

Mermin inequalities for perfect correlations

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Any n -qubit state with n independent perfect correlations is equivalent to a graph state. We present the optimal Bell inequalities for perfect correlations and maximal violation for all classes of graph states with $n \leq 6$ qubits. Twelve of them were previously unknown and four give the same violation as the Greenberger-Horne-Zeilinger state, although the corresponding states are more resistant to decoherence.

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I. INTRODUCTION

In 1989, Greenberger, Horne, and Zeilinger (GHZ) showed that no local hidden variable (LHV) theory can assign predefined local results which agree with the *perfect* correlations predicted by quantum mechanics for separated measurements on $n \geq 3$ distant sites on a system prepared in the n -qubit GHZ state [1]. Mermin converted the n -party GHZ proof into a violation of an n -party Bell inequality [2]. The amount of the violation of Mermin's inequalities, measured by the ratio \mathcal{D} between the quantum value of the Bell operator and its bound in LHV theories, grows exponentially with n . For a given n (with n odd), Mermin's inequality gives the maximal possible violation of any n -party two-setting Bell inequality in quantum mechanics [3].

Can we extend this result to other n -qubit states? The essential ingredient for GHZ-type proofs and Mermin-type Bell inequalities is that they require an n -qubit quantum state, which is a simultaneous eigenstate of n commuting local observables (i.e., a stabilizer state). Any stabilizer state is, up to local rotations, equivalent to a graph state [4] (i.e., a stabilizer state whose generators can be written with the help of a graph [5]). These states are essential in quantum error correction [6] and one-way quantum computation [7]. For a small number of qubits, all classes of nonequivalent graph states under single-qubit unitary transformations are known [5]. For $3 \leq n \leq 6$, there are thirteen different classes of graph states which are nonequivalent to GHZ states. For a given n , some of them are more robust against decoherence than the GHZ state [8].

Bell inequalities for graph states constitute a subject of intense study [9–16]. However, the Mermin inequalities for most of them are unknown. For a given state, the Mermin inequality is the Bell inequality such that (I) the Bell operator is a sum of stabilizing operators of that state (i.e., perfect correlations), and (II) the violation is maximal. If the maximum is obtained for Bell operators with a different number of terms, then we will choose the one with the lowest number, since the other inequalities contain this inequality and require more measurements. For some graph states, the Mer-

min inequality is not unique due to the symmetries of the graph.

This definition is motivated by the relation between the original GHZ proof [1] and the Mermin inequality [2]. The aim of this paper is to introduce the Mermin inequalities for *all* graph states (or, equivalently, for all pure states with n independent perfect correlations) with $n < 7$ qubits.

The graph state $|G\rangle$ is the unique n -qubit state that satisfies $g_i|G\rangle = |G\rangle$, for $i = 1, \dots, n$, where g_i are the generators of the stabilizer group of the state, defined as the set $\{s_j\}_{j=1}^{2^n}$ of all products of the generators. The perfect correlations of the graph state are

$$\langle G|s_j|G\rangle = 1 \quad \text{for } j = 1, \dots, 2^n. \quad (1)$$

The g_i 's are obtained with the help of a graph G . For instance, the n -qubit GHZ state is associated to the star-shaped graph in which qubit 1 is connected to all the other qubits (see the graphs 1, 2, 3, 5, and 9 in Fig. 1). This means that $g_1 = X_1 \otimes_{i=2}^n Z_i$ and $g_i = X_1 \otimes Z_i$ for $i \neq 1$; X_i , Y_i , and Z_i denote the Pauli matrices acting on the i th qubit (see [5] for more details).

There are many possible GHZ-type proofs for a given graph state associated to a connected graph of $n \geq 3$ qubits. All of them have the same structure. Any LHV theory assigning predefined values -1 or 1 to X_i , Y_i , and Z_i in agreement with the quantum predictions given by Eq. (1) must satisfy

$$s_j = 1 \quad \text{for } j = 1, \dots, 2^n. \quad (2)$$

However, if we choose a suitable *subset* of q predictions from the set (2), and assume predefined values, either -1 or 1 , then for some choices it happens that, at most, only $p < q$ of these predictions are satisfied. For the remaining $q - p$ quantum predictions, the prediction of the LHV theory is the opposite: $s_j = -1$. This difference can be reformulated as a violation of the Bell inequality

$$\beta \leq 2p - q, \quad (3)$$

where the Bell operator β is the sum of the stabilizing operators of the chosen subset. According to Eq. (1), the graph state satisfies

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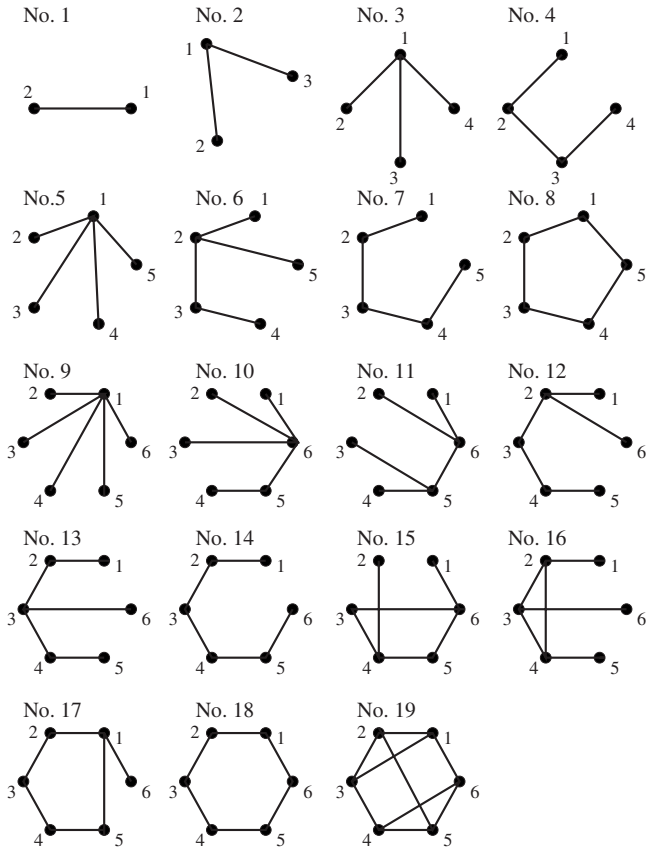


FIG. 1. Graphs representing all classes of n -qubit graph states, with $2 \leq n \leq 6$, that are nonequivalent under single-qubit unitary transformations and graph isomorphism. The figure is taken from Ref. [5].

$$\langle G | \beta | G \rangle = q. \tag{4}$$

Therefore, $|G\rangle$ violates the inequality (3) by an amount $\mathcal{D} = q / (2p - q)$. For the GHZ proof with n odd, the maximum contradiction, measured by q/p , and the maximum violation of the Bell inequality, measured by \mathcal{D} , is obtained when $q = 2^{n-1}$ and $p = 2^{n-2} + 2^{(n-3)/2}$. This is Mermin's inequality [2]. If we take a different subset of stabilizing operators, then we can have a violation of a Bell inequality, but usually not the maximum one.

Specifically, for a given graph state associated to a connected graph of $n \geq 3$ qubits, if we consider the Bell operator consisting of the whole set of stabilizing operators, then we always have a violation of a Bell inequality [10], but not the maximum one. A violation occurs because that Bell operator contains a simpler Bell operator giving the maximum violation.

Why are we interested in those Bell inequalities with the maximum \mathcal{D} ? \mathcal{D} is the measure of nonlocality used in Refs. [2,3,10]. For graph states and stabilizer Bell inequalities, it is well defined, easily computable, and the two practical measures of nonlocality, the resistance to noise and the detection efficiency for a loophole-free Bell experiment, are connected to \mathcal{D} .

(i) In actual experiments, instead of a pure state $|G\rangle$, we usually have a noisy one, $\rho = \mathcal{V} |G\rangle \langle G| + (1 - \mathcal{V}) \mathbb{1} / 2^n$, where $\mathbb{1}$ is the identity matrix in the Hilbert space of the whole system. \mathcal{D} is related to the minimum value of \mathcal{V} required to actually observe a violation of the Bell inequality $\mathcal{V}_{\text{crit}}$. For graph states and stabilizer Bell inequalities, if \mathcal{D} increases, then $\mathcal{V}_{\text{crit}}$ decreases. Specifically, a simple calculation gives that $\mathcal{V}_{\text{crit}} = 1 / \mathcal{D}$.

TABLE I. Mermin inequalities for all graph states of $n < 6$ qubits.

Graph state	g_i	$\beta \leq 2p - q$	Settings	\mathcal{D}
2 (GHZ ₃)	$g_1 = X_1 Z_2 Z_3$ $g_i = Z_1 X_i$ for $i \neq 1$	$g_1(1 + g_2)(1 + g_3) \leq 2$	2-2-2	2
3 (GHZ ₄)	$g_1 = X_1 Z_2 Z_3 Z_4$ $g_i = Z_1 X_i$ for $i \neq 1$	$g_1(1 + g_2 g_3 + g_2 g_4 + g_3 g_4) \leq 2$ and $g_1 \rightarrow g_1 g_2$ $g_1(1 + g_i)(1 + g_j) \leq 2$ and $g_1 \rightarrow g_1 g_k$	1-2-2-2 2 - 2(i) - 2(j) - 1(k)	2
4 (LC ₄)	$g_1 = X_1 Z_2, g_4 = Z_3 X_4$ $g_i = Z_{i-1} X_i Z_{i+1}$ for $i = 2, 3$	$(1 + g_1)g_2(1 + g_3) \leq 2$ and $g_3 \rightarrow g_3 g_4$ $(1 + g_1)g_2(g_3 + g_4) \leq 2$ and $g_3 \rightarrow g_3 g_4$ $g_i \rightarrow g_{i+1}$	2-2-2-1 2-2-1-2	2
5 (GHZ ₅)	$g_1 = X_1 Z_2 Z_3 Z_4 Z_5$ $g_i = Z_1 X_i$ for $i \neq 1$	$g_1(1 + g_2)(1 + g_3)(1 + g_4)(1 + g_5) \leq 4$	2-2-2-2-2	4
6 (Y ₅)	$g_1 = X_1 Z_2, g_5 = Z_2 X_5$ $g_2 = Z_1 X_2 Z_5$ $g_3 = Z_2 X_3 Z_4$ $g_4 = Z_3 X_4$	$g_2[(1 + g_1 + g_5)(1 + g_3 + g_3 g_4) + (1 + g_1 g_5)g_4]$ $+ (g_1 + g_5)g_3(1 + g_4) \leq 7$ $g_2 \rightarrow g_2 g_4$ $\beta \rightarrow g_4 \beta$ and 32 nonsymmetric more	3-3-3-3-2 3-3-3-3-3	$\frac{15}{7}$
7 (LC ₅)	$g_1 = X_1 Z_2, g_5 = Z_4 X_5$ $g_i = Z_{i-1} X_i Z_{i+1}$ for $i = 2, 3, 4$	$(1 + g_1)[(1 + g_2)g_3(1 + g_4) + g_2 g_4](1 + g_5) \leq 8$	3-3-3-3-3	$\frac{5}{2}$
8 (RC ₅)	$g_i = Z_{i-1} X_i Z_{i+1}$	$\gamma + \sum_{i=1}^5 g_i g_{i+1} \leq 9$ $\gamma + g_j g_{i+1} + g_i g_{i+2} + g_{i-1} g_{i+1}$ $+ g_{i-2} g_i g_{i+1} g_{i+2} + g_{i-2} g_{i-1} g_i g_{i+1} \leq 9$ $\gamma = \frac{1}{2} [\prod_{i=1}^5 (1 + g_i) - \prod_{i=1}^5 (1 - g_i)]$ and 105 more	3-3-3-3-3 3-3-3-3-3	$\frac{7}{3}$

TABLE II. Symmetric Mermin inequalities for all graph states of $n=6$ qubits.

Graph state	g_i	$\beta \leq 2p - q$	Settings	\mathcal{D}
9 (GHZ ₆)	$g_1 = X_1 Z_2 Z_3 Z_4 Z_5 Z_6$ $g_i = Z_1 X_i$ for $i \neq 1$	$g_1(1 + \sum_{i \neq j \neq 2} 1 g_i g_j + \sum_{i \neq j \neq k \neq l \neq 1} g_i g_j g_k g_l) \leq 4$ and $g_1 \rightarrow g_1 g_2$	1-2-2-2-2-2 1-2-2-2-2-2	4
10	$g_i = X_i Z_6$ for $i=1, 2, 3$ $g_4 = X_4 Z_5$ $g_5 = Z_4 X_5 Z_6$ $g_6 = Z_1 Z_2 Z_3 Z_5 X_6$	$(1+g_1)(1+g_2)(1+g_3)(1+g_5)g_6 \leq 4$ $g_5 \rightarrow g_4 g_5$ $(1+g_1)(1+g_2)(1+g_3)(g_4+g_5)g_6 \leq 4$ $g_5 \rightarrow g_4 g_5$	2-2-2-1-2-2 2-2-2-1-2-2 2-2-2-2-1-2 2-2-2-2-1-2	4
11 (H_6)	$g_1 = X_1 Z_6, g_2 = X_2 Z_6$ $g_3 = X_3 Z_5, g_4 = X_4 Z_5$ $g_5 = Z_3 Z_4 X_5, g_6 = Z_1 Z_2 X_6$	$g_1(1+g_2)(1+g_3)(1+g_4)(1+g_5)g_6 \leq 4$ $g_1 \leftrightarrow g_2$ (i.e., permute them) $(1+g_1)(1+g_2)g_3(1+g_4)g_5(1+g_6) \leq 4$ $g_3 \leftrightarrow g_4$	1-2-3-3-3-2 2-1-3-3-3-2 3-3-1-2-2-3 3-3-2-1-2-3	4
12 (Y_6)	$g_1 = X_1 Z_2, g_6 = Z_2 X_6$ $g_2 = Z_1 X_2 Z_3 Z_6, g_3 = Z_2 X_3 Z_4$ $g_4 = Z_3 X_4 Z_5, g_5 = Z_4 X_5$	$(1+g_1)g_2(1+g_3)g_4(1+g_5)(1+g_6) \leq 4$	2-2-1-2-2-2	4
13 (E_6)	$g_1 = X_1 Z_2, g_5 = Z_4 X_5$ $g_2 = Z_1 X_2 Z_3, g_4 = Z_3 X_4 Z_5$ $g_3 = Z_2 X_3 Z_4 Z_6, g_6 = Z_3 X_6$	$(1+g_3+g_3 g_6)[(1+g_1)g_2+g_4(1+g_5)$ $+ (1+g_1)g_2 g_4(1+g_5) \leq 8$ and 37 more	2-3-3-3-2-2	3
14 (LC ₆)	$g_1 = X_1 Z_2, g_6 = Z_5 X_6$ $g_i = Z_{i-1} X_i Z_{i+1}$ for $i=2, 3, 4, 5$	$(1+g_1)g_2(1+g_3)(1+g_4)g_5(1+g_6) \leq 4$	2-2-3-3-2-2	4
15	$g_1 = X_1 Z_6, g_2 = X_2 Z_4$ $g_3 = X_3 Z_4 Z_6, g_5 = Z_4 X_5 Z_6$ $g_4 = Z_2 Z_3 X_4 Z_5, g_6 = Z_1 Z_3 Z_5 X_6$	$(g_3+g_5)(1+g_1)(1+g_2)(1+g_4)(1+g_6)$ $+ (1+g_3 g_5)(g_4+g_2 g_4+g_6+g_1 g_6) \leq 16$ and 6 more	3-3-3-3-3-3	$\frac{5}{2}$
16	$g_1 = X_1 Z_2, g_5 = Z_4 X_5$ $g_2 = Z_1 X_2 Z_3 Z_4, g_4 = Z_2 Z_3 X_4 Z_5$ $g_3 = Z_2 X_3 Z_4 Z_6$ $g_6 = Z_3 X_6$	$g_3(1+g_1+g_2+g_1 g_2+g_4+g_5+g_4 g_5)(1+g_6)$ $+ (1+g_1)g_2(1+g_5+g_6)+g_4(1+g_5)(1+g_1+g_6)$ $+ (1+g_1)g_2 g_4(1+g_5) \leq 12$ and 3 more	3-3-3-3-3-3	3
17	$g_1 = X_1 Z_2 Z_6$ $g_2 = Z_1 X_2 Z_3, g_5 = Z_1 Z_4 X_5$ $g_3 = Z_2 X_3 Z_4, g_4 = Z_3 X_4 Z_5$ $g_6 = Z_1 X_6$	$[g_1(1+g_2 g_5)(g_3+g_4)$ $+ (1+g_1)(g_2+g_5)(1+g_3 g_4)(1+g_6) \leq 8$ $\beta \rightarrow g_3 \beta$	3-3-3-3-3-3 3-3-3-3-3-3	3
18 (RC ₆)	$g_i = Z_{i-1} X_i Z_{i+1}$	$\sum_{i=1}^6 s_i - 1 - \sum_{i=1}^6 g_i - g_1 g_3 g_5 - g_2 g_4 g_6 \leq 19$	3-3-3-3-3-3	$\frac{55}{19}$
19	$g_1 = X_1 Z_2 Z_3 Z_6, g_4 = Z_3 X_4 Z_5 Z_6$ $g_2 = Z_1 X_2 Z_3 Z_5, g_5 = Z_2 Z_4 X_5 Z_6$ $g_3 = Z_1 Z_2 X_3 Z_4, g_6 = Z_1 Z_4 Z_5 X_6$	$g_1 g_4 + g_3 g_6 + g_1 g_3 g_4 g_6 + g_2(g_4+g_6+g_4 g_6)$ $+ g_5(g_1+g_3+g_1 g_3) + (g_2+g_5)(g_3 g_4+g_1 g_6$ $+ g_1 g_3 g_4 g_6)$ $+ g_2 g_5 [g_1 g_4(1+g_3+g_6) + g_3 g_6(1+g_1+g_4)] \leq 9$	3-3-3-3-3-3	$\frac{7}{3}$

(ii) An open problem in fundamental physics is achieving a loophole-free Bell experiment. A particularly important problem is the *detection loophole* [17]. \mathcal{D} is related to the minimum detection efficiency required for a loophole-free Bell experiment η_{crit} . For graph states and stabilizer Bell inequalities, if \mathcal{D} increases, then η_{crit} decreases. Specifically, for GHZ states and the Mermin inequality with n odd, $\eta_{\text{crit}} = [2 + (\ln 2 / \ln \mathcal{D})] / 4$ [18].

(iii) In addition, \mathcal{D} provides the relevant parameters of the underlying GHZ-type proof: q and p . Any GHZ-type proof can be converted into an n -party quantum pseudotelepathy game in which a team assisted with a graph state always wins, while a team with only classical resources wins only with probability p/q [19]. Therefore, the higher \mathcal{D} , the lower p/q and the higher quantum advantage.

The knowledge of the Mermin inequalities for all graph states is important for

(a) *Quantum information*. Graph states are essential for quantum information tasks. Mermin inequalities are useful tools for their experimental analysis. For instance, in recent experiments preparing 6-qubit graph states \mathcal{V} is around 0.5 [20–22], thus Bell inequalities with $\mathcal{D} > 2$ are required to observe violation. We will show that for all 6-qubit graph states, Mermin inequalities have $\mathcal{D} > 2$.

(b) *Nonlocality vs decoherence experiments*. For GHZ states, \mathcal{D} increases exponentially with n [2]. However, GHZ states' entanglement lifetime under decoherence decreases with n [8]. Therefore, a fundamental limitation seemingly exists to observe macroscopic violations of Bell inequalities with GHZ states. A natural question is: Does this limitation

also hold for other types of graph states? What happens to those graph states whose lifetime under decoherence does *not* decrease with n [8]? To answer these questions we need to know how \mathcal{D} scales with n within a family of graph states, and which graph states have higher \mathcal{D} .

II. MERMIN INEQUALITIES FOR GRAPH STATES

For each graph state, our task is to obtain, from all possible Bell operators which are sums of stabilizing operators, those which provide the highest violation. The exhaustive study for $n \geq 6$ becomes computationally difficult because the number of potential Bell operators to test scales like 2^{2^n} . However, if we restrict our attention to Bell operators with the same symmetry as the underlying graph, this investigation is still computationally feasible for $n=6$.

In Table I we present all the Mermin inequalities for all graph states with $2 < n < 6$ qubits. In Table II we present the Mermin inequalities possessing the same symmetry as the underlying graph for all graph states with $n=6$ qubits. In both tables we have followed the classification and the labeling of the qubits of Fig. 1 (taken from Ref. [5]). LC_n (RC_n) denotes the n -qubit linear (ring) cluster state [10], Y_5 denotes the 5-qubit graph state associated to the graph “ Y ”, H_6 the 6-qubit graph state associated to the graph “ H ”, etc. The quantum prediction for each Bell operator β is q (i.e., the number of terms of β); p is the maximum number of the q perfect correlations that a LHV theory can satisfy; $\mathcal{D} = \frac{q}{2p-q}$ is the violation of the Bell inequality $\beta \leq 2p - q$.

Some of the inequalities in Tables I and II were previously known. For the n -qubit GHZ states with n odd, we recover the original Mermin inequalities [2]. For the n -qubit GHZ states with n even, the original Mermin inequalities are the sum of our two symmetric inequalities (the first two inequalities for the GHZ_4 in Table I and the two inequalities for the GHZ_6 in Table II). Our inequalities have the same

violation as Mermin’s, but only half of the terms. For n even, Ardehali proposed a method giving an additional violation of $\sqrt{2}$ [23]. Ardehali’s inequalities do not use only perfect correlations. Ardehali’s method can be extended to other graph states [24]. Mermin’s inequalities have been tested in the laboratory using 3- [25] and 4-qubit GHZ states [26,27]. Sources of 5- [28] and 6-qubit GHZ states [20,21] already exist.

For the 4-qubit cluster state (LC_4), the Mermin inequalities in Table I contain those introduced in [11,12]. These inequalities have been recently tested in the laboratory [29–31].

However, twelve of the Mermin inequalities in Tables I and II are new. They include those for the RC_5 , important for quantum error correction codes [6]; for the H_6 a universal resource for one-way quantum computation recently prepared in the laboratory [21]; and for the Y_6 , which allows a demonstration of anyonic statistics in the Kitaev model [22,32,33].

A remarkable fact is, that four 6-qubit graph states have the same violation as the GHZ_6 : the graph state no. 10, the H_6 , the Y_6 , and the LC_6 (see Table II). This is interesting because these states are more resistant to decoherence than the GHZ_6 [8]. This proves that the nonlocality vs decoherence ratio of GHZ states is not universal: there are states with similar violations but that are more robust against decoherence.

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