Delayed sudden birth of entanglement

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The concept of time delayed creation of entanglement by the dissipative process of spontaneous emission is investigated. A threshold effect for the creation of entanglement is found where the initially unentangled qubits can be entangled after a finite time despite the fact that the coherence between the qubits exists for all times. This delayed creation of entanglement, which we call sudden birth of entanglement, is opposite to the currently extensively discussed sudden death of entanglement and is characteristic for transient dynamics of one-photon entangled states of the system. We determine the threshold time for the creation of entanglement and find it is related to time at which the antisymmetric state remains the only excited state being populated. It is shown that the threshold time can be controlled by the distance between the qubits and the direction of initial excitation relative to the interatomic axis. This effect suggests an alternative for the study of entanglement and provides an interesting resource for creation on demand of entanglement between two qubits.

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Dynamical creation of entanglement in the presence of a noisy environment and its disentangled properties are problems of fundamental interest in quantum computation and quantum information processing. They have attracted a great deal of attention, especially in connection with the phenomenon of decoherence induced by spontaneous emission resulting from the interaction with the environment which leads to irreversible loss of information encoded in the internal states of the system and thus is regarded as the main obstacle in practical implementations of entanglement. Contrary to intuition that spontaneous emission should have a destructive effect on entanglement, it has been shown that under certain circumstances this irreversible process can in fact entangle initially unentangled qubits [1], thus implying a kind of quantum coherence induced in the emission. This effect has been studied for identical qubits coupled to a common multimode vacuum field or coupled to a damped single-mode cavity field and has a simple explanation in terms of the collective nature of the spontaneous emission from a system of qubits being located within a transition wavelength of each other or coupled to a single-mode cavity field. In this terminology, the system can be represented in terms of the collective (Dicke) symmetric and antisymmetric states which decay with significantly different rates [2,3]. Both states are maximally entangled states, but the entanglement results solely from the trapping properties of the antisymmetric state of the system. More precisely, with the initially only one qubit excited, a part of the initial population is trapped in the antisymmetric state from which it cannot decay or may decay much slower than the populations of the remaining states. This is the reason why the system decays to an entangled long-living mixed state involving only the antisymmetric and the ground states of the system. In this way, an entanglement persisted over a long time is obtained dynamically via spontaneous emission. The degree of the entanglement such generated is determined by the population of the antisymmetric state that with the initially only one atom excited approaches a steady state value of one-half.

Apart from the constructive effect of spontaneous emission on entanglement, it has been shown that some entangled states of two qubits can have interesting decoherence properties that two initially entangled qubits can reach separability abruptly in a finite time which is much shorter than the exponential decoherence time of spontaneous emission [4]. This drastic nonasymptotic feature of entanglement has been termed as the "entanglement sudden death," and is characteristic of the dynamics of a special class of initial two-photon entangled states. In fact, the effect shows up only if specific initial two-photon coherences are created between the qubits. A recent experiment by Almeida *et al.* [5] with correlated horizontally and vertically polarized photons has shown evidence of the sudden death of entanglement under the influence of independent environments. The required initial two-photon coherence was created by the parametric down-conversion process. Considering the present interest in understanding of the decoherence in entangled qubits, it presents a fascinating example of a dynamical process in which spontaneous emission affects entanglement and coherences in very different ways. Although the sudden death feature is concerned with the disentangled properties of spontaneous emission there can be interesting "sudden" features in the temporal creation of entanglement from initially independent qubits. If such features exist, they would provide an interesting resource for creation on demand of entanglement between two qubits.

In this paper we show that a "sudden" feature in the temporal creation of entanglement exists in a dissipative time evolution of interacting qubits. We term this feature as delayed (sudden) birth of entanglement, as it is opposite to the sudden death of entanglement, and show the feature arises dynamically with initially separable qubits. The delayed creation of entanglement is not found in the small sample Dicke model which ignores the evolution of the antisymmetric state. It is also not found in a system with initially only one qubit excited. For this, an initial entanglement and the experimentally difficult individual addressing of qubits are not

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required. The initial conditions considered here include both qubits inverted which can be done using a standard technique of a short π pulse excitation. The second initial condition considered here involves excitation by a short $\pi/2$ pulse which leaves qubits separable but simultaneously prepared in the superposition of their energy states. We carry our considerations in the context of concurrence and two-level atoms interacting through the vacuum field and analyze how the concurrence evolves in time. We determine the threshold time for creation of entanglement and discuss the dependence of the magnitude of the entanglement on the distance between the qubits and direction of excitation relative to the interqubit axis. Related calculations have appeared involving entanglement creation via spontaneous emission [1]. However, these calculations studied a limited set of initial conditions and as such these calculations miss the feature of delayed birth of entanglement which depends on specific initial conditions of both qubits. The sudden birth of entanglement deserves more careful study, especially in view of its fundamental importance in a controled creation of entanglement on demand in the presence of a dissipative environment.

The usual way to identify entanglement between two qubits in a mixed state is to examine the concurrence, an entanglement measure that relates entangled properties to the coherence properties of the qubits [6]. For a system described by the density matrix ρ, the concurrence C is defined as

$$\mathcal{C}(t) = \max[0, \lambda_1(t) - \lambda_2(t) - \lambda_3(t) - \lambda_4(t)], \qquad (1)$$

where $\{\lambda_i(t)\}$ are the square roots of the eigenvalues of the non-Hermitian matrix $\rho(t)\tilde{\rho}(t)$ with

$$\tilde{\rho}(t) = \sigma_y \otimes \sigma_y \rho^*(t) \sigma_y \otimes \sigma_y, \qquad (2)$$

and σ_y is the Pauli matrix. The range of concurrence is from 0 to 1. For unentangled (separated) atoms $\mathcal{C}(t)=0$, whereas $\mathcal{C}(t)=1$ for the maximally entangled atoms.

The density matrix, which is needed to compute $\mathcal{C}(t)$ and written in the basis of the separable product states $|1\rangle = |g_1 g_2\rangle$, $|2\rangle = |e_1 g_2\rangle$, $|3\rangle = |g_1 e_2\rangle$, $|4\rangle = |e_1 e_2\rangle$ is in general composed of sixteen nonzero density matrix elements. However, in the case of the simple dissipative evolution of the system without any initial coherences between the qubits and without the presence of coherent excitations, the density matrix takes a simple block diagonal form

$$\rho(t) = \begin{pmatrix} \rho_{11}(t) & 0 & 0 & 0 \\ 0 & \rho_{22}(t) & \rho_{23}(t) & 0 \\ 0 & \rho_{32}(t) & \rho_{33}(t) & 0 \\ 0 & 0 & 0 & \rho_{44}(t) \end{pmatrix}, \qquad (3)$$

in which we put all the coherences, except the one-photon coherences $\rho_{23}(t)$ and $\rho_{32}(t)$, equal to zero. As we will see the zeroth coherences remain zero for all time, they cannot be created by spontaneous decay. However, the coherences $\rho_{23}(t)$ and $\rho_{32}(t)$ can be created by spontaneous emission even if they are initially zero.

For a system described by the density matrix (3), the concurrence has a simple analytical form

$$\mathcal{C}(t) = \max\{0, \tilde{\mathcal{C}}(t)\}, \qquad (4)$$

with

$$\tilde{\mathcal{C}}(t) = 2|\rho_{23}(t)| - 2\sqrt{\rho_{11}(t)\rho_{44}(t)}. \qquad (5)$$

It is evident there is a threshold for the coherence at which the system becomes entangled. Thus, the nonzero coherence $\rho_{23}(t)$ is the necessary condition for entanglement, but not in general a sufficient one since there is also a rather subtle condition of a minimum coherence between the qubits.

Alternatively, we may study conditions for entanglement by writing the concurrence (5) in terms of the maximally entangled Dicke symmetric $|s\rangle = (|2\rangle + |3\rangle)/\sqrt{2}$ and antisymmetric $|a\rangle = (|2\rangle - |3\rangle)/\sqrt{2}$ states

$$\tilde{\mathcal{C}}(t) = \sqrt{[\rho_{ss}(t) - \rho_{aa}(t)]^2 - [\rho_{sa}(t) - \rho_{as}(t)]^2} - 2\sqrt{\rho_{11}(t)\rho_{44}(t)}, \qquad (6)$$

which shows the threshold for entanglement depends on the distribution of the population between the entangled and separable states. Notice that the threshold depends on the population of the upper state $|4\rangle$. Thus, no threshold features can be observed in entanglement creation by spontaneous emission for qubits initially prepared in a single photon state.

In addition to the threshold phenomenon, there is also an evident competition between the symmetric and antisymmetric states in creation of entanglement. We see that the best for creation of entanglement through the one-photon states is to populate either symmetric or antisymmetric states, but not both simultaneously. Thus, one could expect a large entanglement can be created when one of the two entangled states is excluded from the dynamics and remains unpopulated for all times.

However, we demonstrate a somewhat surprising result in which entanglement cannot be created by spontaneous emission in the Dicke model which excludes the dynamics of the antisymmetric state. The Dicke model is composed of three states in a ladder configuration: the upper state $|4\rangle$, the intermediate symmetric state $|s\rangle$, and the ground state $|1\rangle$. Physically, the Dicke model represents two qubits confined to a region much smaller than the resonant wavelength [2]. In this case, the time evolution of the density matrix elements under the spontaneous emission is determined by the following density matrix elements [3]

$$\rho_{44}(t) = \rho_{44}(0)e^{-2\gamma t},$$

$$\rho_{ss}(t) = \rho_{ss}(0)e^{-2\gamma t} + 2\gamma t \rho_{44}(0)e^{-2\gamma t},$$

$$\rho_{aa}(t) = \rho_{aa}(0), \qquad (7)$$

which shows that the antisymmetric state does not participate in the spontaneous dynamics of the system. The population of the antisymmetric state remains constant in time. As a result, if the system is prepared in the antisymmetric state it stays there for all times. We are, however, interested in the dynamical creation of entanglement by spontaneous emission from separable states to entangled states.

In the Dicke model, the only entangled state which participates in the spontaneous dynamics is the symmetric state

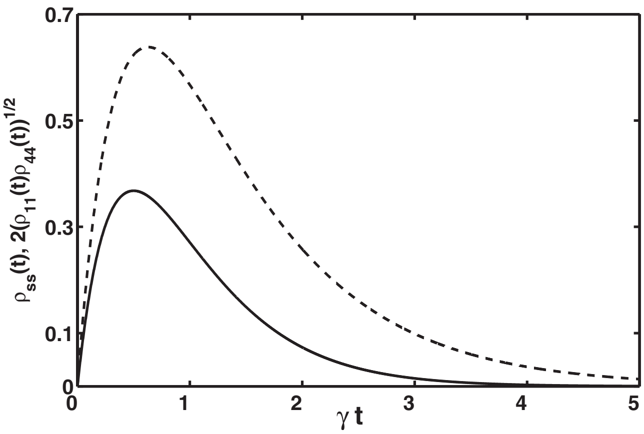

FIG. 1. The time evolution of the population $\rho_{ss}(t)$ (solid line) and the threshold factor $2\sqrt{\rho_{11}(t)\rho_{44}(t)}$ (dashed line) for initially both qubits inverted, $\rho_{44}(0)=1$.

$|s\rangle$, so let us see if one can create entanglement by spontaneous emission that can populate the symmetric state from the upper state $|4\rangle$. Figure 1 shows the time evolution of the population $\rho_{ss}(t)$ and the threshold factor $2\sqrt{\rho_{11}(t)\rho_{44}(t)}$ for the initially fully inverted qubits. We see that the threshold factor overweights the population $\rho_{ss}(t)$ for all times, which indicates that despite a large population of the symmetric state, no entanglement is created. Thus, we may conclude that spontaneous emission cannot create entanglement in the Dicke model where only the symmetric state participates in the atomic dynamics.

We now turn to the system of two qubits which are separated by distances comparable to the resonant wavelength. In this case, the antisymmetric states fully participates in the spontaneous dynamics and the time evolution of the density matrix elements under the spontaneous emission and for an arbitrary initial state is given by [3]

$$\rho_{44}(t) = \rho_{44}(0)e^{-2\gamma t},$$

$$\rho_{ss}(t) = \rho_{ss}(0)e^{-(\gamma+\gamma_{12})t} + \rho_{44}(0)e^{-2\gamma t}\frac{\gamma+\gamma_{12}}{\gamma-\gamma_{12}}(e^{(\gamma-\gamma_{12})t}-1),$$

$$\rho_{aa}(t) = \rho_{aa}(0)e^{-(\gamma-\gamma_{12})t} + \rho_{44}(0)e^{-2\gamma t}\frac{\gamma-\gamma_{12}}{\gamma+\gamma_{12}}(e^{(\gamma+\gamma_{12})t}-1),$$

$$\rho_{sa}(t) = \rho_{sa}(0)e^{-(\gamma+2i\Omega_{12})t}, \quad (8)$$

and $\rho_{11}(t)=1-\rho_{44}(t)-\rho_{ss}(t)-\rho_{aa}(t)$. Note that the full solution for the density matrix elements exhibits the effect of the cooperative damping γ_{12} and the dipole-dipole interaction Ω_{12}.

We consider spontaneous creation of entanglement in the system initially prepared in a separable state. The entanglement depends, of course, on the initial state of the system. We consider two examples of initial states. As the first example, consider a state which covers a broad class of initial states in which the qubits are prepared in the superposition of their energy states

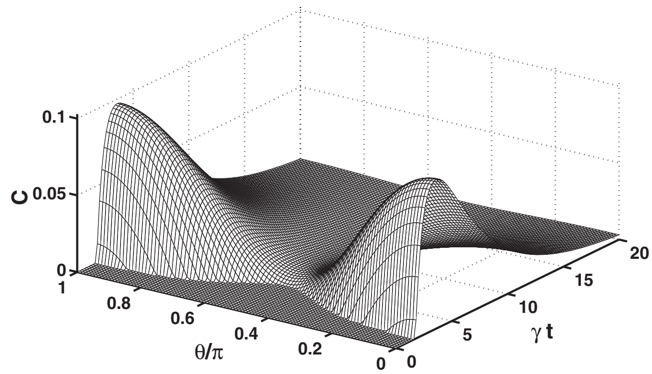

FIG. 2. The time evolution of the concurrence and its dependence on the direction of excitation relative to the interatomic axis for $r_{12}/\lambda=0.25$ and the polarization of the atomic dipole moments $\vec{\mu}\|\vec{r}_{12}$.

$$|\Psi_0\rangle = \frac{1}{2}(|g_1\rangle + ie^{i\vec{k}\cdot\vec{r}_1}|e_1\rangle) \otimes (|g_2\rangle + ie^{i\vec{k}\cdot\vec{r}_2}|e_2\rangle), \quad (9)$$

where $\vec{k}$ is the wave vector of the excitation field. The initial state $|\Psi_0\rangle$ is separable and can be created in practice by an incident $\pi/2$ pulse excitation of each qubit. In this case, the initial values of the density matrix elements are

$$\rho_{sa}(0) = \frac{1}{4}i\sin\vec{k}\cdot\vec{r}_{12}, \quad \rho_{ss}(0) = \frac{1}{4}(1+\cos\vec{k}\cdot\vec{r}_{12}),$$

$$\rho_{aa}(0) = \frac{1}{4}(1-\cos\vec{k}\cdot\vec{r}_{12}), \quad \rho_{44}(0) = \frac{1}{4}, \quad (10)$$

which shows that a particular initial state depends on the distance between the qubits and the direction of excitation relative to the interatomic axis.

Figure 2 shows the concurrence as a function of time and the angle θ between the excitation direction and the vector r_{12} connecting the atoms. It is seen there is no entanglement at earlier times independent of the direction of excitation, and *suddenly* at some finite time an entanglement starts to build up. However, no entanglement builds up if the system is initially excited in the direction perpendicular to the interatomic axis. One can see from Eq. (10) that in this case the system is excited through the symmetric state. Thus, similar to the Dicke model, entanglement in the system cannot be created by an excitation of the system through the symmetric state. A large entanglement is created only if the system is excited in the direction of the interatomic axis. This means that it is crucial for the entanglement creation by spontaneous emission to be accomplished an excitation through the antisymmetric state.

The above conclusion is supported by the analysis of the time evolution of the population of the excited states of the system which is illustrated in Fig. 3. It is quite evident from the figure that at the time $t\approx 4/\gamma$ when the entanglement starts to build up, the antisymmetric state is the only excited state of the system being populated. This effect is attributed to the slow decay rate of the antisymmetric state. The state

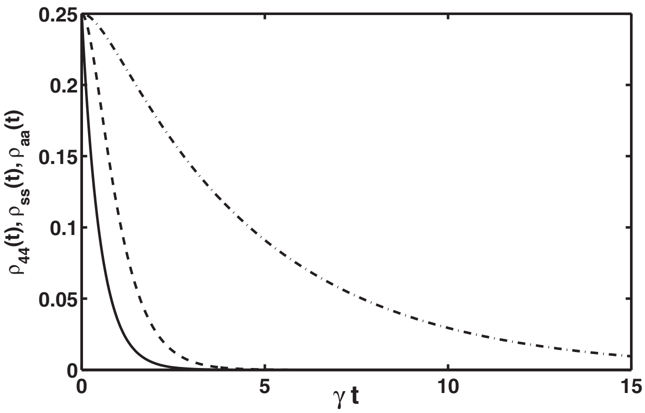

FIG. 3. The time evolution of the populations $\rho_{44}(t)$ (solid line), $\rho_{ss}(t)$ (dashed line), and $\rho_{aa}(t)$ (dashed-dotted line) for θ=0, r_{12}/λ =0.25, and $\vec{\mu} \| \vec{r}_{12}$.

decays on the time scale of $(\gamma - \gamma_{12})^{-1}$ that is much shorter than the decay time of the symmetric and the upper states.

In the second example, we consider the qubits initially prepared in their excited states, which can be realized in practice by a short π pulse excitation. In this case

$$\rho_{44}(0) = 1, \quad \rho_{sa}(0) = \rho_{ss}(0) = \rho_{aa}(0) = 0. \tag{11}$$

Figure 4 illustrate the concurrence as a function of time and the distance between the qubits. Similar to the first example, illustrated in Fig. 2, there is no entanglement at earlier times, but suddenly at some finite time an entanglement starts to build up. However, it happens only for a limited range of the distances r_{12}. It is easy to show that the "islands" of entanglement seen in Fig. 4 appear at distances for which γ_{12} is different from zero.

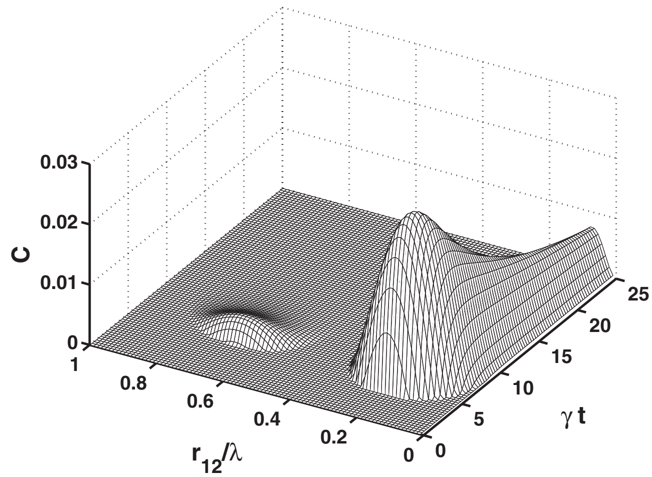

FIG. 4. The time evolution of the concurrence and its dependence on the distance between two initially inverted qubits.

One can easily show that similar to the first example, the entanglement seen in Fig. 4 decays out on a time scale $(\gamma - \gamma_{12})^{-1}$ which is the time scale of the population decay from the antisymmetric state.

In summary, we have predicted an interesting phenomenon of delayed (sudden) birth of entanglement which initially separable qubits become entangled via spontaneous emission after a finite time. In contrast to the sudden death phenomenon that involves two-photon entangled states, the sudden birth involves one-photon entangled symmetric and antisymmetric states. We have demonstrated that the participation of the antisymmetric state in the dynamics is crucial for creation of entanglement in the systems.

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