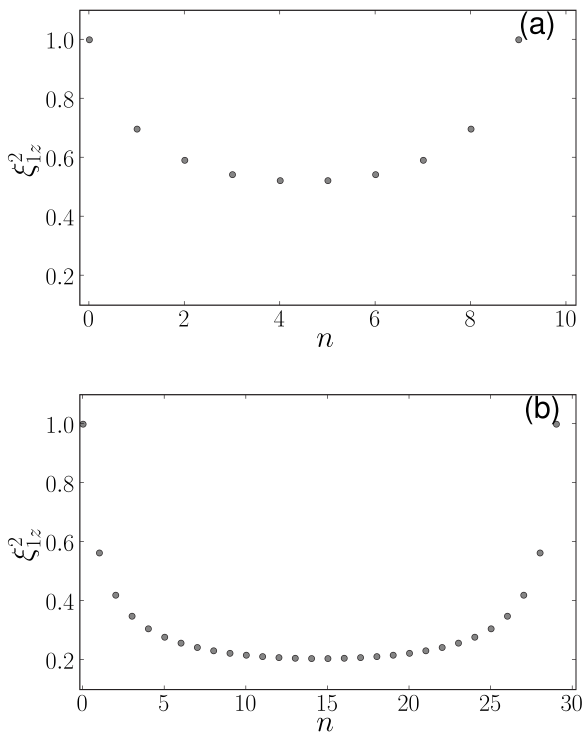

FIG. 4. Plot of ξ^2_{1z} versus n for (a) $J=5$ for $\theta=2.0$ and (b) $J=15$ for $\theta=2.8$.

it is always the case that $\langle(\Delta\hat{J}_z^2)\rangle < \langle\hat{J}_z^2\rangle$. The reason for this behavior is easy to understand. As we apply the angular momentum lowering operator, we are removing Dicke states and thus redistributing the occupation probabilities to within the remaining states, whose numbers reduce with every lowering operation. This effect can be seen in Fig. 5 where we plot $P_M^{(J,n-)}$ versus M for (a) $J=5$ for $\theta=2.0$ and (b) $J=15$ for $\theta=2.4$ corresponding to maximum squeezing as seen in Fig. 1 for each case. Starting with the binomial distribution for the spin-coherent state ($n=0$), we notice that, as expected, the distribution migrates toward the lower values of M as n increases. Furthermore, because of the loss Dicke states, the distribution narrows for increasing n, indicating a decrease in the variance $\langle(\Delta\hat{J}_z)^2\rangle$ over its value for the spin-coherent state. Squeezing in $\hat{J}_z$ has been discussed in the context of a pair of Bose-Einstein condensates by Dunningham *et al.* [14] where they call it *relative number squeezing*.

That spin squeezing occurs in two of the squeezing parameters simultaneously might seem to imply a violation of the angular momentum uncertainty relations. However, no such violation occurs as can be demonstrated numerically by evaluating the quantity

$$\frac{1}{J^2}\xi^2_{1x}[\xi^2_{1y}(\langle\hat{J}_x\rangle^2 + \langle\hat{J}_z\rangle^2) + \xi^2_{1z}(\langle\hat{J}_x\rangle^2 + \langle\hat{J}_y\rangle^2)] \geq 1, \quad (18)$$

and the two additional expressions obtained by the cyclic permutations of x, y, and z. These relations arise from the angular momentum uncertainty relations and the definitions in Eq. (16).

We now briefly address the issue of a physical mechanism that could generate our spin-squeezed states. Two systems immediately come to mind: a collection of atoms in a cavity and a collection of trapped ions in the same vibrational mode.

We consider first the case of cavity QED. The relevant interaction Hamiltonian, in the interaction picture, is given by

$$\hat{H}_I = \hbar\Omega(\hat{a}\hat{J}_+ + \hat{a}^\dagger\hat{J}_-), \quad (19)$$

where the operators $\hat{a}$ and $\hat{a}^\dagger$ describe the quantized cavity field. This is the so-called Tavis-Cummings model [15], an extension of the Jaynes-Cummings model [16] to the case of many atoms. As for generating spin squeezing by repeated lowering operations on a spin-coherent state, there are at least two ways this could be done. We first assume that a collection of N atoms, prepared in an ACS by coherent excitation with a classical (laser) field, is loaded into a cavity which initially contains no photons. We further assume that the cavity is connected to a photodetector by an optical fiber. We suppose that the atom-cavity system can rapidly be tuned in and out of resonance by the application of a small Stark shift or a minute change in the cavity dimensions. When tuned out of resonance, no transitions can occur. We assume that at the beginning, the cavity tuning is out of resonance and that the cavity field is initially in the vacuum state $|0\rangle_a$. The initial state of the system is thus $|\zeta,J\rangle|0\rangle_a$. When tuned into resonance for a short time t, the atom-cavity field evolution is described by

$$|\psi(t)\rangle \approx [1 - it\Omega(\hat{a}\hat{J}_+ + \hat{a}^\dagger\hat{J}_-)]|\zeta,J\rangle|0\rangle_a$$
$$= |\zeta,J\rangle|0\rangle_a - it\Omega(\hat{J}_-|\zeta,J\rangle)|1\rangle_a. \quad (20)$$

We assume that after time t the system is again tuned out of resonance, thus halting the evolution. If a photon is detected, the atomic state is reduced to $|\zeta,J;1-\rangle \sim \hat{J}_-|\zeta,J\rangle$. The procedure can be repeated so that after the detection of n photons sequentially, the atomic state is reduced to $|\zeta,J;n-\rangle$. Note that it would be difficult to implement the generation of spin-squeezed states via the action of the *raising* operator owing to problems with the necessity of preparing the cavity initially in, say, a single-photon state and of the subsequent determination of the cavity field in the vacuum state required for state reduction.

An alternative cavity QED approach would be to have a collection of atoms prepared in a ACS and then projected as a group through a sequence of n cavities prepared in vacuum states. The collection would need to be velocity selected so that the interaction times in each cavity would be short. One photon from each cavity needs to be detected in order to produce a spin-squeezed state.

In the case of trapped ions, one could begin by loading a trap with N ions and cool them to the zero-point energy of the center-of-mass vibration mode of their collective center of mass. The ions could then be prepared into a spin-coherent state by the global application of a resonant laser beam to all ions in a such manner so as to not change the center-of-mass motional state. By the application of lasers to

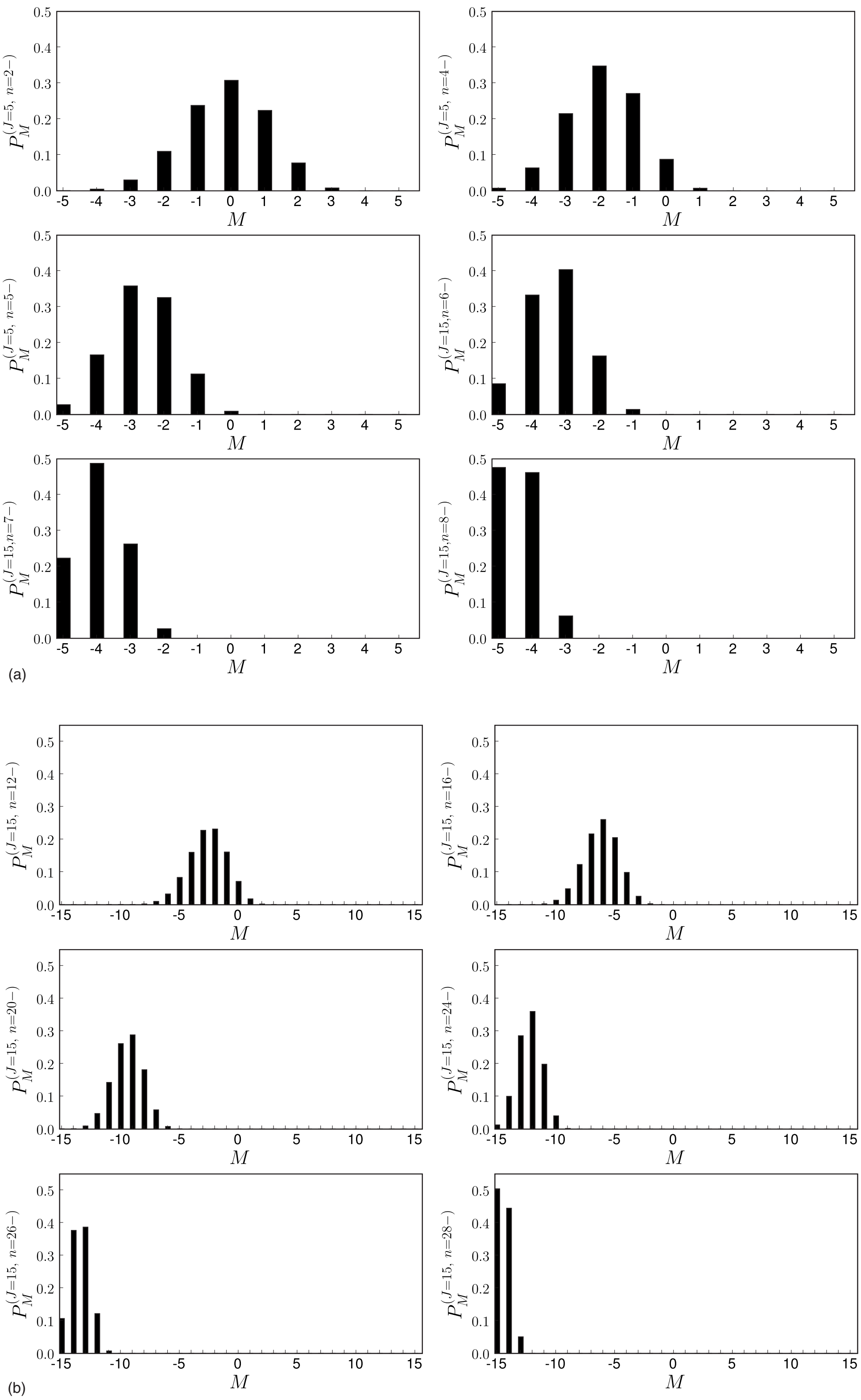

FIG. 5. A sequence of plots of $P_M^{(J,n-)}$ versus M for (a) $J=5$ for $\theta=2.0$ and (b) $J=15$ for $\theta=2.4$ corresponding to maximum squeezing as seen in Fig. 1 for each case, for different values of n as indicated.

obtain the appropriate vibrational sideband tuning [17], it is possible to obtain interactions, in the interaction picture, of either the form

$$\hat{H}_I = \hbar\Omega(\hat{J}_+\hat{a} + \hat{J}_-\hat{a}^\dagger) \tag{21}$$

or

$$\hat{H}_I = \hbar\Omega(\hat{J}_+\hat{a}^\dagger + \hat{J}_-\hat{a}), \tag{22}$$

where now the operators $\hat{a}$ and $\hat{a}^\dagger$ represent the vibrational mode for the center of mass. The former interaction is again the Tavis-Cummings model [14], while the latter constitutes the anti–Tavis-Cummings model. The rotating-wave approximation has been made for both interactions. Wineland *et al.* [3] have studied the spin squeezing generated by these interactions by continuous evolution from an initial state where all the atoms are in their ground states and the vibrational mode is in a coherent state. But for our purposes, we start with the atoms in an ACS and the vibrational mode in the ground state $|0\rangle$. Furthermore, the evolution is not continuous as the interaction needs to be switched on and off and requires an intermediate detection of a single vibrational phonon for motion of the center of mass of the collection of ions during the period when the interaction is turned off. One has a bit more flexibility within the ion trap scenario as compared with the cavity QED case, as either interaction could be used to generate spin-squeezed states, both creating a phonon in the vibrational motion of the center of mass; the former interaction lowers on the ACS while the latter cause the ACS to be raised. The latter interaction is not available in cavity QED. But which interaction form is used is immaterial as multiple applications of either lead to the same degree of spin squeezing, as mentioned above. Once a spin-squeezed state is obtained, one can apply the Ramsey method of separated oscillatory fields [18] to perform high-resolution frequency measurements [3].

In summary, we have presented a possible method of generating spin-squeezed states in a collection of two-level atoms or ions prepared in a spin-coherent state by the use of multiple actions of the angular momentum ladder operators on the initial state.

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[1] M. Kitagawa and M. Ueda, Phys. Rev. A **47**, 5138 (1993).

[2] See, for example, (a) G. S. Agarwal and R. R. Puri, Phys. Rev. A **49**, 4968 (1994); (b) L. Vernac, M. Pinard, and E. Giacobino, *ibid.* **62**, 063812 (2000); (c) H. Prakash and R. Kumar, Eur. Phys. J. D **42**, 475 (2007).

[3] D. J. Wineland, J. J. Bollinger, W. M. Itano, F. L. Moore, and D. J. Heinzen, Phys. Rev. A **46**, R6797 (1992); D. J. Wineland, J. J. Bollinger, W. M. Itano, and D. J. Heinzen, *ibid.* **50**, 67 (1994).

[4] A. André, A. S. Sørensen, and M. D. Lukin, Phys. Rev. Lett. **92**, 230801 (2004); D. Oblak, P. G. Petrov, C. L. G. Alzar, W. Tittel, A. K. Vershovski, J. K. Mikkelsen, J. L. Sørensen, and E. S. Polzik, Phys. Rev. A **71**, 043807 (2005).

[5] A. Sørensen, L.-M. Duan, J. I. Cirac, and P. Zoller, Nature (London) **409**, 63 (2001). See also A. S. Sørensen and K. Mølmer, Phys. Rev. Lett. **86**, 4431 (2001).

[6] R. H. Dicke, Phys. Rev. **93**, 99 (1954).

[7] F. T. Arecchi, E. Courtens, R. Gilmore, and H. Thomas, Phys. Rev. A **6**, 2211 (1972).

[8] J. M. Radcliffe, J. Phys. A **4**, 313 (1971).

[9] G. S. Agarwal and K. Tara, Phys. Rev. A **43**, 492 (1991). See also V. V. Dodonov, M. A. Marchiolli, Ya. A. Korennoy, V. I. Man'ko, and Y. A. Moukhin, *ibid.* **58**, 4087 (1998); A. Biswas and G. S. Agarwal, *ibid.* **75**, 032104 (2007).

[10] A. Ourjoumtsev, R. Tualle-Brouri, J. Laurat, and P. Grangier, Science **312**, 83 (2006). See also M. Dakna, T. Anhut, T. Opatrný, L. Knöll, and D.-G. Welsch, Phys. Rev. A **55**, 3184 (1997).

[11] A. Zavatta, S. Viciani, and M. Bellini, Science **306**, 660 (2004); A. Zavatta, S. Viciani, and M. Bellini, Phys. Rev. A **72**, 023820 (2005).

[12] See M. Sargent III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, MA, 1974), p. 400.

[13] See S. J. D. Phoenix, S. M. Barnett, and P. L. Knight, J. Mod. Opt. **40**, 979 (1993); J. I. Cirac and P. Zoller, Phys. Rev. A **50**, R2799 (1994).

[14] J. A. Dunningham, K. Burnett, and M. Edwards, Phys. Rev. A **64**, 015601 (2001).

[15] M. Tavis and F. W. Cummings, Phys. Rev. **170**, 379 (1968).

[16] E. T. Jaynes and F. W. Cummings, Proc. IEEE **51**, 89 (1963).

[17] See Ref. [3] and J. J. Bollinger, W. M. Itano, D. J. Wineland, and D. J. Heinzen, Phys. Rev. A **54**, R4649 (1996) for a discussion of realizing these interactions in ion traps.

[18] N. F. Ramsey, *Molecular Beams* (Oxford University Press, London, 1963), p. 124. For discussions of the Ramsey method in the current context, see Ref. [3] and A. André and M. D. Lukin, Phys. Rev. A **65**, 053819 (2002).